NOTES AND VIEWS

ON MAHALANOBIS MODELS*

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1. Professor Mahalanobis' models used in connexion with India's Second Five Year Plan have been examined in a large number of studies. Some of these are appreciative while many others are critical; some at a superficial level while others at considerable depth. In view of this, it is difficult to add new and illuminating points to this body of literature. However, since there is little purpose in traversing old grounds when one has nothing to add, we shall concentrate on a few points that are relatively novel and use this as an excuse for presenting, briefly, not only his well known models but also one of his little known ideas which has some relevance in describing growth if not in planning it.

2. In his two-sector model, the unduplicated output of the economy is conceived to be originating in two sectors, one producing all investment goods and the other producing all consumer goods. This implies allocation of all intermediate consumption between the two sectors ensuring that the sector outputs are final goods, I for the former sector and C, for the latter, adding up neatly to national income, Y, in an economy conceived as closed, for simplicity. Observe that the similar allocation by Marx related to total output and not unduplicated output, and the sectors represented investment and intermediate goods as against final consumer goods. It is not assumed that investment every year is split up—in the ratio $\lambda_i : \lambda_c (\lambda_i + \lambda_c = 1)$ between investment goods and consumer goods sectors. With subscript t standing for the year, we can, therefore, write

$$I_t + C_t = Y_t,$$

$$I_{t+1} - I_t = \beta_i \lambda_i I_t,$$

$$C_{t+1} - C_t = \beta_c \lambda_c I_t$$

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where β_t, β_c are reciprocals of respective sectoral marginal capital output ratios, also assumed to be constants. The above equations lead to

$$(1) \qquad Y_t = Y_0 \left[1 + \alpha_0 \frac{\lambda_t\beta_t + \lambda_c\beta_c}{\lambda_t\beta_t} \left\{ (1+\lambda_t\beta_t)^t - 1 \right\} \right]$$

where $\alpha_0 = I_0/Y_0$ giving the pattern of growth of national income over time.

3. Normally $\beta_t < \beta_c$ and hence (1) depicts a higher rate of growth of Y for smaller λ_t for the first few years. But when one takes a longer time horizon, a larger λ_t systematically gives a higher rate of growth of Y. It can be shown that

$$(2) \qquad \text{Lt } \alpha_t = \lim_{t \to \infty} \left(I_t/Y_t \right) = \frac{\lambda_t\beta_t}{\lambda_t\beta_t + \lambda_c\beta_c}$$

and some limiting values are given below by way of illustration when $\alpha_t = 6$ p.c., $\beta_t = 0{\cdot}25$ and $\beta_c = 0{\cdot}50$.

$\lambda_t =$	0·1	0·3	0·5
$(\lambda_t\beta_t)/(\lambda_t\beta_t + \lambda_c\beta_c)$ p.c. =	5·3	17·7	33·3

When α is higher than the limiting value, α_t's will gradually decline towards it. When, however, it is smaller than the limiting value, α_t's will increase until the limit is reached. On the basis of such considerations, it was thought that λ_t should be raised from about 10 to 33 per cent. A larger rise would have given a higher long period rate of growth, but this would not have been practically feasible and would have given too large a limiting rate of investment, and figures given above bring out the point made, β_t, β_c being assigned realistic values.

4. Haldane's study of maxima of function (1) was not very interesting, because with realistic values of β_t and $\beta_c (\beta_c > \beta_t) Y_t$, at a point of time, decreases monotonically with increasing λ_t for the first few years, say 0 to T. Then for a brief period, $T+1$ to $T+t$, a true maximum exists for each year and can be obtained by Haldane's method. But beyond $T+t$, the function uniformly increases with λ_t. Since T and t are both small in comparison with reasonable time horizons of perspective plans, where β_t and β_c are realistic the quest for a maxima is not illuminating.

5. Mahalanobis' 4-sector model was an allocation model while his two sector model just described was a growth model. With aggregate employment (N), net investment (A) and income to be generated (E) during the coming plan period being given, the problem is to

allocate these by some meaningful sectors. The sectors chosen were investment goods industries (i) and three types of consumer goods industries, factory type (C_1), hand type including agriculture (C_2) and services of all sorts (C_3). With small n's standing for sectoral employments with suitable subscripts and introducing new parameters θ giving investment per worker, we can now write

$$N = n_i + n_1 + n_2 + n_3$$
$$A = n_i\theta_i + n_1\theta_1 + n_2\theta_2 + n_3\theta_3$$
$$E = \beta_i n_i\theta_i + \beta_1 n_1\theta_1 + \beta_2 n_2\theta_2 + \beta_3 n_3\theta_3$$

where β's are reciprocals of capital-output ratios as before. With realistic values of θ's and β's obtained from available statistics, and using the fact that a third of the investment should be in the investment goods sector by virtue of the findings based on the two-sector model, it is now possible to solve the system of equations (3) and obtain the sectoral allocation of the main aggregates. With available values of β's and θ's for individual activities within a main sector, their levels were chosen more or lesson a trial and error basis on various considerations taking care to see that aggregation does not violate the solutions obtained at the level of the main sector. However, since not all details were available for several activities ultimately chosen, the above should be regarded more as the logical rather than the actual procedure for obtaining the targets for individual activities.

6. It is necessary to stress four points about the models described. First, they are not supposed to describe the past performance of an economy. Hence the parameters used, though based on past information, are supposed to hold good during a future period. The models are, therefore, strictly planning models rather than growth models. However, in a subsequent study, it has been demonstrated by Professor Mahalanobis that the two-sector model describes past performance during a period also when parameters are estimated on the basis of data relating to the period. Second, the models were devised to solve a concrete problem and it had to be formulated and the estimates of parameters obtained in about three or four months' time. It was not expected that the models would have any lasting value. The fact that they attracted considerable attention demonstrated that they were not as transient as Professor Mahalanobis expected them to be. They did not represent just scaffoldings to be dismantled once the building was erected. Third, the models constituted only a part of the intellectual underpinning of the Second Plan. There were many other observations and recommendation including

even prognostication about proximate behaviour of administrators and anticipation of foreign exchange bottlenecks. It was never thought that the Second Plan would have a smooth sailing once just the requirements of the models were met. It was specifically mentioned that there were several limiting conditions and the Plan would fail unless these were met. Finally, the models are obviously incomplete, and this was partly by design and partly due to the lack of clear understanding of the problem at the time, but Mahalanobis models are capable of being expanded and made more complete. For example, exports and imports can be introduced in the two-sector model and the foreign trade sector may function like the investment goods sector, to some extent. The number of sectors can be increased in the other model, and the system of equations can be used for obtaining a vector of targets which maximizes an objective function, instead of just obtaining one solution, even when it is not a very unstable one for marginal changes in parameters.

7. More recently, Professor Mahalanobis suggested an interesting measure of the growth of per capita real income during a period, while in the past, he was concerned with models intended to bring about an increase in the rate of growth of per capita real income. As this formulation is relatively less known, we propose to conclude this essay with a consideration of this.

8. Given sequences of real incomes, $Y_0, Y_1, \ldots, Y_n$ and population $P_0, P_1, P_2, \ldots, P_n$, subscript '0' depicting the base year and $1, \ldots n$ being n years under consideration, the aggregate growth in income during the period is $\sum_{i=1}^{n} Y_i - nY_o$, the corresponding growth in population being $\sum_{i=1}^{n} P_i - nP_o$. If the additional population has to be provided at the rate of base period per capita income $\left(y_o = \frac{Y_o}{P_o} \right)$, then the new income above this is given by

$$\sum_{i=1}^{n} Y_i - nY_o - \frac{Y_o}{P_o} \left\{ \sum_{i=1}^{n} P_i - nP_o \right\}.$$

This new income obviously has to be located at the centre of the period, *i.e*, on $\left(\frac{n+1}{2} \right)$ th year rather than on the nth year. The

new income per year per head is the above expression divided by $\frac{n+1}{2}\sum_{i=1}^{n} P_i$ and hence the rate of growth of per capita real income on the base period per capita income is obtained by dividing

$$\frac{2}{n+1} \cdot \frac{\sum_{i=1}^{n} Y_i - nY_o - \frac{Y_o}{P_o}\left\{\sum_{i=1}^{n} P_i - nP_o\right\}}{\sum_{i=1}^{n} P_i} \text{ by } \frac{Y_o}{P_o}$$

which reduces to

$$(4) \qquad r = \frac{2}{n+1}\left\{\frac{P_o\sum_{i=1}^{n} Y_i}{Y_o\sum_{i=1}^{n} P_i} - 1\right\}.$$

Professor Mahalanobis called this the consolidated rate of growth of per capita income and compared this with the usually accepted rates of growth obtained from the end points, *i.e.*, from years 0 and n. It will be of interest to study the implications of a rate of growth obtained in this manner. Apart from the advantage that it depends on the entire data and not just the terminal figures, the data enter here in an economically meaningful way. In contrast, when we use the arithmatic average of rates of growth between years 0 and 1, 1 and 2, and so on up to, $n-1$ and n, as a measure of the rate of growth during the period $(0, n)$, we no doubt use the full information. But one is not convinced why arithmatic rather than geometric average has been used. The use of geometric average here takes us back to terminal points and the use of arithmatic average simply appears as a mechanical dodge to use the whole information. An illustration of the computation of the consolidated rate is given in the Appendix table.

Appendix

Consolidated rate of growth of national income between 1950-51 and 1960-61 at 1948-48 factor cost

Sr. No.		*National income at 1948-49 prices (Rs. abja)*	*Population in million*	*Per capita national income at 1948-49 prices (in Rs.)*
(0)	(1)	(2)	(3)	(4)
1.	1950-51	88·5	357·6	247·5
2.	1951-52	91·0	363·6	250·3
3.	1952-53	94·6	370·0	255·7
4.	1953-54	100·3	376·8	266·2
5.	1954-55	102·8	383·9	2o7 8
6.	1955-56	104·8	391·4	267·8
7.	1956-57	110·0	399·2	275 6
8.	1957-58	108·9	407·4	267 3
9.	1958·59	116·5	415·9	280·1
10.	1959-60	118·6	424·8	279·2
11.	1960-61	127·5	434·1	293·7
12.	Actual total for 10-year-period	1075·0	3967·1	
13.	Total for 10 years at 1950-51 level	885·0	3576 0	
14.	Difference : Increase in 10 years	190·0	391·1	
15.	Amount required for increase in population	96·8		
16.	Net increase over 10-year-period	93·2		
17.	Consolidated increase : total period			23·5
18.	Consolidated increase : per person per year			4·3
19.	Consolidated increase : rate per person per year (per cent)			1·74
20.	Total interval rate of increase	44·1		18·7
21.	Interval rate of increase per year	3·7		1·7

Note. abja=100 crores (Rupees).