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SOME ACCEPTANCE CRITERIA FOR SINGLE SAMPLING MULTIATTRIBUTE PLANS

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SUMMARY. Several acceptance criterion are considered for the comparison of cost and discriminating power. A linear cost model for non-destructive testing for discrete prior distribution of process average has been formulated. The general practice of formulating sampling plans separately for each of the characteristics have been reviewed with some proposed alternatives.

1. Introduction

There are many situations in industry where products are inspected for more than one attribute characteristics. Often in these situations it is reasonable to assume that a defect with respect to any of the characteristics occur independently of others. For example a metal closure may be inspected for surface defects such as 'print peeling', 'off centre', 'dirty facing' and functional defects such as 'damaged' or, for example, steel tapes may be inspected for two quality characteristics such as surface finish (whether good/bad) and coil demension (whether off specification or not). The defects in rubber tread and fabric prep in cycle tyre can also be considered as independent. We consider the problems of acceptance sampling in these situations.

We suppose that there are r attribute characteristics for a product. A defect with respect to any of the characteristics occurs independently of others. The situation also permits us to take a sample of size n from a lot of size N and inspect for all the r characteristics in any order. This of course implies that no inspecting is destructive. If x_i be the number of defects of i-th kind in the sample

$$P(x_1, x_2, \ldots, x_r) = \prod_{i-1}^{r} P(x_i). \qquad \ldots \quad (1)$$

For a process average $(p_1, p_2, \ldots, p_r)$ the probability of obtaining x_i can be expressed as

$$P(x_i) = b(x_i, n, p_i) = \binom{n}{x_i} p_i^{\,x_i}(1-p_i)^{n-x_i}. \qquad \ldots \quad (2)$$

We assume p_i's are small enough to assume poisson condition, i.e.

$$b(x_i, n, p_i) \simeq g(x_i, m_i) = e^{-m_i} m_i^{x_i}/(x_i)\,!$$

Here $$m_i = n.p_i. \qquad \ldots \quad (3)$$

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The purpose of the present study is to compare the effect of different acceptance criterions on *OC* function and cost. However most of the results are restricted to the case of $r = 2$.

2. Sampling scheme

Scheme A. We first examine the following acceptance criterion and observe some of the properties of the *OC*. Plan $A\ (n, c_1, c_2, \ldots, c_r)$ have the acceptance criterion :

Accept if $x_{(i)} \leqslant c_i$; reject otherwise

where
$$x_{(i)} = \sum_{j=1}^{i} x_j,\ i = 1, 2, \ldots, r. \qquad \ldots \quad (4)$$

The *OC* function under poisson condition is given by

$$PA(c_1, c_2, \ldots, c_r : m_1, m_2, \ldots, m_r) = \sum_{x_1=0}^{c_1} g(x_1, m_1) \sum_{x_2=0}^{c_2-x_{(1)}} g(x_2, m_2) \ldots \sum_{x_r=0}^{x_r-x_{(r-1)}} g(x_r, m_r) \qquad \ldots \quad (5)$$

Theorem 1: *For $i < j$, the discriminating power of the OC for the i-th characteristic is more than or equal to that for the j-th characteristic.*

Proof: To compare the change in *OC* function for changes in p_i we compare the PA_i obtained by differentiation —(5) with respect to m_i. Thus

$$-PA_i' = PA(c_1, c_2, \ldots, c_r : m_1, m_2, \ldots, m_r)$$
$$-PA(c_1, c_2, \ldots, c_i-1, c_{i+1}-1, \ldots c_r-1 : m_1 \ldots m_r) \text{ for } c_i > 0 \qquad \ldots \quad (6)$$

and

$$-PA_i' = PA(c_1, c_2, \ldots, c_r : m_1, m_2, \ldots, m_r) \text{ for } c_i = 0 \qquad \ldots \quad (7)$$

Thus the *OC* function is a decreasing function of p_i. *OC* function decreases with decrease in c_i. And $c_i = 0$ implies $c_j = 0$ for all $j < i$. It therefore follows from (6) and (7)

$$-PA_i' > -PA_{i+1}'. \qquad \ldots \quad (8)$$

This proves the theorem.

The above property of the plan A therefore allows us to order the characteristics in the order of relative discriminating power.

It is also worthnoting that although the sample size has been kept same for all the characteristics the poisson *OC* would satisfy (8) for different sample sizes for different characteristics.

Scheme C *and* D. Sampling scheme C is the natural extension of single sampling for single attribute. In industry it is a general practice to determine (n_i, c_i) pairs for sample size and acceptance number for each i. For mathematical convenience we consider the case $n_i = n$ for all i and define the C kind sampling scheme. $C(n, c_1, c_2, \ldots, c_r)$ with the acceptance criterion :

Accept if $x_i \leqslant c_i$ for all i ; reject otherwise. Under poisson condition the OC can be expressed as

$$PC(c_1, c_2, \ldots, c_r : m_1, m_2, \ldots, m_r) = \prod_1^r G(c_i, m_i) \qquad \ldots \quad (9)$$

$$G(c_i, m_i) = \sum_{x_i=0}^{c_i} g(x_i, m_i). \qquad \ldots \quad (10)$$

The design and optimality of multi-attribute sampling plans have so far been considered for such plans whose OC can be expressed by (9) under poisson condition. However it is difficult to design such plans which would satisfy (8). Thus sampling scheme A has atleast one logic in its favour. Before we proceed to compare the cost of A and C we consider one more sampling scheme. We define D kind sampling scheme $D(n, c)$ with the acceptance criterion.

Accept if $\sum_{i=1}^{r} x_i \leqslant c$; reject otherwise. Note that

$$PD(c : m_1, m_1, \ldots, m_r) = G\left(c, \sum_{i=1}^{r} m_i\right) \qquad \ldots \quad (11)$$

Theorem 2: *For $r=2$ we define a plan $B(n, c_1, c_2)$ with acceptance criterion. Accept if $x_2 \leqslant c_1, x_1+x_2 \leqslant c_2$; and reject otherwise.*

$$PA(c_1, c_1+c_2 : m_1, m_2)+PB(c_2, c_1+c_2 : m_1, m_2)-PC(c_1, c_2 : m_1, m_2)$$
$$= PD(c_1+c_2 : m_1, m_2) \qquad \ldots \quad (12)$$

Proof : $PB(c_2, c_1+c_2, m_1, m_2)-PC(c_1, c_2, m_1, m_2)$

$$= \sum_{x_2=0}^{c_2} P(x_2)\, P(x_1 \leqslant c_1+c_2-x_2) - \sum_{x_2=0}^{c_2} P(x_2) \sum_{x_1=0}^{c_1} P(x_1)$$

$$= \sum_{x_2=0}^{c_2} P(x_2) \sum_{x_1=c_1+1}^{c_1+c_2-x_2} P(x_1)$$

$$= \sum_{c_1+1}^{c_1+c_2} P(x_1)\, P(x_2 \leqslant c_1+c_2-x_1)$$

$$\therefore \text{LHS of (12)} = \sum_{x_1=0}^{c_1+c_2} P(x_1)\, P(x_2 \leqslant c_2-x_1)$$

$$= P(x_1+x_2 \leqslant c_1+c_2) = \text{RHS of (12)}.$$

We will use this result for cost comparison in section 4.

3. Cost model

Let A be the set of x_i's for which we declare the lot as acceptable and $\bar{A}$ be the complementary set. X_i denotes the number of defectives of i-th characteristic in the lot. Let the costs be

$$C(x) = n\, S_0 + \sum_{i=1}^{r} x_i S_i + (N-n)\, A_0 + \sum_{i=1}^{r} (X_i - x_i) A_i$$

and

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_r) \in A \quad \ldots \quad (12)$$

$$C(x) = n\, S_0 + \sum_{i=1}^{r} x_i s_i + (N-n)\, R_0 + \sum_{i=1}^{r} (X_i - x_i) R_i$$

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_r) \in \bar{A}. \quad \ldots \quad (13)$$

The interpretations of cost parameters are as follows. S_0 is the cost of inspection per item in the sample for all the characteristics put together. S_i the cost proportional to the number of defectives in the sample. The cost of acceptance, A_0 associated with remainder of the lot is usually negligible or zero. A_i is the cost of accepting an item containing defective for i-th characteristic. We assume the loss due to use of defective item is additive over all the characteristics. This means if an item contains more than one defects, say for $i = 1$ and 2 the loss will be the sum of damages for both the characteristics put together. The assumption is reasonably valid under many situations. However proportion of items containing more than one category of defects will usually be small.

Costs of rejection consists of a part $\Sigma(N-n)R_0$ proportional to the number of items in the remainder of the lot and another part $\Sigma(X_i - x_i)\, R_i$ proportional to the number of defective item rejected. If rejection means sorting, R_0 will give the sorting cost/item for all category of defects put together. R_1 denotes the additional cost for items found with defective of i-th category (for example, cost of repair) and is additive over different category of defects.

Moskewitz *et al.* (1984) have considered similar cost model when decisions are taken separately for each of the characteristics. However the above formulation is due to the author reported in 1980. (See reference) and is applicable to any acceptance criterion. Denoting the hypergeometric probability by

$$P\ (x_1/X_1) = \binom{n}{x_i}\binom{N-n}{X_i - x_i} \Big/ \binom{N}{X_i} \text{ for } i = 1, 2, \ldots, r. \quad \ldots \quad (14)$$

The average cost for lot of size N with $(X_1, \ldots, X_r)$ defects become

$$\sum_{x \epsilon A} c(x) \prod_{1}^{r} P(x_i/X_i) + \sum_{x \epsilon \bar{A}} c(x) \prod_{1}^{r} P(x_i/X_i). \qquad \ldots \quad (15)$$

If the lot quality is distributed as binomial, i.e.

$$P(X_i) = \binom{N}{X_i} p_i^{X_i} (1-p_i)^{N-X_i} \; i = 1, \ldots, r. \qquad \ldots \quad (16)$$

The process average $(p_1, p_2, \ldots, p_r)$ is denoted as $\boldsymbol{p}$ then average cost of p can be easily shown as

$$K(N, n, \boldsymbol{p}) = n(S_0 + \Sigma \, S_i \, p_i) + (N-n) \, [A_0 + \Sigma \, A_i \, p_i) \, P(\boldsymbol{p}) + (R_0 + \Sigma \, R_i \, p_i) Q(\boldsymbol{p})] \qquad \ldots \quad (17)$$

$P(\boldsymbol{p})$ denotes the average probability of acceptance at $\boldsymbol{p}$ and $Q(\boldsymbol{p}) = 1 - P(\boldsymbol{p})$. If there are q states for the process average such that at j-th state

$$\boldsymbol{p}(j) = (p_1^{(j)}, \ldots, p_r^{(j)}) \qquad \ldots \quad (18)$$

with probability w_j and

$$\sum_{j=1}^{q} w_j = 1. \qquad \ldots \quad (19)$$

Then the overall average cost become

$$K(N, n) = \sum_{j} K(N, n, \boldsymbol{p}^{(j)}) \; w_j. \qquad \ldots \quad (20)$$

Equation (20) is a general cost model for r characteristics assuming independence. For $r = 1$ and $q = 2$ the model is identical to the cost model developed by Hald (1965) for discrete prior distribution.

For our present discussion we will consider the case of two quality characteristics $r = 2$ and $q = 2$. Introducing the cost functions for $j = 1, 2$

$$K_s \, (\boldsymbol{p}^{(j)}) = S_0 + \sum_{i=1}^{2} \; S_i \, p_i^{(j)} \qquad \ldots \quad (21)$$

$$K_a \, (\boldsymbol{p}^{(j)}) = A_0 + \sum_{i=1}^{2} \; A_i \, p_i^{(j)} \qquad \ldots \quad (22)$$

$$K_r \, (\boldsymbol{p}^{(j)}) = R_0 + \sum_{i=1}^{2} \; R_i \, p_i^{(j)} \qquad \ldots \quad (23)$$

$$K_m \, (\boldsymbol{p}^{(j)}) = \min \, [K_a \, (p^{(j)}), K_r \, (p^{(j)})] \qquad \ldots \quad (24)$$

we assume

$$K_s(\boldsymbol{p}^{(j)}) > K_m(\boldsymbol{p}^{(j)}).$$

Let $$K_a(\boldsymbol{p}^{(1)}) < K_r(\boldsymbol{p}^{(1)}) \text{ and } K_a(\boldsymbol{p}^{(2)}) > K_r(\boldsymbol{p}^{(2)})$$

then $$K_m(\boldsymbol{p}^{(j)}) = K_a(\boldsymbol{p}^{(j)}) \quad \text{for } j = 1$$

$$= K_r(\boldsymbol{p}^{(j)}) \quad \text{for } j = 2. \qquad \ldots \quad (25)$$

Further let K_s, K_a, K_r and K_m denote the expected value of the corresponding cost function defined in (21), (22), (23) and (24).

Denoting $K = K(N, n)/N$ sampling inspection should only be taken recourse to if

$$K - K_m < \min\,[K_a - K_m,\ K_r - K_m].$$

The regret function $R(N, n)$ is expressed as

$$R(N, n) = [K(N, n) - K_M(N, n)]/(K_s - K_m) \qquad \ldots \quad (26)$$

$K_M(N, n)$ is the average minimum unavoidable cost. This works out to be

$$R(N, n) = n + (N - n)\,[\nu_1\, Q(\boldsymbol{p}^{(1)}) + \nu_2 p(\boldsymbol{p}^{(2)})] \qquad \ldots \quad (27)$$

where $$\nu_j = w_j\,|\,K_a(\boldsymbol{p}^{(j)}) - K_r(\boldsymbol{p}^{(j)})\,|\,/(K_s - K_m) \text{ for } j = 1, 2.$$

If $R_0 = S_0$ and $R_i = S_i$ for all i ; $K_s = K_r$ and $\nu_1 = 1$.

4. Comparison of costs

We will consider the case for $r = 2$, $q = 2$. Let

$$\boldsymbol{p}^{(1)} = (p_1, p_2)$$

and $$\boldsymbol{p}^{(2)} = (p_1', p_2').$$

Let $p = p_1 + p_2$, $p' = p_1' + p_2'$, $m' = np'$, $m = np$, $\rho = p_1/p$ and $\rho' = p_1'/p'$.

4.1 *Comparison of scheme* A *and* D. For a given N the optimal plan $A(n, c_1^*, c_2^*)$ satisfies the following inequality

$$RA_{c_2}(n, c_1, c_2 - 1) \leqslant 0 < RA_{c_2}(n, c_1, c_2) \qquad \ldots \quad (28)$$

RA ( ) denotes the regret function values. Under poisson condition

$$\frac{\nu_2 g(c_2, m')\, B(c_1, c_2, \rho')}{\nu_1\, g(c_2, m)\, B(c_1, c_2, \rho)} \leqslant 1 < \frac{\nu_2\, g(c_2+1, m')\, B(c_1, c_2+1, \rho')}{\nu_1\, g(c_2+1, m)\, B(c_1, c_2+1, \rho)} \qquad \ldots \quad (29)$$

and an optimal $D(n, c_2)$ satisfies

$$RD_{c_2}(n, c_2 - 1) \leqslant 0 < RD_{c_2}(n, c_2) \qquad \ldots \quad (30)$$

which under poission condition works out as

$$\nu_2\, g(c_2, m')/\nu_1\, g(c_2, m) \leqslant 1 \leqslant \nu_2\, g(c_2+1, m')/\nu_1\, g(c_2+1, m).$$

Here $$B(c, n, p) = \sum_{x=0}^{c} b(x, n, p). \qquad \ldots \quad (31)$$

Theorem 3 : *For an optimal plan $D(n, K)$ the $RA\ (n, K-1, K) \leqslant RD\ (n, k)$ if*

$$(p_1'/p_1)^K > (p'/p)^{K+1}. \qquad \ldots \quad (32)$$

Proof: $$[RD\ (n, K)-RA\ (n, K-1, K)]/(N-n)$$

$$= (\nu_2 e^{-m'}(m_1')^K/K\ !)-\nu_1(e^{-m}\, m_1^K/K\ !).$$

$$= \nu_2\, g(K+1, m')\, [(K+1)\, (m_1')^K/(m')^{K+1}]$$

$$-\nu_1\, g(K+1, m)\, [(K+1)\, m_1^K/(m)^{K+1}].$$

Since (n, k) satisfies (30) a sufficient condition for $RD-RA \geqslant 0$ is given by (32).

Usually $p_1' > p_1$ and $p_2' > p_2$. If now $\rho \neq \rho'$ then (32) implies $(p_1'/p_1) > (p_1'/p_2)$ i.e. $\rho < p'$.

It is worthnoting that K is an increasing function of N. And if for some $K = K_0$ (say) the inequality (32) is satisfied then for all $K > K_0$, (32) will be satisfied. Thus for $\rho \neq \rho'$ we can order the characteristics suitably to formulate an A plan cheaper than the optimal D plan for sufficiently large lots.

Theorem 4 : *For $\rho = \rho'$ there always exist a plan $D(n, K)$ cheaper than the optimal plan A for any given lot size.*

Proof: To prove this we first show that for $\rho = \rho'$ and for all $i = 2, 3 \ldots K$

$$[RA(n, K-1, K) \geqslant RD(n, K)] \Longrightarrow [RA(n, K-i, K) > RA(n, K-1, K)] \qquad \ldots \quad (33)$$

Let $a = \nu_1\, e^{-m}\, m_1^K$ and $b = \nu_2\, e^{-m'}\, (m_1)^K$ then LHS inequality implies $a \geqslant b$. Now

$$[RA\ (n, K-i, K)-RA(n, K-1, K)]/(N-n)$$

$$= \nu_1 \sum_{j=1}^{i-1} g(K-j, m_1)\, G(j,\ m_2)-\nu_2 \sum_{j=1}^{i-1} g(K-j, m_1')\, G(j, m_2)$$

the j-th term of this series when multiplied by $(K-j)$! gives

$$a \sum_{x=0}^{j} (1/x\ !)\, (m_2/m_1)^x\, m_1^{x-j}-b \sum_{x=0}^{j} (1/x\ !)\, (m_2'/m_1')\, (m_1')^{x-j}.$$

Since $a > b$ and $m_2/m_1 = m_2'/m_1'$ and $m_1' > m_1$ the j-th term is a positive quantity and thus (33) holds.

Next we note that the optimal plan A satisfies (29) which is identical to (30) for $\rho = \rho'$. Thus

$$\nu_2\, g(K, m) \leqslant \nu_1\, g(K,m).$$

This implies

$$RD(n, K) \leqslant RA\,(n, K-1, K). \qquad \ldots \quad (34)$$

From (33) and (34) we conclude that the plan $D(n, K)$ is cheaper than the optimal plan $A(n, K-1, K)$ and hence the theorem is proved.

Thus for $\rho = \rho'$ we prefer D *plan to* A *plan. And for $\rho \neq \rho'$ we prefer* A *plan to* D *plan.*

4.1 *Numerical example* 1 : Note that under poisson condition the OC function of a plan $D(n, c)$ at (p_1, p_2) is given by that of a single sampling plan for single characteristics at $p = p_1 + p_2$. The regret function is dependent on only four parameters p, p', ν_1, ν_2. Hence the tabulated optimal plans for single characteristic with the above parameter, can be used as optimal $D(n, c)$ by matching p, p', ν_1, ν_2 values.

Let $(p_1, p_2) = (.002, 005)$ and $(p_1', p_2') = (.02, .03)$, $\nu_1 = 1$, $\nu_2 = 0.7$. Using the table of optimal single sampling plan $(r = 1)$ for $p = .007$, and $p' = .05, \nu_1 = 1, \nu_2 = 0.7$ (See Hald, 1965) for lot size $(N) = 6000$, we get $n = 285$, and $C = 6$. The exact value of $D(n^*, K^*)$ under poisson condition works out as $D(286, 6)$.

The regret function value (equation 27) $RD(286, 6) = 359.3678$. Here $\rho \neq \rho'$. To verify the inequality (32) we note that for $K > 6$ plan $A(n, K^*-1, K)$ will have lesser regret function value. The actual $RA(286, 5, 6)$ works out as 359.2855.

Example 2 : We choose $(p_1, p_2) = (0028, .0042)$ and $(p_1', p_2') = (02, .03)$, $N = 6000$, $\nu_1 = 1, \nu_2 = 0.7$. Here $\rho = \rho'$. The optimal A plan works out as $A(286,5,6)$. To see that this is optimal we work out $RA(287, 5, 6) = 359.5697$, $RA(285,5,6) = 359.5410$ and $RA(286, 5, 6) = 359.5306$. $RA(286, 4, 6) = 362.9882$, $RA(286,3,6) = 396.5396$, $RA(286,2,6) = 597.0290$, $RA(286,1,6) = 1397.9816$, $RA(286, 0, 6) = 3438.4856$. Also, $RD(286, 6) = 359.3678$. Thus the inequalities (33) and (34) are verified.

4.2 *Comparison of C kind plan with other plans.*

Theorem 5 : *For a given lot size*

$$RA(n, c_1, c_1+c_2)+RB(n, c_2, +c_2)-RC(n, c_1, c_2) = RD(n, c_1+c_2). \quad \ldots \quad (36)$$

Proof: From (12) we obtain

$$QA(c_1, c_1+c_2, m_1, m_2)+QB(c_2, c_1, m_1, m_2)-QC(c_1, c_2, m_1, m_2)$$
$$= QD(c_1+c_2, m_1, m_2) \quad \ldots \quad (37)$$

$$RA(n, c_1, c_1+c_2) = n+(N-n)\,[\nu_1\, QA(c_1, c_1+c_2, : m_1, m_2)$$
$$+\nu_2 PA(c_1, c_1+c_2, : m_1', m_2')]$$

and similar expression holds for regret function of the corresponding B, C and D plan. Combining (37) and (12) we immediately get (36).

4.3 *Comparison of plan C and D. Case* 1 : $\rho = \rho'$. We shall first show that it is possible to construct an equivalent plan $D(n_0, K_0)$ for any given $C(n, c_1, c_2)$ such that the OC function will have approximately same values at (p_1, p_2) and at (p_1', p_2').

Since $QC(c_1, c_2, : m_1, m_2)$ is a function of m alone for a given c_1, c_2, and ρ. we denote this as $QC(m)$. $QC(m)$ has the same properties as a distribution function. We shall call $-PC'(m)$ the OC density and $-\int m^r\, dPC(m) = E(m^r)$ the OC moments of order r. We shall now prove that

Theorem 6 :
$$E(m) = \sum_{x_1=0}^{c_1} \sum_{x=x_1}^{c_2+x_1} b(x_1, x, \rho) \quad \ldots \quad (38)$$

$$E(m^2) \sum_{x_1=0}^{c_1} \sum_{x=x_1}^{c_2+x_1} 2(x+1)\, b(x_1, x, \rho). \quad \ldots \quad (39)$$

Proof:
$$PC(m) = G(c_1, m\rho).\quad G(c_2, m(1-\rho))$$
$$= \sum_{x_1=0}^{c_1} g(x_1, m.\rho) \sum_{x=x_1}^{c_2+x_1} g(x_2, m(1-\rho))$$

where $x = x_1+x_2$. Using

$$g(x_1, m_1)\, g(x_2, m_2) = g(x, m)\, b(x_1, x, \rho)$$

we get

$$-PC'(m) = -Sg'(x, m)\, b(x_1, x, \rho),$$

S denotes the summation with respect to x_1 and x over the domain indicated.

Since

$$\int_0^\infty m\, g'(x, m)\, d\, m = -1$$

$$\int_0^\infty m^2 g'(x, m)\, d\, m = -2(x+1)$$

the result follows.

We can however express these in terms of binomial moments. Let

$$B_K(c, n, p) = \sum_{x=0}^{c} B_{K-1}(x, n, p) \text{ and } B_0(c, n, p) = b(c, n, p)$$

then

$$E(m) = (\rho')^{-1}(c_2+1) - (\rho\rho')^{-1} B_2(c_2+c_1+3, \rho)$$

$$\begin{aligned} E(m^2) = {} & 2(c_2+2)(c_2+1)\rho'^{-1} - 2\rho\rho'(c_2+c_1+3)\, B_2(c_2, c_2+c_1+2, \rho') \\ & +2(\rho\rho')^{-1}(\rho^{-1}-\rho'^{-1})\, B_3(c_2, c_2+c_1+3, \rho') \\ & -2(\rho\rho')^{-1}(\rho^{-1}-\rho'^{-1})\, B_3(c_2, c_2+2, \rho'). \end{aligned}$$

The derivation of these expression has been omitted. From the Theorem 6 it follows that for $\rho = \rho'$ a simple and rather accurate approximation of the *OC* of any *C* kind plan can be obtained from the *OC* of the plan $D(n_0, K_0)$ by equating the mean and the variance. This gives

$$K_0+1 = E^2(m)/V(m) \quad \ldots \quad (40)$$

$$n_0 = n\, E(m)/V(m) \quad \ldots \quad (41)$$

so that K_0 and n/n_0 are uniquely determined from the given (c_1, c_2, ρ). Usually n/n_0 comes out to be < 1, clearly

$$RD(n_0, K_0) \leqslant RC(n, c_1, c_2).$$

Case 2 : $\rho < \rho'$.

Theorem 7 : *Let* $F(m, c_1, c_2) = G(c_1, m\rho)\ G(c_2, m(1-\rho)) - G(c_1, m\rho')\ G(c_2, m(1-\rho'))$ *Then* $F(m, c_1, c_2) \leqslant 0$ *for* $c_1 \geqslant c_2$.

Proof : If $c_1 = c_2 = c$, then

$$\frac{d}{d\rho} G(c, m\rho)\ G(c, m(1-\rho))$$

$$= -mg(c, m\rho)\, G(c, m(1-\rho)) + mG(c, m\rho)\, g(c, m(1-\rho)).$$

The ratio of absolute values of the 2nd term to the 1st term

$$= \left[\sum_{x=0}^{c} (m\rho)^{x-c}/x\,!\right] \Big/ \left[\sum_{x=0}^{c} m(1-\rho)^{x-c}/x\,!\right],$$

If $\rho < .5$ this ratio is greater than 1 and $G(c_1, m\rho)\, G(c_2, m(1-\rho))$ is an increasing function of ρ and hence $F(m, c_1, c_2) \leqslant 0$.

$$F(m, c+1, c)-F(m, c, c)$$

$$= g(c+1, m\rho)\, G(c, m\,(1-\rho))-g(c+1, m\rho')\, G(c, m(1-\rho')).$$

The ratio of absolute values of 1st term to 2nd term for $c_1 = c+1$, and $c_2 = c$

$$\leqslant \left(\sum_{x=0}^{c_2} (m\rho')^{x-c_1}/x\,! \right) \Big/ \left(\sum_{x=0}^{c_2} (m\rho)^{x-c_1}/x\,! \right)$$

$$\leqslant 1.$$

Thus $F(m, c+1, c) - F(m, c, c) \leqslant 0$. This proves the theorem.

Theorem 8 : *For* $c_2 \geqslant c_1$, $F(m, c_1, c_2)$ *undergoes atmost one change of sign from -ve to+ve. Writing*

$$G(c_1, m\rho')\, G(c_2, m(1-\rho'))$$

$$\simeq G(c_1, m\rho)\, G(c_2, m(1-\rho))-m(\rho'-\rho)\, g(c_1, m\rho)\, G(c_2, m(1-\rho))$$

$$+m(\rho'-\rho)\, g(c_2, m(1-\rho))\, G(c_1, m\rho)$$

we note

$$= F(m)/A = \sum_{i=0}^{c_1} m^{c_2-i} [\{(1-\rho)^{c_2-i} \rho^{c_1} /(c_2-i)\,!\, c_1\,!\}$$

$$-\{\rho^{c_1-i} (1-\rho)^{c_2}/ (c_1-i)\,!\, c_2!\}]$$

$$+ \sum_{x=c_2-c_1-1}^{o} \{m(1-\rho)^x/x\,!\}\, \rho^{c_1}/c_1\,!$$

where $$A = (e^{-m}\, m^{c_1})\, m(\rho'-\rho) > 0.$$

We note

(a) The 2nd series function is > 0.

(b) If for any values of i, the coefficient of m^{c_2-i} in the first series$=0$, then

$$[(1-\rho)/\rho]^i = (c_2\,!\,(c_1-i)\,!\,/(c_2-i)\,!\,c_1\,!) < (c_2-i+1/c_1-i+1)^i.$$

Thus $(1-\rho)/\rho < (c_2- i +1)/(c_1-i+1) < (c_2-i)/(c_1-i)$. Hence the coefficient of $m^{c_2-i-1} > 0$.

(c) For $i = 0$, the coefficient of $m^{c_2-i} = 0$.

From these it follows that $F(m)$ undergoes atmost one change of sign and $F(m) = 0$ will have atmost one real positive root. This proves the theorem.

We now construct two D kind plans using (c_1, c_2, ρ) and (c_1, c_2, ρ') and call them $D(n_0, K_0)$ and $D(n_0', K_0')$. If now

(a) $F(m) \leqslant 0$ for $m = n(p_1+p_2)$ and for $m = n(p_1'+p_2')$ then

$$QD(K_0 : n_0(p_1+p_2)) = QC(c_1, c_2 : m_1, m_2)$$

$$PD(K_0 : n_0(p_1'+p_2')) \leqslant PC(c_1, c_2 : m_1', m_2')$$

for no $\leqslant n$, $RD(n_0, K_0) \leqslant RC(n, c_1, c_2)$.

(b) $F(m) \leqslant 0$ for $m = n(p_1+p_2)$ and $F(m) > 0$ for $m = n(p_1'+p_2')$ then

$$QD(K_0, n_0(p_1+p_2)) \leqslant QC(c_1, c_2, : m_1, m_2)$$

$$PD(K_0, n_0(p_1+p_2)) \leqslant PC(c_1, c_2 : m_1', m_2')$$

For $n_0 \leqslant n$, $RD(n_0, K_0) \leqslant RC(n, c_1, c_2)$.

(c) If $F(m) > 0$ for $m = n(p_1+p_2)$ and $F(m) > 0$ for $m = n(p_1'+p_2')$ then for both $D(n_0, K_0)$ and $D(n_0', K_0')$ the regret function will be higher than the regret function of the corresponding C plan.

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