Optimal Subsidies and Taxes When Some Factors Are Traded

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I

Trade taxes are widely used to collect revenue. The levy, instead of a uniform consumption tax, would secure revenue without welfare loss, if factor supplies are inelastic. It is, however, often not feasible, particularly in underdeveloped countries, to tax the consumption of much of home output. It is therefore of practical importance to determine the optimal policies for collecting specified revenue through trade taxes. Yet, while the optimum tariff for the exercise of national monopoly power has been extensively and elegantly analyzed in the literature, there has been little discussion of the optimum revenue tariff.[1] Our first task in this paper is to analyze this problem, taking trade in intermediate goods into account. As has been pointed out by Bhagwati in his invaluable survey (1964, p. 3), the assumption that only final goods are traded has been a central limitation of trade theory in a real world in which a large portion of world trade consists of intermediate goods.

Consider a small country confronted by given world prices[2] and endowed with a single factor, labor (L), which is inelastic in supply, immobile

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[1] Johnson (1951–52) sets out the formula for the maximum revenue tariff when national monopoly power exists. The problem discussed by Meade (1955, pp. 186–99) and Kindleberger (1963, pp. 223–24) is the optimal mix of consumption, production, and trade taxes when these taxes can be levied on some but not on all commodities. Kemp (1964, p. 183) sets before the reader the problem of determining the optimal set of tariffs to raise given revenue, but provides no solution.

[2] There is thus no scope for the exercise of national monopoly power. We choose units of quantity for all tradables such that world prices are unity.

between countries, and always fully employed.[3] This country produces a good x solely for export, makes all it needs of a consumer good c_d, and relies wholly on imports for supplies of another consumer good c_i. Quantities of x, c_d, and c_i are denoted by X, C_d, and C_i. The production of x or c_d requires both labor and a wholly imported factor, metal, the quantum of metal imports being M. Trade is assumed to be balanced,[4] so $X = C_i + M$. The production functions for x and c_d exhibit constant returns to scale and are twice continuously differentiable and concave. They are $f(m_x)$ and $g(m_d)$, where m_x and m_d are metal/labor ratios in the production of x and c_d.[5] The utility function $U(C_d, C_i)$ is increasing in both arguments, twice continuously differentiable and concave.[6]

Under free trade, technique in export production will be such that the marginal value product of metal at world prices is unity. This determines the wage rate (w); using asterisks as superscripts to denote the free trade values of variables, we write w^* for the free trade wage rate. Technique in making c_d is uniquely determined, given the factor/price ratio $w^*/1$, and so is p, the price of a unit of c_d. The labor L_d devoted to making c_d, will be such that the ratio of the marginal utilities of the two goods equals the product/price ratio; $U_i/U_d = 1/p$ in equilibrium, where U_i and U_d are marginal utilities derived from the consumption of c_i and c_d, respectively.

The government now has to collect revenue (R) equal to a specified fraction (π) of the wage bill through trade taxes alone for redistribution to consumers.[7] Revenue collection entails production cost due to departure from free trade techniques in production and/or consumption cost because the product/price ratio confronting consumers is not at the free trade level.[8] Our problem is to determine the optimal tax policy which secures specified revenue with the least welfare loss, taking both production and consumption costs of the tariff into account. We first prove that it is optimal to maintain the free trade technique in export production; if, initially, metal used to make exports is taxed while exports are free of tax or subsidy, welfare can be raised by abolishing the tax on metal used to make exports and adjusting taxes on imports for home use to make up the revenue loss. We next show that the widely accepted rule of thumb that taxes on imports for home use should be uniform is erroneous.

We write t_x and t_d for the rates of tax on imports of metal to make x and c_d, respectively; t_i for the rate of tax on imports of c_i; and t for the rate

[3] The total supply of labor by choice of units is unity.

[4] This assumption is relaxed in Appendix A.

[5] It is also assumed that the marginal physical product of a factor in the production of either good tends to zero (infinity) as its relative quantity tends to infinity (zero).

[6] It is further assumed that the marginal utility of either good tends to infinity (zero) as the consumption of that good tends to zero (infinity), and the marginal utility of either good increases as the consumption of the other good increases.

[7] We assume that revenue is redistributed to consumers, for the sake of analytical simplicity.

[8] Johnson (1960, 1964) elucidates the production and consumption costs of a tariff.

of subsidy on exports of x. Negative t_x, t_d, and t_i denote subsidy rates, and negative t denotes a tax rate.

The marginal conditions for profit maximization are:

$$(1 + t)f'(m_x) = 1 + t_x, \tag{1}$$

$$(1 + t)[f(m_x) - m_x f'(m_x)] = w, \tag{2}$$

$$pg'(m_d) = 1 + t_d, \tag{3}$$

and

$$p[g(m_d) - m_d g'(m_d)] = w. \tag{4}$$

As $C_d = L_d g(m_d)$ and $C_i = X - M = (1 - L_d)[f(m_x) - m_x] - L_d m_d$, the utility function can be written in the form

$$U\{L_d g(m_d), (1 - L_d)[f(m_x) - m_x] - L_d m_d\}. \tag{5}$$

The marginal condition for utility maximization is

$$\frac{U_i}{U_d} = \frac{1 + t_i}{p}. \tag{6}$$

Given the assumptions regarding $U, C_i > 0$ and $0 < L_d < 1$. For a feasible solution, we must also have

$$1 + t,\ 1 + t_x,\ 1 + t_d,\ 1 + t_i,\ p,\ w,\ m_d,\ m_x \geqq 0. \tag{7}$$

The revenue constraint is

$$-t(1 - L_d)f(m_x) + t_x(1 - L_d)m_x + t_d L_d m_d + t_i\{(1 - L_d)[f(m_x) - m_x] - L_d m_d\} = \pi w = R. \tag{8}$$

We have six equations, (1) through (4) and (6) and (8), and nine unknowns. We use the equations to solve uniquely for t, t_x, t_d, t_i, p, and w in terms of m_d, m_x, and L_d. Then, feasible values of m_d, m_x, and L_d are those for which (7) holds.

Given $m_d \geqq 0$, $m_x \geqq 0$, $1 > L_d > 0$, and $(1 - L_d)[f(m_x) - m_x] - L_d m_d > 0$, equations (1) through (4) and (6) determine $(1 + t_d)/p$, w/p, $(1 + t_x)/(1 + t)$, and $(1 + t_i)/p$ uniquely. Hence, if choice of m_d, m_x, and L_d results in non-negative w with (8) being met, then (7) is also met. Now (8) can be rewritten[9] as (8′) on the following page.

[9] The equation was derived thus:

$$(1 + t_d)L_d m_d = \frac{w L_d g(m_d)}{g(m_d) - m_d g'(m_d)} - wL_d$$

from (3) and (4),

$$[(1 + t_x)m_x - (1 + t)f(m_x)](1 - L_d) = -w(1 - L_d)$$

from (1) and (2), and

$$1 + t_i = \frac{U_i}{U_d} \cdot \frac{w}{g(m_d) - m_d g'(m_d)}$$

from (4) and (6).

Substitute in (8) after adding and subtracting $(1 - L_d)[f(m_x) - m_x] - L_d m_d$ to its left-hand side, and thereafter substitute C_d for $L_d g(m_d)$ and C_i for $(1 - L_d)[f(m_x) - m_x] - L_d m_d$.

$$\frac{w\left\{\frac{U_i}{U_d}\cdot C_i + C_d - (1+\pi)[g(m_d) - m_d g'(m_d)]\right\}}{g(m_d) - m_d g'(m_d)} = 0. \qquad (8')$$

The term $g(m_d) - m_d g'(m_d)$—the marginal physical product of labor in industry c_d—is always finite and positive. If (8') is to be met with positive w, the expression in braces in the numerator, for which we shall write N, must be equal to zero.[10]

We wish to maximize the utility function (5) subject to $N = 0$, $1 > L_d > 0$, $m_d \geqq 0$, $m_x \geqq 0$, $C_i > 0$. Suppose that for some non-negative L_d, m_d, and m_x these constraints are met (m_x being different from m_x^*). Given these values of L_d and m_d and therefore of C_d, we can maximize C_i, and hence welfare, by changing m_x to m_x^*, given that $C_i = (1 - L_d)[f(m_x) - m_x] - L_d m_d$. If N continues to be non-negative,[11] the optimal value of m_x is clearly m_x^*.

But suppose that N becomes negative. This must be because the fall in U_i/U_d when C_i is raised with C_d held constant outweighs the rise in C_i; $g(m_d) - m_d g'(m_d)$ does not change for we have not altered m_d. In that case, we can hold m_d and C_i constant when we change m_x to m_x^*. Less labor is now needed in industry x to secure the net exports to pay for unchanged C_i and the metal hitherto used to make c_d; we can raise L_d and therefore C_d. Then U_i/U_d is higher, as C_i is unchanged while C_d is larger, and N is greater. We have therefore raised welfare while meeting the revenue constraint.This completes our proof that it is optimal to maintain the free trade technique in export production.

Let us now set $t = t_x = 0$ so that $w = w^*$, and consider the optimal values of t_d and t_i.[12] For this purpose we use Figure 1, in which OT (OT_1) is the quantity of c_i (c_d) that can be consumed if there is no consumption of the other consumer good and if the free trade technique is used to make c_d. The slope of TT_1 is the free trade product/price ratio, and under free trade the consumption point is P. One way of collecting revenue is to set $t_d = 0$, thus maintaining the free trade technique in producing c_d, and to set t_i at the level that secures the specified revenue. Consumption will then be at Q, the point on TT_1 at which an indifference curve is tangent to a

[10] The solution $w = 0$ must be ruled out, for it can be verified from (1) through (4) that if $w = 0$ all factor and product prices are zero, and no determinate equilibrium exists.

[11] If $N > 0$, more than specified revenue will be collected, and the excess can be returned to taxpayers.

[12] Choice of w is arbitrary because (8) is homogeneous of degree 1 in the variables $1 + t$, $1 + t_x$, $1 + t_d$, $1 + t_i$, p, and w. If the proportionate change in $1 + t$, $1 + t_x$, $1 + t_d$, and $1 + t_i$ is λ, it is clear from (1) through (4) and (6) that the quantities of each good produced and consumed do not change and that the proportionate change in p and w is λ. The quantities t, t_x, t_d, and t_i change to $\lambda(1 + t) - 1$, $\lambda(1 + t_x) - 1$, $\lambda(1 + t_d) - 1$, and $\lambda(1 + t_i) - 1$. As the levy of a uniform tax *cum* subsidy at the rate $\lambda - 1$ does not change revenue, the proportionate change in revenue is also λ.

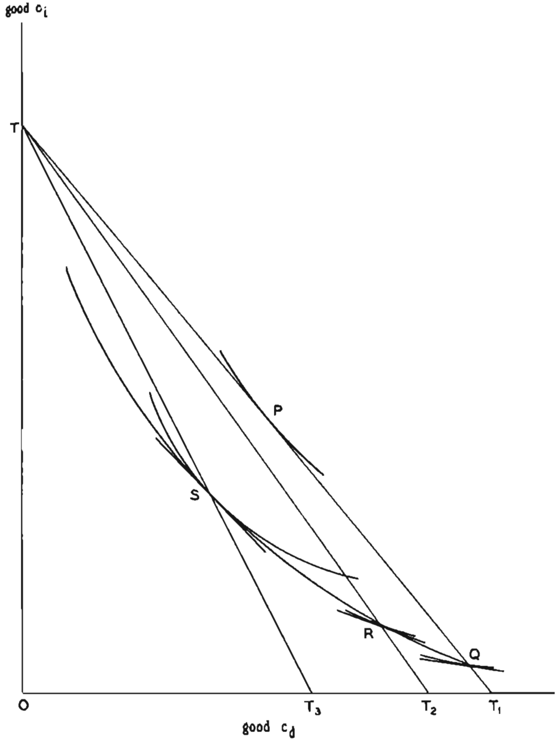

FIG. 1

price line with the slope $p^*/1 + t_i$. This policy entails consumption cost but no production cost, and the welfare loss resulting from its adoption sets a ceiling to the welfare cost of revenue collection. It may, however, be possible to raise welfare by levying a tax at a low rate on imports of metal to make c_d, thus accepting some production cost, and lowering t_i to the extent consistent with collection of the specified revenue. The quantity of c_d that can be produced and consumed if there is no consumption of c_i is now OT_2, and consumption will be at the point R on TT_2 at which an indifference curve is tangent to a price line which is steeper than the price line with consumption at Q. Because of the change in the slope of the price line, R may lie on a higher indifference curve than Q, as in Figure 1, though TT_2 is inside OTT_1. By continuing to lower t_i and to raise t_d while meeting the revenue constraint, we can trace the curve QRS—the locus of points such as Q and R—which is the consumption frontier subject to the revenue constraint. The optimal consumption point is S, at which an indifference curve is tangent to QRS; beyond this point, the gain in welfare due to lower t_i reducing consumption cost is less than the welfare loss resulting from the higher production cost entailed by larger t_d.

There is no particular reason why the optimal values of t_i and t_d should be the same. It is often asserted that a uniform tax on all imports for home use is optimal, because this equates the marginal utilities derived from alternative uses of foreign exchange. For this proposition to be valid, foreign exchange availability must be invariant with respect to tax changes. In general, however, the allocation of domestic factors to export production and hence foreign exchange availability vary when tax rates are changed, and this is why duties on imports for home use are unlikely to be uniform in the second-best revenue tariff.[13,14]

II

We now turn to problems of output diversification. Underdeveloped countries often wish to diversify output because of the uncertainties considered to be associated with production of a limited number of commodities, mainly for export. They desire to make new items, whether for export or for the home market. Sometimes, however, the objective is the autarkic one of producing new items for home consumption only. We discuss the optimal means of achieving these alternative objectives, assuming that any revenue needed can be raised in a non-distortionary way and that the policy instruments available are output subsidies and trade and consumption taxes. We use the model of Section I with a modification. We now assume that x alone is made under free trade, c_d and c_i being imported, and we allow for the possibility of producing c_i at home, the production function for this good having the same characteristics as those for x and c_d.[15] Objective 1 is output worth θ at world prices of c_d and/or c_i *for*

[13] A counterexample to prove that optimal duties on imports for home use need not be equal is provided in Appendix B.

[14] Kindleberger (1963, pp. 223–24) defines a tariff for revenue only as being one that has consumption cost but does not affect production. He concludes that a uniform tax on imports is not optimal, because it is likely to distort production. The definition is unacceptable, because while production changes are not an objective of the revenue tariff, there is no reason why they should not be accepted if, as a result, the welfare loss due to revenue collection is minimized.

[15] The unit of quantity for c_d has a world price of unity.

home use or for export, while the alternative objective 2 is production worth θ at world prices of c_d and/or c_i *for home use alone*. We prove the following. (1) Given objective 1, the optimal policy is to hold factor prices at the free trade level and to subsidize output of that good other than x (which we assume to be c_d) in whose production the unit cost is least, so as to maintain the free trade product/price ratio in consumption.[16] (2) If at least θ of c_d is consumed at home under policy (1), an equivalent result is secured by levy of an import duty on c_d and c_i at the uniform rate just sufficient to induce production of c_d. Either this policy or policy (1) is also optimal to secure objective 2. (3) If, however, c_d is exported under policy (1), that policy is superior to alternative policies, given objective 1. The optimal policy to achieve objective 2 is to hold factor prices at the free trade level, to levy import duties on c_d and c_i at rates just sufficient to induce production, and to levy an optimal consumption tax on c_i; use of import duties alone yields lower welfare.

Suppose that the output of c_d is θ and that c_i is not made. With any given L_d, m_d foreign exchange available to import c_i and additional supplies of c_d is $(1 - L_d)[f(m_x) - m_x] - L_d m_d$. Differentiating partially with respect to m_x, we have $m_x = m_x^*$ for a maximum value and we set $w = w^*$. Differentiating partially with respect to m_d, $\partial L_d/\partial m_d = -L_d/[f(m_x^*) - m_x^* + m_d]$ for a maximum value. As $L_d g(m_d) = \theta$,

$$g'(m_d) = -\frac{g(m_d)}{L_d} \cdot \frac{\partial L_d}{\partial m_d} = \frac{g(m_d)}{f(m_x^*) - m_x^* + m_d} = \frac{w^* + pm_d g'(m_d)}{p[f(m_x^*) - m_x^* + m_d]}$$

from (4). Hence, $pg'(m_d)[f(m_x^*) - m_x^*] = w^*$; and as $f(m_x^*) - m_x^* = w^*$, $pg'(m_d) = 1$. The optimal factor prices in the manufacture of c_d are thus w^* and 1. Similarly, if c_i is made, the optimal factor prices will once more be w^* and 1. By assumption, the unit cost u_d of c_d at these factor prices is less than the unit cost u_i of c_i. Production of a unit of c_i instead of a unit of c_d to meet an output constraint reduces foreign exchange available for imports by $u_i - u_d$. A subsidy to specified output of c_d at the rate $u_d - 1$ secures objective 1 at the least cost. If consumers demand at least θ of c_d at the world price ratio, levy of import duty on c_d and c_i at the rate $u_d - 1$ is an equivalent policy.

It may be, however, that c_d is exported when its output is subsidized. In that case, grant of an output subsidy is superior to alternative policies when objective 1 is specified, for levy of import duties and/or consumption taxes result in higher production cost due to c_i being produced and/or consumption cost because of departure from the free trade product/price ratio in consumption.

If the autarkic objective 2 is stipulated, we note that in the optimal solution $p_d/p_i = 1 - u_i + u_d$, where p_d and p_i are the domestic prices of

[16] That an output subsidy is the optimal means of diversifying production when the terms of trade are fixed was shown by Corden (1957). See also Johnson (1965).

the two goods. The resources released when one unit less of c_d is made are inadequate to produce an additional unit of c_i to meet the output constraint and need to be supplemented by a fall of $u_i - u_d$ in imports of c_i. Now, if import duties alone are levied, the duty on c_i must be $u_i - 1$ and its price u_i for market equilibrium when this good is both produced and imported. The price of c_d is its unit cost u_d when it is prohibitively protected. Good c_d is then relatively too expensive from the social point of view, for

$$(1 - u_i + u_d) - \frac{u_d}{u_i} = -\frac{(u_i - u_d)(u_i - 1)}{u_i} < 0,$$

as $u_i > u_d > 1$. It is easily verified that an additional levy of a tax on each unit of c_i consumed at the rate $[(u_i - u_d)(u_i - 1)]/(1 - u_i + u_d)$ secures equality of p_d/p_i and $1 - u_i + u_d$. If only import duties are levied, the product/price ratio will not equal the marginal cost ratio from the social point of view, and welfare will be lower.[17]

In Figure 2, OP is the quantity θ of c_d, OT is the quantity of c_d that can be consumed if there is no consumption of c_i, and the slope of TT_1T_2 is the free trade product/price ratio. If at this price ratio the consumption point is C_1, it is optimal to subsidize output of c_d or to tax import of c_d and c_i at the rate $u_d - 1$, given either objective. If, however, an indifference curve is tangent to TT_1T_2 to the left of T_1, say, at C_2, this consumption point can be reached only if objective 1 is specified and output of c_d is subsidized; this good will then be exported. The consumption frontier when objective 2 is stipulated is TT_1T_3, and levy of optimal import duties and consumption tax secures equilibrium at C_3, where an indifference curve is tangent to T_1T_3. Levy of import duties alone results in consumption at, say, C_4, where an indifference curve cuts the consumption frontier.

III

We now offer some concluding observations.

We have shown that inputs used in export production must be free of duty in the second-best revenue tariff. While it is common practice to grant drawbacks of duty paid on the import content of exports, there are often irrational requirements, such as imported input must be physically embodied in the export good. From the economic point of view there is no

[17] Cooper and Massell (1965) assert that an efficient tariff is one that maximizes national income at world prices subject to specified output of protected goods. This proposition is valid only on their special assumption that the quantities demanded of each good are invariant with reference to relative prices, which eliminates the consumption cost of protection.

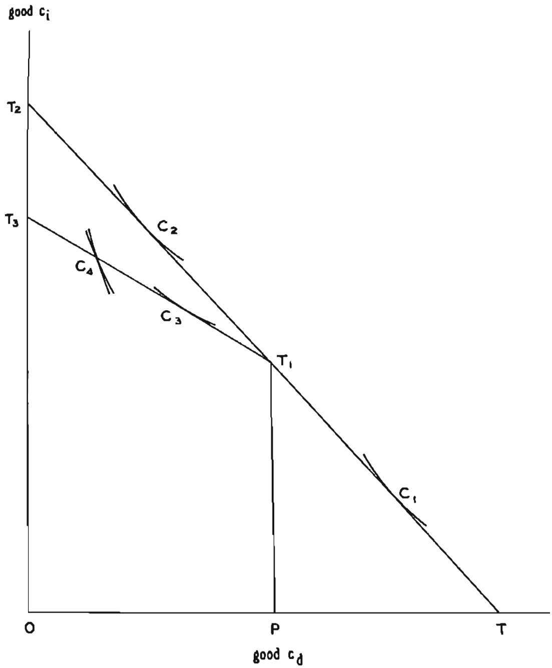

FIG. 2

difference between fuel consumed in making a product and components built into it.[18]

We have also proved that optimal revenue duties on imports for home use will in general not be uniform. This does not, of course, mean that complex tariffs in the real world, which have been framed largely in response to pressures of interested groups, are optimal. What our conclusion

[18] There are, no doubt, practical difficulties in verifying the import content of exports. Production for export in free trade zones is a convenient solution. High tariff countries are likely to find that the setting up of such zones expands exports. In many cases, sample inspection of exports at the frontier will suffice to determine the production process and hence import content. While no system can be perfect, the objective should be to make imported inputs available to exporters at world prices to the fullest extent practicable.

suggests is that revenue tariffs should be based on objective empirical research.

When countries claim that they need trade taxes to collect revenue, they should be under an obligation to show that alternative means of raising revenue are inferior. It is remarkable that while multiple exchange rate and import control systems are subject to international review, countries can levy any tariffs they like on items not bound in GATT negotiations. As a result, many countries, some of which are very rich, levy tariffs which probably both reduce national welfare unnecessarily and worsen international income distribution.

If a government deficit-finances expenditure and the central bank makes an equal profit by setting multiple exchange rates for different categories of trade transactions, this is equivalent to the levy of a revenue tariff and our conclusions apply fully. If, on the other hand, government expenditure is deficit financed and quantitative restrictions are applied to imports, identical results can be secured only if the level of expenditure is the same as when tariffs are levied. For this to happen, import licensees must save all their windfall profits. And as they probably will not do so, a secondary round of price increases which induces further savings through a change in income distribution or other effects is likely to take place. Needless to say, exporters should be allowed to import inputs freely if quantitative restrictions are irrationally preferred to tariffs or multiple exchange rates. In practice, import control systems often impede exports, because exporters have to use high-cost domestic substitutes for imported materials.

When a country wishes to diversify production, output subsidies—and not protection—are called for. When the objective is the irrational one of autarky, not only protective duties but also consumption taxes will be ingredients of the optimal policy. Imported inputs used by protected industries must be free of duty. Taxation of such inputs is suboptimal, because domestic factors are then diverted from making exports to wasteful substitution of imported inputs.

Appendix A

In this appendix we relax the assumption that trade is balanced. Assume that the magnitude of a specified capital inflow (Δ) or outflow ($-\Delta$) is independent of the trade policy followed. Loans or gifts are received from or made to foreigners only by the private sector. We show that specified revenue can be collected without departure from free trade first-best if $\Delta < 0$ or if $\Delta > 0$ and the revenue requirement at free trade prices is less than the capital inflow.

Consider the levy of a uniform import tax (subsidy) and export subsidy (tax) at the rate $t(-t)$. Then $R = t(M + C_1 - X) = t\Delta$. The revenue required is πw; and as $w = w^*(1 + t)$, $\pi w^*(1 + t) = t\Delta$. But $1 + t$ has to be positive for feasibility. Hence, $t > 0$ if $\Delta > 0$, and $t < 0$ if $\Delta < 0$.

The solution for t is $t = \pi w^*/(\Delta - \pi w^*)$. Hence, the condition $t < 0$ will be met for all $\Delta < 0$. But if $\Delta > 0$, $t > 0$ only if $\pi w^* < \Delta$.

The crucial assumption in the above proof, that the magnitude of capital flows is independent of the trade policy followed, is of course unlikely to hold in the real world. It may be noted, however, that levy of a uniform tax *cum* subsidy by, say, a capital-importing country does not alter the profit rate on investment in that country. Tax on the capital inflow is offset by subsidy on capital servicing charges. The amount that the foreign investor must provide in his currency to set up a given plant will, on the other hand, be higher. At any rate, levy of such a uniform tax *cum* subsidy can usefully be considered by countries receiving significant amounts from abroad by way of capital inflow or migrants' remittances.

If $\Delta > 0$ and $\pi w^* \geqq \Delta$, only a second-best solution is possible. Our analysis of the balanced trade case can easily be extended to show that the free trade technique must be maintained in export production in this second-best solution.

Appendix B

In this appendix we provide a counterexample to show that duties on imports for home use need not be uniform in the second-best revenue tariff. We assume that trade is balanced. Consider the following utility and production functions:

$$U = (C_d^{-\rho} + C_t^{-\rho})^{-1/\rho} \tag{B1}$$

where $\rho \geqq -1$, and

$$G = L_d^{1-\alpha} M_d^{\alpha} \tag{B2}$$

where $0 < \alpha < 1$. No specific production function need be assumed for x. It can easily be shown that if $\rho = 0$, that is, if the elasticity of substitution in the utility function is unity, then optimal $t_d = t_t$. We now prove that if the elasticity of substitution in the utility function is less than unity, then t_d is less than t_t in the optimal solution. Our method is to suppose that $t_d = t_t$ in an optimal solution and to prove that this is impossible, as welfare can be raised while meeting the revenue constraint by reducing t_d and raising t_t.

Partially differentiating (B1) with respect to C_d and C_t, we ascertain that $U_t/U_d = (C_d/C_t)^{\rho+1} = (1 + t_t)/p$ from (6). If $t_d = t_t$ in the optimal solution, $(1 + t_t)/p = (1 + t_d)/p = g'(m_d)$ from (3). Therefore,

$$\frac{U_t}{U_d} = \left(\frac{C_d}{C_t}\right)^{\rho+1} = \frac{1 + t_t}{p} = \frac{1 + t_d}{p} = g'(m_d). \tag{B3}$$

Now change t_d, L_d, C_t, t_t, and M_d by δ, γ, μ, ϵ, and η, respectively. By the balance of payments constraint, $-(1 - L_d - \gamma)w^* + (M_d + \eta) + (C_t + \mu) = 0$, and $-(1 - L_d)w^* + M_d + C_t = 0$; hence, $\mu = -\eta - \gamma w^*$. Now

$$\frac{g(m_d) - m_d g'(m_d)}{g'(m_d)} = \frac{w^*}{1 + t_d}.$$

But

$$\frac{g(m_d) - m_d g'(m_d)}{g'(m_d)} = \frac{(1 - \alpha)m_d}{\alpha},$$

or

$$m_d = \frac{\alpha}{1 - \alpha} \cdot \frac{w^*}{1 + t_d}.$$

This implies $\Delta m_d = -m_d\delta/(1+t_d)$. By definition, $L_d m_d = M_d$. Hence, $\Delta M_d = \eta = m_d \Delta L_d + L_d \Delta m_d$, or $\eta = \gamma m_d - \delta M_d/(1+t_d)$. This implies that

$$\mu = -\gamma(m_d + w^*) + \frac{\delta L_d m_d}{1+t_d}. \tag{B4}$$

For the revenue constraint to be met,

$$(M_d + \eta)(t_d + \delta) + (C_i + \mu)(t_d + \epsilon) = M_d t_d + C_i t_d.$$

Ignoring second-order terms,

$$\delta M_d + t_d\left(\gamma m_d - \frac{\delta L_d m_d}{1+t_d}\right) + C_i\epsilon + t_d\left[-\gamma(m_d + w^*) + \frac{\delta L_d m_d}{1+t_d}\right] = 0,$$

or $\epsilon = (\gamma t_d w^* - \delta M_d)/C_i$.

Now from (B3), $(\rho + 1)(\log C_d - \log C_i) = \log(1 + t_i) - \log p$. Hence,

$$(\rho+1)\left(\frac{\Delta C_d}{C_d} - \frac{\Delta C_i}{C_i}\right) = -\frac{\Delta p}{p} + \frac{\Delta t_i}{1+t_i}. \tag{B5}$$

Now $C_d = L_d g(m_d) = L_d m_d^\alpha$, and

$$\Delta C_d = m_d^\alpha \Delta L_d + \alpha L_d m_d^{\alpha-1}\Delta m_d$$

$$= \gamma m_d^\alpha + \alpha L_d m_d^{\alpha-1}\left(-\frac{m_d\delta}{1+t_d}\right) = m_d^\alpha\left(\gamma - \frac{\alpha\delta L_d}{1+t_d}\right).$$

Therefore,

$$\frac{\Delta C_d}{C_d} = \frac{\gamma}{L_d} - \frac{\alpha\delta}{1+t_d}. \tag{B6}$$

From (B4) and (B6),

$$\frac{\Delta C_d}{C_d} - \frac{\Delta C_i}{C_i} = \gamma\left(\frac{1}{L_d} + \frac{m_d + w^*}{C_i}\right) - \frac{\delta}{1+t_d}\left(\alpha + \frac{L_d m_d}{C_i}\right). \tag{B7}$$

Now

$$p = \frac{w^*}{g(m_d) - m_d g'(m_d)} = \frac{w^*}{1-\alpha}\cdot m_d^{-\alpha},$$

and

$$\frac{\Delta p}{p} = -\frac{\alpha\Delta m_d}{m_d} = \frac{\alpha\delta}{1+t_d}.$$

$$\frac{\Delta t_i}{1+t_i} = \frac{\epsilon}{1+t_d} = \frac{\gamma t_d w^* - \delta L_d m_d}{C_i(1+t_d)}.$$

Therefore,

$$-\frac{\Delta p}{p} + \frac{\Delta t_i}{1+t_i} = \frac{1}{1+t_d}\left[\frac{\gamma t_d w^*}{C_i} - \delta\left(\alpha + \frac{L_d m_d}{C_i}\right)\right]. \tag{B8}$$

Substituting from (B7) and (B8) into (B5) and rearranging terms,

$$\gamma\left[\frac{\rho+1}{L_d} + \frac{(\rho+1)(m_d+w^*)}{C_i} - \frac{t_d}{1+t_d}\cdot\frac{w^*}{C_i}\right] = \frac{\rho\delta}{1+t_d}\left(\alpha + \frac{L_d m_d}{C_i}\right).$$

Now

$$(\rho+1)\left(\frac{1}{L_d} + \frac{m_d + w^*}{C_i}\right) = \frac{\rho+1}{L_d C_i}(C_i + L_d m_d + w^* L_d);$$

and as trade is balanced, $C_i + L_d m_d + w^* L_d = C_i + M_d + w^* L_d = w^*$.

Hence,

$$\gamma\left[\frac{w^*(\rho + 1)}{L_d C_l} - \frac{w^* t_d}{C_l(1 + t_d)}\right] = \frac{\rho\delta}{1 + t_d}\left(\alpha + \frac{L_d m_d}{C_l}\right).$$

Therefore,

$$\gamma = \frac{\rho\delta L_d C_l\left(\alpha + \frac{M_d}{C_l}\right)}{w^*(1 + t_d)\left[(\rho + 1) - \frac{t_d L_d}{1 + t_d}\right]} \tag{B9}$$

Now $\Delta U = U_d \Delta C_d + U_l \Delta C_l$. But

$$\Delta C_l = -\gamma(m_d + w^*) + \frac{\delta L_d m_d}{1 + t_d},$$

and

$$\begin{aligned}\Delta C_d &= g'(m_d)\Delta M_d + [g(m_d) - m_d g'(m_d)]\Delta L_d \\ &= g'(m_d)\left[\gamma m_d - \frac{\delta L_d m_d}{1 + t_d}\right] + \gamma[g(m_d) - m_d g'(m_d)] \\ &= \gamma g(m_d) - \frac{\delta L_d m_d g'(m_d)}{1 + t_d}.\end{aligned}$$

Therefore,

$$\begin{aligned}\Delta U &= U_d\left[\gamma g(m_d) - \frac{\delta L_d m_d g'(m_d)}{1 + t_d}\right] + U_l\left[-\gamma(m_d + w^*) + \frac{\delta L_d m_d}{1 + t_d}\right] \\ &= \gamma[U_d g(m_d) - U_l(m_d + w^*)] + \frac{\delta L_d m_d}{1 + t_d}[U_l - U_d g'(m_d)].\end{aligned}$$

But as $U_l/U_d = g'(m_d)$, the second term above is zero. Hence,

$$\begin{aligned}\Delta U &= \gamma U_d g'(m_d)\left[\frac{g(m_d) - m_d g'(m_d)}{g'(m_d)} - w^*\right] \\ &= \gamma U_d g'(m_d)\left(\frac{w^*}{1 + t_d} - w^*\right) \\ &= -\frac{t_d \gamma U_d w^* g'(m_d)}{1 + t_d}.\end{aligned} \tag{B10}$$

Thus, ΔU is positive if γ is negative. If $\rho > 0$, we know from (B9) that γ is negative if δ is negative, for $(\rho + 1)$ is larger than $t_d L_d/(1 + t_d)$, the latter term being less than unity. We can raise welfare by lowering t_d and raising t_l; thus, $t_d = t_l$ could not have been optimal.

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