On the Distribution of the Ratio of two Samples drawn at random from Two Uncorrelated Populations of Pearsonian Type III.

Karl Pearson's original form[1] for the Type III distribution can be written as a gamma-function :

$$df(x) = C.e^{-ax}.x^{N}.dx \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

where a and N are the population parameters and

$$C = \frac{a^{N+1}}{N!}$$

Let two uncorrelated Type III populations be represented by

$$\text{(i) } df(x) = C_1.\ e^{-a_1 x}.\ x^{N_1}.\ dx \qquad \ldots\ldots (2 \cdot 1)$$

$$\text{(ii) } df(y) = C_2.\ e^{-a_2 y}.\ y^{N_2}.\ dy \qquad \ldots\ldots (2 \cdot 2)$$

The joint distribution of x and y is given by,

$$df\ (x,y) = (C_1 C_2).\ e^{-(a_1 x + a_2 y)}.\ x^{N_1}.\ y^{N_2}.dxdy \qquad \ldots\ldots (3)$$

Let

$$u = \frac{x}{y}$$

where x and y are two random samples from the two populations.

For a given value of y, equation (3) may be written in the form,

$$df(x,y) = (C_1 C_2).u^{N_1}.du.e^{-(a_1 u + a_2)y}.y^{N_1+N_2+1}.dy$$

Integrating over y from 0 to ∞, we get

$$df = (C_1 C_2).\ (N_1+N_2+1)!\ \frac{u^{N_1}}{(a_1 u + a_2)^{N_1+N_2+2}}.du$$

Substituting the values of C_1 and C_2, the equation becomes

$$df = \frac{(N_1+N_2+1)!}{N_1!\ N_2!}\ a_1^{N_1+1}.\ a_2^{N_2+1}.\ \frac{u^{N_1}}{(a_1 u + a_2)^{N_1+N_2+2}} du \ldots\ldots (4)$$

which is the required distribution of the ratio of two samples from two Type III populations.

It is interesting to note that, Fisher's Z-distribution[2] and Student's t-distributions[3,4] are merely particular cases of the more general distribution given in (4). Fuller details will be published shortly in *Sankhya : The Indian Journal of Statistics.*

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1. Karl Pearson : *Phil. Trans, 180* A.
2. R. A. Fisher : *Proceedings of Int. Math. Congress,* Toronto, 1924.
3. "Student" : *Biometrika, 6 :* 1-25.
4. R. A. Fisher : *Metron 5,* (No.3), 3-17.