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STATISTICAL NOTES FOR AGRICULTURAL WORKERS*

No. 3†.—AUXILIARY TABLES FOR FISHER'S Z-TEST IN ANALYSIS OF VARIANCE.

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The interpretation of results of field experiments designed on the model of Fisher's "Latin Square" or "Randomized Block" depends ultimately on his z-test for the significance of the difference of two variances. Let $v_1=s_1^2$, and $v_2=s_2^2$ be two variances based on n_1 and n_2 degrees of freedom ($v_1 > v_2$). Then "z" is defined by the equation

$$z = \frac{1}{2} \log_e \left(\frac{v_1}{v_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The 5 per cent. and one per cent. points for the distribution of "z" are given in Fisher's Table VI (Statistical Methods for Research Workers, 1930, pp. 212—215).

It is, of course, absolutely necessary to use natural logarithms (*i.e.*, to the base "e") in connexion with Fisher's Table VI. With suitable tables of natural logarithms the procedure would be quite simple and straightforward. But such tables are not always available, and usually the common logarithms (to the base "10") have to be converted into natural logarithms. This is apt to cause trouble to field

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* We are receiving a large number of enquiries of a statistical nature from agricultural workers in different parts of India. Many of these enquiries are of considerable general interest, and it is proposed to publish notes on selected topics from time to time. These notes will deal mainly with statistical methods and procedure, and it is not intended that they should always contain new matter—Ed.

† Nos. 1 and 2 of this series appeared in this Journal, Vol. II, Parts I and II respectively.

workers not familiar with the use of natural logarithms. In fact, I have often been asked whether something could not be done to eliminate the necessity of using natural logarithms.

For ordinary routine analysis of field trial data, it is possible to construct auxiliary tables with the help of which the z-test can be applied without recourse to natural logarithms. I am giving here six such tables, two for working with ordinary logarithms (to the base 10), two for working directly with the ratio of standard deviations and two for the ratio of variances, one table in each set corresponding to the 5 per cent. and the other to the 1 per cent. points in Fisher's z-table.

Table I (5 per cent. probability) and Table II (1 per cent. probability) are for working with common logarithms (to the base "10"), and have been obtained directly from Fisher's Table VI (pp. 224—226) by multiplying "z" by $2 \log_{10}{}^{e} = 0{\cdot}86858896$*.

Illustration I.—In Fisher's Example 41 (Analysis of Variation in Experimental Field Trials (1) pp. 205—209), the two observed variances (mean squares) are $v_1 = 3{\cdot}967$ and $v_2 = 0{\cdot}727$ based on $n_1 = 11$ and $n_2 = 24$ degrees of freedom. The common logarithms to the base 10 are

$$\log_{10} 3{\cdot}967 = 0{\cdot}59846$$
$$\log_{10} 0{\cdot}727 = \bar{1}{\cdot}86153$$

$$\text{Difference} = 0{\cdot}73693 = 2\,Z'$$

Looking up Table I, we notice that for $n_1 = 8$ and $n_2 = 24$, the 5 per cent. value of $2\,Z'$ is 0·3720. Further from Table II, for $n_1 = 8$ and $n_2 = 24$, the one per cent. point is 0·5267. The observed value of '$2\,Z'$' is 0·7369 for $n_1 = 11$ and $n_2 = 24$. The difference between the two observed variances is therefore definitely significant.

Illustration II.—In Fisher's Example 38 (Homogeneity of Small Samples, [1932] $v_1 = 60{\cdot}25$ ($n_1 = 24$), $v_2 = 31{\cdot}62$ ($n_2 = 3$). We find

$$\log_{10} 60{\cdot}25 = 1{\cdot}77996$$
$$\log_{10} 31{\cdot}65 = 1{\cdot}50037$$

$$\text{Difference} = 0{\cdot}27959 = 2\,Z'$$

In Table I, we find that for $n_1 = 24$ and $n_2 = 3$, $2\,Z' = 0{\cdot}9364$. The observed difference is, therefore, quite insignificant.

* The tabled values therefore give $2\,Z' = \log_{10} (s_1^2/s_2^2)$

The use of even common logarithm tables may be dispensed with if we use Table III (5 per cent. probability) and Table IV (1 per cent. probability) which give directly the value of the ratio of the variances, $x = (v_1/v_2) = (s_1^2/s_2^2)$

Illustration III.—In illustration I, we have $v_1 = 3{\cdot}967$ ($n_1 = 11$), and $v_2 = 0{\cdot}727$ ($n_2 = 24$). By direct division we find that the observed value of $v_1/v_2 = 5{\cdot}457$. In Table III (5 per cent. probability) we notice that for $n_1 = 8$, and $n_2 = 24$, expected $(v_1/v_2) = 2{\cdot}355$. Also in Table IV (1 per cent. probability) for $n_1 = 8$, $n_2 = 24$, $v_1/v_2 = 3{\cdot}363$. The observed difference is, therefore, definitely significant.

Illustration IV.—In the second illustration we had $v_1 = 60{\cdot}25$ ($n_1 = 24$), $v_2 = 31{\cdot}65$ ($n_2 = 3$). The observed ratio of v_1/v_2 is $1{\cdot}904$. In Table III (5 per cent. probability) the expected value of v_1/v_2 for $n_1 = 24$ and $n_2 = 3$ is $8{\cdot}638$. The observed difference is quite insignificant.

It will be noticed that in using Tables III and IV, it will usually be unnecessary to find v_1/v_2 with any great accuracy. For example in illustration III, we can see by mere inspection that the observed ratio of v_1/v_2 is greater than 5, and hence must be significant. So also in illustration IV, it is easy to see that v_1/v_2 is less than 2, and hence insignificant.

Finally, Table V and Table VI give the values of $y = S_1/S_2$ (*i.e.*, the ratio of the two standard deviations) at 5 per cent. and one per cent. levels of significance respectively.

Illustration V.—It is found [Mahalanobis, 1931] that the standard deviation for head-length for the Rabai tribe of East Africa is given by $S_1 = 7{\cdot}25$ cm. ($n_1 = 12$), and for the Nandi tribe is $S_2 = 4{\cdot}06$ cm. ($n_2 = 13$). The observed ratio of $(S_1/S_2) = 1{\cdot}786$.

In Table V (5 per cent. probability) for $n_1 = 12$ and $n_2 = 13$ the expected value of (S_1/S_2) is $1{\cdot}614$. But in Table VI (one per cent. probability) the expected value of (S_1/S_2) is $1{\cdot}9901$. We conclude that the probability of the difference being real lies between 0·05 and 0·01, and may be considered significant.

I need scarcely add that, just as in using Fisher's Tables, n_1 must refer to the larger of the two variances.

The Tables were computed by Babu Sudhir Kumar Banerjee and Babu Jitendra Mohan Sen Gupta in the Statistical Laboratory of the Presidency College with the aid of a grant from the Imperial Council of Agricultural Research.

TABLE I.

Five per cent. points of the distribution of $2z' = \log_{10}(s_1^2/s_2^2)$.

Values of n_2	Values of n_1: 1	2	3	4	5
1	2·2080	2·2999	2·3339	2·3514	2·3620
2	1·2674	1·2787	1·2825	1·2844	1·2855
3	1·0056	·9801	·9673	·9599	·9549
4	·8870	·8417	·8190	·8054	·7963
5	·8200	·7624	·7332	·7154	·7033
6	·7772	·7112	·6773	·6565	·6422
7	·7475	·6755	·6382	·6150	·5990
8	·7257	·6493	·6092	·5841	·5668
9	·7090	·6290	·5869	·5602	·5418
10	·6959	·6130	·5692	·5413	·5219
11	·6852	·6001	·5548	·5259	·5057
12	·6765	·5894	·5429	·5131	·4921
13	·6691	·5804	·5328	·5023	·4808
14	·6627	·5727	·5243	·4931	·4710
15	·6573	·5661	·5168	·4851	·4626
16	·6527	·5603	·5104	·4782	·4552
17	·6485	·5553	·5047	·4720	·4487
18	·6448	·5508	·4997	·4665	·4429
19	·6415	·5468	·4952	·4617	·4378
20	·6386	·5432	·4911	·4573	·4331
21	·6360	·5399	·4875	·4533	·4289
22	·6335	·5370	·4842	·4498	·4251
23	·6314	·5343	·4812	·4465	·4216
24	·6294	·5318	·4784	·4435	·4184
25	·6276	·5296	·4758	·4407	·4154
26	·6258	·5275	·4735	·4382	·4128
27	·6243	·5256	·4714	·4358	·4102
28	·6229	·5238	·4693	·4336	·4079
29	·6215	·5221	·4675	·4316	·4057
30	·6203	·5206	·4657	·4297	·4037
60	·6022	·4984	·4406	·4023	·3744
∞	·5845	·4765	·4158	·3751	·3452

TABLE I—*contd.*

Five per cent. points of the distribution of $2z^1 = \log_{10}(s_1^2/s_2^2)$—contd.

	Values of n_1					
	6	8	12	24	∞	
1	2·3692	2·3782	2·3872	2·3963	2·4054	Values of n_2
2	1·2862	1·2872	1·2881	1·2890	1·2899	
3	·9514	·9407	·9147	·9364	·9308	
4	·7898	·7811	·7717	·7615	·7504	
5	·6946	·6829	·6700	·6558	·6400	
6	·6318	·6177	·6020	·5845	·5645	
7	·5873	·5712	·5532	·5328	·5092	
8	·5540	·5364	·5163	·4935	·4665	
9	·5281	·5092	·4875	·4624	·4325	
10	·5075	·4874	·4643	·4373	·4045	
11	·4906	·4696	·4452	·4165	·3810	
12	·4766	·4546	·4292	·3989	·3610	
13	·4647	·4420	·4156	·3838	·3437	
14	·4546	·4312	·4038	·3708	·3285	
15	·4457	·4217	·3936	·3594	·3151	
16	·4379	·4134	·3846	·3493	·3031	
17	·4312	·4062	·3767	·3404	·2924	
18	·4251	·3997	·3696	·3324	·2826	
19	·4197	·3939	·3632	·3251	·2737	
20	·4148	·3886	·3575	·3186	·2655	
21	·4104	·3839	·3522	·3126	·2581	
22	·4064	·3796	·3475	·3071	·2512	
23	·4027	·3757	·3431	·3021	·2448	
24	·3994	·3720	·3391	·2975	·2388	
25	·3963	·3686	·3354	·2932	·2332	
26	·3934	·3656	·3321	·2892	·2280	
27	·3904	·3627	·3288	·2855	·2231	
28	·3883	·3601	·3259	·2821	·2185	
29	·3860	·3576	·3231	·2789	·2142	
30	·3838	·3552	·3206	·2759	·2101	
60	·3530	·3216	·2827	·2305	·1428	
∞	·3290	·2874	·2437	·1811	··	

Table II.

One per cent. points of distribution of $2z^1 = \log_{10}(s_1^2/s_2^2)$.

Values of n_2	Values of n_1: 1	2	3	4	5
1	3·6077	3·6989	3·7327	3·7501	3·7607
2	1·9934	1·9957	1·9964	1·9967	1·9970
3	1·5330	1·4888	1·4692	1·4580	1·4508
4	1·3263	1·2553	1·2225	1·2035	1·1909
5	1·2111	1·1230	1·0813	1·0566	1·0400
6	1·1381	1·0384	·9903	·9614	·9418
7	1·0880	·9798	·9270	·8946	·8728
8	1·0515	·9369	·8803	·8455	·8216
9	1·0237	·9043	·8446	·8077	·7823
10	1·0019	·8785	·8164	·7777	·7510
11	·9844	·8576	·7935	·7534	·7256
12	·9699	·8405	·7747	·7334	·7045
13	·9578	·8261	·7589	·7164	·6868
14	·9475	·8139	·7453	·7020	·6716
15	·9387	·8034	·7338	·6896	·6586
16	9310	·7942	·7236	·6787	·6471
17	·9243	·7862	·7148	·6692	·6371
18	·9183	·7791	·7068	·6607	·6282
19	·9130	·7728	·6998	·6533	·6202
20	·9083	·7671	·6936	·6465	·6130
21	·9040	·7619	·6879	·6403	·6066
22	·9001	·7573	·6827	·6348	·6007
23	·8966	·7531	·6780	·6298	·5954
24	·8933	·7492	·6738	·6251	·5906
25	·8904	·7457	·6698	·6209	·5860
26	·8877	·7425	·6662	·6170	·5819
27	·8852	·7394	·6628	·6134	·5780
28	·8828	·7366	·6597	·6100	·5745
29	·8807	·7340	·6568	·6069	·5712
30	·8787	·7316	·6541	·6040	·5681
60	·8498	·6970	·6155	·5622	·5236
∞	·8218	·6633	·5777	·5211	·4706

TABLE II—*contd.*

One per cent. points of distribution of $2z^1 = \log_{10}(s_1^2/s_2^2)$—*contd.*

Values of n_2	Values of n_1: 6	8	12	24	∞
1	3·7679	3·7768	3·7857	3·7948	3·8039
2	1·9971	1·9972	1·9975	1·9977	1·9978
3	1·4458	1·4392	1·4322	1·4248	1·4170
4	1·1821	1·1703	1·1576	1·1439	1·1292
5	1·0282	1·0114	·9951	·9762	·9552
6	·9276	·9085	·8875	·8641	·8376
7	·8568	·8351	·8108	·7835	·7520
8	·8042	·7803	·7533	·7226	·6865
9	·7636	·7378	·7085	·6748	·6345
10	·7313	·7039	·6726	·6362	·5920
11	·7049	·6762	·6432	·6044	·5566
12	·6831	·6532	·6186	·5775	·5264
13	·6646	·6336	·5978	5·547	·5004
14	·6489	·6170	·5798	·5350	·4777
15	·6353	·6025	·5642	·5178	·4577
16	·6234	·5898	·5506	·5026	·4398
17	·6130	·5787	·5384	·4890	·4238
18	·6037	·5688	·5277	·4791	·4093
19	·5953	·5600	·5180	·4661	·3961
20	·5879	·5520	·5093	·4563	·3840
21	·5811	·5448	·5014	·4473	·3730
22	·5750	·5382	·4943	·4392	·3627
23	·5694	·5322	·4877	·4316	·3533
24	·5642	·5267	·4816	·4247	·3446
25	·5595	·5217	·4761	·4183	·3363
26	·5552	·5170	·4709	·4124	·3287
27	·5512	·5126	·4662	·4069	·3215
28	·5475	·5086	·4617	·4018	·3148
29	·5440	·5049	·4577	·3969	·3083
30	·5408	·5014	·4538	·3925	·3024
60	·4940	·4507	.3973	·3254	·2043
∞	·4475	·3999	·3394	·2530	0

Table III.

Five per cent. points of the distribution of the ratio of variance, $x = (s_1^2/s_2^2)$.

	Values of n_1					
	1	2	3	4	5	
1	161·45	199·50	215·72	224·57	230·17	Values of n_2
2	18·512	19·000	19·163	19·248	19·298	
3	10·129	9·552	9·276	9·118	9·014	
4	7·710	6·945	6·591	6·388	6·257	
5	6·607	5·786	5·410	5·192	5·050	
6	5·987	5·143	4·756	4·534	4·388	
7	5·591	4·732	4·347	4·121	3·972	
8	5·317	4·459	4·067	3·838	3·688	
9	5·117	4·256	3·863	3·633	3·482	
10	4·965	4·103	3·708	3·478	3·326	
11	4·844	3·982	3·587	3·357	3·204	
12	4·747	3·885	3·490	3·259	3·106	
13	4·667	3·805	3·410	3·179	3·025	
14	4·600	3·739	3·344	3·112	2·958	
15	4·543	3·683	3·287	3·056	2·901	
16	4·494	3·634	3·239	3·007	2·853	
17	4·451	3·592	3·197	2·965	2·810	
18	4·414	3·555	3·160	2·928	2·773	
19	4·381	3·522	3·127	2·895	2·740	
20	4·351	3·493	3·098	2·866	2·711	
21	4·325	3·467	3·072	2·840	2·685	
22	4·301	3·443	3·049	2·817	2·661	
23	4·279	3·422	3·028	2·795	2·640	
24	4·260	3·403	3·009	2·777	2·621	
25	4·242	3·385	2·991	2·759	2·603	
26	4·225	3·369	2·975	2·743	2·587	
27	4·210	3·354	2·961	2·738	2·572	
28	4·196	3·340	2·947	2·714	2·558	
29	4·183	3·328	2·934	2·702	2·545	
30	4·171	3·316	2·922	2·690	2·534	
60	4·001	3·151	2·758	2·525	2·368	
∞	3·841	2·996	2·605	2·372	2·214	

TABLE III—*contd.*

Five per cent. points of the distribution of the ratio of variance, $x = (s_1^2/s_2^2)$—contd.

Values of n_2	Values of n_1: 6	8	12	24	∞
1	233·97	238·89	243·91	249·04	254·32
2	19·329	19·371	19·414	19·453	19·496
3	8·941	8·844	8·744	8·638	8·527
4	6·164	6·041	5·912	5·774	5·628
5	4·950	4·818	4·678	4·527	4·365
6	4·284	4·144	4·000	3·841	3·669
7	3·866	3·725	3·574	3·410	3·230
8	3·580	3·438	3·284	3·116	2·928
9	3·374	3·230	3·073	2·900	2·707
10	3·217	3·072	2·913	2·737	2·538
11	3·094	2·948	2·788	2·609	2·405
12	2·999	2·848	2·686	2·506	2·296
13	2·915	2·767	2·604	2·420	2·207
14	2·848	2·699	2·534	2·349	2·131
15	2·790	2·641	2·475	2·288	2·066
16	2·741	2·591	2·424	2·235	2·010
17	2·699	2·548	2·381	2·190	1·961
18	2·661	2·510	2·342	2·150	1·917
19	2·629	2·477	2·308	2·114	1·878
20	2·599	2·447	2·278	2·083	1·843
21	2·573	2·421	2·250	2·054	1·812
22	2·549	2·397	2·226	2·028	1·783
23	2·528	2·375	2·203	2·005	1·757
24	2·508	2·355	2·183	1·984	1·733
25	2·490	2·337	2·165	1·965	1·711
26	2·474	2·321	2·148	1·947	1·691
27	2·459	2·305	2·132	1·930	1·672
28	2·445	2·292	2·118	1·915	1·654
29	2·432	2·278	2·104	1·901	1·638
30	2·421	2·266	2·092	1·887	1·622
60	2·254	2·097	1·918	1·700	1·389
∞	2·099	1·938	1·752	1·517	1·000

TABLE IV.

One per cent. points of the distribution of the ratio of variance, $x = (s_1^2/s_2^2)$.

Values of n_2	Values of n_1: 1	2	3	4	5
1	4052·1	4999·0	5403·5	5625·1	5764·1
2	98·495	99·008	99·167	99·237	99·305
3	34·117	30·815	29·459	28·709	28·236
4	21·200	18·001	16·693	15·978	15·521
5	16·258	13·274	12·059	11·391	10·966
6	13·744	10·924	9·779	9·149	8·746
7	12·246	9·546	8·452	7·846	7·460
8	11·259	8·649	7·591	7·006	6·631
9	10·561	8·022	6·992	6·423	6·057
10	10·044	7·560	6·552	5·994	5·636
11	9·647	7·205	6·217	5·668	5·317
12	9·330	6·927	5·953	5·412	5·064
13	9·074	6·701	5·740	5·205	4·862
14	8·862	6·514	5·563	5·035	4·695
15	8·683	6·359	5·417	4·893	4·556
16	8·532	6·227	5·292	4·772	4·437
17	8·400	6·112	5·185	4·669	4·336
18	8·285	6·013	5·092	4·579	4·248
19	8·184	5·926	5·010	4·501	4·170
20	8·096	5·849	4·938	4·431	4·103
21	8·017	5·780	4·875	4·368	4·042
22	7·944	5·719	4·816	4·314	3·988
23	7·881	5·663	4·765	4·264	3·939
24	7·823	5·614	4·718	4·218	3·895
25	7·770	5·568	4·676	4·177	3·855
26	7·722	5·527	4·637	4·140	3·818
27	7·677	5·488	4·601	4·106	3·785
28	7·636	5·453	4·568	4·074	3·754
29	7·597	5·421	4·538	4·045	3·726
30	7·563	5·390	4·510	4·018	3·699
60	7·077	4·978	4·126	3·649	3·339
∞	6·635	4·605	3·782	3·320	3·017

TABLE IV—*contd.*

One per cent. points of the distribution of the ratio of variance, $x = (s_1^2/s_2^2)$—contd.

Values of n_2	Values of n_1: 6	8	12	24	∞
1	5859·4	5981·4	6105·8	6234·2	6366·5
2	99·325	99·365	99·425	99·464	99·504
3	27·910	27·489	27·053	26·597	26·122
4	15·208	14·800	14·374	13·930	13·271
5	10·672	10·266	9·888	9·467	9·019
6	8·466	8·101	7·718	7·313	6·880
7	7·191	6·840	6·458	6·074	5·650
8	6·371	6·029	5·641	5·279	4·859
9	5·802	5·467	5·111	4·730	4·311
10	5·386	5·057	4·706	4·327	3·909
11	5·069	4·745	4·397	4·021	3·620
12	4·820	4·500	4·156	3·780	3·361
13	4·620	4·302	3·961	3·586	3·165
14	4·456	4·140	3·800	3·427	3·005
15	4·318	4·004	3·666	3·294	2·869
16	4·201	3·889	3·553	3·181	2·753
17	4·102	3·791	3·455	3·083	2·653
18	4·015	3·706	3·370	3·014	2·566
19	3·939	3·631	3·296	2·925	2·489
20	3·871	3·565	3·231	2·859	2·421
21	3·811	3·506	3·173	2·801	2·360
22	3·759	3·453	3·121	2·749	2·305
23	3·710	3·406	3·074	2·702	2·256
24	3·666	3·363	3·031	2·659	2·210
25	3·627	3·324	2·993	2·620	2·169
26	3·591	3·288	2·958	2·585	2·132
27	3·558	3·256	2·925	2·551	2·096
28	3·528	3·226	2·896	2·522	2·064
29	3·499	3·198	2·869	2·494	2·034
30	3·474	3·173	2·843	2·469	2·006
60	3·119	2·823	2·496	2·115	1·601
∞	2·802	2·511	2·182	1·791	1·000

TABLE V.

Five per cent. points of the distribution of the ratio of standard deviations, $y = (s_1/s_2)$

Values of n_2	Values of n_1: 1	2	3	4	5
1	12·706	14·124	14·688	14·986	15·171
2	4·3025	4·3591	4·3776	4·3873	4·3929
3	3·1826	3·0907	3·0456	3·0195	3·0024
4	2·7766	2·6353	2·5674	2·5274	2·5013
5	2·5764	2·4054	2·3258	2·2787	2·2472
6	2·4468	2·2678	2·1810	2·1293	2·0946
7	2·3646	2·1765	2·0839	2·0293	1·9929
8	2·3060	2·1117	2·0166	1·9591	1·9203
9	2·2621	2·0631	1·9654	1·9060	1·8660
10	2·2282	2·0255	1·9257	1·8649	1·8238
11	2·2009	1·9955	1·8940	1·8322	1·7900
12	2·1788	1·9711	1·8682	1·8053	1·7623
13	2·1604	1·9507	1·8466	1·7830	1·7393
14	2·1447	1·9336	1·8287	1·7642	1·7200
15	2·1314	1·9190	1·8130	1·7480	1·7034
16	2·1200	1·9062	1·7997	1·7341	1·6889
17	2·1098	1·8951	1·7880	1·7218	1·6763
18	2·1009	1·8853	1·7777	1·7110	1·6651
19	2·0930	1·8767	1·7684	1·7015	1·6553
20	2·0859	1·8690	1·7602	1·6930	1·6464
21	2·0796	1·8619	1·7528	1·6852	1·6385
22	2·0738	1·8556	1·7461	1·6783	1·6313
23	2·0687	1·8498	1·7402	1·6720	1·6248
24	2·0649	1·8447	1·7346	1·6663	1·6188
25	2·0596	1·8399	1·7294	1·6610	1·6133
26	2·0555	1·8355	1·7248	1·6562	1·6083
27	2·0518	1·8314	1·7206	1·6515	1·6037
28	2·0485	1·8276	1·7165	1·6474	1·5994
29	2·0452	1·8241	1·7129	1·6436	1·5954
30	2·0423	1·8210	1·7095	1·6400	1·5917
60	2·0003	1·7750	1·6608	1·5891	1·5389
∞	1·9599	1·7308	1·6140	1·5402	1·4879

TABLE V—*contd.*

Five per cent. points of the distribution of the ratio of standard deviations, $y=(s_1/s_2)$—contd

	Values of n_1				
	6	8	12	24	∞
1	15·296	15·456	15·618	15·781	15·948
2	4·3965	4·4013	4·4061	4·4106	4·4154
3	2·9901	2·9740	2·9571	2·9391	2·9200
4	2·4826	2·4579	2·4315	2·4030	2·3724
5	2·2249	2·1950	2·1628	2·1276	2·0892
6	2·0697	2·0364	1·9999	1·9599	1·9153
7	1·9682	1·9302	1·8906	1·8467	1·7971
8	1·8923	1·8543	1·8121	1·7651	1·7110
9	1·8368	1·7971	1·7529	1·7030	1·6453
10	1·7937	1·7526	1·7068	1·6545	1·5931
11	1·7591	1·7170	1·6696	1·6153	1·5507
12	1·7310	1·6878	1·6390	1·5828	1·5153
13	1·7074	1·6635	1·6137	1·5557	1·4854
14	1·6876	1·6428	1·5919	1·5325	1·4597
15	1·6705	1·6250	1·5733	1·5126	1·4373
16	1·6557	1·6096	1·5571	1·4951	1·4177
17	1·6428	1·5962	1·5430	1·4798	1·4002
18	1·6313	1·5844	1·5304	1·4662	1·3844
19	1·6213	1·5738	1·5192	1·4540	1·3704
20	1·6122	1·5642	1·5092	1·4431	1·3576
21	1·6040	1·5558	1·5001	1·4332	1·3460
22	1·5966	1·5481	1·4920	1·4242	1·3354
23	1·5898	1·5411	1·4844	1·4160	1·3255
24	1·5838	1·5346	1·4776	1·4085	1·3164
25	1·5781	1·5287	1·4714	1·4016	1·3080
26	1·5729	1·5233	1·4657	1·3951	1·3002
27	1·5682	1·5183	1·4602	1·3892	1·2929
28	1·5638	1·5138	1·4553	1·3838	1·2861
29	1·5596	1·5094	1·4505	1·3786	1·2797
30	1·5558	1·5053	1·4464	1·3738	1·2737
60	1·5014	1·4480	1·3847	1·3040	1·1787
∞	1·4486	1·3922	1·3237	1·2318	1·000

Values of n_2

TABLE VI.

One per cent. points of the distribution of the ratio of standard deviations, $y = (s_1/s_2)$.

Values of n_2	Values of n_1: 1	2	3	4	5
1	63·657	70·704	73·508	75·001	75·922
2	9·9244	9·9503	9·9582	9·9617	9·9652
3	5·7741	5·5511	5·4276	5·3581	5·3138
4	4·6043	4·2427	4·0857	3·9972	3·9307
5	4·0322	3·6433	3·4726	3·3750	3·3115
6	3·7073	3·3052	3·1271	3·0247	2·9573
7	3·4994	3·0898	2·9072	2·8011	2·7314
8	3·3555	2·9408	2·7552	2·6469	2·5751
9	3·2498	2·8323	2·6443	2·5343	2·4611
10	3·1693	2·7494	2·5597	2·4483	2·3741
11	3·1464	2·6842	2·4933	2·3807	2·3057
12	3·0544	2·6319	2·4398	2·3264	2·2504
13	3·0123	2·5886	2·3958	2·2814	2·2049
14	2·9770	2·5523	2·3587	2·2439	2·1667
15	2·9467	2·5216	2·3275	2·2120	2·1344
16	2·9209	2·4953	2·3004	2·1845	2·1064
17	2·8982	2·4722	2·2876	2·1608	2·0824
18	2·8783	2·4522	2·2565	2·1398	2·0610
19	2·8608	2·4344	2·2383	2·1215	2·0421
20	2·8454	2·4184	2·2222	2·1050	2·0255
21	2·8315	2·4042	2·2078	2·0901	2·0105
22	2·8188	2·3914	2·1946	2·0770	1·9969
23	2·8072	2·3798	2·1828	2·0649	1·9848
24	2·7969	2·3693	2·1721	2·0538	1·9737
25	2·7874	2·3596	2·1624	2·0438	1·9634
26	2·7787	2·3509	2·1533	2·0346	1·9540
27	2·7707	2·3427	2·1449	2·0263	1·9455
28	2·7632	2·3352	2·1372	2·0184	1·9375
29	2·7563	2·3282	2·1302	2·0111	1·9302
30	2·7500	2·3217	2·1236	2·0045	1·9232
60	2·6602	2·2311	2·0311	1·9102	1·8272
∞	2·5759	2·1460	1·9447	1·8219	1·7371

TABLE VI—*contd.*

One per cent. points of the distribution of the ratio of standard deviations, $y=(s_1/s_2)$—contd.

Values of n_2	Values of n_1				
	6	8	12	24	∞
1	76·547	77·339	78·140	78·957	79·790
2	9·9662	9·9682	9·9712	9·9732	9·9752
3	5·2830	5·2430	5·2013	5·1572	5·1110
4	3·8997	3·8470	3·7913	3·7322	3·6693
5	3·2608	3·2040	3·1446	3·0768	3·0033
6	2·9096	2·8402	2·7782	2·7042	2·6230
7	2·6816	2·6154	2·5434	2·4645	2·3769
8	2·5241	2·4554	2·3805	2·2977	2·2043
9	2·4087	2·3382	2·2608	2·1747	2·0761
10	2·3208	2·2488	2·1693	2·0801	1·9770
11	2·2515	2·1782	2·0970	2·0053	1·8930
12	2·1955	2·1212	2·0385	1·9443	1·8333
13	2·1494	2·0740	1·9901	1·8879	1·7791
14	2·1109	2·0346	1·9494	1·8513	1·7333
15	2·0780	2·0011	1·9149	1·8150	1·6937
16	2·0497	1·9721	1·8849	1·7835	1·6593
17	2·0253	1·9470	1·8587	1·7559	1·6289
18	2·0037	1·9249	1·8358	1·7360	1·6019
19	1·9846	1·9054	1·8159	1·7102	1·5777
20	1·9676	1·8879	1·7975	1·6910	1·5560
21	1·9523	1·8724	1·7812	1·6736	1·5363
22	1·9387	1·8582	1·7667	1·6580	1·5183
23	1·9261	1·8454	1·7533	1·6436	1·5029
24	1·9148	1·8338	1·7411	1·6307	1·4869
25	1·9045	1·8232	1·7300	1·6187	1·4729
26	1·8950	1·8134	1·7198	1·6077	1·4599
27	1·8863	1·8043	1·7104	1·5976	1·4479
28	1·8782	1·7961	1·7017	1·5882	1·4368
29	1·8707	1·7884	1·6937	1·5793	1·4262
30	1·8638	1·7812	1·6861	1·5713	1·4164
60	1·7660	1·6802	1·5800	1·4544	1·2652
∞	1·6740	1·5847	1·4782	1·3382	1·1000

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