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How to discriminate shapes using the shape vector

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Abstract: A scalar transform to describe a 2-dimensional shape is proposed. The transform, called the shape vector, does not depend on the position, size and orientation of a region, and it is shown how the shape vector can be used in discriminating shapes.

Key words: Region, centre of gravity, radius of a region, shape vector, shape distance.

Introduction

Shape discrimination is a major problem in Pattern Recognition. There are two types of description on which this discrimination is based. One involves structural descriptions of a shape while the other describes a shape in terms of scalar measurements. Structural descriptions of a shape are usually made on the basis of medial axis transforms, Freeman chain codes, decomposition into simpler subsets etc. [1–6]. Among scalar transforms are moments, Fourier coefficients [7–9], etc.

In this paper we will propose a scalar transform to describe a 2-dimensional shape. This transform will be called the shape vector. As we will see later, the shape vector does not depend on the position, size and orientation of a region in the plane. Hence, it can be used to discriminate shapes of 2-dimensional regions.

In Section 2 we give the definitions of the shape vector, the shape distance and the shape similarity measures. An algorithm for extraction of the shape vector from a 2-dimensional region is given in Section 3. In Section 4 we discuss some properties of the shape vector and propose an extension of the shape vector to 3-dimensional solids.

2. The shape vector of a region

A *region* A is a compact and connected subset in the 2-dimensional plane such that the closure of the interior of A is A itself. The shape of a region is what remains of a region after disregarding its size, position and orientation. In other words, two regions have the same shape if we can make them coincide exactly by translation, dilation and rotation. Dilation involves uniform change of scale along the x and y axes. Clearly, all circles have the same shape. So do all squares. But the shape of a rectangle (or an ellipse) depends on the ratio of the lengths of its two axes.

Now, if a transform of a region is to represent the shape of the region, it has to be independent of the size, position and orientation of the region. We will construct such a transform now.

The *centre of gravity* of a region A is defined to be the point $C=(c_1, c_2)$ where

$$c_1=\int_A x\,\mathrm{d}a/\text{area }(A),$$

$$c_2=\int_A y\,\mathrm{d}a/\text{area }(A),$$

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where ∫ means Riemann integration over A, x and y indicate the coordinates of points in A.

The *radius* r of a region A is the distance of the farthest point of A from its centre of gravity. In other words,

$$r = \sup_{(x,y)\in A} \{(c_1 - x)^2 + (c_2 - y)^2\}^{1/2}.$$

Note that the radius of a region A is invariant under translation, dialation and rotation.

We define n concentric rings of equal width around the centre of gravity (c_1, c_2) in the following way: The k-th ring T_k is the set

$$\left\{(x, y) \in \mathbb{R}^2 : (k-1)\frac{r}{n} < \{(c_1 - x)^2 + (c_2 - y)^2\}^{1/2} \leq \frac{kr}{n}\right\} \quad \text{for } k = 1, 2, \ldots, n.$$

$\mathbb{R}^2$ is the Euclidean plane.

It is clear that the region A is a subset of $\bigcup_{k=1}^{n} T_k$ and in fact, a proper subset unless A is a circle. The area of T_k is $2\pi(2k-1)r^2/n^2$. Now, for each ring T_k we define the proportion of the black portion (i.e. of the region A) present in it as the area of $A \cap T_k$ divided by the area of T_k. Let this proportion for the k-th ring be v_k where $0 \leq v_k \leq 1$. The n-dimensional vector $(v_1, v_2, \ldots, v_n)$ is called the *shape vector* of the region A and is denoted by $V_n(A)$.

Thus, V_n describes a mapping that transforms a region in the plane into a point in the n-dimensional Euclidean space and hence brings about a great deal of reduction in the dimensionality of the description of a shape. From the construction of V_n, it is clear that the mapping V_n is not unique in general. That is, two regions with different shapes may give rise to the same shape vector (Figure 1). However, for a limited class of shapes the mapping is often unique and can be used for pattern discrimination.

The shape vector of a region is $(1, 1, \ldots, 1)$ if and only if the region is a circular disc. The shape vectors of some standard shapes are given in Table 1.

A shape distance D can be defined for 2-dimensional regions on the basis of the shape vector as

$$D(s_1, s_2) = \frac{1}{n}\sum_{k=1}^{n} |v_k^1 - v_k^2|$$

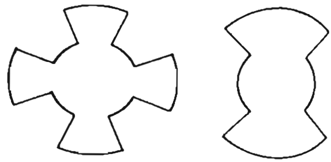

Figure 1. The two regions above have different shapes but their shape vectors are the same for any value (even) of n. In fact, in the shape vector the first $n/2$ entries will be 1 while the last $n/2$ entries will be 0.5.

where $(v_1^1, v_2^1, \ldots, v_n^1)$ and $(v_1^2, v_2^2, \ldots, v_n^2)$ are the shape vectors of the shapes s_1 and s_2 respectively.

Note that the value of D depends on the value of n. But for any fixed value of n and for any shapes s_1, s_2 and s_3,

(i) $D(s_1, s_2) \geq 0$;

(ii) $D(s_1, s_2) = 0$ if $s_1 = s_2$;

(iii) $D(s_1, s_2) = D(s_2, s_1)$; and

(iv) $D(s_1, s_2) + D(s_2, s_3) \geq D(s_1, s_3)$.

D does not define a metric on the set of shapes since the converse of (ii) above is not true. D defines a pseudometric on shapes. A different and more general pseudometric among shapes on the basis of the shape vector is the square root of $(V^1 - V^2)^t A (V^1 - V^2)$ where A is a symmetric positive definite matrix and V^1 and V^2 are shape vectors of two shapes.

A shape similarity measure μ between two shapes of 2-dimensional regions can be defined as

$$\mu(s_1, s_2) = 1 - D(s_1, s_2).$$

Since D lies between 0 and 1, so does μ. If two shapes resemble each other closely, the value of μ is high. The shape vector of a circular disc is $(1, 1, \ldots, 1)$. So the degree of circularity of an arbitrary shape with the shape vector $(v_1, v_2, \ldots, v_n)$ is

$$1 - \frac{1}{n}\sum_{k=1}^{n}(1 - v_k) = \frac{1}{n}\sum_{k=1}^{n} v_k$$

The values of the similarity measure μ between some standard shapes are given in Table 2. The computations are done using the algorithm given in the next section.

Table 1
Shape vectors ($n = 8$) for some standard shapes

		V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
Circle		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Square		1.00	1.00	1.00	1.00	1.000	0.89	0.35	0.09
Ellipse	(1.1)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.61
	(1.2)	1.00	1.00	1.00	1.00	1.00	1.00	0.91	0.36
	(1.5)	1.00	1.00	1.00	1.00	1.00	0.80	0.44	0.20
	(2.0)	1.00	1.00	1.00	1.00	1.00	0.42	0.27	0.13
	(2.5)	1.00	1.00	1.00	0.74	0.44	0.30	0.20	0.10
	(3.0)	1.00	1.00	0.92	0.52	0.35	0.24	0.16	0.08
	(4.0)	1.00	1.00	0.59	0.36	0.25	0.18	0.12	0.06
Rectangle	(1.1)	1.00	1.00	1.00	1.00	1.00	0.87	0.36	0.09
	(1.2)	1.00	1.00	1.00	1.00	1.00	0.78	0.39	0.09
	(1.5)	1.00	1.00	1.00	1.00	0.88	0.60	0.43	0.10
	(2.0)	1.00	1.00	1.00	0.91	0.59	0.45	0.37	0.14
	(2.5)	1.00	1.00	0.99	0.66	0.46	0.36	0.30	0.16
	(3.0)	1.00	1.00	0.87	0.52	0.38	0.30	0.25	0.16
	(4.0)	1.00	0.99	0.58	0.37	0.28	0.23	0.19	0.14

The ratio of the major axis to the minor axis for the ellipses and rectangles are given within brackets.

Table 2
Values of μ between some standard shapes ($n = 8$)

		Circle	Square	Ellipse							Rectangle						
				1.1	1.2	1.5	2.0	2.5	3.0	4.0	1.1	1.2	1.5	2.0	2.5	3.0	4.0
Circle		1.00	0.79	0.95	0.91	0.81	0.68	0.60	0.53	0.44	0.79	0.78	0.75	0.68	0.62	0.56	0.47
Square		0.79	1.00	0.84	0.88	0.96	0.88	0.80	0.74	0.65	0.99	0.98	0.94	0.87	0.81	0.75	0.67
Ellipse	1.1	0.95	0.84	1.00	0.96	0.85	0.73	0.65	0.58	0.49	0.84	0.83	0.80	0.73	0.67	0.61	0.52
	1.2	0.91	0.88	0.96	1.00	0.90	0.78	0.69	0.62	0.53	0.88	0.87	0.84	0.77	0.71	0.65	0.56
	1.5	0.81	0.96	0.85	0.90	1.00	0.88	0.79	0.73	0.64	0.97	0.98	0.94	0.88	0.81	0.75	0.67
	2.0	0.68	0.88	0.73	0.78	0.88	1.00	0.91	0.85	0.76	0.89	0.89	0.93	0.96	0.92	0.87	0.79
	2.5	0.60	0.80	0.65	0.69	0.79	0.91	1.00	0.94	0.84	0.81	0.81	0.85	0.92	0.96	0.93	0.86
	3.0	0.53	0.74	0.58	0.62	0.73	0.85	0.94	1.00	0.91	0.74	0.75	0.78	0.85	0.92	0.96	0.92
	4.0	0.44	0.65	0.49	0.53	0.64	0.76	0.84	0.91	1.00	0.65	0.66	0.69	0.76	0.83	0.88	0.96
Rectangle	1.1	0.79	0.99	0.84	0.88	0.97	0.89	0.81	0.74	0.65	1.00	0.99	0.94	0.88	0.81	0.75	0.67
	1.2	0.78	0.98	0.83	0.87	0.98	0.89	0.81	0.75	0.66	0.99	1.00	0.96	0.89	0.82	0.76	0.68
	1.5	0.75	0.94	0.80	0.84	0.94	0.93	0.85	0.78	0.69	0.94	0.96	1.00	0.92	0.85	0.80	0.71
	2.0	0.68	0.87	0.73	0.77	0.88	0.96	0.92	0.85	0.76	0.88	0.89	0.92	1.00	0.93	0.87	0.79
	2.5	0.62	0.81	0.67	0.71	0.81	0.92	0.96	0.92	0.83	0.81	0.82	0.85	0.93	1.00	0.94	0.86
	3.0	0.56	0.75	0.61	0.65	0.75	0.87	0.93	0.96	0.88	0.75	0.76	0.80	0.87	0.94	1.00	0.91
	4.0	0.47	0.67	0.52	0.56	0.67	0.79	0.86	0.92	0.96	0.67	0.68	0.71	0.79	0.86	0.91	1.00

3. Algorithm to compute the shape vector

Let a digital region be represented as a binary matrix $M(i,j)$. That is, $M(i,j) = 0$ or 1.

Step 1. The total number of 1-pixels in the picture is computed as $\text{NUM} = \sum_i \sum_j M(i,j)$. The coordinates of the centre of gravity of the picture are computed as

$$c_1 = \sum_{M(i,j)=1} i/\text{NUM} \quad \text{and} \quad c_2 = \sum_{M(i,j)=1} \text{NUM}.$$

We do not truncate or round off c_1 and c_2 to integers.

Step 2. The boundary of the binary picture is scanned. Let B be the set of boundary pixels. Then the radius r of the binary picture is obtained from

$$r^2 = \max_{(i,j)\in B} [(i-c_1)^2 + (j-c_2)^2].$$

Step 3. The matrix M is extended such that the set $\{(i,j): (i-c_1)^2 + (j-c_2)^2 \le r^2\}$ is contained in the extended matrix. Obviously, for the additional pixels (i,j), $M(i,j)=0$. Now, each of the 0-pixels is classified into one of the n concentric rings or outside in the following way: a 0-pixel (i,j) belongs to the k-th ring if $(k-1)r^2/n < (i-c_1)^2 + (j-c_2)^2 \le Kr^2/n$ and outside if $(i-c_1)^2 + (j-c_2)^2 > r^2$. Each of the 1-pixels is classified into one of the n concentric rings in a similar way. Note that no 1-pixel falls outside. thus for each of the n concentric rings, we know the numbers of 0-pixels and 1-pixels in it.

Step 4. Finally, the shape vector $(v_1, v_2, \ldots, v_n)$ is obtained as v_k = (number of 1-pixels in the k-th ring)/(total number of 0-pixels and 1-pixels in the k-th ring).

4. Conclusion

It is to be noted that the description of a shape in terms of its shape vector has a basic inadequacy. That is, it is not information preserving. For an information preserving description, the description becomes better with increasing value of n and asymptotically determines the shape uniquely. it is true that the set of different shapes giving rise to the same shape vector reduces as n increases, but does not tend to a singleton in general as n goes to infinity. For example, in Figure 1, two different shapes give rise to the same shape vector however large the value of n(even) is. Thus, the shape vector is not an information preserving description. However, the shape vector is easy to compute and may be used for shape discrimination for a class of limited number of shapes.

The extension of shape vectors of 2-dimensional regions to the shape vector of a 3-dimensional solid A is (c_1, c_2, c_3) where $c_1 = \int_A x\,\mathrm{d}a/\text{volume}(A)$, $c_2 = \int_A y\,\mathrm{d}a/\text{volume}(A)$ and $c_3 = \int_A z\,\mathrm{d}a/\text{volume}(A)$. The radius r of A is

$$\sup_{(x,y,z)\in A} ((c_1-x)^2 + (c_2-y)^2 + (c_3-z)^2)^{1/2}.$$

Instead of concentric rings T_k in 2-dimensions, we consider concentric rings p_k in 3-dimensions where

$$p_k = \left\{(x,y,z)\in\mathbb{R}^3 : (k-1)\frac{r}{n} < ((c_1-x)^2 + (c_2-y)^2 + (c_3-z)^2)^{1/2} \le \frac{Kr}{n}\right\}.$$

For each ring p_k, v_k is defined as the ratio of the volume of $(A \cap p_k)$ to the volume of p_k. The shape distance and the shape similarity measure can be defined for 3-dimensions in a similar fashion.

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