Short Communications

A Note on Paroush's Comments Over the Ces Production Function*

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Arrow, Chenery, Minhas and Solow (ACMS) have developed the Constant Elasticity of Substitution (CES) production function of the form :

$$V=\gamma\left[\delta K^{-\rho}+(1-\delta)L^{-\rho}\right]^{-\frac{1}{\rho}} \quad \ldots(1)$$

where V is value added, L is labour input, K is capital input and

$\sigma=\frac{1}{1+\rho}$ =constant elasticity substitution [1]. Paroush has commented that assumption of competitive market condition has been made in developing equation (1) by ACMS [2]. It is, however, true that ACMS have approached empirically in developing the production function wherein they have not shown explicitly that the constraint of market condition is not necessary to derive the CES family of production functions. But if we note carefully, we find that the assumption of competitive market condition as made by ACMS has really been utilised to determine the third parameter, ρ, of the equation (1), while the development of equation (1) is perfectly general. Following the definitions of ACMS, we have

$$\sigma=-\frac{f'(f-xf')}{xff''},$$

where,
$$y=\frac{V}{L}=f\left(\frac{K}{L}\right)=f(x)$$

or,
$$\sigma=\frac{(f-xf')\frac{dy}{dx}}{y\frac{d}{dx}(f-xf')}=\frac{(f-xf')}{d(f-xf')}\cdot\frac{dy}{y}$$

or
$$\log y=\log a+\sigma\log\left(y-x\frac{dy}{dx}\right) \quad \ldots(2)$$

ACMS have really developed the equation (1) from a relation of this form with σ replaced by b, where

$$\log y=\log a+b\log w,\ w=\text{wage rate.}$$

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* Views expressed in this note are author's personal views.

They have shown that under competitive market condition $\sigma = b$. But as they have developed equation (1) from equation (2) [see equation (9), page 230 in reference 1], there is no loss of generality in the CES production function as developed by ACMS.

It should be further noted that more general nature of the family of CES production function, removing the restriction on degree of return to scale, would be of the form :

$$V = \gamma \left[\delta K^{-\rho} + (1-\delta) L^{-\rho} \right]^{-\frac{\lambda}{\rho}} \quad \text{.. (3)}$$

where the additional λ is scale parameter which may have a value less than, equal to or more than unity corresponding to, diminishing, constant or increasing return to scales respectively. The question of equilibrium, particularly for $\lambda > 1$, is being studied separately and the results will be communicated in a subsequent paper. This general function (3) can easily be deduced as a corollary to the theorem proved by Paroush in his note [2], which can be stated as follows :

> Every production function, V, homogeneous in K and L of degree λ, with constant elasticity of substitution σ, is a "mean value in K and L of order $1 - \frac{1}{\sigma}$ $(= -\rho)$" raised to the power λ.

It should be noted that if the competitive market condition is assumed to be true as is done by ACMS, both λ and ρ can now be empirically determined. For, under that condition, we get the following empirical correlation (4) as significant

$$\log V = a_1 + a_2 \log C + a_3 \log L, \quad \text{...(4)}$$

where C is total labour cost.

Then we have $\rho = \frac{a_3}{a_2}$ and $\lambda = \frac{a_3}{1 - a_2}$ · Substituting these values of λ and ρ in equation (3), other two parameters δ and γ can easily be determined like those in CES production function of ACMS.

References

[1] Arrow, K.J., Chenery, H.B., Minhas B.S. and Solow, R.M. (1961),
"Capital-Labour Substitution and Economic Efficiency".
The Review of Economics and Statistics, *43*, 225-248,

[2] Paroush, J. (1964)
"A Note on the CES Production Function"
Econometrica, *32*, 213-214.