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A measure of edge ambiguity using fuzzy sets

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Abstract: Algorithms for providing a quantitative measure of edge ambiguity are described through fuzzy measures in a set. An index is defined whose value is maximum for a grey tone image and decreases as the fuzziness in detecting edges decreases. The inherent fuzzifying property of the S-function is found to enable one not to use the INT operator for contrast enhancement and to save the time of computation greatly as measured by the index value. The index value is also found to increase with standard deviation of injected random noise.

Key words: Image processing, fuzzy sets, edge ambiguity.

1. Introduction

The object of an ege detector is to detect the presence and location of changes in grey levels in an image. When an image is processed for extracting edges/contours of its various regions, it is ultimately up to the viewers to judge its quality for a specific application and how well a particular method works. The process of evaluation of edge-enhancement quality (or edge ambiguity) therefore becomes a subjective one which makes the definition of a 'good edge-detected image' an elusive standard for comparison of algorithm performance.

The present work is an attempt to make this evaluation task somewhat objective by providing a quantitative measure of edge ambiguity in an image. The fuzzy measures, namely the index of fuzziness (Kaufmann, 1975), the entropy (De Luca and Termini, 1972) and the index of nonfuzziness are used here in defining an index of edge ambiguity. The membership functions for implementing these measures are made here position dependent to incorporate the spatial relationship among the grey levels. The index value is seen to decrease as edge ambiguity decreases.

Two contrast enhancement algorithms using the S-function and with/without the INT operator (Zadeh, 1975) are also presented here. The comparison of their performance is made on the basis of the index value when an X-ray image of a wrist is considered as input. The effect of noise on the index value is also studied in a part of the experiment.

2. Definitions

Let $X = \{\mu_X(x_{mn}) = \mu_{mn}/x_{mn},\ m = 1,2,\ldots,M;\ n = 1,2,\ldots,N\}$ be the fuzzy set representation of the pattern corresponding to an $M \times N$, L-level image array, where $\mu_X(x_{mn})$ or $\mu_{mn}/x_{mn}(0 \le \mu_{mn} \le 1)$ denotes the grade of possessing some property μ_{mn} (as defined in the next section) by the (m,n)th pixel intensity x_{mn}. Let $\underset{\sim}{X} = \{\mu_{\underset{\sim}{X}}(x_{mn})\}$ be similarly defined as the nearest ordinary plane to X, such that $\mu_{\underset{\sim}{X}}(x_{mn}) = 0$ if $\mu_X(x_{mn}) \le 0.5$ and is equal to 1 for $\mu_X(x_{mn}) > 0.5$.

The linear index of fuzziness $\gamma_l(X)$ and entropy $H(X)$ of the image X are defined as

$$\gamma_l(X) = \frac{2}{MN} \sum_m \sum_n |\mu_X(x_{mn}) - \mu_{\underset{\sim}{X}}(x_{mn})| \quad (1a)$$

$$= \frac{2}{MN} \sum_m \sum_n \mu_{X \cap \bar{X}}(x_{mn}). \quad (1b)$$

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$$H(X) = \frac{1}{MN \ln 2} \sum_m \sum_n S_n(\mu_X(x_{mn})), \tag{2a}$$

with Shannon's function

$$S_n(\mu_X(x_{mn})) = -\mu_X(x_{mn}) \ln \mu_X(x_{mn}) - (1-\mu_X(x_{mn})) \ln (1-\mu_X(x_{mn})). \tag{2b}$$

Let us define another measure called 'index of nonfuzziness' $\eta(X)$ as

$$\eta(X) = \frac{1}{MN} \sum_m \sum_n |\mu_X(x_{mn}) - \mu_{\bar{X}}(x_{mn})| \tag{3}$$

where $\bar{X}$ is the complement of X.

$\gamma_l(X)$ $(0 \le \gamma_l(X) \le 1)$ defines the amount of fuzziness present in the μ_{mn} plane of X by measuring the linear distance between the fuzzy property plane X and its nearest ordinary plane $\underline{X}$. $X \cap \bar{X}$ is the intersection between fuzzy image planes $X = \{\mu_{mn}/x_{mn}\}$ and $\bar{X} = \{(1-\mu_{mn})/x_{mn}\}$. $\mu_{X\cap\bar{X}}(x_{mn})$ denotes the degree of membership of x_{mn} to such a property plane $X \cap \bar{X}$ so that

$$\mu_{X\cap\bar{X}}(x_{mn}) = \mu_{mn} \cap \bar{\mu}_{mn} = \min\{\mu_{mn}, (1-\mu_{mn})\} \quad \text{for all } (m,n).$$

The term 'entropy' $(0 \le H(X) \le 1)$, on the other hand, measures the ambiguity in X by using Shannon's function in the property plane but its meaning is quite different from that of the classical entropy because no probabilistic concept is needed to define it. Both $\gamma(X)$ and $H(X)$ have the property that they increase monotonically in the interval $[0, 0.5]$ and decrease monotonically in $[0.5, 1]$ with a maximum (=unity) at $\mu = 0.5$ in the fuzzy property plane of X.

The index of nonfuzziness $(0 \le \eta(X) \le 1)$, as its name implies, measures the amount of nonfuzziness in μ_{mn} plane of X by computing its distance from its complement plane. Unlike $\gamma(X)$ and $H(X)$, its value decreases monotonically in $[0, 0.5]$ and monotonically increases in $[0.5, 1]$ with a minimum (=zero) at $\mu = 0.5$.

3. Enhancement algorithm

The contrast intensification operator INT (Zadeh, 1975) on a fuzzy set A generates another fuzzy set $A' = \text{INT}(A)$, the membership function of which is

$$\begin{aligned}\mu_{A'}(x) &= \mu_{\text{INT}(A)}(x) \\ &= 2(\mu_A(x))^2, \quad 0 \le \mu_A(x) \le 0.5 &\text{(4a)} \\ &= 1 - 2(1-\mu_A(x))^2, \quad 0.5 \le \mu_A(x) \le 1. &\text{(4b)}\end{aligned}$$

This operation reduces the fuzziness of a set A by increasing the values of $\mu_A(x)$ which are above 0.5 and decreasing those which are below it. Let us define operation (4) by a transformation T_1 of the membership function $\mu(x)$.

In general, each μ_{mn} in the image X may be modified to μ'_{mn} to enhance the image in the property plane by a transformation function T, where

$$\mu'_{mn} = T_r(\mu_{mn}) = \begin{cases} T'_r(\mu_{mn}), & 0 \le \mu_{mn} \le 0.5, \quad \text{(5a)} \\ T''_r(\mu_{mn}), & 0.5 \le \mu_{mn} \le 1, \quad \text{(5b)} \\ r = 1, 2, \ldots. \end{cases}$$

As r increases, contrast around the cross-over point (value of x_{mn} for which $\mu(x_{mn}) = 0.5$) increase and fuzzines in μ_{mn} plane as measured by equations (1) to (3) would decrease. In the limiting case, as $r \to \infty$, T_r produces a two-level (binary) image. It is to be noted that corresponding to a particular operation of T' one can use any of the multiple operations of T'' and vice versa to attain a desired amount of enhancement.

Property plane

All the operations described above are restricted to the fuzzy property plane. To enter this domain from the spatial x_{mn} plane, we define an expression of the form

$$p_{mn} = \mu_{mn} = G(x_{mn}) = (1 + (|\hat{x} - x_{mn}|/F_d)^{F_e})^{-1}, \tag{6}$$
$$m = 1, 2, \ldots, M; \; n = 1, 2, \ldots, N.$$

This represents the S-type function $G_s(x_{mn})$ for $\hat{x} = x_{\max}$, maximum level $(L-1)$ in X and a π-type function $G_\pi(x_{mn})$ for $\hat{x}$ = some arbitrary level l_c, $0 < l_c < x_{\max}$. F_e and F_d are two positive constants (called fuzzifiers) and their values are determined from the cross-over points in the enhancement operations.

Suppose x_c is the cross-over point (threshold level) for an S-type function. Then we have from equation (6)

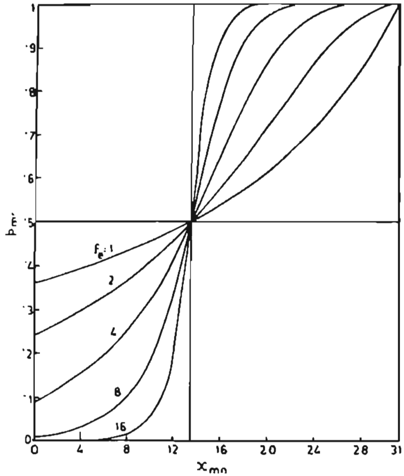

Figure 1. Plot of equation (9) for different values of F_e.

$$p_{mn} = G_s(x_c) = 0.5 = (1 + (|x_{\max} - x_c|/F_d)^{F_e})^{-1} \quad (7)$$

or

$$F_d = (x_{\max} - x_c) \Big/ \left(\left(\frac{1}{0.5} \right) - 1 \right)^{1/F_e} = x_{\max} - x_c. \quad (8)$$

In terms of the cross-over point, equation (6) can therefore be written as

$$p_{mn} = G_s(x_{mn}) = \frac{1}{1 + \left[\dfrac{x_{\max} - x_{mn}}{x_{\max} - x_c} \right]^{F_e}}. \quad (9)$$

A plot of this equation for different values of F_e is shown in Figure 1 for a 32-level image when x_c is set at 13.5. The algorithm using ambiguity measure for automatic selection of x_c without refering to the histogram is available in (Pal et al., 1983).

It is to be noted from equation (9) that for $x_{mn} = 0$, p_{mn} $(= \mu_{mn})$ has a finite positive value, say α, where

$$\alpha = (1 + (x_{\max}/(x_{\max} - x_c))^{F_e})^{-1}. \quad (10)$$

So the μ_{mn} plane becomes restricted in the interval $[\alpha, 1]$ instead of $[0, 1]$. After enhancement, the enhanced p'_{mn} plane (as obtained with equation (5)) may contain some regions where $\mu'_{mn} < \alpha$ due to the transformation T'. The algorithm includes a provision for constraining all the $p'_{mn} < \alpha$ values to α so that the inverse transformation

$$x'_{mn} = G_s^{-1}(\mu'_{mn}), \quad \alpha < \mu_{mn} \leq 1, \quad (11)$$

will allow those corresponding x'_{mn} values to have grey level zero.

Furthermore, the higher the value of F_e, the smaller is the value of α (as shown in Figure 1) and the greater is the resemblance with the standard S-function (Zadeh, 1975). Since the standard S-function does not have the provision of altering its cross-over point, we can not use it for contrast enhancement problem.

Let us now give two algorithms for automatic enhancement of contrast among successive regions in an image.

Algorithm 1.

Step 1. Apply the G_s transformation (equation (9)) to obtain the p_{mn} or μ_{mn} plane from the x_{mn} plane for a particular value of x_c and F_e.

Step 2. Modify the p_{mn} plane by r $(r = 1, 2, \ldots)$ successive applications of the INT operator $T_r(\mu_{mn})$ (equation (5)) to result in a contrast intensified property plane μ'_{mn}.

Step 3. Obtain an enhanced version $X' = \{x'_{mn}\}$ of the image X using inverse function G_s^{-1}.

Algorithm 2.

Step 1. Apply the G_s transformation to obtain the μ_{mn} plane from the x_{mn} plane for a particular value of x_c and F_e $(= F''_e$, say).

Step 2. Reduce the value of F_e to F'_e $(F'_e \ll F''_e)$ and apply the G_s^{-1} transformation on the μ_{mn} plane to result in an enhanced-contrast image plane x'_{mn}.

The higher the difference between F''_e and F'_e, the greater would be the contrast around x_c. Unlike Algorithm 1, Algorithm 2 does not need an INT operator for enhancement and therefore the time of computation can be reduced greatly.

4. Index for edge ambiguity

A measure of edge ambiguity in X may be defined as

$$\delta(X) = [1 - I(X)]^{\beta} \tag{12}$$

where $I(X)$ stands for $\gamma(X)$ or $H(X)$ or $(1-\eta(X))$ and β is a positive constant.

For the contrast enhancement problem we considered fuzziness only in the grey levels. But for computing $I(X)$, we consider fuzziness in the spatial domain and make the membership function $\mu_X(x_{mn})$ of the (m,n)th pixel in X dependent on its local distribution such that

$$\mu_X(x_{mn}) = \frac{0.5}{1 + 1/N_1 \sum_Q |x_{mn} - x_{ij}|}, \tag{13}$$
$$(i,j) \in Q, \ (i,j) \neq (m,n).$$

Q is a set of N_1 neighbouring coordinates of the point (m,n).

From equation (13) we see that if all the pixels have the same intensity then $x_{mn} = x_{ij}$ for all (m,n), $\mu(x_{mn}) = 0.5$ for all (m,n) and $I(X) = 1$. The measure of edge ambiguity would therefore be zero, as there is no edge in the image. An image with dissimilar grey levels would have a higher $\delta(X)$ value.

Since with increase in the value of r (equation (5)) the contrast among successive regions in X increases, the dissimilarity in grey levels would decrease bacause the pixels in a region would tend to possess similar intensity level. The value of $\delta(X)$ would therefore decrease with increase in r. Or, in other words, the higher the contrast among different regions in X, the less is the difficulty (ambiguity) in taking decision regarding edges (contours) or in detecting edges and hence the lower is the value of $\delta(X)$.

Therefore, if one applies an edge detection operator and computes the $\delta(X)$ value on the edge detected output, a decrease in edge ambiguity (i.e., conversion of grey tone edges to their two tone versions) would automatically correspond to a decrease in the value of $\delta(X)$.

5. Implementation

Figure 2 shows the radiograph of a part of the wrist containing a radius and a part of two small carpal bones. The digitised version of the picture is represented by an array of 128×145 $(= M \times N)$ dimension having 256 $(= L)$ grey levels. Figure 3 (Pal and King, 1983) shows the contours of different regions of Figure 2. These contours were obtained by applying a contrast enhancement technique (as described before) and then 'min' edge detector. Figures 3(a), 3(b) and 3(c) correspond to $r = 2$, 4 and 8 of the T_r operator (equation (5)), respectively.

Table 1 illustrates the value of $\delta(X)$ corresponding to Figures 2 and 3 when $I(X)$ in equation (12) stands for $\gamma_l(X)$ which reflects a measure of edge ambiguity decreases with a minimum of 0.10025 for two tone edges. As a typical illustration, we have demonstrated only three edge detected outputs of Figure 2. The algorithm described in (Pal and King, 1983) is similar to that of Algorithm 1 presented in Section 3 with the exception that the p_{mn} plane was extracted using

$$p_{mn} = G_s(x_{mn}) = \left(1 + \frac{|x_{max} - x_{mn}|}{F_d}\right)^{-F_e}. \tag{14}$$

The slight modification of equation (14), given in equation (6), is found to reduce the number (r) of

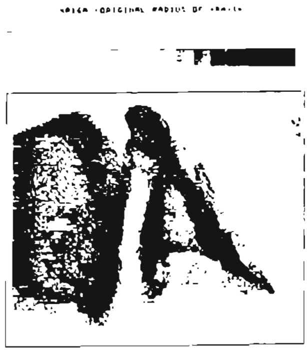

Figure 2. Input X-ray image.

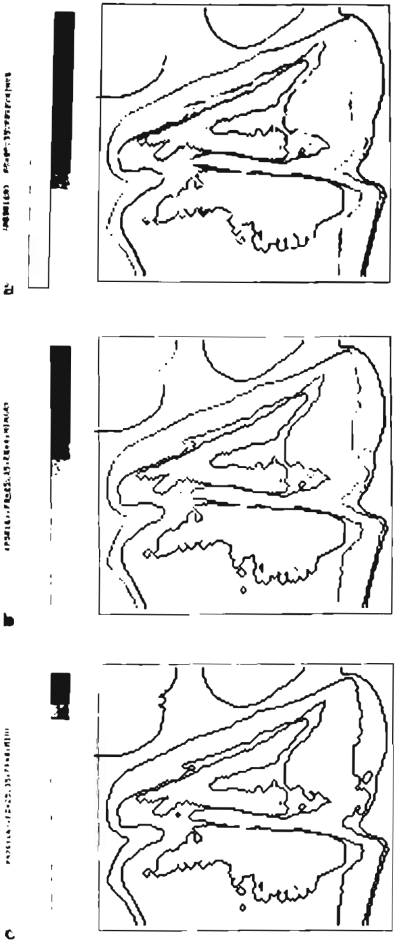

Figure 3. Edge detected output; (a) $r=2$, (b) $r=4$, (c) $r=8$.

Table 1
$\delta(X)$ value for the agorithm in (Pal and King, 1983)

r	$\delta(X)$
2	0.32187
4	0.17542
5	0.12828
6	0.11493
7	0.10749
8	0.10221
10	0.10025

$\delta(X)$ for output image (Figure 2) = 0.53369

Table 2
$\delta(X)$ value for Algorithm 1

r	$\delta(X)$
2	0.28152
3	0.16050
4	0.10932
5	0.10045
6	0.10025

Table 3
$\delta(X)$ value for Algorithm 2

F_e^-	F_e'	$\delta(X)$
40	4	0.17494
40	2	0.14251
40	1	0.11660
40	0.5	0.10850
40	0.25	0.10254
40	0.125	0.10025
60	4	0.14347
60	2	0.12184
60	1	0.11159
60	0.5	0.10361
60	0.25	0.10118
60	0.125	0.10025

Table 4
$\delta(X)$ value for $\sigma = 4$ with zero mean

r	Algorithm 1	Algorithm[a]
2	0.39521	0.43738
3	0.23709	0.34490
4	0.16380	0.25175
5	0.15081	0.18929
6	0.15022	0.16812
7	⋮	0.15887
8	⋮	0.15231

[a]Algorithm by Pal and King (1983).

the INT operation to attain a desired edge detected output. This is shown in Table 2 where the value of $\delta(X)$ is always seen to be smaller than that in Table 1 for a particular value of r. This superiority is because of the greater fuzzifying property and contrast steepness of equation (6) (Figure 1) as compared to equation (14).

Table 3 shows the value of $\delta(X)$ for different combinations of F_e'' and F_e' when Algorithm 2 is considered as enhancement operation. The greater the difference between F_e'' and F_e', the lower is the ambiguity in detecting edges and the smaller is the $\delta(X)$ value. Since we need no more INT operations here, the time of computation is also further reduced for a desired output.

The average time of computation by EC 1033 is found to be 2 mins 34 secs and 2 mins 28 secs for Algorithms 1 and 2 respectively as compared to 2 mins 38 secs required for the one in Pal and King (1983).

In support of our above mentioned claim for noisy images and to demonstrate the effect of noise on edge detected output, the experiment was also conducted by making Figure 2 corrupted by random noise with zero mean and various standard deviations (σ). Tables 4 and 5 show as a typical illustration, the $\delta(X)$ values corresponding to three algorithms when $\sigma = 4$ with mean zero. The decrease in the value of $\delta(X)$ with increase of r is also found to hold good for noisy image. It is also seen that for a particular value of r or (F_e'', F_e'), $\delta(X)$, as expected, increases as standard deviation of noise injected on X increases. For example, consider Algorithm 1 with $r = 6$. $\delta(X)$ increases from 0.10025 to 0.10456, 0.11429, 0.12941 and 0.15022 as σ increases from 0 to 1, 2, 3 and 4 respectively. This increase in $\delta(X)$ value is because of the contributions of the spurious wiggles which appeared in the edge-detected output due to noise.

Table 5
$\delta(X)$ value for Algorithm 2 when $\sigma = 4$ with zero mean

F_e''	F_e'	$\delta(X)$
60	4	0.21585
60	2	0.17713
60	1	0.16279
60	0	0.15523
60	0.25	0.15117
60	0.125	0.15022

6. Conclusion

Fuzzy measures in a set have been used to define an index of edge ambiguity which is found to provide, on a global level, a quantitative measure of the amount of difficulty (ambiguity) in detecting contours of various regions in an image.

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