ON SOME ECONOMETRIC MODELS FOR INDIA

1. INTRODUCTION

The imperative need for the estimation of the Keynesian parameters such as the income multiplier has been felt increasingly in the recent years. This is specially so in the case of countries like India which have embarked on a programme of centralised planning for economic development. This paper contains the results of some econometric studies undertaken in this field in the Indian economic scene. Three simple models have been considered and analysed, the models differing in their ability to describe the actual situation and to adequately represent the economic forces that are operating. Section Two contains a discussion on the models used in this study and in each case the parameters are estimated with their margins of error indicated wherever it is deemed desirable. The theoretical basis for the estimation of the structural parameters is presented in the Appendix. The use of these models as a tool for prediction is demonstrated in Section Four.

2. THE THREE MODELS

2.A. The first model considered here is one truncated in terms of a complete system of mutually interdependent variables in that a major part of national product is treated as autonomous, and only private consumption expenditure is derived from a relation with income. The variables used in the model are :

- GNP : gross national product
- Y : national income
- Y_d : disposable income
- T_e : autonomous expenditure consisting of gross capital formation and government consumption
- C_p : private consumption expenditure

The variables are in rupees, per capita, deflated to 1948-49 prices. The effect of population change over time is eliminated by considering

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per capita figures in the place of aggregates and price effect is removed by deflation. The model consists of the following relationships :

$$GNP = C_p + T_e \quad , \text{ by definition}$$

$$\left.\begin{aligned} Y_d &= \delta + \gamma(GNP) \\ C_p &= \beta + \alpha Y_d \end{aligned}\right\} \text{ by assumption}$$

The first relation is definitional while the other two are behaviouristic equations which have been assumed to be linear for the sake of simplicity. We shall consider T_e as exogenous (explanatory) and derive the endogenous (explained) variables in terms of T_e. The structural parameters are not directly estimated by the least squares method, but some simple functions of these parameters are readily estimated from the reduced forms of the equation on the application of regression methods. The resulting equations are

$$C_p = 187.6016 + 1.0284\ T_e$$
$$Y_d = 179.6485 + 1.6976\ T_e$$

The marginal propensity to consume (α) as estimated from this model is 0.6058.

2.B. The second model is the Keynesian system where no lags are involved. However, as in 2(A), correction for changes in price and population has been taken into account. The model postulates a simple linear relation between consumers expenditure and income and assumes that the relationship is stable over time. Stated in symbols,

$$Y = C + Z$$
$$C = \alpha\ Y + \beta$$

Here Y stands for the per capita income in rupees at constant prices, and C for consumers' expenditure per capita also in constant rupees. Lastly, Z represents the residual which is an autonomous part of the income, per capita, deflated. Of these variable, C and Y are regarded as endogenous variables and are derived in terms of the exogenous variable, Z. The reduced form in this case is the following :

$$C = 204{\cdot}6122 + 2{\cdot}2974\ Z$$
$$Y = 204{\cdot}6122 + 3{\cdot}1980\ Z$$

The marginal propensity to consume (α) is given by 0·7184. The investment multiplier, M, is given by 1/(1–0·7184), that it, 3·5511, and its 5% confidence interval is found to be (–3·05, 12·04). However, the multiplier is known to be greater than or equal to unity and hence the admissible confidence interval will be written as (1, 12·04). This step is justified when we observe that the equation for solving M is obtained by minimising a quadratic function of M in the range $M \geqslant 1$. The marginal propensity to consume will consequently lie in the interval $0 < \alpha \leqslant 0{\cdot}92$. One point is clear that, although the interval is too large for α, the marginal propensity does not exceed 0·92. We note here that the estimate of marginal propensity is more realistic than in the previous model.

2.C. Now the treatment of Z as an exogenous variable is evidently not a wholesome procedure. To consider Z as wholly endogenous may also appear as another extreme, apart from the fact that this will complicate the estimation problems. So the best course of action seems to be to split up Z into two components, one exogenous and the other endogenous. This is the basis of the third model which can be explicitly stated as follows.

$$Y = C + Z$$
$$Z = Q + (\gamma Y + \delta)$$
$$C = \alpha Y + \beta$$

The variables Y, C and Z retain their meaning as in 2(B). The new variable, Q, is the exogenous part of Z while $Z-Q$ is regarded as endogenous and assumed to depend on the present level of income in a linear form. More specifically, Q refers to the capital formation on the government account. It may be noted that one more endogenous variable is added to the system discussed in 2(B) but the model remains closed since a new equation has also been brought in. The parameters to be estimated in this model are four in number.

The reduced form is given below :

$$C = 218{\cdot}2967 + 3{\cdot}5910\ Q$$
$$Y = 225{\cdot}1460 + 5{\cdot}0056\ Q$$

The marginal propensity (α) in this model is estimated to be 0·7173 and M, the multiplier becomes 3·5373. One will observe that much improvement has not taken place in the estimate of α over the estimate obtained in model B. Now, the parameter, γ, can be interpreted as a measure of incentive to invest in the private sector and is estimated to be 0·0828. The questions relating to the joint confidence region for the structural parameters, α and γ, are briefly discussed in the Appendix.

3. Indian Data

At the outset it has to be stated that the time series data used in this study are inadequate. The reason for this also is not far to seek. Fairly accurate figures of Indian national aggregates are available only from 1948-49 onwards, when the National Income Committee appointed after independence submitted its first report. Moreover, all the relevant data are not obtainable for the recent years in the form necessary for our analysis. The necessary information has been culled from a number of official and unofficial sources and is presented in Table 1 below :

A brief explanation of the manner of computation of data for the present study is not out of place at this stage. The "Estimates of National Income (1948-55)" published by the Central Statistical Organization contains the figures relating to the total as well as per capita net output at current prices and also at 1948-49 prices. From these series, the population and price index are derived (lines 1 and 2). In fact, the same series of price index has been used to deflate both consumption and investment figures. The use of a single series for both types of aggregates is somewhat unreasonable from theoretical considerations, but, as is well known, no separate official series of deflators for consumption and investment are yet available. The series for Y is obtained from the White Paper referred to above (line 3). The disposable income Y_d is defined here as national income less all direct taxes and miscellaneous fees (line 4). This is different from the private disposable income in that it contains the business reserves and similar corporate incomes. Information regarding the net investment (Z) is derived from an unofficial source (line 5).[1] The

[1] Baldev Kumar, (1957). "Estimates of Domestic Fixed Capital Formation in India, 1948-49 to 1954-55." Preliminary Conference on Research in National Income.

TABLE 1

	Year						
	1948-49	1949-50	1950-51	1951-52	1952-53	1953-54	1954-55
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1. population (millions)	350·8	354·8	359·3	363·6	368·7	373·8	378·1
2. price index (1948-49=100)	100·0	102·2	107·7	109·6	103·8	104·5	93·6
3.* Income (Y)	246·9	248·7	246·3	250·2	256·6	268·6	271·8
4. disposable income (Y_d)	237·5	239·9	237·5	241·6	247·4	259·7	261·9
5. net investment (Z)	12·6	14·4	13·4	15·1	15·4	16·1	21·2
6. consumers expenditure (C)	234·3	234·3	232·9	235·1	241·2	252·5	250·6
7. net capital formation on government account (Q)	4·4	5·3	5·1	5·3	5·7	6·5	10·3
8. gross national product (GNP)	257·1	259·8	257·1	261·5	269·0	281·5	286·4
9. autonomous expenditure (T_s)	34·2	36·8	35·3	37·7	39·8	41·5	50·4
10. private consumption expenditure (C_p)	222·9	223·0	221·8	223·8	229·2	240·0	236·0

* Lines (3) to (10) are in rupees, per capita, deflated to 1948-49 prices.

consumers' expenditure (C) is obtained as a residual of Y after deduction of Z (line 6). The net capital formation on the government account (Q) has been extracted from the unofficial source cited above (line 7). The gross national product, GNP, is national income plus depreciation and the latter is taken from Baldev Kumar's paper (line 8). The autonomous expenditure (T_e) consists of government consumption and gross investment the former being obtained from the White Paper. For our analysis the net output of Government administration has been treated as government consumption expenditure (line 9). The gross national product less the autonomous expenditures gives us the private consumption expenditure (C_p) (line 10).

4. Some Forecasts

Econometric models are used in two ways : (a) in studying the past data and their inter-relationships and (b) in the prediction of important economic aggregates. For either purpose, it is necessary that the model be simple yet realistic in as far as it takes into account as much information as is feasible. This is achieved by considering more variables and more equations instead of working with a few variables related by a few relationships. The first aspect has been examined in the earlier sections. In this section we will consider these models as an aid in predicting the endogenous variables for different levels of the exogenous variables. The reduced form of the model which explains the endogenous variables in terms of the exogenous variables are of particular use in this connection. The logic of such a prediction procedure is that the exogenous variables can be determined in advance by the governments and other major business bodies. Moreover, this enables one to choose the appropriate levels of the exogenous variable consistent with the desired targets. Some forecasts are presented in Tables 2, 3 and 4. In each case five levels are chosen for the exogenous variable. The variables are in rupees, per capita, deflated to 1948-49 prices.

TABLE 2

Projection of private consumption and disposable income..

autonomous expenditures, (T_e)	private consumption (C_p)	disposable income (Y_d)
(1)	(2)	(3)
50	239·0	264·5
55	244·2	273·0
60	249·3	281·5
65	254·4	290·0
70	259·6	298·5

TABLE 3

Projection of National Income and Consumption.

investment (Z)	national income (Y)	consumption (C)
(1)	(2)	(3)
20	268·6	250·6
24	281·4	259·7
28	294·2	268·9
32	306·9	278·1
40	332·5	296·5

TABLE 4

Projection of National Income and Consumption.

capital formation on government account (Q)	national income (Y)	consumption (C)
(1)	(2)	(3)
10	275·2	254·2
15	300·2	272·2
20	325·2	290·1
25	350·3	308·1
30	375·3	326·0

It will be seen that the projections show considerable agreement with the available observations. This can be verified by comparing the 1954-55 figures given in Table 1 with the first line in each of the tables 2, 3 and 4.

5. Conclusion

A mention may be here made of the earlier attempts at estimating the marginal propensity to consume in India. In this connection the contribution of Narasimham requires special attention.[2] Using ordinary regression methods, he derived the marginal propensity to consume for labour incomes, farm incomes and non-corporate business incomes as 1·00, 0·96 and 0·86 respectively. Also these values were arrived at after fixing arbitrarily the parameters pertaining to certain types of income. In fact, even the value of unity for the marginal propensity to consume from labour income was fixed arbitrarily after it was found to be as small as 0·22 a priori. The presence of such difficulties in the estimation of the marginal propensity to consume in India persuades the authors to believe that the analysis in the present paper has been a fruitful one inasmuch as the estimates obtained are reasonable without any arbitrary assumption regarding the value of the parameters.

Appendix

The underlying principle of the present analysis is that the relationships of the model, excepting the definitional identities, are subject to random disturbances, but there are no observational errors. For purposes of statistical inference a complete specification of the distribution of the random variables characterising the shocks in the equations has to be made. For illustration, we shall consider the model C which states,

$$\begin{aligned} Y_t &= C_t + Z_t \\ Z_t &= Q_t + (\gamma Y_t + \delta + v_t) \\ C_t &= \alpha Y_t + \beta + u_t \end{aligned} \qquad (1)$$

where the suffix t stands for the year of observation. The assumptions

[2] *A Short-term Planning Model for India*, Amsterdam, North-Holland Publishing House, 1957.

regarding the distributions of the shock terms u_t and v_t are the following.

$$
\begin{aligned}
&u_t \sim N(O, \sigma_{uu}) \quad \text{independent of } t \\
&v_t \sim N(O, \sigma_{vv}) \quad \text{"} \\
&E(u_t v_{t'}) = \begin{cases} \sigma_{uv} \text{ independent of } t, \text{ if } t = t' \\ O \quad \text{"} \quad , \text{ if } t \neq t' \end{cases}
\end{aligned}
\tag{2}
$$

As in the case of all econometric models, the first endeavour is to get the reduced form of the model. After this step, the usual procedure is to obtain the least squares estimates for the parameters of the reduced form, and these estimates are best unbiased under the conditions stated in (2).

Following Haavelmo,[3] the reduced form for the structure in (1) is :

$$
\begin{aligned}
C_t &= A_1 Q_t + A_o + U_t \\
Y_t &= B_1 Q_t + B_o + V_t
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
A_o &= \frac{\alpha\delta + (1-\gamma)\beta}{1-\alpha-\gamma} \qquad & B_o &= \frac{\beta+\delta}{1-\alpha-\gamma} \\
A_1 &= \frac{\alpha}{1-\alpha-\gamma}, & B_1 &= \frac{1}{1-\alpha-\gamma} \\
U_t &= \frac{\alpha v_t + (1-\gamma) u_t}{1-\alpha-\gamma} & V_t &= \frac{u_t + v_t}{1-\alpha-\gamma}
\end{aligned}
\tag{4}
$$

From the least squares estimates for A's and B's, consistent estimates of the basic parameters, α, β, γ and δ are obtained as follows :

(5)

$$
\begin{aligned}
\hat{\alpha} &= \frac{m_{oq}}{m_{yq}} \\
\hat{\beta} &= \frac{m_{yq} m_o - m_{oq} m_y}{m_{yq}} \\
\hat{\gamma} &= \frac{m_{yq} - m_{oq} - m_{qq}}{m_{yq}} \\
\hat{\alpha} &= \frac{m_y (m_{oq} + m_{qq}) - m_{yq} (m_o + m_q)}{m_{yq}} \cdot
\end{aligned}
\tag{5}
$$

[3] *Studies in Econometric Method* (1953). Cowles Commission for Research in Economics, Monograph 14. John Wiley & Sons., Inc., New York. pp. 75-93.

where $$m_r = \frac{1}{N} \sum_{t=1}^{N} r_t$$

$$m'_{rs} = \frac{1}{N} \sum_{t=1}^{N} \left(r_t - m_r\right)\left(s_t - m_s\right)$$

N being the number of observations.

We define the variance co-variance matrix, $\sum$, by

$$\sum = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{pmatrix}$$

where Σ_{11}, Σ_{12}, and Σ_{22} are respectively the Var(U), cov(U, V) and var(V) and are estimated unbiasedly by s_{11}, s_{12} and s_{22} Hence

$$s_{11} = \frac{N}{N-2} \cdot \frac{m_{qq}\, m_{cc} - m^2_{cq}}{m_{qq}}$$

$$s_{12} = \frac{N}{N-2} \frac{m_{qq}\, m_{cy} - m_{yq}\, m_{cq}}{m_{qq}}$$

$$s_{22} = \frac{N}{N-2} \frac{m_{qq}\, m_{yy} - m^{2*}_{qy}}{m_{qq}} \quad \cdots \cdots \cdots \quad (6)$$

Let $\begin{pmatrix} s^{11} & s^{12} \\ s^{12} & s^{22} \end{pmatrix}$ be a matrix inverse to $\begin{pmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{pmatrix}$

The confidence region for (α, γ) can be obtained by reference to a statistic defined by Hotelling's

$$T^2 = N m_{qq}\Big[s^{11} (A_1 - \hat{A}_1)^2 + 2s^{12} (A_1 - \hat{A}_1)(B_1 - \hat{B}_1) + s^{22} (B_1 - \hat{B}_1)^2\Big] \quad (7)$$

$\hat{A}_1$ and $\hat{B}_1$ being the best linear unbiased estimates of A_1 and B_1 respectively: Clearly

$$\hat{A}_1 = \frac{m_{cq}}{m_{qd}} \quad , \quad \hat{B}_1 = \frac{m_{yq}}{m_{qr}}$$

It is known that $\frac{N-3}{2(N-2)} T^2$ has an F-distribution with 2 and

(N—3) degrees of freedom. Hence

$$\frac{N-3}{2(N-2)} \quad T^2 \leqslant F_{0 \cdot 05}\,(2,\, N-3)$$

gives us the confidence region for A_1 and B_1. Now

$$\alpha = \frac{A_1}{B_1} \quad \text{and} \quad \gamma = 1 - \frac{1 + A_1}{B_1} \tag{9}$$

Therefore, one can easily derive the confidence region for (α, γ) since the transformation (A_1, B_1) to (α, γ) is one-to-one. The resulting region is not, however, an ellipse.

For, let, in general,

$$Q\,(x,y) = a_{11}x^2 + 2a_{12}xy + a_{22}y^2 \leqslant k$$

represent the confidence region for (x, y). The confidence region for (ξ, η) related to (x, y) by

$$x = \frac{\xi}{1-\xi-\eta}, \qquad y = \frac{\eta}{1-\xi-\eta}$$

is given by

$$a_{11}\xi^2 + 2a_{12}\xi + a_{22} \leqslant k(1-\xi-\eta)^2 \tag{10}$$

since the transformation is non-singular. It may, however, be noted that the transformation is always defined because $\xi + \eta < 1$, since in the present case ξ stands for the propensity to consume and η for the propensity to invest in the private sector. From (10) it will be seen that the confidence region for (ξ, η) is not an ellipse unless $a_{11} < 0$ which, in fact, is not true.

From the data presented in Table 1 we obtain the following sample means and the matrix of corrected sum of product of the variables Y. C and Q.

	Y	C	Q
mean	255·58	240·18	·6·08
Y	681·612	514·287	116·992
C		406·988	88·928
Q			28·872

We obtain the following results.

$\hat{\alpha} = 0\cdot7178,$ $\hat{\beta} = 56\cdot7814$

$\hat{\gamma} = 0\cdot0828,$ $\hat{\delta} = 3\cdot2555$

From (7) and (8) above the confidence region for A_1 and B_1 is given by

$$[\,0.3799\,(A_1 - 3.5910)^2 - 0.7464\,(A_1 - 3.5910)(B_1 - 5.0056) + 0.4177\,(B_1 - 5.0056)^2\,] \leqslant 0.7423$$

The same lines of analysis can be adopted for the other models also with some minor modification.

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