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Methods of computation of suspended load from bed materials and flow parameters

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ABSTRACT

Computation of the grain-size distribution of the suspended load above a sand bed must take into consideration: (1) sorting processes from the bed to the bed layer and (2) sorting between the bed layer and suspension. Grain-size distributions of the bed layers above sand beds of three different types have been computed in this work, both by the Einstein and the Gessler methods. Using these as references, suspended load distributions have been obtained in each case by the Rouse suspension equation. A new formula has also been developed in partial modification of Hunt's method for *direct* computation of bed load and suspended load from a bed's grain-size distribution and flow parameters.

Comparison of the computed data with actual observations in laboratory flumes show that no one method is particularly superior to the others, but the present method is advantageous because it affords *direct* computation of the suspended load from a bed's grain-size distribution, without going through an intermediate stage (bed load). The possible sources of error in each of the methods have been discussed.

INTRODUCTION

Controlled experiments in laboratory flumes have shown earlier that grain-size distributions of sediments suspended in flowing water bear definite relationship with the bed material, flow velocity and height of suspension above the bed. These studies have also shown that during the flow a sorting mechanism is initiated immediately above the bed (at or near the bed layer) and the grain-size distribution of the bed layer influences the size distribution of the suspension above (Sengupta, 1979). Any attempt to compute theoretically the grain-size distribution of the suspended load above a sand bed must therefore take into consideration a two-stage sorting process: (1) sorting from the bed to the bed layer, and (2) sorting between the bed layer and the suspension above.

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Several well-known methods for computation of the bed load are available (see Graf, 1971, chapter 7, for a general discussion). Using one of these formulae for computation of the bed load, the suspended load can be computed with the help of the well-known Rouse equation, using the distribution at the upper boundary of the bed layer as the reference. A new formula, developed in this article, in partial modification of a method suggested by Hunt (1954), affords direct computation of the bed load and the suspended load from the bed's grain-size distribution, given the flow parameters.

Grain-size frequency distributions of the suspended loads above sand beds of three different types have been computed in this paper with the help of the Rouse equation, using the bed layer distributions obtained independently by the Einstein (1950) and Gessler (1965) methods as references. The suspended load distributions above each of the three sand beds have also been computed with the help of the modified Hunt method developed in this work. The efficiency of each of these methods has been tested by comparing the computed data with actual observations on grain-size distributions of suspended load samples collected in close-circuit hydraulic flumes under known flow conditions.

EXISTING METHODS

The bed load equations of Einstein (1950) and Gessler (1965) and the suspension equation of Rouse (1938) are briefly discussed below.

Einstein's bed load equation

Einstein (1950) developed a bed load formula which relates the rate of bed load transport (Φ_*) to properties of the grain and of the flow causing the movement (Ψ_*)

$$\Phi_* = \frac{i_B q_B}{i_b \rho_s g}\left\{\frac{\rho}{(\rho_s-\rho)\, g D^3}\right\}^{\frac{1}{2}} \tag{1}$$

and

$$\Psi_* = \xi Y \Psi(\beta_1/\beta_x)^2. \tag{2}$$

This bed-load equation expresses the equilibrium conditions of the exchange of bed particles between the bed layer and the bed (see Appendix A for key to symbols).

In equation (2) ξ was designated by Einstein as the 'hiding factor' and was defined as a function of D/X. This definition assumed that the small particles hide between the larger ones or within the laminar sublayer. The correction factor Y describes the change of lift coefficient in mixtures with various roughness. It is a function of $k_{s/\delta}$, where k_s, the representative grain diameter of the bed material, is given by 'that size of which 65% of the mixture (by weight) is finer' (Einstein, 1950).

Gessler's bed load equation

Gessler (1965), made the following assumptions for studying the problem of bed-load transport; (1) the turbulent fluctuations of the bed shear stress are distributed according to the normal-error law, and (2) a grain starts to move when the 'effective (instantaneous) bottom shear stress' on it exceeds a critical value which is a function of the grain size and the grain Reynolds number (Re_*).

According to a Gaussian distribution, the probability of a grain being eroded from the bed is given by

$$\begin{aligned} p &= 1-q\left\{\frac{\tau_0}{(\gamma_s-\gamma_f)\, D} < T_c\right\} \\ &= 1-\frac{1}{\sigma_1\sqrt{2\pi}}\int_{-\infty}^{T_c/\bar{T}-1} \exp(-t^2/2\sigma_1^2)\mathrm{d}t \end{aligned} \tag{3}$$

where $\frac{\tau_0-\bar{\tau}_0}{\bar{\tau}_0}$ follows a Gaussian distribution with mean zero and variance σ_1^2 and

$$\bar{T} = \frac{\bar{\tau}_0}{(\gamma_s-\gamma_f)\, D}. \tag{4}$$

For a given grain size, T_c is constant. In this method, R_{e*} being known, the values of T_c could be read directly from the modified Shields' curve provided by Gessler (1965, fig. 8).

The bed load (concentration at the bed layer) for a sand bed was obtained by multiplying the bed concentration by the probabilities for the different grain sizes as obtained by equation (3).

The suspension equation

The equation commonly used for predicting the suspension concentration in a turbulent flow, developed from the work of Schmidt (1925) and others in Germany (popularized in the English-speaking world by Rouse, 1938, and hence is commonly referred to as the Rouse equation). It is a particular case of the diffusion equation in which the upward transport of material due to turbulent mixing is equated to the sediment settling velocity,

$$\frac{S_y}{S_a} = \left(\frac{d-y}{y}\cdot\frac{a}{d-a}\right)^{c(\phi)/(\chi u^*)} \tag{5}$$

With the help of this well-known equation it is possible to calculate the concentration of material of a particular grain size at any level (S_y) in the channel, provided the concentration at a reference level (S_a) is known.

A METHOD FOR DIRECT COMPUTATION OF SUSPENDED LOAD

In a uniform flow, where the concentration varies only with the vertical coordinate y throughout the depth and the diffusion coefficients of sediment and water are assumed to be the same (i.e. $\epsilon_s = \epsilon_m$), the concentration equation for sediment is of the form (see Hunt, 1954)

$$\epsilon_s \frac{\partial S_y}{\partial y}+(1-S_y)\, cS_y = 0. \tag{6}$$

For fully developed turbulent flow, the momentum diffusion coefficient ϵ_m for water is given by

$$\epsilon_m = \frac{\tau}{\rho \mathrm{d}u/\mathrm{d}y}, \tag{7}$$

where τ is the shear stress at any point in the fluid and is of the form

$$\tau = \tau_0(1-y/d). \tag{8}$$

Combining equations (7) and (8), and taking $\epsilon_s = \epsilon_m$, the diffusion coefficient for sediment ϵ_s can be written as

$$\epsilon_s = \tau_0(1-y/d)/\rho\frac{\mathrm{d}u}{\mathrm{d}y}. \tag{9}$$

To find ϵ_s, we take the von Kármán velocity distribution used by Hunt (1954) as follows

$$\frac{u-U_{max}}{u_*} = \frac{1}{\chi}\left[(1-y/d)^{1/2}+B\ln\left\{\frac{B-(1-y/d)^{1/2}}{B}\right\}\right] \tag{10}$$

which satisfies the boundary condition $u = U_{max}$ at $y = d$ at the free surface and B is a constant. Our determination of B is different from that of Hunt. We determine B from the condition that $u = 0$ at $y = k_s$, the roughness of the bed (see Schlichting, 1968). Since k_s is extremely small in comparison with the depth of the flow d, the ratio k_s/d is neglected for higher powers of k_s/d. Under this assumption and after making some simplifications (Ghosh, Mazumder & Sengupta, 1979), the constant B is found out as

$$B = 1-\frac{1}{2}\frac{k_s}{d}+\exp\left[-1-\frac{\chi U_{max}}{u_*}\right]. \tag{11}$$

If we put $B = 1$ in equation (10), the velocity distribution coincides with that of von Kármán (1930). Hunt (1954) investigated the values of B experimentally. He also pointed out that the agreement between equation (10) and the observed velocities is close throughout the depth even for the observations made nearest to the bed, at a height of about 0·1 inch (0·25 cm).

The velocity distribution (equation 10) with B given by equation (11), is plotted against y/d for various values of maximum velocities (U_{max}) above the three sand beds (Fig. 1). It is seen that the agreement between the observed and expected velocities is very close throughout the vertical height y.

Since the velocity distribution (equation 10) is valid only down to $y = 0{\cdot}25\ \mathrm{cm} = y_1$ (say), a linear velocity distribution is assumed below this height, $y = y_1$ and down to the point $y = k_s$, where the velocity is assumed to be zero (Jobson & Sayre, 1970). The linear velocity profile is of the form

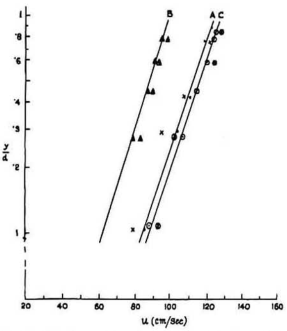

Fig. 1. Velocity profiles above the sand beds used for flume experiments. A, bed 2 (×, observed; ● computed) B, bed 3 (△, observed; ▲ computed) C, bed 5 (⊗ observed; ⊙ computed).

$$u = \frac{u_{y_1}}{(y_1-k_s)}(y-k_s), \quad y_1-k_s \neq 0, \tag{12}$$

where $u_{y=y_1}$ is the extrapolated velocity from equation (10) with equation (11).

Using equations (9), (10) and (12), one obtains the expressions for sediment diffusion coefficient:

$$\epsilon_s = 2\chi d u_*(1-y/d)\{B-(1-y/d)^{1/2}\} \tag{13}$$

and for the linear velocity profile

$$\epsilon_s = \frac{u_*^2}{u_{y_1}}(y_1-k_s)(1-y/d). \tag{14}$$

Integrating equation (6) from k_s to y using equations (13) and (14), one obtains:

$$\log_e\frac{S_y(1-S_{k_s})}{S_{k_s}(1-S_y)} = K_1\log_e\frac{d-y_1}{d-k_s} + K_2\log_e\left[\frac{(1-y/d)^{1/2}}{(1-y_1/d)^{1/2}}\cdot\frac{B-(1-y_1/d)^{1/2}}{B-(1-y/d)^{1/2}}\right], \tag{15}$$

where

$$K_1 = \frac{c(\phi)\,\mathrm{d}u_{y_1}}{u_*^2(y_1-k_s)}, \quad K_2 = \frac{c(\phi)}{B\chi u_*}.$$

The first term of the right side of expression (15) is found by integrating from k_s to y_1 using equation (14). The second term is obtained by integrating from y_1 to y using equation (13), which is the same as that of Hunt. In a compact form, equation (15) can be written as

$$\frac{S_y}{1-S_y} = \frac{S_{k_s}}{1-S_{k_s}}f(y,\phi) \tag{16}$$

where

$$f(y,\phi) = \left(\frac{d-y_1}{d-k_s}\right)^{K_1}\left[\frac{(1-y/d)^{\frac{1}{2}}}{(1-y_1/d)^{\frac{1}{2}}}\cdot\frac{B-(1-y_1/d)^{\frac{1}{2}}}{B-(1-y/d)^{\frac{1}{2}}}\right]^{K_2}.$$

Let

$$x_1 = \frac{S_{k_s}}{1-S_{k_s}} f(y,\phi) \quad \text{and} \quad x_1' = \frac{S_{k_s}'}{1-S_{k_s}'} f(y,\phi). \quad (17)$$

If we assume $S_{k_s} = \alpha S_{k_s}'$, then from equations (16) and (17), S_y can be written as

$$S_y = \frac{x_1}{1+x_1} \sim \frac{\alpha x_1'}{1+x_1'}. \quad (18)$$

It is reasonable to expect $\alpha \simeq 1$ because at the bed boundary the amount of sediment is relatively large compared to water. Even if α is not close to one, equation (18) will hold approximately, if S_{k_s}' and the product $S_{k_s}' f$ are small. In the sand beds used, S_{k_s}' is small compared to one.

Hence

$$S_y' = \frac{S_y(\phi)}{\Sigma_\phi S_y(\phi)} = \frac{x_1'(\phi)}{1+x_1'(\phi)} \Big/ \Sigma_\phi \frac{x_1'(\phi)}{1+x_1'(\phi)}. \quad (19)$$

With the help of equation (19), it is easy to calculate the suspension concentration of a given grain having the settling velocity $c(\phi)$ at any height $y \geqslant y_1$ above the bed, if the relative concentration S_{k_s}' of the particle at the bed is known. If $y = y_1$, then equations (16) to (19) give the average concentration of sediments of different sizes in the bed layer (S_{bl}').

EXPERIMENTAL STUDIES

The efficiencies of the different theoretical methods for computation of the bed load and suspended load discussed above have been studied by comparing the computed grain-size frequency distributions with those observed in laboratory flumes under known hydraulic conditions. Two closed circuit laboratory flumes, one designed at the Uppsala University, and the other at the Indian Statistical Institute, Calcutta were used for this purpose. The equipment used and the techniques of velocity measurement, sample collection and analyses employed have been described earlier (Sengupta, 1979 and Ghosh *et al.* 1979).

Grain-size distributions of suspended loads over six different sand beds were studied during the Uppsala and Calcutta experiments at various heights above the bed (Sengupta, 1975, 1979 and Ghosh *et al.* 1979). Of these, the results of the experiments over three types of sand beds at a

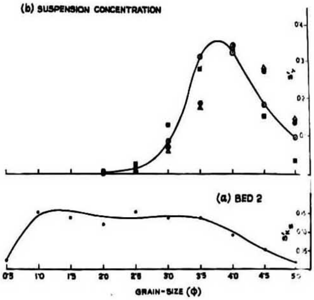

Fig. 2. Bed 2, grain-size distributions (relative concentrations): (a) in the bed (S_{k_s}'); (b) (S_y' at y = 23·3 cm) - ○ observed, ⊗ computed by the present method, △ computed- (Gessler and Rouse equations), ■ computed (Einstein and Rouse equations).

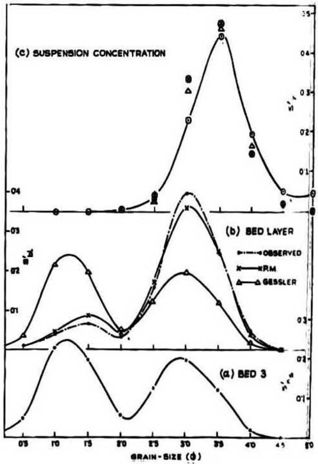

Fig. 3. Bed 3, grain-size distributions (relative concentration): (a) in the bed (S_{k_s}'); (b) in the bed layer (S_{bl}') - ●, observed; ×, present method △, Gessler method; (c) in suspension (S_y' at y = 17·5 cm) – ⊙ observed, ⊙ computed by the present method, △ computed (Gessler and Rouse equations).

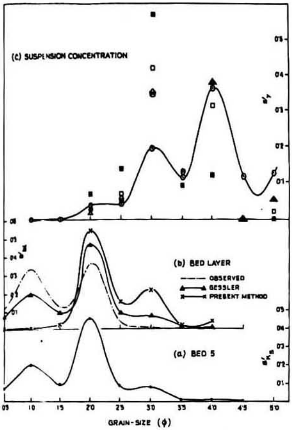

Fig. 4. Bed 5, grain-size distributions (relative concentrations): (a) in the bed (S'_{ks}); (b) in the bed layer (S'_{bl}); (c) in suspension (S'_y at $y = 18{\cdot}0$ cm) – ⊙ observed, ⊗ computed by the present method, △ computed (Gessler and Rouse equations), ■ computed (Einstein equation, with ξ-correction and the Rouse equation), □ computed (Einstein equation, without ξ-correction and the Rouse equation).

height of approximately 20 cm and at flow velocities varying between 98 and 126 cm sec^{-1} are discussed in this article. The following three sand beds (numbered 2, 3 and 5) were chosen for the present study because of their widely different grain-size distribution patterns: nearly uniform (bed 2), bimodal (bed 3) and positively skewed with slight bimodality (bed 5). The bed, bed load and suspended load distributions are graphically presented in Figs 2, 3 and 4. The observed suspended load distributions above these sand beds are tabulated in Table 5.

A record of the bed conditions at different flow velocities was maintained. The experiments were always started with a smooth, flat bed. Bed forms were generated when the competence velocity was exceeded. As expected, the dimensions of the bed forms depended not only on the flow velocity, but also on the grain-size distribution of the bed material. Wavelengths of the bed forms increased with an increase in flow velocity, but at a velocity exceeding about 120 cm sec^{-1}, the bed forms were replaced by nearly flat beds in most of the cases.

Concomitantly with development of the bed forms, jets of sediments were ejected out of ripple crests and went into suspension. This happened while the ripples were moving fast, around the whole channel, so that the contribution to the suspended load came from the *whole bed*, throughout the flume channel. With the passage of time however, the process seemed to reach a steady state, with the suspension concentration reaching a

Table 1. Grain-size frequency distributions of the bed materials used for the experiments

Size class D		Bed 2		Bed 3		Bed 5	
(ϕ)	(mm)	Weight w_b (kg)	Relative conc. S'_{k_s}	Weight w_b (kg)	Relative conc. S'_{k_s}	Weight w_b (kg)	Relative conc. S'_{k_s}
0·0	0·991	—	—	1·6050	0·0054	—	—
0·5	0·701	3·9690	0·0192	14·9850	0·0500	5·5880	0·0745
1·0	0·495	30·5720	0·1477	67·4890	0·2250	15·0240	0·2003
1·5	0·351	27·4440	0·1326	59·3000	0·1977	7·1920	0·0959
2·0	0·246	24·0350	0·1161	15·3890	0·0513	33·7980	0·4506
2·5	0·175	31·1220	0·1504	37·6320	0·1254	6·3800	0·0851
3·0	0·124	27·9490	0·1350	59·8740	0·1996	5·5970	0·0746
3·5	0·088	27·0360	0·1306	37·1380	0·1238	0·5890	0·0079
4·0	0·061	19·0530	0·0920	5·8650	0·0196	0·7650	0·0102
4·5	0·043	10·9710	0·0530	0·6510	0·0022	0·0070	0·0001
> 4·5*	< 0·032	4·8390	0·0234	0·0720	0·0002	0·0610	0·0008
Total		206·9900		300·0000		75·0000	

* Rounded off to 5·0 for computational purpose.

Table 2. Computed values for the bed loads (Einstein method) and suspended loads (Rouse method) for beds 2 and 5

	Bed 2				Bed 5			
	Bed load (S'_{bl})		Suspended load S'_v		Bed load (S'_{bl})		Suspended load (S'_v)	
	With	Without	At $y = 23{\cdot}3$ cm		With	Without	At $y = 18{\cdot}0$ cm	
Size class (ϕ)	ξ-correction (1)	ξ-correction (2)	From (1)	From (2)	ξ-correction (3)	ξ-correction (4)	From (3)	From (4)
0·5	0·0268	0·0180	—	—	0·0751	0·0656	—	—
1·0	0·2560	0·1723	—	—	0·2563	0·1589	—	—
1·5	0·2275	0·1650	—	—	0·1195	0·0646	0·0003	0·0001
2·0	0·1197	0·1396	0·0006	0·0003	0·4475	0·5330	0·0700	0·0298
2·5	0·1616	0·1633	0·0212	0·0101	0·0633	0·0914	0·1401	0·0725
3·0	0·0972	0·1286	0·1223	0·0762	0·0357	0·0728	0·5755	0·4210
3·5	0·0657	0·1069	0·2630	0·2014	0·0017	0·0064	0·0897	0·1240
4·0	0·0359	0·0636	0·4218	0·3515	0·0010	0·0070	0·1207	0·3154
4·5	0·0081	0·0309	0·1394	0·2493	0·0000	0·0001	0·0009	0·0036
>4·5	0·0016	0·0119	0·0318	0·1113	0·0000	0·0004	0·0023	0·0336

'saturation point'. This was confirmed by the results of repeated sampling of the suspended particles from fixed heights, which showed little change in proportion of grain sizes, irrespective of the presence or absence of ripples in the bed immediately below the sampling point. While developing a theoretical model for suspension concentration therefore, it was felt that the influence of bed form can be safely ignored when a steady state has been reached in suspension. This situation simulates suspension transportation in natural streams within reasonable limits.

COMPUTATION OF BED LOAD AND SUSPENDED LOAD

Bed loads and suspended loads were computed for three different sand beds (numbered 2, 3 and 5) using the different methods discussed in the earlier section.

Einstein equations, both with and without ξ-correction, (equations 1 and 2) were used to derive bed layer distributions for beds 2 and 5. Suspension concentrations above each of these bed loads were obtained by the Rouse equation, using the respective bed loads as references. The results are shown in Table 2.

The Gessler equation (equation 3) was used for deriving the bed layer distributions above beds 2, 3 and 5. Suspension concentrations for the respective beds were obtained with the help of the Rouse equation (5), using the respective bed loads as references. The results are shown in Table 3.

Rouse's equation is believed to be valid only within the zone of suspension. Following Einstein (1950) we assume that this zone starts from the upper boundary of the bed layer. Actual computation based on this assumption shows that the use of the upper boundary of the bed layer as a reference yields results which are comparable to the experimental data.

The method developed in this article (equation 10) was utilized to compute the suspended loads above all the three beds *directly* from the grain-size distributions of the respective sand beds. The results are shown in Table 4. The bed load distributions for these sand beds were also computed with the help of equations (16) to (19) when $y = y_1$. Grain-size distributions of the bed loads are presented graphically in Figs 2, 3 and 4.

COMPARISON OF COMPUTED AND OBSERVED DATA

The trends of the observed and computed suspended load distributions, as seen in Figs 2, 3 and 4, generally agree, but the actual values show marked discrepancies in some cases.

To obtain a quantitative idea of these discrepancies, the weighted relative error between the computed and the observed values were computed by the following formula,

$$E = \sqrt{\Sigma \frac{(S'_c - S'_o)^2}{S'^2_o} \cdot S'_o} = \sqrt{\Sigma \frac{(S'_c - S'_o)^2}{S'_o}}$$

where

S'_c = computed suspension concentration,

S'_o = observed suspension concentration.

Table 3. Computed values for the bed loads (Gessler method) and suspended loads (Rouse method) for beds 2, 3 and 5

	Bed 2		Bed 3		Bed 5	
Size class (ϕ)	Bed load (S'_{bl})	Suspended load (S'_y) $y = 23{\cdot}3$ cm	Bed load (S'_{bl})	Suspended load (S'_y) $y = 17{\cdot}5$ cm	Bed load (S'_{bl})	Suspended load (S'_y) $y = 18{\cdot}0$ cm
0·5	0·0189	—	0·0546	—	0·0737	—
1·0	0·1467	—	0·2239	—	0·1995	—
1·5	0·1322	—	0·1975	0·0001	0·0959	0·0001
2·0	0·1161	0·0002	0·0514	0·0011	0·4515	0·0208
2·5	0·1506	0·0077	0·1258	0·0334	0·0854	0·0557
3·0	0·1354	0·0592	0·2004	0·3052	0·0750	0·3567
3·5	0·1310	0·1723	0·1244	0·4637	0·0079	0·1267
4·0	0·0924	0·3383	0·0197	0·1685	0·0102	0·3799
4·5	0·0532	0·2789	0·0022	0·0250	0·0001	0·0057
>4·5	0·0235	0·1435	0·0002	0·0031	0·0008	0·0546

Table 4. Bed loads and suspended loads above beds 2, 3 and 5 computed by the present method

	Bed 2		Bed 3		Bed 5	
Size class (ϕ)	Bed load (S'_{bl})	Suspended load (S'_y) at $y = 23{\cdot}3$ cm	Bed load (S'_{bl})	Suspended load (S'_y) at $y = 17{\cdot}5$ cm	Bed load (S'_{bl})	Suspended load (S'_y) at $y = 18{\cdot}0$ cm
0·5	—	—	—	—	0·0003	—
1·0	0·0024	—	0·0393	—	0·0105	—
1·5	0·0128	—	0·0895	0·0001	0·0221	0·0001
2·0	0·0388	0·0002	0·0404	0·0012	0·5304	0·0301
2·5	0·1196	0·0079	0·1638	0·0368	0·1387	0·0528
3·0	0·1849	0·0619	0·3620	0·3393	0·2119	0·3554
3·5	0·2356	0·1804	0·2537	0·4526	0·0305	0·1227
4·0	0·2120	0·3390	0·0454	0·1460	0·0504	0·3784
4·5	0·1329	0·2721	0·0053	0·0213	0·0005	0·0052
>4·5	0·0609	0·1385	0·0006	0·0027	0·0048	0·0555

Differences between the computed and observed values are shown in Table 6. As seen from this table, none of the methods for computing the suspended load can be regarded as the best for all the beds.

For bed 2, the smallest error is obtained when the bed load concentration is computed by the modified Einstein method (without ξ-correction), whereas for bed 3, Gessler's method gives the smallest error. For bed 5 the best results have been obtained by the present method, whereas Einstein's method (with ξ-correction) gives the largest error. On the whole, all the methods give errors of the same order (excepting Einstein's method for bed 5) and it seems that the present method is as good as any other method for suspended load computation. The real advantage of the present method therefore is that it allows direct computation of suspended load from the bed without going through an intermediate stage, namely, computation of the bed load.

Separate computations show that the error in computation of the suspended load from a reference level in suspension is much smaller than the error of computation of suspended load directly from the bed. This is true both for the Rouse suspension equation and for the present method. For both methods, generally, a larger error is noticed at the lower levels, i.e. in the vicinity of the bed.

DISCUSSION

Although the trends of the grain-size distributions obtained by the application of the Rouse suspension equation on the bed loads obtained by the different methods (Einstein and Gessler) generally agree with the observed data, the actual figures hardly tally. The following might partly explain the discrepancy

Table 5. Flow parameters and observed grain-size distributions of the suspended loads above the sand beds

	Bed 2	Bed 3	Bed 5
d (cm)	30·0	30·0	30·0
H (cm)	25·0	20·0	20·0
h' (cm)	~1·7	~2·5	~2·0
U_{max}(cm sec^{-1})	121·3	97·8	126·0
k_s (cm)	0·0297	0·0451	0·0518
y (cm) = H–h'	23·3	17·5	18·0
V (lit.)	5·0	5·0	5·0
J	0·0020	0·0020	0·0022
Temp. (°C)	19·0	19·0	27·0

Sample no.	Average of III2-121-25-25B		Average of VII-93-20A-B		Average of 5-124-20A-D(w)	
D (ϕ)	w_y (g)	S'_y	w_y (g)	S'_y	w_y (g)	S'_y
1·0	—	—	0·0042	0·0005	0·0045	0·0053
1·5	0·0320	0·0011	0·0085	0·0009	0·0025	0·0030
2·0	0·0953	0·0032	0·0328	0·0035	0·0298	0·0354
2·5	0·3957	0·0132	0·2890	0·0307	0·0363	0·0431
3·0	2·4384	0·0812	2·1576	0·2279	0·1650	0·1959
3·5	9·1444	0·3058	4·1956	0·4428	0·0945	0·1122
4·0	9·5582	0·3196	1·8688	0·1970	0·3058	0·3630
4·5	5·3201	0·1786	0·5001	0·0527	0·0950	0·1128
> 4·5	2·9026	0·0972	0·4196	0·0440	0·1090	0·1294

Table 6. Errors between computed and observed suspended loads above beds 2, 3 and 5

Bed no.	Height y (cm)	Einstein with ξ-corr.	Einstein without ξ-corr.	Gessler method	Present method
2	23·3	0·34	0·27	0·39	0·36
3	17·5	—	—	0·29	0·36
5	18·0	1·18	0·68	0·54	0·53

Note: suspended loads have been computed by the application of the Rouse suspension equation (5), using the respective computed bed layer concentration as a reference level distribution. The specific method used for computation of a particular bed load is indicated above each column.

between the observed results and the values computed by Einstein's method.

(1) Given that a particle is lifted, Einstein assumed that the length of the jump is directly proportional to the grain diameter (L/D = constant). But in reality the jump length is expected to decrease with an increase of grain size. Moreover, as Yang & Sayre (1971) have shown experimentally, the jump length follows a gamma distribution.

(2) Einstein's assumption that the bed layer is two grain diameters thick is also questionable (see Crickmore, 1967).

(3) Einstein assumed that the finest grains have a tendency 'to loosely fill the pores between the larger particles'. On the basis of this assumption he suggested that the finest 10% (by weight) of the bed material may be excluded for computational purposes. He also introduced a correction factor (ξ-correction) in his equation to reduce the effect of the fine particles.

The thickness of the sand beds laid down on the flume base during the present series of experiments never exceeded a few centimetres. At high flow velocities these sand beds wholly migrated downstream in the form of ripples, thereby exposing each grain to the water flow above. Hence no grain could remain permanently hidden in the bed in the manner presumed by Einstein. In fact, the computation of relative suspension loads (in the cases of beds 2 and 5) without Einstein's ξ-correction gives better results than that obtained with ξ-correction (Table 6).

Einstein's hiding factor may, however, be applicable to natural streams, where the sand beds are too thick to be wholly removed by water current, and only the grains in the topmost part of the bed are subjected to repeated exposure to the flowing water.

The main drawback in the application of Gessler's method seems to arise from extrapolation of the critical shear stress values (T_c) for the grains in his fig. 8. In the present case extrapolation below a Re_* value of 10 gives nearly the same value of T_c irrespective of grain size, whereas it appears from Shields' curve and its modifications (Miller, McCave & Komar, 1977) that T_c should increase slightly with a decrease of Re_*. Use of the modified versions of Shields' curve might give better results in such cases.

Einstein's assumption that the standard deviation of the fluctuating lift force is a universal constant ($\sigma_1 = 0{\cdot}5$) was questioned by Gessler. σ_1 was found to be 0·57 by Gessler in his experiments. In the absence of the necessary experimental facilities the same value has been used in the present experiments although the actual value of σ_1 might have been different.

The method developed here for computation of the suspended loads directly from the bed's grain-size distribution produces results which are generally comparable to those obtained by the application of the Rouse equation on Gessler's bed load values (except for bed 3, where Gessler's method gives slightly better results). As has been mentioned earlier, the computation of suspension concentrations using a reference level located *within* the zone of suspension gives a better result than computation of suspension concentration from the bed. This is true both for the Rouse equation and the present method. The reasons for location of larger errors in the vicinity of the bed may be sought in the assumed validity of the diffusion equation in this zone, which still remains to be proved. The existence of a perfectly linear velocity distribution in the bed layer zone, as has been assumed in the present case, also needs critical examination.

SUMMARY AND CONCLUSIONS

This paper reviews some of the well-known methods available for computation of the bed load and suspended load and also discusses a new method developed with a view to computing both the bed load and suspended load directly from the bed's grain-size distribution, given the flow parameters. The methods discussed are those of Einstein (1950) and Gessler (1965) for bed load computation and the equation developed by Rouse (1938) after Schmidt (1925) and others for computation of suspended load. Moreover, a modification of Einstein's bed load equation (without ξ-correction) has been proposed.

The new formula developed in this paper is based on a partial modification of Hunt's (1954) work. The modification proposed is essentially in the form of a change in the velocity profile. While Hunt's equation is valid only for the zone having a logarithmic velocity distribution, our method uses this distribution down to $y = 0{\cdot}25$ cm only and then assumes a linear velocity distribution down to the bed to have a more realistic simulation of the actual velocity pattern. Using this modified velocity distribution, two concentration equations for water and sediment have been set up as in Hunt. Solving these two equations, the sediment concentration at any height above a bed can be obtained in terms of flow parameters and bed materials.

The efficiency of each of these methods has been tested by comparing the computed data with actual observations on the grain-size distributions of suspended load samples collected in laboratory flumes under known hydraulic conditions. The results obtained from three different sand beds have been discussed.

For each of the three beds the trends of the suspended load's grain-size distribution patterns have been computed by the application of the Rouse equation on the bed loads obtained by: (1) Einstein's method (with and without ξ-correction), and (2) Gessler's method. For each of the three beds suspension concentrations have also been computed independently by the present method. The results obtained by the different methods are generally of the same order. Quantitative estimates of the errors between the observed and the computed values indicate that no one method can be claimed to be particularly superior to the others. The possible sources of errors in each of these methods have been discussed. The real advantage of this method is that it affords a direct computation of the suspended load from a bed's grain-size distribution without going through an intermediate stage (bed load) as required by the other methods.

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APPENDIX A: EXPLANATION OF NOTATION

a Any reference level above bed.
c Settling velocity of a particle.
d Initial depth of water as measured from the flume base.
D Grain diameter.
g Acceleration due to gravity.
H Height of sample collection point above the flume base.
h' Average height of a sand ripple at the bed during collection of the suspended load sample.
i_b Proportion of bed sediment in a given grain size.
i_B Proportion of the bed load in a given grain size.
J Energy slope.
k_s Same height as the bed roughness as defined by Einstein (1950).
p Probability of a grain being eroded.
q_B Bed load rate in weight per unit time and width.
R_H Hydraulic radius.
S_v Concentration of material per unit volume.
S_a Relative concentration of sediment at a reference level (a).
S_{bl} $\frac{w_{bl}(\phi)}{\Sigma_\phi w_{bl}(\phi)}$, observed relative concentration distribution at a bed layer.
S_y $\frac{w_y(\phi)}{\Sigma_\phi w_y(\phi)}$, relative concentration distribution at height y above the bed.
S_{k_s} $\frac{w_b(\phi)}{\Sigma_\phi w_b(\phi)}$, concentration distribution at k_s.
t' Variable of integration.
T Dimensionless shear stress.
$\bar{T}$ Dimensionless average bottom shear stress.
T_0 Dimensionless critical shear stress.

u Time average velocity of water in the flow direction.
u_* Shear velocity $\sqrt{\tau_0/\rho}$.
U_{max} Maximum flow velocity.
$u_{y=y_1}$ Time average velocity at $y_1 = 0{\cdot}25$ cm above the bed.
V Volume of water sample collected.
w_b Grain-size frequency distribution ('weight frequency') in the bed.
w_{bl} Grain-size frequency distribution ('weight frequency') at the bed layer.
w_s Grain-size frequency distribution (weight frequency) of the suspended load at y.
X Characteristic grain size in mixture.
y Vertical height above the bed.
α Constant of proportionality.
$(\beta_1/\beta_x)^2$ Correction factor.
γ_s, γ_f Specific weights of solid and fluid respectively.
δ Laminar sublayer thickness.
ϵ_m Water diffusion coefficient.
ϵ_s Sediment diffusion coefficient.
ξ Hiding factor of grain in mixture.
ρ, ρ_s Densities of water and solid respectively.
σ_1 Standard deviation of Gaussian distribution.
τ Shear stress at any point in the fluid.
τ_0 Bottom shear stress, $\sqrt{JR_B g}$.
$\bar{\tau}_0$ Average bottom shear stress.
ϕ $-\log_2 D$, a measure of size.
Φ_* Dimensionless measure of bed-load transport.
χ Von Kármán constant (0·4).
Ψ Intensity of shear on particle.
Ψ_* Intensity of shear for individual grain size.

APPENDIX B: DEFINITIONS

Bed distribution: grain-size frequency distribution (weight frequencies) of the particles available in the bed.

Bed layer: a 'flow layer' immediately above the bed. In the present work, its thickness is assumed to be 2–3 times the maximum grain diameter available in the bed, so that movement of all sizes of the available particles is possible within this layer. For the sand beds used for this work the thickness amounts to 0·25 cm.

Bed load: weight of the particles moving in the bed layer. This motion occurs by rolling, sliding and sometimes also by jumping.

Suspended load: weight of the particles moving in suspension above the bed layer.