Status of the scaling-in-the-mean hypothesis in inclusive processes

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The aim of this paper is to test the validity of the proposed scaling-in-the-mean hypothesis in some inclusive processes. On the basis of a new multiparticle production model based on a certain lepton-hadron relation it has been successfully concluded that the scaling-in-the-mean hypothesis is as fundamental as or more fundamental than Feynman scaling in the small-p_T region and that there is no contradiction between these two as suggested by some other authors.

I. INTRODUCTION

The scaling-in-the-mean hypothesis has now become an important issue in high-energy physics. Dao *et al.*[1] were the first to propose a new scaling law in terms of longitudinal and transverse momenta for semi-inclusive cross sections and showed that the data for pp collisions between 13 and 300 GeV/c are consistent with it for a wide range of multiplicities. Expressed mathematically, the form of the proposed scaling is

$$\frac{\bar{p}_{xN}}{N\sigma_N}\frac{d\sigma_N}{dp_x}=\phi_x^{s1}\left(\frac{p_x}{\bar{p}_{xN}}\right) , \qquad (1)$$

where $x = L$ and T (longitudinal and transverse directions, respectively). The function ϕ_x^{s1} is independent of both energy and multiplicity. Subsequently it was observed[2] in the reaction $pp \to NX$ at 19 GeV/c that scaling in the mean seems to hold not only for the semi-inclusive cross sections but also for the inclusive ones and that the scaling functions are approximately equal in both cases, i.e.,

$$\frac{\bar{p}_x}{\langle N\rangle\sigma}\frac{d\sigma}{dp_x}=\phi_x^{1}\left(\frac{p_x}{\bar{p}_x}\right) , \qquad (2)$$

and

$$\phi_x^{s1}(y)\simeq\phi_x^{1}(y) . \qquad (3)$$

The validity of the scaling-in-the-mean hypothesis for inclusive processes and the near equality of scaling-in-the-mean variables for both inclusive and semi-inclusive processes were studied from the theoretical angle by Ernst and Schmitt.[3] This property has been made use of in our work for which we have taken the liberty of using the terms for inclusive processes in a carefree way.

However, the theoretical basis of the scaling in the mean for multiparticle production processes has been questioned by several authors. For example, Yaes[4] pointed out that asymptotic validity of scaling in the mean for $p_L \gg p_T, m$ of the produced particles is inconsistent with either Koba-Nielsen-Olesen (KNO) scaling or Feynman scaling. However, the new CERN ISR data[5] at $x = 0$, although giving no indication of the onset of Feynman scaling, are consistent with the validity of scaling in the mean. This leads Ernst and Schmitt[6] to make the bold suggestion that scaling in the mean is more fundamental than Feynman scaling which we substantiate theoretically in this paper. However, Ernst and Schmitt in their work tried to establish the theoretical validity of scaling in the mean by rejecting the content of Feynman scaling and relying wholly on KNO scaling. But it is well known that derivation of KNO scaling is itself based on the supposed validity of Feynman scaling, as the authors themselves have pointed out. So it becomes very hard to accept the argument of Ernst and Schmitt in favor of KNO scaling while practically renouncing the parent Feynman scaling in the same breath. This anomaly renders the foundation of their work logically untenable, although the conclusion they arrived at seems to be quite sound. Besides, the validity of KNO scaling in an asymptotic region is not beyond question. In fact, since the publication of Koba-Nielsen-Olesen's work, numerous papers have been published on semi-inclusive scattering from which it emerges that the experimental data can be fitted by various distributions and even in terms of a scaling parameter other than $z = n/\langle n\rangle$. Over and above all these facts, the validity of Feynman scaling itself is not at all beyond a shadow of a doubt. On the contrary the breakdown of Feynman scaling at (i) superhigh energies, (ii) large transverse momenta, (iii) high energies and high transverse momenta, and (iv) the central region ($x = 0$) is nearly established. Under all these limiting conditions the Feynman scaling can, at best, be considered to be only approximately valid. Against this background of various scaling proposals and their subsequent states, the connectedness and compatibility of the scaling-in-the-mean hypothesis with approximate Feynman scaling are to be carefully studied.

In this paper, we shall show the following in the light of a multiparticle production model based on a certain kind of lepton-hadron relation. (i) Scaling in the mean is a valid scaling law and is independent of any other type of scaling. In other words this is not in contradiction with the approximately valid nature of the Feynman scaling. (ii) There is no need to link up the question of the scaling-in-the-mean hypothesis with KNO scaling of any form or with any parameter. (iii) It is valid for the process $pp \to CX$, where $C = \pi^\pm$, $K^\pm$, and $\bar{p}$ but not for the process $pp \to pX$. (iv) The observed universality of scaling in the mean is theoretically justifiable.

II. THE MODEL AND THE METHOD

According to the model of hadrons of our concern recently proposed by one of the authors (P.B.) (Ref. 7), the muonic leptons (μ^+, ν_μ, μ^-) are taken as the fundamental constituents of hadrons where the internal quantum numbers such as isospin, strangeness, and baryon number can be related to the internal angular momentum of the constituents, and there is a geometrical origin of the SU_3 symmetry of hadrons when the leptonic constituents are taken to be bound by a harmonic-oscillator potential. According to this scheme strong interactions can be interpreted in the following way. Any two constituents (muon-antimuon pair) can form a π-meson cluster, and strong interactions involving no exchange of hypercharge are caused by the interaction of the pion in the incident hadron with pions in the target hadron.

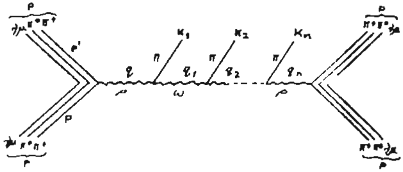

FIG. 1. Multiple production of pions in pp scattering in the present scheme.

According to this model, the configurations of a proton and a neutron are given by

$$p = (\mu^+\nu_\mu\pi^0\nu_\mu) = (\pi^+\pi^0\nu_\mu),$$

$$n = \frac{1}{(\alpha^2+\beta^2)^{1/2}}[\alpha(\nu_\mu\bar{\nu}_\mu\pi^0\nu_\mu) + \beta(\mu^-\mu^+\pi^0\nu_\mu)]$$

$$= \frac{1}{(\alpha^2+\beta^2)^{1/2}}[\alpha(\pi^0\pi^0\nu_\mu) + \beta(\pi^-\pi^+\nu_\mu)].$$

On the basis of this model of nucleons multiple pion production by pp scattering can be depicted according to Fig. 1. According to this multiparticle production scheme,[8] a pion in the structure of the incident proton emits a ρ which then emits ω and π, ω again emits ρ and π and the chain continues. Although this scheme appears to be like a multiperipheral model, actually it is not so since in the multiperipheral model, the momentum transfer along the chain is always bounded, but here this is not so and the virtual mesons may take an infinitely large momentum. Besides, the model itself suggests no peripherality as only the core pions can interact according to this picture.

The structure function here is given by[8]

$$W^{\mu\nu} = \frac{1}{2\pi}\frac{P_0 p_0}{M^2}\sum_n \int\int \frac{d^3P'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} d^4x \exp[i(P+q-P')x] \frac{1}{[(P'-P)^2-m^2]^2} T^\nu(P',P)T_\mu(P',P)(\tfrac{1}{2} f_{\rho\omega\pi})^{2n} \times \frac{1}{n!}\prod_{l=1}^{n} \int \frac{d^3k_l}{2k_{l0}(2\pi)^3}\exp(-ik_l x), \quad (4)$$

where

$$T^\mu(P',P) = (P'-P)_\mu . \quad (5)$$

The invariant amplitude $A(\nu, q^2) = g_{\mu\nu}W^{\nu\mu}$ is thus given by using the notation $a_+ = \frac{1}{2}(a_0 + a_3)$, $a_- = a_0 - a_3$. Thus

$$A_n = -\frac{1}{16\pi^2}\int\int \frac{M^2}{P_0' p_0'}\frac{d^3P'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} F(q'^2, P_T'^2)d^4k(2\pi)^4\delta(P+q-P'-K)\theta(K_-)\theta(K^2)\left(\frac{f_{\rho\omega\pi}^2}{16\pi^2}\right)^n \frac{1}{n!(n-1)!(n-2)!}\left(\frac{K^2}{4}\right)^{n-2} \quad (6)$$

$$= -\frac{1}{128\pi^3} 8\pi M^2 \int d^2P_T' F(P_T'^2, q'^2) d^2p_T' \left(\frac{f_{\rho\omega\pi}^2}{16\pi^2}\right)\frac{1}{2P_+K_-}\frac{1}{n!(n-1)!(n-1)!}\left(\frac{f^2 K_{max}^2}{64\pi^2}\right)^{n-1}, \quad (7)$$

ere

$$q'^2 = (P' - P)^2 = -\frac{P_T'^2 + M^2/\omega^2}{1 - 1/\omega},$$

$$K = \sum k_i,$$

d

$$2P_+' K_- = (P' + K)^2 \simeq s\,.$$

here $K_{max}^2 < s^\epsilon$ for any positive number ϵ, however small. If we sum over all n

$$A(\nu, q^2) = \sum A_n = -\frac{1}{128\pi^5} 2M^2 \int d^2P_T' d^2p_T' F(P_T'^2, q'^2) \frac{f_{\rho\omega\pi}^2}{16\pi^2} \frac{1}{s} \frac{\exp(3\rho^{2/3})}{\sqrt{3}\,\rho^{2/3}}, \tag{8}$$

re

$$\rho = \frac{f^2 K_{max}^2}{64\pi^2}\,.$$

: average multiplicity is given by

$$\langle n\rangle = \frac{\sum nA_n}{\sum A_n} = \frac{\sum nA_n}{A} \underset{s\to\infty}{\longrightarrow} \rho^{1/3}\,. \tag{9}$$

T ing into consideration $K_{max}^2/m_\pi^2 \simeq s/1\ \text{GeV}^2$ and sidering all possible inclusive diagrams $\pi^+\pi^+ \to \pi^- X$, $\pi^+\pi^- \to \pi^0 X$, $\pi^+\pi^0 \to \pi^+ X$, $\pi^+\pi^0 \to \pi^- X$, $\pi^0\pi^+ \to \pi^+ X$, $\pi^0\pi^+ \to \pi^- X$, $\pi^0\pi^+ \to \pi^0 X$, $\pi^0\pi^0 \to \pi^+ X$, and $\pi^0\pi^0 \to \pi^- X$, in Fig. 1, we find the average multiplicity for variety of pion[7] in pp scattering

$$\langle n\rangle_{pp} \simeq 9\left(\frac{f_{\rho\omega\pi}^2}{64\pi^2}\right)^{1/3} \left(\tfrac{4}{25}\right)^{1/3} s^{1/3}\,. \tag{10}$$

factor $\frac{4}{25}$ appears here owing to the fact that interacting pions in proton carry only $\frac{2}{5}$ of the energy.

st and Schmitt[8] were the first to show that g in the mean might be valid through a violation of Feynman scaling at $x = 0$ if and only if nd $\langle n\rangle \sim s^{1/3}$. This point was disputed by yanagi[9] who studied whether such a choice be allowed or not and concluded in the negative. The thrust of Yousuyangi's arguments rests fact that such a strong energy dependence of ge multiplicity ($\langle n\rangle \sim s^{1/3}$) reaches almost to inematical limit ($\langle n\rangle \leq \sqrt{s}/2m$) and is reportedly consistent with the present high-energy data. work is just a refutation of Yousuyanagi's both from theoretical results and experimental findings. According to the present model evident from Eq. (10), the dependence of ge multiplicity on $s^{1/3}$ is not a matter of but a dynamically derived result. This that the obtained $s^{1/3}$ dependence of $\langle n\rangle$ es well with reported experimental results also been shown by us.[6]

is a fact that the power law behavior implies a breakdown of Feynman scaling. However, for bounded transverse momenta and not very high energies (where $\langle n\rangle \sim \frac{1}{3}\ln s$ behavior is roughly consistent with the data) we get approximate scale invariance from our model. Thus approximate scaling behavior of multiparticle processes is, according to this picture, a consequence of bounded p_T. In the following we show that contrary to the hypothesis of Ernst and Schmitt, one can derive scaling in the mean even with the assumption of approximate scale invariance of the structure function.

III. TESTING OF SCALING IN THE MEAN IN SOME INCLUSIVE PROCESSES

First, we consider the case of $pp \to \pi^- X$ as a model example. From Eq. (8), we get[10] for $pp \to \pi^- X$ at small p_T and at $s = 10^3\ \text{GeV}^2$

$$E\frac{d^3\sigma}{dp^3} \simeq 88.87 \exp\left[-7.68\left(\frac{p_T^2 + \mu_\pi^2}{1-x}\right)\right] e^{-8.33x}, \tag{11}$$

where μ_π = pion mass in GeV, x = Feynman scaling variable $= 2p_L/\sqrt{s}$, p_T = transverse momentum of the observed pion in GeV, and $d^3\sigma/dp^3$ is in mb/GeV^3. Expression (11) gives a nice fit to the experimental curves for the inclusive cross sections at small values of x.

Now the scaling-in-the-mean hypothesis suggests that

$$\frac{\bar{p}_{LN}\bar{p}_{TN}}{N\sigma_N}\frac{d\sigma_N}{dp_L dp_T} = \phi\left(\frac{p_L}{\bar{p}_{LN}}, \frac{p_T}{\bar{p}_{TN}}\right), \tag{12}$$

where N is the number of produced charged pions and

$$\bar{p}_{xN} = \frac{1}{N\sigma_N}\int d^3p\,|p_x|\,\frac{d\sigma_N}{d^3p}, \quad x = T, L, \tag{13}$$

where the normalization condition is given by

$$\int d^3p\,\frac{d\sigma_N}{d^3p} = N\sigma_N\,. \tag{14}$$

From Eq. (8), we find for $x=0$

$$N\sigma_N = \int d^3p \, \frac{d\sigma_N}{d^3p} = A_N \ln p_L \,, \tag{15}$$

where A_N is an N-dependent term.

$$\bar{p}_{LN} = \frac{1}{N\sigma_N} \int d^3p \, p_L \frac{d\sigma_N}{d^3p} \simeq \frac{p_L}{\ln p_L} \tag{16}$$

and

$$\bar{p}_{TN} = \frac{1}{N\sigma_N} \int d^3p \, p_T \frac{d\sigma_N}{d^3p} \simeq a \text{ (const)}. \tag{17}$$

It may be remarked that at high energies the average transverse momentum is actually known to converge to a value which is independent of N: $\bar{p}_{TN} \xrightarrow[s\to\infty]{} a > 0$ (Refs. 6, 9). Now we write for $x=0$

$$\frac{\bar{p}_{LN}\bar{p}_{TN}}{N\sigma_N} \frac{d\sigma_N}{dp_L\, dp_T} = \frac{p_L}{\ln p_L} \frac{a}{A_N \ln p_L} \frac{A_N e^{-7.661(p_T^2+\mu_\pi^2)}}{p_L}$$

$$= \frac{a e^{-7.661(p_T^2+\mu_\pi^2)}}{\ln^2 p_L} \,. \tag{18}$$

Now substituting for $1/\ln^2 p_L = \bar{p}_{LN}{}^2/p_L{}^2$ as derived from Eq. (13) wè find

$$\frac{\bar{p}_{LN}\bar{p}_{TN}}{N\sigma_N} \frac{d\sigma_N}{dp_L\, dp_T} = \phi\left(\frac{p_L}{\bar{p}_{LN}}, p_T\right), \tag{19}$$

where $\bar{p}_{TN}$ in the right-hand side is a small constant. Thus we find that the validity of scaling in the mean can be supported[11] without fully abandoning Feynman scaling at small p_T. This argument is irrespective of the slight breaking of the Feynman scaling for the inclusive processes [as is evident from Eq. (8)] even for nonzero values of x.

So far we confined ourselves exclusively to the case of $pp\to\pi^- X$ which gives the general approach to the problem. Now we consider the cases $pp\to\pi^+ X$, $K^\pm X$, $pp\to\bar{p}X$, and $pp\to pX$. In our own model-dependent way we have deduced dynamically the expressions for the inclusive cross sections for all the above-mentioned cases, the details of which are given in a separate paper.[10] The relevant results are

$$E\frac{d^3\sigma}{dp^3}\bigg|_{pp\to K^- X} = 0.393\sigma_{in} \exp\left[-8.7\left(\frac{p_T{}^2+\mu_K{}^2}{1-x}\right)\right] e^{-1.6x},$$

$$E\frac{d^3\sigma}{dp^3}\bigg|_{pp\to \bar{p} X} = 53.4\sigma_{in} \exp\left[-7.38\left(\frac{p_T{}^2+\mu_{\bar{p}}{}^2}{1-x}\right)\right] e^{-3.1x}, \tag{20}$$

$$E\frac{d^3\sigma}{dp^3}\bigg|_{pp\to pX} = \sigma_{in} \frac{x}{6.25 p_T{}^2 + M^2(1-x)^2} \,.$$

It has also been shown in a model-dependent way in the same paper that

$$\frac{\sigma_{inc}(pp\to\pi^+ X)}{\sigma_{inc}(pp\to\pi^- X)} = 1 + 2.7 s^{-1/3}$$

and

$$\frac{\sigma_{inc}(pp\to K^+ X)}{\sigma_{inc}(pp\to K^- X)} = 1 + 10.96\, s^{-1/3}, \tag{21}$$

where s is in GeV^2.

The above expressions were compared with experimental results by the same authors and were found to be in excellent agreement for small values of x and p_T. At $x=0$ and $s=10^3/\text{GeV}^2$ Eqs. (20) and (21) become of the form:

$$E\frac{d^3\sigma}{dp^3}\bigg|_{pp\to\pi^+ X} = C'_{\pi^+} e^{-7.66 p_T{}^2},$$

$$E\frac{d^3\sigma}{dp^3}\bigg|_{pp\to K^- X} = C_{K^-} e^{-8.7 p_T{}^2},$$

$$E\frac{d^3\sigma}{dp^3}\bigg|_{pp\to K^+ X} = C'_{K^+} e^{-8.7 p_T{}^2}, \tag{22}$$

$$E\frac{d\sigma^3}{dp^3}\bigg|_{pp\to\bar{p}X} = C_{\bar{p}} e^{-7.38 p_T{}^2},$$

$$E\frac{d\sigma^3}{dp^3}\bigg|_{pp\to pX} = 0 \,.$$

As all the above expressions except (22) are similar in form to the case of $pp\to\pi^- X$, the validity of scaling in the mean in all such cases (except $pp\to pX$) is, according to this model, beyond any question or shadow of doubt. For $pp\to pX$, as the inclusive cross section vanishes at $x=0$, the question of validity of the scaling-in-the-mean hypothesis for this process does not naturally arise. Even for small nonzero values of x the scaling-in-the-mean hypothesis will not be valid in the present scheme as calculations have shown which is in agreement with Yaes's predictions or conclusions.[4,12]

IV. CALCULATION OF THE SCALING FUNCTIONS FOR THE PROCESS $pp\to\pi^- X$

Since according to the scaling-in-the-mean hypothesis the distributions considered are independent of multiplicity, energy, and initial state there should be one dominant mechanism for pion production at high energies and one should be able to describe the process in terms of only a small number of parameters. In order to arrive at this Ernst and Schmitt[13] applied an information-theoretic approach as the only means because of their idea that "universality probably means that not much dynamics is involved" and "whenever dynamics can be neglected, statistics might be applicable." But ours is a model which can explain universality from the very dynamics of interactions and so the statistical approach might not be the only means of understanding universality. In this section, we derive the actual values for the scaling functions themselves in our own model-dependent way and compare them with experimental results.

The scaling functions are defined by

$$\phi_L = \frac{\bar{p}_{xN}}{N\sigma_N}\frac{d\sigma_N}{dp_x} = \phi_x\left(\frac{p_x}{\bar{p}_{xN}}\right), \quad x = L, T \tag{23}$$

where ϕ_x is a universal function independent of s and N.

By making use of basic relations for inclusive cross sections given by Eqs. (20) and (21) at $x = 0$, we get

$$\frac{d\sigma_N}{dp_L} = \pi\frac{A_N}{p_L}\int e^{-7.68 p_T^2}\, dp_T^2, \tag{24}$$

where $A_N = 88.87$. From Eqs. (15), (16), and (24) we get

$$\frac{\bar{p}_{LN}}{N\sigma_N}\frac{d\sigma_N}{dp_L} = \frac{\pi}{\ln^2 p_L}\,\frac{\Gamma(1)}{7.68} \simeq \frac{0.41}{x_L^2}, \tag{25}$$

where $x_L = p_L/\bar{p}_{LN}$. The plot $\phi_L(x)$ versus x_L is shown in Fig. 2 and found to be in nice agreement with the experimental points. The solid line is our theoretical plot. Similarly, by using Eq. (17) we get the expression for the transverse-momentum scaling function given by

$$\phi_T(x_T) = \frac{\bar{p}_{TN}}{N\sigma_N}\frac{d\sigma_N}{dp_T} = 2\pi a e^{-7.68 a^2 x_T^2} a x_T, \tag{26}$$

where $a = \bar{p}_{TN}$, a small constant < 1, and $p_T = \bar{p}_{TN} x_T$ with $x_T = p_T/\bar{p}_{TN}$. Thus finally we arrive at the form for the transverse-momentum scaling functions from Eq. (26)

$$\phi_T(x_T) \simeq 2\pi a^2 x_T e^{-7.68 a^2 x_T^2}. \tag{27}$$

The nature of the theoretical curve and the experimental points are shown in Fig. 3. The agreement is pair for values of $x_T < 3$. In drawing our theoretical curve we have used $a = 0.3$.

In this connection we have also put to a test a new scaling form proposed by Nakagawa *et al.*[14] The form is

$$\phi'_L(x) = \frac{\langle n\rangle^{\alpha} p_L}{N\sigma_N}\frac{d\sigma_N}{dp_L}. \tag{28}$$

Inserting the relevant equations, the form of the

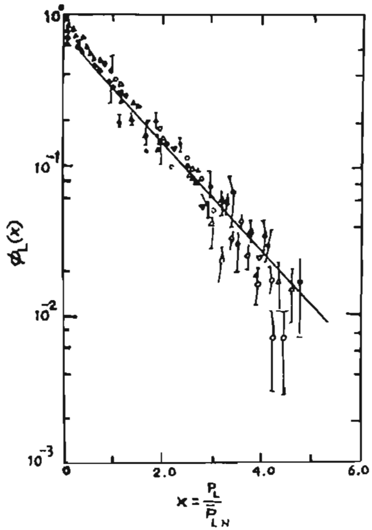

FIG. 2. Comparison of the data of Dao *et al.* (Ref. 1) with our theoretically deduced scaling function ϕ_L [Eq. (25)] $pp \rightarrow \pi^- X$. The data points refer to the various energies and various prongs and the solid line is our theoretical result.

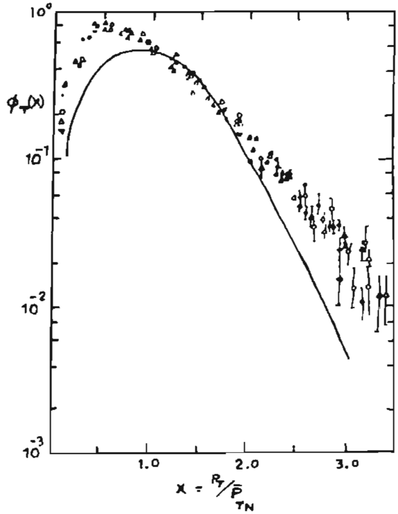

FIG. 3. Comparison of the data of Dao *et al.* (Ref. 1) with our theoretically derived scaling function ϕ_T [Eq. (27)] $pp \rightarrow \pi^- X$. The data points refer to the various energies and various prongs and the solid line represnets our theoretical values.

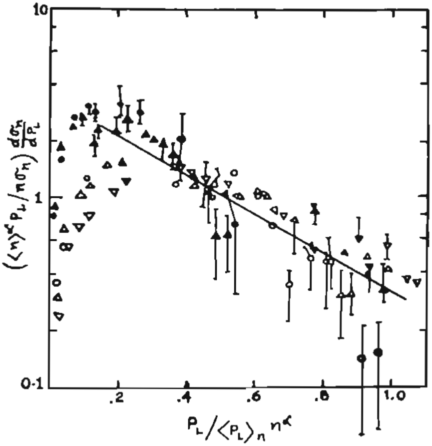

FIG. 4. Plot of $\langle n\rangle^\alpha p_L/N\sigma_N)d\sigma_N/dp_L$ versus $p_L/\langle p_L\rangle\langle n\rangle^\alpha$ for $pp\to\pi^- X$ with $\alpha = 1$. The solid line is our theoretical prediction.

new scaling function in this model becomes

$$\phi_L'(x)=\frac{\langle n\rangle^\alpha p_L}{A_N \ln p_L}\,\frac{\pi A_N}{p_L}\int e^{-7.86 p_T^2}d^2p_T\,, \qquad (29)$$

where α is an energy-dependent variable, the value of which ranges from small values to 1.

Thus finally we get the expression for the new scaling function

$$\phi_L'(x)=\frac{0.41}{p_L/\langle p_L\rangle\langle n\rangle^\alpha}=\frac{0.41}{x}\,, \qquad (30)$$

where x is the new variable given by $x=p_L/\langle p_L\rangle\langle n\rangle^\alpha$. The plot is given in Fig. 4 and compared to experimental results with reasonable success.

V. COMMENTS AND CONCLUSIONS

The entire work has here been conducted without taking into consideration any concept or result of KNO scaling and so, according to the present model, the question of the validity of the scaling-in-the-mean hypothesis is not in any way directly related to the propositions of KNO scaling. Of course, scaling in the mean has its own limitations. First, its validity—both theoretical and experimental—is restricted to the $x=0$ region. Second, the hypothesis seems to be true for only some selective nondiffractive processes since in diffractive events the multiplicity will be small, but the leading particles will carry off almost all the available energy for which $\langle p_L\rangle$ will not be large and the necessary conditions $\langle p_L\rangle \gg \langle p_T\rangle$ break down. This apart, the asymptotic validity of scaling in the mean is yet to be tested. Besides, the question of scaling in the mean for transverse momentum is even more difficult to resolve than for longitudinal momenta. The data seem to indicate that the average value of transverse momentum turns out to be a constant, independent of both s and n, and so at this stage scaling in the mean for transverse momentum would convey no additional information at asymptotic energies.[12]

Regarding the prescription of a new type of scaling-in-the-mean variable by Nakagawa *et al.*, despite fair agreement, we have serious reservations for several reasons: (i) α is energy dependent and an arbitrary variable which can hardly be linked up with physically tangible terms in connection with multiple production phenomena. (ii) The introduction of the new variable does not in any way mark any conceptual development. (iii) It renders the form clumsy. (iv) The authors are silent on the status and fate of scaling in the mean with regard to the transverse-momentum variables.

It has been nicely pointed out by Ernst and Schmitt[13] that "a characteristic feature of asymptotic dynamics should be a kind of universality which should manifest itself in a universal scaling function for different collision processes, i.e., collision processes with different particles in the initial state. The idea behind this is that, at different high-energy collisions, nearly identical systems are formed and that the system partially forgets the initial state in the subsequent decay. The first experimental support to this idea probably came from an experiment by Angelov *et* who showed that the distributions in $p_L/\bar{p}_{LN}$ and $p_T/\bar{p}_{TN}$ for both pp and πp scattering were almost identical. It is worthwhile to mention here that the present scheme of multiparticle production mechanism lends a strong support to and gives a dynamical explanation of the universal behavior of the average charged multiplicity. The same will be true of scaling in the mean also, because reaction mechanisms and multiple-production features will be almost similar in all the processes of the pp, πp, Kp, γp, ep, νp, μp, etc. Thus, so far as this universality is concerned, scaling in the mean according to the present model stands on the same footing as Feynman scaling.

Finally, so far as the relationship between scaling of semi-inclusive and inclusive reactions is concerned, our conclusions are somewhat the same as that of Yotsuyanagi *et al.*[16] on two scores

(i) The semi-inclusive scaling is compatible with the data of the inclusive reactions except in the $pp \to pX$ case. (ii) KNO scaling is not an ideal one and only a "temporary accident" as far as scaling in the mean holds. Furthermore, the present model does in no way justify the tagging of scaling in the mean with KNO scaling of any of its proposed forms.

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