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INEQUALITY AS A DETERMINANT OF MALNUTRITION AND UNEMPLOYMENT: POLICY*

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In the predecessor to this article (Dasgupta and Ray, 1986; hereafter D–R), we developed a theory which provides a link between persistent involuntary unemployment and the incidence of undernourishment, relates them in turn to the production and distribution of income and thus ultimately to the distribution of assets. The theory is founded on the much-discussed observation that at low levels of nutrition-intake there is a positive relation between a person's nutrition status and his ability to function; or to put it at once more generally and more specifically, a person's consumption-intake affects his productivity.

The central idea which we pursued in D–R is that unless an economy in the aggregate is richly endowed with physical assets it is the assetless who are vulnerable in the *labour market*. Potential employers – or speaking metaphorically, the 'market' – find attractive those who enjoy non-wage income, for in effect they are cheaper workers. Put another way, those who enjoy non-wage income can undercut those who do not, and if the distribution of assets is highly unequal even competitive markets are incapable of absorbing the entire labour force: the assetless are too expensive to employ in their entirety, as there are too many of them (See D–R, theorem 2.)

A simple example may help. Suppose each person requires precisely 2000 calories per day to be able to function: anything less and a person's productivity is nil, anything more and his productivity is unaffected. Consider two people, one of whom has no non-wage income while the other enjoys 1500 calories per day of such income. The first person needs a full 2000 calories of wages per day in order to be employable, the latter only 500 calories per day. It is for this reason the assetless is disadvantaged in the labour market. To be sure, if employers compete for the service of people with assets their wages will be bid up and the consequent analysis will be a great deal more complicated than the

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We would urge readers, before embarking on a reading of this article, to skim through Dasgupta and Ray (1986), where the motivation behind the problem analysed here is spelt out in detail and where references to earlier work in the field are provided.

corresponding analysis of a monopsonistic labour market. In D–R we provided this analysis and we showed the precise way in which asset advantages translate themselves into employment advantages. But this suggests strongly that certain patterns of egalitarian asset redistributions may result in greater employment and indeed greater aggregate output. The purpose of this article is to confirm such possibilities, and to explore in some detail public policy measures which ought to be considered in the face of massive market-failure of the kind identified in D–R. In the following section we will reintroduce the notation and redefine certain terms. Section II will contain the heart of our analysis of public policy options. Section III presents our main conclusions. As in our predecessor essay, proofs are relegated to the Appendix.

I. NOTATION AND THE MODEL

We distinguish labour-*time* from labour-*power* and observe that it is the latter which is an input in production. Consider a person who works in the economy under analysis for a fixed number of 'hours' – the duration of the analysis. Denote the labour power he supplies over the period by λ and suppose that it is functionally related to his consumption, I, in the manner of the bold-faced curve in Fig. 1.

Two factors, *land* and *labour-power*, are involved in the production of *rice*. Land is homogeneous, workers are not. Denoting by T the quantity of land and by E the *aggregate* labour-power employed in production (i.e. the sum of individual labour powers employed) let $F(E, T)$ be the output of rice, where the aggregate production function $F(E, T)$ is assumed to be concave, twice differentiable, constant-returns-to-scale, increasing in E and T, and displaying diminishing marginal products. Total land in the economy is fixed, and is $\hat{T}$. Aggregate labour power in the economy is, of course, endogenous.

We represent a large population, normalised at unity, by the unit interval $[0, 1]$, so that each person has a label n, where n is a number between 0 and 1. A person with label n is called an n-person, and we assume that the quantity of land he owns is $\hat{T}t(n)$. We assume that there are a great many landless persons. Thus we suppose that there is some number $\underline{n} > 0$ such that all persons labelled between 0 and $\underline{n}$ are landless, and that $t(n)$, the proportion of aggregate land n-person owns, is an increasing function of n beyond $\underline{n}$. (See fig. 2 in D–R).

A person either does not work in the production sector or works for one unit of time. There are competitive markets for both land and labour power. Let r denote the rental rate on land. Then n-person's non-wage income is $r\hat{T}t(n)$. Each person has a reservation wage which must as a minimum be offered if he is to accept a job in the competitive labour market. $\bar{w}(R)$ denotes the reservation wage, where R denotes non-wage income. In our model $R = r\hat{T}t(n)$. We take it that $\bar{w}$ is constant for all n in the range 0 to $\underline{n}$ and that thereafter it is an increasing function of n (see fig. 3 in D–R). In Fig. 1, $\hat{I}$ is the *efficiency-wage* of a landless person. We take it that the reservation wage of a landless person is less than $\hat{I}$. This is a crucial assumption, and we made much use of it earlier.

To present our results in a sharp form we will suppose that the curvature

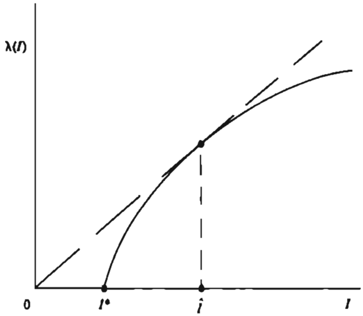

Fig. 1.

of the λ function in Fig. 1 is great at $\hat{I}$. We will therefore be justified in referring to $\hat{I}$ as the food-adequacy standard. A person consuming less than $\hat{I}$ is thus malnourished.

Finally define

$$w^*(n, r) \equiv \arg\min\{w/\lambda[w + r\hat{T}t(n)],\ \text{s.t.}\ w \geqslant \bar{w}[r\hat{T}t(n)]\}, \tag{1}$$

and

$$\mu^*(n, r) \equiv w^*(n, r)/\lambda[w^*(n, r) + r\hat{T}t(n)]. \tag{2}$$

Equation (1) defines the *efficiency-wage rate* of an n-person, and equation (2) defines his *efficiency-piece rate*. (See fig. 2 below for a typical functional form of $\mu^*(n, r)$ for a given value of r.) The reader can obtain a detailed account of these functions in D–R.

In our earlier paper we defined a competitive equilibrium allocation in the economy under review and proved its existence (definition 2 and theorem 1 in D–R). Stated verbally, a competitive equilibrium is an allocation sustained by a land rental rate $\tilde{r}$, a piece rate $\tilde{\mu}$, a set of employed persons, $\tilde{G}$, and a wage rate $\tilde{w}(n)$ on offer to an n-person belonging to the set $\tilde{G}$, such that: (i) $\tilde{r}$ equals the marginal product of land and $\tilde{\mu}$ equals the marginal product of aggregate labour power employed; (ii) a person whose efficiency piece rate falls short of $\tilde{\mu}$ finds employment; (iii) a person whose efficiency piece rate exceeds $\tilde{\mu}$ supplies no labour and is not on demand either; and (iv) employers are indifferent between employing and not employing a person whose efficiency piece rate equals $\tilde{\mu}$. We noted that equilibrium is compatible with the presence of widespread involuntary unemployment and the incidence of undernourishment. In particular, we showed that given the land distribution $t(n)$, if the aggregate quantity of land in the economy is neither too small nor too large, equilibrium entails rationing in the labour market: a fraction of the landless find employment at its efficiency wage $\hat{I}$, while the remaining fraction are disfranchised and

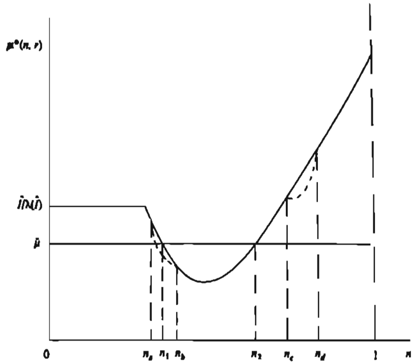

Fig. 2. Partial land reform: n-persons between n_a and n_b gain land, and rentiers between n_c and n_d lose land.

suffer their reservation wage, which is nutritionally inadequate (regime 2 in section II.5 of D–R). We also noted that if the aggregate quantity of land is small all the landless and the marginal landholders are malnourished and unemployed (regime 1 in section II.4 of D–R). Finally, it was noted that if the aggregate quantity of land is large there is no involuntary unemployment in equilibrium and the landless receive a wage in excess of their efficiency wage (regime 3 in section II.6 of D–R).

Fig. 2 illustrates an equilibrium outcome in Regime 1. (The efficiency piece rate, $\mu^*(n, \tilde{r})$ as a function of n, is given by the unbroken U-shaped curve.) The equilibrium piece rate, $\tilde{\mu}$, is less than the efficiency piece rate of the landless, $\hat{l}/\lambda(\hat{l})$. Persons labelled between 0 and n_1 are unemployed, as are people labelled between n_2 and 1, but the latter group choose not to work: their reservation wages are too high.

In what follows we will, as in our earlier paper, denote equilibrium values of economic variables by tildes. Thus $\tilde{r}$, $\tilde{\mu}$ and so on denote equilibrium values of the rental rate, the piece rate, and so forth.

In our earlier paper we studied the implications of aggregate asset accumulation in the economy in question. The distribution of assets was held fixed (theorem 3 in D–R). In this essay we study, for the most part, the implication of asset redistribution.

II. PUBLIC POLICY

Growth, seen as a means of removing poverty and unemployment, has long dominated the development literature. We want now to argue that in certain circumstances it is the *inequality* in the distribution of assets which is the *cause* of poverty and malnutrition and thus in turn involuntary unemployment. To analyse this we will, in this Section, hold the aggregate quantity of land fixed and alter the land distribution.[1] But first we must check that redistributive policies are the only ones that are available. This is confirmed by

THEOREM 1. *Under the conditions postulated, a competitive equilibrium is Pareto-efficient.*

Proof. See Appendix.

II.1. *Partial Land Reforms*

A variety of different redistribution schemes can be studied. For ease of exposition, we will first look at a simple, though important case: that of land transfers from the landed gentry (i.e. those who do not enter the labour market because their reservation wage is too high) to those who are involuntarily unemployed. Such redistributions need not (and, in general, will not) lead to full equalisation of asset holdings. To distinguish them from *full land redistributions* (to perfect equality) which we shall discuss below, call them *partial land reforms*.

In Fig. 2, a partial land reform, where land is transferred to some of the unemployed as well as those 'on the margin' of being unemployed, is depicted.[2] The diagram displays the changes evaluated at the original equilibrium $(\tilde{\mu}, \tilde{r})$. People between n_a and n_b gain land; for them, the $\mu^*(\cdot, \tilde{r})$ function shifts downward; that is, their efficiency-piece-rate is lowered. The losers, between n_c and n_d, also experience a downward shift in $\mu^*(\cdot, \tilde{r})$, but for entirely different reasons – their reservation wages have been lowered.

Of course, a new equilibrium will now be established, one with a different wage schedule and rental rate. Can the two be compared? A partial answer is given in

THEOREM 2. *Suppose that for each parametric specification, the competitive equilibrium is unique.*[3] *Then a partial land reform of the kind just described necessarily leads to at least as much output in the economy (strictly more, if $\mu^*(n, \tilde{r})$ is of the form in Fig. 2)*

Proof. See Appendix.

The result implies that there is no necessary conflict between equality-seeking moves and aggregate output in a resource-poor economy. Such redistributions have three effects. First, the unemployed become more attractive to employers as their non-wage income rises. Second, those among the poor who are

[1] It should be emphasised that although we will talk of land redistribution, consumption redistribution – via lump-sum food transfers – is all that the model requires.

[2] Fig. 2 looks at a land reform in regime 1; clearly, the case of regime 2 can be similarly analysed. (See sections II.4 and II.5 in D–R.)

[3] The assumption of a unique competitive equilibrium can be dropped, but then one would have to look at the stable equilibria. We avoid these to rule out unnecessary technical complications.

employed are more productive to the extent that they, too, receive land. Finally, by taking land away from the landed gentry, their reservation wages are lowered, and if this effect is strong enough, this could induce them to forsake their state of voluntary unemployment and enter the labour market. For all these reasons, the number of employed efficiency units in the economy rises, pushing it to a higher-output equilibrium.

Note, however, that Theorem 2 is silent on how the *set* of employed persons changes. Do previously unemployed persons necessarily find employment? Does the *number* of involuntarily unemployed fall?

Unfortunately, the answer to this question can go either way. There is a natural tendency for employment to rise, because of the features mentioned above. However – and this is characteristic of all *partial* (as opposed to *full*) land reforms – there is a 'displacement effect' at work, whereby newly productive workers are capable of displacing previously employed, less productive workers in the labour market. (An example is given in Dasgupta and Ray, 1984, using an economy in regime 2.)

II.2. *Full Land Reform*

This displacement effect cannot exist in the case of *full* land reforms, our final object of analysis. So as to highlight the detrimental effects of unequal land distributions we assume in what follows that the economy is productive enough to feed everyone adequately. To make this precise assume that land is socially managed and that there is complete equality in the treatment of all. If I denotes the consumption level of each person under such a scheme, $\lambda(I)$ is the labour-power of the representative person, and aggregate output is $F[\lambda(I), \hat{T}]$. For such an allocation to be viable, there must be a solution (in I) to the equation

$$I = F[\lambda(I), \hat{T}]. \qquad (3)$$

It is easy to see that if there is a solution to (3), in general there are two.[1] Concentrate on the *larger* of the two solutions and call it $I(\hat{T})$, and let $\hat{T}_1$ be the smallest value of $\hat{T}$ such that $I(\hat{T}) = \hat{I}$. Thus at $\hat{T}_1$ we have a formalisation of the idea that the economy is productive enough (just about) to feed all adequately (i.e. at the level of the food adequacy standard $\hat{I}$).

To set the stage, we first state:

THEOREM 3. *Let* $[\hat{T}, I(\hat{T})]$ *be an equal division solution. Then, if reservation wages are low enough,*[2] *this is achievable as a competitive equilibrium under full equality of land distribution.*

Proof. See Appendix.

To complete our analysis of full land redistributions, we will show that for each size of land in some range above $\hat{T}_1$, there are unequal distributions of that land that sustain involuntary unemployment and malnourishment (i.e.

[1] This excludes the 'tangency case' where there is exactly one solution. One can show that the smallest $\hat{T}$ for which a solution to (3) exists involves an $I(\hat{T}) < \hat{I}$. So $\hat{T}_1$, to be described below is uniquely defined.

[2] It helps to think of the reservation wage function as being identically zero in the relevant range, for this final section, as its presence adds nothing to the development of our basic point.

equilibrium is in either Regime 1 or 2; see D–R, even though full redistributions are associated with full employment and no malnourishment.

THEOREM 4. *There exists an interval* $(\hat{T}_1, \hat{T}_2)$ *such that if* $\hat{T}$ *is in this interval, full redistributions yield competitive equilibria with full employment and no malnourishment. Moreover for each such* $\hat{T}$, *there are unequal land distributions which give rise to involuntary unemployment and malnourishment.*

Proof. See Appendix.

In other words, we have identified a class of cases, namely, a range of moderate land endowments, where *inequality* of asset ownership can be pinpointed as the *basic* cause of involuntary unemployment and malnourishment. In such circumstances judicious land reforms, or food transfers, can *increase* output and *reduce* both unemployment and the incidence of undernourishment. Indeed, if land were equally distributed the market mechanism would sustain this economy in regime 3 (section II.6 in D–R) in which undernourishment and unemployment are things of the past.

Finally, note that it is perfectly possible that unequal distribution of 'adequate' aggregate land (in the sense of Theorem 4) leads the economy to, say, regime 2 unemployment. In this case, as we have observed, partial land reforms may well have perverse effects on employment. At the same time, full land redistributions lead to full employment (Theorem 4). This observation suggests that in some cases partial reform movements may not serve the desired purpose as well as a more aggressive, total, redistributive policy.

Our last result deals with 'rich' economies, for the sake of completeness. Theorem 5, below, states that for all land endowments greater than or equal to $\hat{T}_2$ (see the statement of Theorem 4), inequality in asset holdings cannot lead to malnourishment and involuntary unemployment through the mechanism highlighted in this paper.[1]

THEOREM 5. *For all* $\hat{T} \geqslant \hat{T}_2$, *there is no land distribution which involves involuntary unemployment or malnutrition.*

Proof. See Appendix.

III. CONCLUSION

In this article and its predecessor we have analysed the implications of the effect of nutrition on a person's capacity to function on employment, production and distribution. Our approach has been very much 'pure theory', but this is because a proper theoretical foundation for these links was not available to us. It has been customary in welfare economics to view the distribution of consumption as a primary social good, and to locate public policies that promote both it and other social goods. Our purpose has been to provide a complementary analysis: the *instrumental* value of redistribution policies. We have so chosen our model economy that asset redistributions (or equivalently,

[1] This statement should *not* be taken to mean that there is no connection between inequality and unemployment in resource-rich economies, only that the causal chain running through *our* analysis is not of the first importance for rich economies.

food transfers) are the only public policies worth considering, (see Theorem 1 above). Theorem 2 goes some way towards showing how certain Lorentz-improving asset redistributions result in lower aggregate unemployment and greater aggregate output. The analysis, however culminates in Theorem 4, where this idea is really nailed down: an economy which is moderately endowed and capable of employing everyone and feeding everyone adequately will fail to do so if the distribution of assets is highly unequal. It follows that asset-redistribution policies, or food transfer programmes, can be highly potent as regards aggregate output and employment in moderately endowed economies.

The theory of unemployment we have offered here is, we believe, a descendant of classical theories. And in Theorem 5 we provide the link between our theory and that of the now-standard competitive one by showing that the chain connecting asset distribution and aggregate employment is snapped if the economy is richly endowed in assets. But some of the most influential doctrines today concerning material prospects for less developed countries would seem to be based on the efficacy of the market mechanism. We would not argue that there is anything wrong in planners trying 'to get prices right'. But as Theorem 4 makes plain, this may be far from the most potent option available, for even if one were to get them right (as in Theorem 4) the market mechanism could be an unmitigated disaster.

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APPENDIX

Competitive equilibrium has been defined formally in D–R (see definition 2). $\hat{G}$ is the set of persons who, in equilibrium, are employed and $\hat{w}(n)$ is the wage rate of an n-person who is employed. Finally, $\nu(n)$ is the Lebesgue measure on $[0, 1]$.

Proof of Theorem 1. Let $\{\hat{r}, \tilde{\mu}, \hat{G}, \hat{w}(n)\}$ be an equilibrium. It sustains a 'utility' schedule $\hat{Y}(n)$ given by

$$\left.\begin{aligned} \hat{Y}(n) &= \hat{w}(n) + \hat{r}\hat{T}l(n) && (\text{for } n \in \hat{G}), \\ &= \bar{w}[\hat{r}\hat{T}l(n)] + \hat{r}\hat{T}l(n) && (\text{for } n \notin \hat{G}). \end{aligned}\right\} \tag{4}$$

Let $\hat{I}(n)$ be the income accruing to n-person from the economy under review. Then $\hat{I}(n) = \hat{Y}(n)$ for $n \in \hat{G}$ and $\hat{I}(n) = \hat{r}\hat{T}l(n)$ for $n \notin \hat{G}$.

Suppose $[\hat{r}, \tilde{\mu}, \hat{G}, \hat{w}(n)]$ is *not* Pareto-efficient. Then there is a set $A \subset [0, 1]$, with $\nu(A) = 1$, and a feasible 'utility' schedule $Y(n)$ for $n \in [0, 1]$ such that

$Y(n) \geqslant \hat{Y}(n)$ on $n \in A$, and a set B, with $\nu(B) > 0$, such that $Y(n) > \hat{Y}(n)$ for $n \in B$. We want to show that this cannot be; that any such $Y(n)$ is infeasible.

Let G be the set of persons who are employed in the economy at this Pareto-superior allocation. Let $\hat{G}^c \equiv A - \hat{G}$ and $G^c \equiv A - G$, and write $C \equiv \hat{G} \cap G^c$, $D \equiv \hat{G}^c \cap G$, $K \equiv \hat{G}^c \cap G^c$ and $J \equiv \hat{G} \cap G$. We then have

$$\left.\begin{aligned} Y(n) &= \bar{w}[I(n)] + I(n) \quad (\text{for } n \in G^c), \\ &= I(n) \quad\quad\quad\quad\quad (\text{for } n \in G), \end{aligned}\right\} \tag{5}$$

where $I(n)$ is the income given to n-person from the economy at this Pareto-superior allocation. Let $E \equiv \int_G \lambda[I(n)]\, d\nu(n)$. We wish to show that

$$\int_A I(n)\, d\nu(n) > F(E, \hat{T}). \tag{6}$$

Now, note that $C \cup D \cup K \cup J = A$. Moreover,

$$I(n) \geqslant \hat{r}\hat{T}t(n) \quad (\text{for } n \in K). \tag{7}$$

(This follows from the fact that $\bar{w}'(R) \geqslant 0$ for all $R \geqslant 0$.)

It is possible to show that $E > \hat{E}$ (see Dasgupta and Ray, 1984, appendix). Consider first $n \in J$. The additional consumption that he enjoys in the Pareto-superior allocation is $I(n) - \hat{I}(n)$. We show that this is no less than his contribution to the addition in output $F(E, \hat{T}) - F(\hat{E}, \hat{T})$.

We begin by noting that this latter contribution does not exceed $F_E(\hat{E}, T)\, \lambda'[\hat{I}(n)]\, [I(n) - \hat{I}(n)]$.[1] From equation (1) and $\lambda[\hat{I}(n)] > 0$, we know that

$$\lambda[\hat{I}(n)] \geqslant \hat{w}(n)\, \lambda'[\hat{I}(n)]. \tag{8}$$

Using (8) and conditions (iv) and (v) of definition 2 in D–R we conclude that n's contribution to the addition in aggregate output cannot exceed $I(n) - \hat{I}(n)$. In particular, if the latter is positive the increase in his consumption *exceeds* his contribution to additional output.

Next consider $n \in D$. It follows from (1) above and condition (ii) of definition 2 in D–R that

$$F_E(\hat{E}, \hat{T}) \leqslant w/\lambda[w + \hat{R}(n)] \quad (\text{for } w \geqslant \bar{w}[\hat{R}(n)]), \tag{9}$$

where $\hat{R}(n) = \hat{r}\hat{T}t(n)$. But by hypothesis $I(n) - \hat{I}(n) \geqslant \bar{w}[\hat{R}(n)]$, (see (4) and (5)). And so (9) applies for $w = I(n) - \hat{I}(n)$. It follows that the contribution of this n-person to the addition in output is less than or equal to the left hand side of

$$F_E(\hat{E}, \hat{T})\, \lambda[I(n)] = F_E(\hat{E}, \hat{T})\, \lambda[I(n) + \hat{R}(n) - \hat{I}(n)] \leqslant I(n) - \hat{I}(n), \tag{10}$$

(since $\hat{R}(n) = \hat{I}(n)$ for $n \in \hat{G}^c$). Moreover, if $\nu(D) > 0$, it follows from the *strict* concavity of F in E, and from (10) that the contribution to the increase in total output by all $n \in D$ is *less* than the increase in the total consumption of all $n \in D$.

Next note that if $n \in K$, he works in neither allocation, and so he adds nothing to the increase in production. Suppose then that $C = \Phi$. We are then done,

[1] This follows from the strict concavity of $F(E, \hat{T})$ as a function of E.

because by hypothesis $\nu(B) > 0$. So we must have $\nu(B \cap D) > 0$, or $\nu(B \cap J) > 0$, or $\nu(B \cap K) > 0$. Under any of these circumstances the total increase in output must fall short of the total increase in consumption. It follows that the allocation $Y(n)$ is infeasible.

If, on the other hand, $C \neq \Phi$, the argument is a little bit more complicated. For details see Dasgupta and Ray (1984), Appendix. ■

Proof of Theorem 2. As in the proof of theorem 1, in D–R, construct a correspondence $M'(r)$ as in equation (26) there, corresponding to the new land distribution $t'(n)$ after a partial land reform. In what follows we use primes on all relevant variables (functions) corresponding to the new equilibrium. We also borrow other notation from that theorem.

Note first $\min M'(r) > \min M(r)$. To see this observe that in moving from $t(n)$ to $t'(n)$ none of the people who were previously employed *loses* land. Moreover, the gain in landholding among some who were involuntarily unemployed pushes their $\mu^*(n, \hat{r})$ down to $\mu'^*(n, \hat{r})$, below $\hat{\mu}$. This means that $\nu[B'(r) - B(r)] > 0$. Hence $\min M'(r) > \min M(r)$. A similar argument establishes that $\nu[G'(r) - G(r)] > 0$ and so $\max M'(r) > \max M(r)$.

If $\hat{E} \in M'(\hat{r})$, then the original labour power-output configuration continues to be the unique equilibrium. Otherwise

$$E(r) = \hat{E} < \min M'(\hat{r}). \tag{11}$$

By virtue of the properties established in the proof of theorem 1 (D–R) of the correspondence $M(r)$, we conclude that there is $\hat{r}' > \hat{r}$ and $\hat{E}' \in M'(\hat{r}')$ such that $\hat{E}' = E(\hat{r}')$. But since $E(r)$ is an increasing function $\hat{E}' > \hat{E}$. From $\hat{E}'$ construct the new (unique) equilibrium as in the proof of theorem 1 in D–R. This equilibrium thus sustains greater output.

Finally, observe that if the original equilibrium was in regime 1, $\hat{C} = \bar{B}(\hat{r})$ and so $\hat{C} \subseteq B'(\hat{r})$, where $\bar{B}(\hat{r})$ is the closure of $B(\hat{r})$. But this proves that (11) must hold and therefore that the new equilibrium sustains higher aggregate output. ■

Proof of Theorem 3. Let $[\hat{T}, I(\hat{T})]$ be an equal division solution. Define $\hat{\mu} = F_E\{\lambda[I(\hat{T}), \hat{T}\}$ and $\hat{r} = F_T\{\lambda[I(\hat{T})], \hat{T}\}$. Choose the reservation-wage function low enough so that, in particular $\bar{w}(\hat{r}\hat{T}) < \lambda[I(\hat{T})]\hat{\mu}$ and let $\hat{C} = [0, 1]$, with $\hat{w}(n) \equiv \hat{w} \equiv \lambda[I(\hat{T})]\hat{\mu}$ for all $n \in [1, 0]$. (Recall that by the equal distribution postulate, $t(n) = 1$). It remains to check that this allocation satisfies the conditions of definition 2 in D–R. (See Dasgupta and Ray, 1984, Appendix.) ■

Proof of Theorem 4. Define $\hat{T}_0$ as the minimum value of $\hat{T}$ for which equation (3) has a positive solution. It is easy to check that

$$F_E\{\lambda[I(\hat{T}_0)], \hat{T}_0\}\lambda'[I(\hat{T}_0)] = 1. \tag{12}$$

By equations (3) and (12) and Euler's Theorem we have

$$\begin{aligned} I(\hat{T}_0) = F\{\lambda[I(\hat{T}_0)], \hat{T}_0\} &= F_T \hat{T}_0 + F_E \lambda[I(\hat{T}_0)] \\ &> F_E \lambda[I(\hat{T}_0)] = \lambda[I(\hat{T}_0)]/\lambda'[I(\hat{T}_0)]. \end{aligned} \tag{13}$$

From (13) we conclude that $I(\hat{T}_0) < \hat{I}$ and since $I(\hat{T})$ is an increasing and unbounded function of $\hat{T}$, we have $\hat{T}_1$, well defined and $\hat{T}_1 > \hat{T}_0$. Given this last we have, from (12), that for all $\hat{T} \geqslant \hat{T}_1$,

$$F_B\{\lambda[I(\hat{T})], \hat{T}\}\lambda'[I(\hat{T})] < 1. \tag{14}$$

Moreover, $I(\hat{T}) > \hat{I}$ for $\hat{T} > \hat{T}_1$. Now define

$$\mu(\hat{T}) \equiv F_B\{\lambda[I(\hat{T})], \hat{T}\}. \tag{15}$$

We may now use (14) to show that $\mu(\hat{T})$ is an increasing and unbounded function of $\hat{T} \geqslant \hat{T}_1$. But $\mu(\hat{T}_1) < \hat{I}/\lambda(\hat{I})$. Therefore there exists $\hat{T}_2 > \hat{T}_1$ such that $\mu(\hat{T}_2) = \hat{I}/\lambda(\hat{I})$.

Finally, we will show that for $\hat{T} \in [\hat{T}_1, \hat{T}_2)$ equal distribution of $\hat{T}$ generates equilibria involving full employment and an absence of malnutrition while there exist *unequal* distributions of $\hat{T}$ which generate equilibria in regime 1 or 2 (see D–R). The first part of the claim follows trivially from Theorem 3 and the fact that $I(\hat{T}) > \hat{I}$ for $\hat{T} > \hat{T}_1$. We establish the second part now.

Let $\hat{T} \in [\hat{T}_1, \hat{T}_2]$. Consider the equilibrium resulting from an equal distribution of this. It is in Regime 3. Let $\hat{r}$, $\hat{\mu}$ and $\hat{w}$ be the rental rate on land, the piece rate and the wage rate, respectively. Then clearly

$$\hat{w}/\lambda(\hat{w}+\hat{r}\hat{T}) = \hat{\mu} > \min\,[w/\lambda(w+\hat{r}\hat{T})] \quad (w \geqslant \bar{w}(\hat{r}\hat{T})). \tag{16}$$

We conclude that $M(r)$ in equation (26) of D–R is a singleton at $\hat{r}$. It also follows that in a small neighbourhood of $\hat{r}$, say $[r_a, r_b]$, $M(r)$ remains a singleton. Since $\hat{\mu} = \hat{\mu}(\hat{r}) < \hat{I}/\lambda(\hat{I})$, we can also ensure that $\mu(r_a) < \hat{I}/\lambda(\hat{I})$.

Now let δ be a small positive number and define a 'slightly' unequal land distribution $t(n, \delta)$ as:

$$\begin{aligned} t(n, \delta) &= 0 && (\text{for } n \in [0, \delta]), \\ &= 2(n-\delta)/(2-3\delta)\,\delta && (\text{for } n \in [\delta, 2\delta]), \\ &= 2/(2-3\delta) && (\text{for } n \in [2\delta, 1]). \end{aligned} \tag{17}$$

Now choose a small positive number δ_0 such that $t(n, \delta)$ in (17) is well-defined for all $\delta \in [0, \delta_0)$. In Theorem 3 and the first part of the theorem being proved reservation wages were chosen to be sufficiently small so that they were not a binding constraint. Choose δ_0 small enough so that they remain non-binding for all $r \in [r_a, r_b]$. Now define a corresponding $M(r, \delta)$ on $[r_a, r_b] \times [0, \delta_0]$ analogous to equation (26) in D–R, for the land distribution $t(n, \delta)$. Then notice that $M(r, 0) = M(r)$. It is easy to verify that if r_a, r_b and δ_0 are chosen suitably $M(r, \delta)$ is a singleton, (i.e. a function). It is also continuous in δ at $\delta = 0$. We conclude that for δ close to zero but positive, there is $\hat{r}(\delta) \in [r_a, r_b]$ so that

$$M[\hat{r}(\delta), \delta] = M[\hat{r}(\delta)]. \tag{18}$$

It is a simple matter to check that this is an equilibrium. But because $\hat{r}(\delta) \geqslant r_a$ and $\mu(r_a) < \hat{I}/\lambda(\hat{I})$, it must be true that the new equilibrium piece rate, $\mu[\hat{r}(\delta)]$, is less than $\hat{I}/\lambda(\hat{I})$. ■

Proof of Theorem 5. Suppose not. Then for some $\hat{T} \geqslant \hat{T}_2$ there exists a land distribution $I(n)$ such that $\tilde{\mu} < \hat{I}/\lambda(\hat{I})$. By definition of $\hat{T}_2$ and the fact that $\mu(\hat{T})$ is an increasing function of $\hat{T}$ for $\hat{T} > \hat{T}_1$ (see proof of Theorem 4 and equation (15)) we have $\mu(\hat{T}) \geqslant \hat{I}/\lambda(\hat{I})$ for $\hat{T} \geqslant T_2$, so that combining all this with the strict concavity of F in E, we note

$$\int_{[0,1]} \lambda[I(n)]\, d\nu(n) > \lambda[I(\hat{T})]. \tag{19}$$

Now consider the maximisation problem

$$\operatorname*{Max}_{I(n)} \int_{[0,1]} \lambda[I(n)]\, d\nu(n), \tag{20}$$

subject to the feasibility constraint

$$\int_{[0,1]} I(n)\, d\nu(n) = F\left\{\int_{[0,1]} \lambda[I(n)]\, d\nu(n),\ \hat{T}\right\}. \tag{21}$$

One can then show (see Dasgupta and Ray, 1984, Appendix) that the solution, $I(n)$ say, is unique and equals $I(\hat{T})$ for n almost everywhere in [0, 1]. But this contradicts (19). ■