Decentralised Longterm Planning—A Frame*

Introduction

1·1. The paper presents an operational frame for the preparation of a longterm plan for the development of the national economy that would ensure consistency of targets and optimal utilization of resources. A longterm plan is understood not just as a set of targets but as a collection of projects, one for the longterm development of each of a number of sectors into which the economy is divided for planning purposes. The central idea underlying the paper is that a balanced and optimal plan understood in this dual sense, physical and statistical, can be worked out only in a collaboration between two sets of workers, the central planners working in the office charged with plan preparation (say, the Planning Commission) setting balanced and optimal targets through the solution of equations arising from a model of general equilibrium and the sectoral planners working in the different administrative bodies charged with production and development (say, the Ministries) preparing projects for the longterm development of the economic sectors coming under their respective jurisdiction. The central planners have to be principally of the nature of econometricians and the sectoral planners of that of industrial economists. The industrial economists will have to be dependent on the econometricians for the targets and the prices with respect to which they have to prepare projects and the criteria they have to use in making choice decisions ; the econometricians on their part will have to be dependent on the industrial economists for all technical information that has to be made use of in the general equilibrium model in the form of functional forms and parametric values. A relation of mutual dependence of the Planning Commission and the Ministries is of course essential to all process of planning : what our paper attempts to do is to formalise the process in order to enable consistency and optimality of targets to be sought with mathematical rigour, even though the entire plan is not to be produced in one piece in a push-button machine through the solution of a single system of equations but is to be built up in a process of successive corrections and adjustments wherein is thrown in the labours of a large number of groups of workers working on different problems in different organizations.

1·2. (We have distinguished three activities in the process of planning as visualised above, namely the central calculations in the Planning Commission for target setting and price (actual and not shadow prices) fixing ; the preparation of projects in the Ministries for the realisation of he targets ; and the exchange of information and decisions between the Planning Commission and the Ministries leading to the successive adjustments and corrections of the targets and the prices on the one hand and of the projects on the other) The scope of the present paper is such as to cover the first and third activities and leave out the work of project building in the Ministries.) The scope thus falls short of the entire planning process but is larger than that of many currently used econometric models of planning which relate only to the central calculations.[1] From this follows the limitation that they can only act as starting points in a process of planning, they can set only preliminary targets for the different activities in the economy. The preliminary targets would necessarily require to be altered subsequently as they would be found to be mutually inconsistent once the industrial economists have calculated the requirements of raw materials, fuel, power, capital goods and labour to implement the projects. These requirements would fail to be matched by the supply targets which are based on input flow and capital coefficients which are of the nature of statistical averages, whereas the industrial economists in the Ministries would appropriately base their estimates on technological information. This problem would not arise if the central calculations were not based on statistically calculated aggregative coefficients but on technological norms. But this would mean either that one uses for central calculations a non-linear model or that the technological norms are assumed to satisfy the condition of constant returns to scale, which is seldom justified. If a non-linear model is to be used, it would call for explicit formulations of production for all the different sectors in terms of simple non-linear mathematical functions with known properties whereas in reality the technologically valid production relations might not allow of any convenient mathematical representation at all. The method proposed by us is meant to provide a way out of this impasse : it neither assumes constant returns to scale nor makes use of a non-linear model with explicit mathematical functions for the different relations between output and input. In our frame no assumptions whatsoever are made with respect to the production functions. An iterative procedure is devised whereby in any given round of calculations the norms used represent the average rates of input to output obtaining at the specific levels of output as following from the previous round of iteration. As a price to be paid for this generality we are not in a position to ensure the convergence of the iterative procedure. The convergence property (or its absence) would naturally depend on the nature of the production functions[2]. Our frame of computation will be useful under certain circumstances and not under others. Whether it is useful or not in a particular planning set up is a problem of empirical investigation into the nature of the production functions which cannot be undertaken in this paper.

1·3. This paper is thus meant to provide a frame for the central co-ordination of the decentralised decision makings that go into the preparation of the sectoral development projects. The allowance for decentralisation is made purposely, for the considerations that have to be gone into in preparing a project for the development of a particular sector are so detailed and so specific to the particular sector that they are best left to the charge of experts with specialised knowledge about the sector. While the frame does not lay down any computational procedure to be followed by the industrial economists in working out the development programmes, it does lay down the criteria of choice to be made use of by the sectoral experts in making decisions regarding the technological, locational, and other aspects of production that go to determine the relation between output and the various inputs. It is another distinguishing feature of our frame that it accomodates the making of technological and other choices. Most computable growth models[3] work with fixed coefficients implying the assumption of a unique technology for each sector. The way the activity analysis approach tries to tackle the situation by allowing for more than one column of coefficients for a sector hardly goes far enough to meet the problem, for it is suitable only to situations where there are just a few *basic* alternative process or activities for each sector.

1·4. Our frame is not only meant for the fixation of production targets but also accomodates the problem of price fixation. The financial balance aspect of planning is also taken care of through the inclusion of indirect and direct tax rates and savings rates in the central calculations.[4]

Definitions

2·1. We shall conceive of a longterm plan as having a dual character, a statistical and physical one, designed so as to lead to (1) the attainment of a given set of objectives within a specific period of time, subject to (ii) the fulfilment of certain feasibility conditions and (iii) the satisfaction of a criterion of optimality.

2·2. Regarded statistically, we shall define a longterm plan as a collection of optimally balanced statistical structures, namely (i) a production and commodities flow structure χ, (ii) a price structure Λ, and (iii) an income structure $\triangle$, all referring to the end year of the plan period. By *balanced* we mean that the quantities occuring in the structures satisfy certain equality aud inequality relations representing feasibility conditions. By *optimal* we mean that they are such as to minimise a particular function in those quantities treated as variables representing cost to the society understood in some appropriate fashion. The relations and the optimality function will be explicitly stated in sections 4 and 5.

2·3. The production structure χ is a statistical picture of the sectoral distribution of productive resources and the pattern of production and consumption activities during the end year of the plan. We consider a conceptual division of the economy into N branches of activities called sectors. We shall distinguish the sectors from the commodities of which we consider a classification into n categories. Within each sector j we make a distinction between the productive facilities that are brought about afresh during the plan period and those that survive from before the commencement of the plan. We do not find it necessary to measure the surviving capital stock ; we measure the former by the vector $(K_{1j}, K_{2j} \ldots K_{nj})$ where K_{ij} stands for the quantity of commodity i in the newly formed capital stock in sector j. We designate by X^o_j and X_j the levels of activity corresponding to the two parts of the capacity in the end year in sector j and by L^o_{sj} and L^o_{sj} the corresponding employment of labour of skill category $s (s = 1, 2 \ldots u)$; also by x^o_{ij} and x_{ij} the production of commodity i by these two activities.[5] The structure also shows the following pattern of utilisation of commodity i : inter-industrial consumption by old units in sector j (X^o_{ij}), and the same by new units (X_{ij}) ; hcusehold consumption (h_i), government consumption (g_i), capital formation (z_i), export (e_i), import m_i, and addition to stock (f_i). The production structure may therefore be said to stand for the set $(K_{ij}, L^o_{sj}, L_{sj}, X^o_{ij}, X_{ij}, X^o_{ij}, X_{ij}, h_i, g_i, z_i, e_i, m_i, f_i, j = 1, 2, \ldots N ; s = 1, 2, \ldots u; i = 1, 2, \ldots n)$ of which all the elements are measured in physical units.

2.4 The price structure covers the following prices for each commodity i : domestic ex-factory price p_{id} exclusive of excise duties; import price p_{im} exclusive of import duties ; and domestic market price p_i inclusive of all commodity taxes. It also includes u wage rates w_s $(s = 1, 2, \ldots\ldots u)$ for labour of u skill categories and a unique rate of return to capital, π. Λ therefore can be represented by the set (p_{id}, p_{im}, p_i, w_s, π ; s = 1,2,......u ; i = 1,2,......n).

2.5 The income structure Δ shows the sectoral pattern of employment and income generation in old and new units of production as well as its distribution among the different modes of utilization of income, namely savings, consumption and tax payment. We do not bring into the picture income distribution among different income groups of household, crucially important though the matter is, because of the absence of any reliable information on income distribution in the Indian economy. The structure Δ therefore can be represented by the set ((y°_j, y_j, c°_j, c_j, s°_j, s_j, d°_j, d_j ; j = 1,2,......N)) where y°_j and y_j are the two parts of income generated in sector j, c°_j and c_j the parts of these income flows that are consumed ; s°_j and s_j the parts that are saved, and d°_j and d_j the parts that are transferred to government as direct tax. The savings s°_j and s_j incorporate both personal savings and corporate savings and both private savings and government savings in so far as the latter accrues in the form of profit of public enterprises.[6]

2.6 In physical terms a longterm plan may be regarded as a collection of sectoral development projects or programmes π_j (j = 1,2,... . N) which would ensure the realisation of the structure X in the end year of the plan T through the most efficient utilization of resources (according to some given criterion of efficiency). Each sectoral programme will consist of decisions regarding :

(i) the degrees of capacity utilisation of old units ;

(ii) the new units of production, along with their capacities, to be set up and the old units to be scrapped ;

(iii) the techniques and production processes to be associated with these units ;

(iv) the way their installation and operation would be phased in time ;

(v) the way they would be located in space.

The decisions will also be co-ordinated to each other so as to represent an optimality (that is, efficient utilization of resources), specific to the sector. All economic calculations for balance and optimality internal to a sector are to be based on the price system Λ.

2.7 Quantitatively, the sectoral development programme π_j for sector j would lay down :

(i) The path of growth of output of sector j in two parts, one for the units of production existing at the beginning of the plan period and designated X°_{jt} and the other for the units brought about in the course of the plan, designated X_{jt}, each for $t=1,2,\ldots\ldots T_j$ where $T_j \geqslant T$ is the time horizon of the sectoral programme for sector j. (We shall explain in paragraph 2.10 the reason for allowing T_j to differ from T). The paths would be such as to reach by year T the targets X°_j and X_j laid down in the structure X, that is, the paths must be such as to satisfy the conditions $X^{\circ}_{jT} = X^{\circ}_j$ and $X_{jT} = X_j$. It is also clear that $X_{jt}=0$ for $t=0$ and that $X^{\circ}_{jt} \leqslant X^{\circ}_{jo}$ for $t \geqslant 0$.[5] We shall treat the ratios

$$(2.1.)\quad r^{o}_j = \frac{\sum_{t=1}^{T} X^{o}_{jt}}{X^{o}_{jT}} \qquad r_j = \frac{\sum_{t=1}^{T} X_{jt}}{X_{jT}}$$

as quantitative characteristics of the paths of growth of output in sector j.

(ii) The commodity composition of output in sector j in each year t from old and new units, x^{o}_{ijt} and x_{ijt} $(i=1,2,\ldots\ldots n)$.

(iii) The pattern of current inputs and fixed capital formation in each year t corresponding to the outputs X^{o}_{jt} and X_{jt} :

intermediate goods	X^{o}_{ijt}	of ith commodity $(i=1,2,\ldots\ldots n)$ for old units
	X_{ijt}	of the same for new units
labour	L^{o}_{sjt}	of sth category of labour $(s=1,2,\ldots\ldots u)$ for old units
	L_{sjt}	of the same for new units
fixed capital	Z_{ijt}	of commodity i in physical units going into fixed capital formation in sector j in the form of new units.

2.8 The commodity production x°_{ijt} and the inputs X°_{ijt} and L°_{sjt} would be entirely determined by the technological features of the stock of capital in year t surviving from before the beginning of the plan period and would not represent any choice made by the planners.[5'] We are assuming that the units of production surviving from the pre-plan period will receive repair services but no replacement of capital goods: that is, in so far as replacement takes place it will be assumed that an old unit vanishes and a new unit takes its place. Hence the old units are not shown to require any input of fixed capital whereas its repair and maintenance are treated as current inputs. The output compsition x_{ijt}, and the input combinations X_{ijt}, L_{sjt} and Z_{ijt}. on the other hand, are determined by the technological characteristics of the new units of production and hence represent technological decisions made by sectoral planners of sector j.[6] If $\frac{x_{ijt}}{X_{jt}}$, $\frac{X_{ijt}}{X_{jt}}$, $\frac{L_{sjt}}{X_{jt}}$ and $\sum_{v=1}^{t} \frac{Z_{ijv}}{X_{jt}}$ are plotted against X_{jt} for the different values of t and extrapolated and interpolated, we generate a set of empirical curves which we may designate by $b_{ij}(X_j)$, $a_{ij}(X_j)$, $l_{ij}(X_j)$ and $k_{ij}(X_j)$ respectively.[7] These functions may be arranged in the matrix form as follows :

$$
2.2 \quad \begin{bmatrix}
b_{11}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots b_{1N}(X_N) \\
\vdots \\
b_{n1}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots b_{nN}(X_N) \\
\vdots \\
a_{11}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{1N}(X_N) \\
\vdots \\
a_{n1}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{nN}(X_N) \\
\vdots \\
k_{11}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots k_{1N}(X_N) \\
\vdots \\
k_{n1}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (k_{nN}(X_N) \\
\vdots \\
l_{11}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots l_{1N}(X_N) \\
\vdots \\
l_{n1}(X_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots l_{nN}(X_N)
\end{bmatrix}
$$

and designate it by $\Omega : X$. We shall call it the technological functions matrix. The column j of this matrix presents the collection of technological functions relevant to sector j. We shall designate this column by $\Omega_j : X_j$ and call it an array of technological functions for sector j. For any given value of X_j $(j=1,2,\ldots\ldots N)$ the functions $b_{ij}(X_j)$, $a_{ij}(X_j)$, $k_{ij}(X_j)$ and $l_{ij}(X_j)$ would take given values, say b_{ij}, a_{ij}, k_{ij} and l_{ij}. The matrix of these elements arranged as in (2.2) will be designated by Ω and called the technological coefficients matrix. Column j of this matrix will be designated by Ω_j and called an array of technological coefficients for sector j. As there can conceivably be various alternative combinations of inputs all leading to the same output level of X_j in the new units of sector j, there is actually a set $(\Omega_j : X_j)$ of alternative arrays from which $\Omega_j : X_j$ is a selection made by the sectoral experts by making use of some criterion of efficiency. It follows that we can also think of a set $(\Omega : X)$ of alternative matrices for the economy from which the matrix $\Omega : X$ is a selection made by the planning authorities in a decentralised fashion.

2.9 It is to be noted that as the statistical structure χ refers only to the end year of the plan, the feasibility and optimality of the projects π_i is ensured only for the end year, but not for the intervening plan years. Thus, it is not certain that the paths of growth of sectoral outputs X^o_{jt} and X_{jt} (J = 1, 2 N and t = 1, 2...T_j) would be in balance with each other[8], in each individual year having been independently worked out by the sectoral experts working in the different Ministries. This difficulty would not be there if the statistical structure would cover not only the end year but each individual plan year. However, if we have not so expanded the coverage of χ that is because of the following two reasons. Firstly calculation of the production structure for each time point t would have multiplied the dimensions of the computational task of the central planners beyond the capacity of the maximum computing capacity that is likely to be available for quite some time in India ; secondly, it appears to us that the time path to be followed by production in any given sector must be a function of various factors technologically specific to that sector so that they may be fully taken into account only by the sectoral experts and hence best left to their charge. The end year levels of activity of the different sectors being in balance, it should always be possible in practice to arrange for the building up and depletion of stocks in the different sectors at such differential rates so as to ensure the matching of demand and supply of commodities and resources during the intervening years. This part of the planning process has been left out of the scope of our frame.

2.10 It is the time lags technologically specific to a particular sector to which reference has been made in the previous paragraph that give rise to the necessity of allowing the time horizon of a sector development programme to differ from that of the longterm plan taken as a whole. Let us take a sector like animal husbandry. Improvement of stocks through cross-breeding, an important element in any such programme, involves certain biologically determined time lags and it might be inconvenient to have a time horizon for the development programme for animal husbandry which is not a common multiple of these time lags. The time taken for re-forestation, replantation, etc also give rise to similar situations in the forestry, plantation etc. sectors.

Objectives and Conditions

3.1 One can consider the planned development of an economy to proceed through a number of well defined and meaningful stages and the approach we take in this paper is to consider as the task of longterm planning the achievement of the transition from one such stage to another. For an underdeveloped country like India, an acceptable first stage of development is a stage which provides to all members of the society a level of living that may be described as a *human* level of living as opposed to the *subhuman* level of living which is the lot of the greater part of the population at present. The level of living that divides the human from subhuman can be worked out from *normative* considerations. The goal may therefore be to attain a stage of development of the productive forces such that it should be possible to provide the population with certain minimum levels of consumption of certain essential consumer commodities and certain minimum levels of social services. The goal should naturally also be to realise such conditions as would ensure the further continuation of the process of development by allowing for an adequate rate of capital formation. A first stage of development must also be such as to make the economy self-sufficient, that is, to remove the necessity of foreign assistance. In the context of the declared socialist motivation of Indian planning, a significant reduction in the inequality of the standard of living may also be regarded as a part of the goal.

3.2 The desired features of the goal can be translated into the following five precise quantitative specifications to characterise the economy in the end year :

Objective 1 : per capita consumption of a number n_o of specified essential consumer commodities should not be less than certain normative minimum quantities q_i for any class of consumers $(i=1, 2 \ldots\ldots\ldots n_o)$.

Objective 2 : specified social services (schools, hospital beds, running water supply etc.) should be available to the entire population at certain normative rates.

Objective 3 : the degree of inequality in the distribution of total consumption expenditure (net of indirect taxes)[9] measured by the Lorentz ratio should equal a number θ lower than the base year value θ^o.

Objective 4 : the ratio of gross fixed capital formation to gross national income should equal or exceed a given value ϕ.

Objective 5 : foreign exchange earnings should equal foreign exchange spendings.

3.3 The feasibility conditions to be satisfied are as follows :

Feasibility Condition 1 : the supply of each commodity should exceed the demand for it in physical quantity so that there can be addition to stocks at a minimum rate greater than or equal to zero.

Feasibility Condition 2 : receipts must equal disbursements for every accounting sector of the economy.

3.4 Given that the goal of longterm planning is the transition from one stage of development to another and given that dependence on foreign economic assistance is one of the most undesirable results and a characteristic symptom of an economy's underdeveloped state, it appears to us that the best criterion for the optimal use of resources is to stipulate that the objectives are to be attained through the minimum utilization of foreign assistance, We shall compare this criterion with three alternative criteria which are often put forward to show that they are less satisfactory than the one proposed. They are the minimisation of current costs of production, the minimisation of fixed capital requirements and the minimisation of foreign exchange requirements. Use of the first criterion would recommend a diversion of investment to low cost modern technology industries which might be highly capital intensive and call for a great deal of imported plant and machinery without ptoviding any means of meeting either the capital needs or the foreign exchange needs. Use of the second would recommend the development of low capital intensive industries which might however raise the cost of production and thus slow down the formation of domestic savings and reduce foreign exchange earnings. Use of the third criterion would recommend a pattern of import substitution which might call for a very high rate of capital formation and at the same time slow down the formation of domestic savings by raising production costs. The criterion chosen by us is a kind of synthesis of these three alternative and partly conflicting criteria, in as much as lessening of dependence on foreign assistance is possible only through the lowering of domestic costs of production as well as of fixed capital requirements and of course of foreign exchange needs: but use of the criterion will prevent any two of the minimisation consideration being completely ignored while pushing the third to its extreme point.

3.5 In case foreign assistance were not to be available at all or available only in a given fixed amount for the entire plan period, the criterion of minimising the resources gap will not obviously do as no such gap, whether minimal or not, may be incorporated in a plan. The equalisation of domestic savings and capital requirements (over and above foreign assistance), would in that case have to be treated as an additional feasibility condition. It appears to us that the most appropriate criterion of optimality under these conditions would be the un-orthodox one of minimising the very length of the transitional period; that is, to make the plan in such a way as to take the economy from one stage to another along the shortest possible path in time. While in the main body of the paper we shall assume the time horizon of planning to be given and use the optimization criterion as suggested in the previous paragraph, we shall sketch an approach towards the solution of the second problem (that of shortest time path) in Appendix 3.

The System of Equations

4.1 The objectives and feasibility conditions can be translated into a system of equalities and inequalities in the elements of the statistical structures X, Λ and Δ as represented by the relations (4.1) to (4.10) below. The first relation makes use of demand functions D_i to ensure that the market prices of commodities $p_1, p_2, \ldots\ldots p_n$ and C_α, the per capita total consumption expenditure (net of indirect taxes) of the lowest α% of the population, will be such as to make their demand for commodity i equal the normative minimum q_i $(i=1, 2, \ldots\ldots n_o)$. It thus represents the condition to be satisfied if Objective 1 is to be attained within the market mechanism.

4.2 The second relation is a translation of Objective 2, achieved by subjecting the public consumption of the different commodities, g_i, to certain lower limits g°_i determined on the basis of the projected expansion of the administrative, defence and social services, the latter conforming to the quantitative implications of Objective 2.

4.3 The third relation is based on the assumption that the distribution of C, per capita consumption expenditure (net of indirect taxes), is distributed according to the lognormal law[10] with mean ρ_1 and or variance ρ_2. It translates the third objective about the Lorentz ratio equalling a given figure lower than the base year figure θ^0, G standing for the in complete normal integral.

4.4 The fourth relation follows from Objective 4, ϕ being the minimum ratio of capital formation to income to be reached by the end plan. y is the gross income generated in the economy and z the gross year of the capital formation. Objective 5 is expressed by (5) wherem_0 stands for the value of net payments to be made in foreign exchange over and above those for financing imports.

4.5 Feasibility condition 1 gets its algebraic expression in (4.6) whereas (4.7) to 4.10) are called for to give expression to Feasibility Condition 2. These latter are called in to ensure that the physical balance ensured by (4.6) is matched by a financial balances based on appropriate relations between the prices, wage rates, direct and indirect tax rates and the rate of return to capital. Relation (4.7) stands for the accounting balance for each individual sector of activity. The receipts (net of indirect taxes) by the sale of commodities by the new units[11] of a sector brought about by fresh investment are shown to be greater than or equal to the total of payments to other sectors for purchases of intermediate products (including trade, commerce transport and other services) and income generated in the sector. (4.8), 4.9) and (4.10) spell out the accounting balance for the income receivers. The part of personal income allocated to consumption must equal the supply of consumer goods in value if the price system is to be maintained. The total domestic savings is made to eqnal in value capital formation in fixed capital and inventories, the resources gap (and therefore net foreign assistance) being assnmed zero (for the end year and not to be confused with the gap for the entire plan perion, assumed positive). The receipts of the public authorities by way of direct taxes on income as well as in the form of indirect tax receipts is made to equal the current expenditures of the public authorities, the profits of government productive enterprises being treated as retained earnings of corporations and therefore merged with the savings generated in the sectors.[12]

4.6 The number of independent variables for which the system of equations (4.1) to (4.10) has to be solved may be reduced by taking into account the definitional relations among them and establishing by choice a few additional ones. These constitute a set of subsidiary equations, numbered (4.11) to (4.24). The first of them, (4.11), applies the tautology of the sum of all the consumer demands equalling the total value of consumption to the lowest α % of consumers. Relation (4.12) is a definition of the total household demand h for commodity i in terms of the demand function D_i and the distribution function F of consumers over per capita total consumption expenditure, C. P is the population in year T. (413) is a definitional identity for C

4.7 Relations (4.14) to (4.17) connect the levels of activity X_j, the quanta of output x_{ij} and the input combinations X_{ij}, L_{ij} and K_{ij} by the definitional functions $b_{ij}(X_j)$, $a_{ij}(X_j)$, $l_{ij}(X_j)$ and $k_{ij}(X_j)$. The functions being defined by the very relations (4.14) to (4.17) are perfectly general in character. In so far as these represent one possible set of quantitative relations between output and input determined by the technological conditions of production, they become identified as an element $Q : X$ of the set $(Q : X)$ of technological functions matrices defined in section 2.8.

Table showing equalities and inequalities to be solved in the central calculations

A. Objectives

(4.1) $D_i(C\alpha, p_1, p_2 \ldots\ldots p_n) = q_i \quad (i = 1, 2, \ldots\ldots\ldots n_o)$

(4.2) $g_i \geqslant g_i^o \quad (i = 1, 2, \ldots\ldots\ldots)$

(4.3) $2G\left(\frac{\rho_2}{\sqrt{2}}\right) - 1 = \theta \leqslant \theta^o$

(4.4) $z = y\phi$

(4.5) $\sum_{i=1}^{n} e_i p_i = \sum_{i=1}^{n} m_i p_{im} + m_o$

B. Feasibility Conditions

(4.6) $\sum_{j=1}^{N} (x^o_{ij} + x_{ij}) + m_i \geqslant \sum_{j=1}^{N} (X^o_{ij} + X_{ij}) + h_i + g_i + z_i + e_i + f_i \quad (i = 1, 2, \ldots\ldots\ldots n)$

(4.7) $\sum_{i=1}^{n} x_{ij} p_{id} = \sum_{i=1}^{n} x_{ij} p_i + y_j \quad (j = 1, 2, \ldots\ldots\ldots N)$

(4.8) $\sum_{j=1}^{N} (c^o_j + c_j) = \sum_{i=1}^{n} h_i p_i$

(4.9) $\sum_{j=1}^{N} (s^o_j + s_j) = \sum_{i=1}^{n} (z_i + f_i) p_i$

$$\left.\begin{array}{l}\sum_{j=1}^{N}(d_j+d^o_j)+\\ +\sum_{i=1}^{n}\sum_{j=1}^{n}(x^o_{ij}+x_{ij})\tau_i+\\ +\sum_{i=1}^{n}m_i\,\tau_{im}\end{array}\right\} = \sum_{i=1}^{n} g_i\,p_i \text{ where } \tau=p_i-p_{id} \text{ and } \tau_{im}=p_i-p_{im}. \tag{4.10}$$

C. Subsidiary Relations

(4.11) $C\alpha \quad = \sum_{i=1}^{n} p_i\ D_i(C\alpha,\ p_1,\ p_2 \ldots\ldots\ldots p_n)$

(4.12) $h_i \quad = P\int_0^1 D_i(C,\ p_1,\ p_2 \ldots\ldots\ldots p_n)\sqrt{\dfrac{C\alpha}{d\,F\,(C\,\rho_1,\ \rho_2)}}$

(4.13) $C_\alpha \quad = \int_0^\alpha C\ dF\ (C,\ \rho_1,\ \rho_2)$

(4.14) $x_{ij} \quad = b_{ij}\ (X_j)\ X_j$

(4.15) $X_{ij} \quad = a_{ij}\ (X_j)\ X_j$

(4.16) $L_{ij} \quad = l_{ij}\ (X_j)\ X_j$

(4.17) $K_{ij} \quad = k_{ij}(X_j)\ X_j$

(4.18) $y \quad = \sum_{j=1}^{N}(y^o_j+y_j) = \sum_{j=1}^{N}(c^o_j+c_j+s_j+d^o_j+d_j)$

(5.19) $y_j \quad = \sum_{s=1}^{u} L_{sj}\ w_s+K_j\pi$

(4.20) $s_j+s^o_j \quad = (y_j-d_j)\ v_j\ +\ (y^o_j-d^o_j)\ v^o_j$

(4.21) $\dfrac{z_i\ \ p_i(d)}{z} \quad = \dfrac{\sum_{j=1}^{N} K_{ij}\ \ p_i(d)}{\sum_{i=1}^{n}\sum_{j=1}^{N} K_{ij}\ \ p_i(d)}$

(4.22) $e_i \quad = e^o_i$

(4.23) $f_i \quad = \left\{ m_i+\sum_{j=1}^{N}(x^o_{ij}\ +\ x_{ij}) \right\}$

$$(4.24)\quad X^{o}_{N} + X_{N} = \sum_{i=1}^{n} m_i \left\{ p_i - p_i(m) - \tau_i(m) \right\}$$

$$= (s^{0}_{N} + s_{N})$$

$$(4.25)\quad p_i = p_i(d) + \tau_i$$

$$(4.26)\quad K_j = \sum_{i=1}^{n} K_{ij}\ p_i$$

$$(4.27)\quad z = \sum_{i=1}^{n} z_i\ p_i(d)$$

4.8 (4.18) is the national accounting identity of total income with its utilization in consumption, savings and direct tax payment. (4.19) is another identity, that of income generated in the new units of production and income received by wage and salary earners and the owners of capital, K_j being the value of capital defined by (2.26). (4.20) relates savings to disposable income through sector specific savings rates, v^0_j and V_j^c. Equations (4.21), (4.22), and (4.23) are introduced to eliminate the degrees of freedom associated with the variables z_i, e_i and f_i which represent three items of demand for commodity i, namely, demand arising from capital formation. export demand and inventory demand. Relation (4.21) stipulates that the proportionate contribution of commodity i to the value of fixed capital formation is the same in the terminal year of the plan as in the entire plan period[13]. (4.22) amounts to an admission of failure to treat export demand in any endogneous way : export projections e^0_j are to be worked out somehow or other and treated as exogenous variables. Relation (4.23) represents a simple decision to make an allowance of a given proportion β_j of the suyply of commodity i for inventory formation.

4.9 Considering that trade in imported goods involves purchase of goods from outside the economy, and considering that the ratio of domestic production price to import price is a crucial magnitude to be taken into account when decisions regarding import substitution have to be taken, we find it convenient to define a sector, the Nth one, as one devoted entirely to the import activity. This definition adds one more equation, namely (4.24). We assume for simplicity that this activity does not require any capital or current cost so that the entire income generated in the sector is saved.

4.10 (4.25), (4.26) and (4.27) are three definitional identities. The first relates with each other ex-factory prices, market prices (net of trade and transport margin) and indirect tax rates. The second defines value of capital stock in terms of the commodities going into the stock. The third is a definition of the value of capital formation in the end year in terms of the commodities going into it.

The Criterion Function

5.1 We shall now undertake a translation into the algebraic language of the criterion of efficiency we have chosen for our plan, namely the minimisation of foreign assistance. One may regard foreign assistance primarily as a problem of foreign exchange shortage or one of the shortage of capital resources. In India both the problems are acute but it appears to us that the shortage of capital resources is the more fundamental cause of our dependence or foreign assistance. In any case, in the plan calculations, estimates of the required volume of foreign assistance are based on the estimates of domestic capital resources and domestic capital requirments. We likewise choose to measure the volume of foreign assistance by the capital resources gap for the entire plan period. There are, however, some difficulties in giving algebraic expression to this resources gap in a fashion suitable for use in connection with the equation system (4.1) to (4.10).

5.2 The measurement of capital requirements does not give rise to any difficulty ; it can be simply written as :

$$(5.1) \quad \sum_{i=1}^{n} \sum_{j=1}^{N} K_{ij} \, p_i$$

involving variables belonging to the structures X and Λ. The measurement of the matching availability of domestic savings, however, involves variables referring to time points t intervening between O to T and therefore not covered by the structures X, Λ, and Δ, and not occuring in the equation system (4.1) to (4.10). Thus, the supply of savings during the entire plan period may be written as :

$$(5.2) \quad \sum_{t=1}^{T} \sum_{j=1}^{N} (s^{o}_{jt} + s_{jt})$$

where s^{o}_{jt} and s_{jt} have the same definitions for year t as s_j^{o} and s_j for year T. In order to reduce (5.2) to a form wherein only variables referring to time t=T are involved we take recourse to the stratagem of assuming that the growth of savings will follow the same path as output, so that we may write :

$$(5.3) \quad \sum_{t=1}^{T} \frac{s^o_{jt}}{s^o_{jT}} = r^o_j \quad \text{and} \quad \sum_{t=1}^{T} \frac{s_{jt}}{s_{jT}} = r_j$$

of course equivalence of (2.1) and (5.3) would only be valid when the price and taxes structure remains unchanged during the course of the plan period and when the new units set up during the period all have the same technological features. Using (5.3) we can write the criterion function as :

$$(5.4) \quad \lambda = \sum_{i=1}^{n} \sum_{j=1}^{N} K_{ij}\, p_i \; - \sum_{j=1}^{N} (s^o_j\, r^o_j + s_j\, r_j)$$

Computational Procedure

6.1 The structures X, Λ and $\triangle$ are to be worked out by the central planners by solving the equation system (4.1) to (4.10), subject to the criterion function (5.4). The equations, it must be recognised, are of a very complicated character, as activity levels (i.e. elements of X) prices (i.e. elements of Λ) and technology (i.e. element of the set Ω : X) are all variables in the equations as well as in the criterion function, occuring frequently in product relations with each other. There is obviously no possibility here of a straightforward application of the linear programming technique. If the latter has been extensively made use of in multisector models of growth, that has been so at the cost of assuming constant prices, unique technology with constants returns to scale and exogenously determined final demand.

6.2 We shall sketch below a sequence of computational steps that may be taken to solve the system when the functions $a_{ij}(X_j)$, $b_{ij}(X_j)$, $l_{ij}(X_j)$ and $k_{ij}(X_j)$ take given values a_{ij}, b_{ij}, l_{ij} and k_{ij}; that is, when the technological functions matrix Ω; X can be replaced by a technological coefficients matrix Ω in the equation system (4.1) to (4.10) aud when r_i takes a given value, The quantities x^o_{ij}, x^o_{ij}, L^o_{ij} and r^o_j referring to the base year stock of capital will be taken as known g^o_i, θ^o, m^o, $p_j(m)$, e^o_i and β_i will also be treated as given. We shall see in the section 8 how this assumption and this computational procedure may be integrated in a larger framewok of calculations.

Step 1 : Put $p_i = p^o_i$, the base year market price of commedity i, for $i = n_o + 1, \ldots\ldots n$ in equations (4.1) and (4.11) and solve for $p_1\, p_2 \ldots\ldots p_{n_o}$ and $C.\alpha$ Let the solutions be $p^1_1, p^1_2 \ldots\ldots p^1_{n_o}$ and $C^1.\alpha$ (We are assuming the existence of unique solutions).

Step 2 : C^1a may now be substituted in (4.13) which together with (4.3) may be solved for ρ_1 and ρ_2. Let the solution be ρ_1^1 and ρ_2^1.

Step 3. The first approximations ρ_1^1, ρ_2^1, p_1^1 $p_2^1 \ldots p^1_{n_o}$ and the base year prices $p^o_{n_o}+1 \ldots p^o_n$ may now be substituted for the respective variables in (4.12) to yield first approximations h_i^1 to h_i (i=1, 2, n).

Step 4 : Gross national income y and gross capital formation z, at factor cost, can now be estimated by using the formulae

$$(6{\cdot}1) \quad (1-\phi)y = P\int_o^1 C\, d\, F(C, \rho_1, \rho_2) + \sum_{i=1}^{n} g^o_i p^o_i(d) + m^o$$

$$(6.2) \quad \phi . y = z.$$

where $p_i^o(d)$ is the base year ex-faciory price of commodity i. Let the solutions be y^1 and z^1.

Step 5 : The flow of commodity i into capital formation can be estimated by using the formulae :

$$(6{\cdot}3) \quad z_i\, p_i^o(d) = \mu_i^o\, z^1$$

where μ_i^o is a proportion representing the share of commodity i in capital formation in the end year anticipated on the basis of past trends.

Step 6. Making use of the relation (4·2), the subsidiary relations (4.14), (4·15), (4.22) and (4·23), and substituting the first approximations h_i^1 and z_i^1 for h_i and z_i and the numbers a_{ij} and b_{ij} for the functions $a_{ij}(X_j)$ and $b_{ij}(X_j)$ we can now write (5·6) as :

$$(6.4) \quad \left(1-\beta_i\right)\left\{ m_i + \sum_{j=i}^{N}\left[x^o_{ij} + X_j b_{ij}\,(X_j)\right]\right\} > \sum_{j=1}^{N}\left\{ X^o_{ij} + X_j a_{ij}(X_j)\right\} + h_i^1 + g_i^o + z_i^1 + e_i^o$$

which can be regarded as a linear inequality in the variables X_j(j=1, 2,... N) and m_i (i=1, 2,...n). There are n inequalities and n+N variables and the problem can be tackled by the linear programming technique, using as objective function λ, now regarded as a linear function only in X_j and m_i by making the following substitutions in (5·4) :

$$(6.5) \quad K_{ij}p_i = X_j\, k_{ij}\, p_i (i=1, 2, \ldots n)$$

$$(6.6) \quad s_j^o = \left\{ \sum_{i=1}^{n} x_{ij}^o\, p_i^o\,(d) - \sum_{i=1}^{n} X_{ij}^o\, p_i \right\}(1-\delta^o_j)\, v^o_j$$

for j=1, 2,...N−1.

$$(6.7)\quad s_j = X_j\left\{\sum_{i=1}^{n} b_{ij}\, p_i^o\,(d) - \sum_{i=1}^{n} a_{ij}\, p_i\right\}(1-\delta_j^o)\, v_j$$

for $j=1, 2, \ldots N-1$.

$$(6.8)\quad s_N^o + s_N = \sum_{i=1}^{n} m_i\left\{p_i - p_i(m) - \tau_i^o)\right\}$$

$$(6.9)\quad p_i = p_i^1 \quad \text{for } i=1, 2, \ldots\ldots n_o$$
$$= p_i^o \quad \text{for } i=1, 2, \ldots\ldots n$$

where δ_j^o is the rate of direct taxation on income generated in sector j in the base year and $\tau_i^o(m)$ the base year rate of indirect taxation on imported good i. Let the solutions be X_j^1 and m_i^1, We shall also approximate to f_j by f_i^1 obtained by substituting X_j^1 and m_i^1 in (4·23).

Step 7. One may obtain first approximations K_j^1 to K_j by writing :

$$(6.10)\quad K_j^1 = X_j^1 \sum_{i=1}^{n^o} k_{ij}\, p_i^1 + X_{ij} \sum_{i=n_o+1}^{n} k_{ij}\, p_i^o$$

Step 8. One may now take up equations (4·7), (4·9), (4·10) together with (4·14) to (4·20), (4·25) and (4·26), substitute in them first approximations X_j^1, m_i^1, p_i^1 and K_{ij}^1 for X_j ($j=1, 2,\ldots\ldots N$), m_i ($i=1, 2,\ldots n$), p_i($i=1, 2,\ldots n_o$) and K_{ij}($j=1, 2,\ldots N$) and the base year wage rates w_s^o for w_s and obtain the following :

$$(6.11)\quad \sum_{i=1}^{n} b_{ij}\, p_i\,(d) \geqslant \sum_{i=1}^{n} a_{ij}\, p_i + \sum_{s=1}^{u} l_{sj}\, w_s^o + K_j^1\pi$$

$$(6.12)\quad s_j^o = \left\{\sum_{i=1}^{n} x_{ij}^o\, p_i\,(d) - \sum_{i=1}^{n} X_{ij}^o\, p_i - d_j^o\right\} v_j^o$$

$(j=1, 2,\ldots N-1)$

$$(6.13)\quad s_j = \left\{X_j^1 \sum_{i=1}^{n} b_{ij}\, p_i\,(d) - X_j^1 \sum_{i=1}^{n} a_{ij}\, p_i - d_j\right\} v_j$$

$(j=1, 2,\ldots N-1)$

$$(6.14)\quad s_N^o + s_N = \sum_{i=1}^{n} m_i^1\left\{p_i - p_i(m) - \tau_i(m)\right\} - (d_N^o + d_N)$$

$$(6.15)\quad \sum_{j=1}^{n} (s_i^o + s_j) = \sum_{i=1}^{n} (z_i^o + f_i^1)\, p_i$$

$$(6.16)\quad \sum_{i=1}^{n} g_i^o\, p_i = \Sigma^N (d_j^o + d_j) + \sum_{j=1}^{N}\sum_{i=1}^{n} (x_{ij}^o + X_j^1 b_{ij})\tau_i$$

$$+ \sum_{i=1}^{n} m_i^1\, \tau_i(m)$$

(6.17) $p_i \qquad = p_i^1$ for $i = 1, 2, \ldots n_o$

This may be treated as a linear system of equations in the market prices p_i $(i = n_0 + 1, \ldots n)$, the cost prices and the indirect tax rates, $p_i(d)$, τ_i and $\tau_i(m)$ $(i = 1, 2, \ldots n)$, the quanta of direct taxes d_j^o and d_j $(j = 1, 2, \ldots N)$, and the rate of return to capital π. This may be treated by the linear programming method, using once more λ as the objective function, now to be regarded as a linear function in these variables by writing s_j^o and s_j as in (6.12), (6.13) aad (6.14) and

$$\sum_{i=1}^{n} K_{ij}\, p_i \text{ as :}$$

$$(6.18) \quad \sum_{i=1}^{n_0} X_j^1\, b_{ij}\, p_i^1 + \sum_{i=n_o+1}^{n} X_j^1\, b_{ij}\, p_i$$

6.3. The eight steps sketched above complete a single round of the iteration to be used in the central calculations. The first approximations arrived at in the first round are to be used in the second round of iteration in every place where in the first round base year figures were used. Thus p_i^1 are to be used for market prices p_i $(i = \overline{n_0 + 1_0} \ldots n($ in steps 1, 3, 6 and 7 in the second round whereas in the first round use was made of the base year prices p_i^o. Again, in steps 4 and 5 the base year cost prices $p_i^o(d)$ are used in the first round ; in the second they may be substituted by the corresponding first approximations $p_i^1(d)$. In the first round a coefficient μ_i^o based on past trends in used to represent the contribution of commodity i to capital formation ; this may now be replaced by the following estimate based on the results of the first round iteration.

$$(6.19) \quad \mu^1_i = \frac{\sum_{j=1}^{N} k_{ij}\, p_i^1\,(d)\, X_j^1}{\sum_{i=1}^{n} \sum_{j=1}^{N} k_{ij}\, p_i^1\,(d)\, X_j^1}$$

Decentralised Technological Choice

7.1. The equation system (4.1) to (4.10) presented in section 4 involves a set of technological functions represented by the matrix $\Omega : X$. In section 6 we have outlined a computational procedure for solving the system when the matrix of $\Omega : X$ can be replaced by a matrix of numbers, Ω ; that is, when the functions b_{ij}, a_{ij}, $l_{ij}(X)_j$ and $k_{ij}(X_j)$ can be replaced by the numbers b_{ij}, a_{ij}, l_{ij}, and k_{ij}. This replacement assumes knowledge of the functional forms (and of course also of the values for the activity levels). Our computational schemes are thus based on an apriori and tentative solution of the problem of technological choice. In case there were only few independent arrays in the set of the technological arrays (Ω_j, X_j) for sector j, the problem of technological choice and that of setting the production and price pattern could have been solved simultaneously in the central calculations themselves by incorporating all the independent arrays of $(\Omega_j : X_j)$ in the matrix $\Omega : X$ as so many columns. This would have been in line with the activity analysis approach. But we are of the view that the technological possibilities open to any sector are such that if one were to enumerate all the possible independent arrays of technological functions (if they are denumerable at all) and provide for each a column in the matrix $\Omega : X$ there might have to be an extremely large number of columns, even an infinity of them.[14] It is this that has prompted us to adopt an approach wherein the central calculations are carried out in terms of a matrix $\Omega : X$ which has only one column for the jth sector, whereas the choice of the array $\Omega_j : X_j$ to be used as column in the matrix $\Omega : X$ from among the set of possible alternative arrays $(\Omega_j : X_j)$ is made in a decentralised fashion by the sectoral planners. We shall now discuss the selection procedure to be used by the sectoral planners.

7.2 Posed in its utmost generality, the problem, of course, is of applying the criterion of efficiency chosen for the plan to the problem of selecting a technological functions matrix Ω ; X from the set of possible alternatives ($(\Omega : X)$) ; that is, to regard λ as written in (5.4) as a function of the elements of the matrix $\Omega : \lambda$, and consider its minimisation for for variation among the elements of the set ($(\Omega : X)$). In the decentralised approach we require criteria λj for the different sectors j which, when simultaneously and decentrally applied, would lead to the same result as would a centrally operated application of λ, if it were at all practicable. It is to be understood that λj has not necessarily the same economic significance for sector j as λ has for the economy as a whole. Thus, if it stands, as in our model, for the capital resources gap for the economy, λj *does not* stand for the same gap for sector j. The criterion, λj is a derivative of the criterion λ for use in sector j and $\sum_{j=1}^{N} \lambda j$ is not neccessarily equal to λ.

7.3 Considering that the sectoral planners are called upon to make their technological decisions in the context of preparing sectoral plans for the realisation of tentative activity targets handed over to them by the central planners, we can reduce the degree of generality of the problem by posing it now not as one of choice of an array of technological functions from the set ((Ωj : Xj)), but of a vector of coefficients from the set ((Ωj)), where Ωj is obtained by giving Xj in Ωj : Xj its target value. We can derive the criterion λj as follows. We have sketched a computational scheme in section 6 for solv.ng for the elements of X and Λ, for givenΩ. This mean that the elements of X and Λ can all be expressed in the ultimate analysis as functions of the elements of Ω. They may thus be eliminated from the expression (5.4), leaving an expression $\lambda(\Omega)$ only in the elements of Ω. When all the elements of the matrix excepting those of column j are treated as given, we derive the function $\lambda(\Omega j)$. This, to be minimised for variations in the arrays belonging to the set ((Ωj)) may be treated as the criterion λj for sector j. In Appendix 2 we illlustrate the numerical evaluation of λj with the help of a two sector model.

7.4 There will be certain sectors j in the economy for which the alternative technologies available are such that the elements of the set ((Ωj)) may be related to each other by some mathematical function or functions. For such sectors it should be possible to reduce the minimisation of $\lambda(\Omega j)$ for variations in Ωj to an exercise in mathematical programming.

7.5 There will be certain other sectors j for which it is not possible to identify or establish any mathematical functional relation as among the arrays of the set ((Ωj)). For such sectors no further advance is possible on purely mathematical lines. The criterion λj will have to be used such sectors as a practical guide to decision making. For such sectors the problem of technology choice is inseparable from the task of actually preparing the development projects πj. We can conceive of a set of alternative projects ((πj)) for sector j each designed to attain the end year activity target and they would imply a set of alternative technogies represented by the matrix ((Ωj)) of technological coefficients valid for the end year level of activity. The sectoral planners will not of course work out separately all the possible alternative projects ((υj)) and retain the one which would imply such a Ωj as to minimise Λ (Ωj). They would take a guess at the right technological choices and prepare a project $\pi^1 j$ implying the coefficients $\Omega^1 j$. They would then try out several technnological changes in the project generating the alternative projects $\pi^2 j$, $\pi^3 j$... etc. such that if the sequence of implied technological coefficients arrays be $\Omega^2 j$, $\Omega^3 j$...... etc. then the sequence $\lambda(\Omega^1 j)$, $\lambda(\Omega^2 j)$, $\lambda(\Omega^3 j$............is a decreasing one. The sectoral planners will stop the process when they find that is not possible to extend the decreasing sequence beyond a certain point.

The Planning Process

8.1 It is now possible for us to give an integrated picture of the formalised planning process incorporating both central calculations and decentralised technological decision making. The process will consist of a series of cycles, each comprising three operations. The three operation in the νth cycle are described in the next paragraph.

8.2 *Operation 1 ;* The central planners pass on to the sectoral planners of sector j a price structure $\Lambda(\nu 1)$, a tentative turget xj (x1) for activity level in the sector based on the structure X (ν) and a criterion λj (ν-1). The structure Λ (ν-1) and X (ν-1) and criterion λj (ν-1) are the products of the previous cycle's work.

Operation 2 : The sectooral planners of sector j draw up a development project πj (ν) for sector j which lays down a path of growth of output Xj_t (ν) in sector j (for t = 1, 2..............Tj) and thus defines a technological functions array $\Omega\nu j$: Xj consisting of the technological curves $b\alpha ij$ (Xj), $a\nu ij$ (Xj) $l\nu_s j$ (Xj) and $k\nu ij$ (Xj) as well as growth coefficient $r\nu j$ and incidentally a vector $\Omega\nu j$: Xj (s-1) of constant coefficients valid for the activity level Xj (ν-1)[15]. The choice of $\Omega\nu j$: Xj is made from the set ((Ωj : Xj)) of available alternative technological functions matrices in such a way that when the vector $\Omega\nu j$: Xj (ν-1) is substituted for Ωj in the criterion λ (Ωj), the latter taken the minimal value for variations in Ωj : Xj (ν-1).

Operation 3 : The central planners solve equations (1) to (10) by using $\Omega\nu j$: Xj (ν-1) for Ωj : Xj and $r\nu j$ for rj ; that is, substituting in the equations for aij (Xj), bij (Xj), lsj (Xj) and kij (Xj) their value at Xj= Xj (ν-1) and putting rj=$r\nu j$ in the criterion function 5.4. The results are a revised production structure X (ν) and revised price structure $\Lambda(\nu)$. Cycle $\nu+1$ of work can now be started off with the help of these two structures.

8.3 Before the first cycle of work can start, there has to be a preliminary round of work consisting of only one operation, namely calculation by the central planners of preliminary structures X (o) and Λ (o) by substituting for bij (Xj), aij (Xj), $l_s j$ (Xj) and kij (Xj) the constant coefficients $b^\circ ij$, $a^\circ ij$, $l^\circ_s j$ and $k^\circ ij$ estimated on the basis of statistical data relating to the base year and which will thus be the same as those holding for the units of production surviving from the pre-plan period. Thus the coefficients $b^\circ ij$ and $o^\circ ij$, could be estimated for the Indian manufacturing industries on the basis of the reports of the Annual Survey of Industries and the coefficients $k^\circ ij$ could be estimated on the basis of project reports of the Planning Commission. Usual econometric exercises with growth of planning models make use only of such coefficients and thus correspond only to the preliminary round of calculations in our scheme of the formalised planning process.

8·4 In the following we give a diagrammatic representation of the process at work. Let us consider sector j and the input of commodity i into sector j. In the zero cycle, we start with an aggregative statistical estimate of the input coefficient of i into j, namely b^o_{ij}, assumed constant for all levels of x_j. This defines a straight line AB in Diagram 1 horizontal

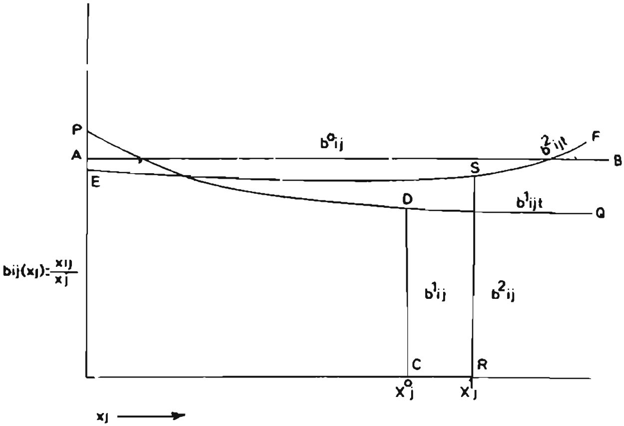

Diagram 1

to the x-axis which measures output level. Using b^o_{ij} we solve a set of equations and derive a preliminary estimate of the output level of sector j, X^o_j, This is handed over to the sectoral planners and the optimal project π_j^1 they prepare define in diagram 2 the curves OM measuring output X^1_{ijt} against t and ON measuring input X_{ij}^1 against t. With the help of these two curves we draw the curve PQ representing $b^1_{ijt} = \frac{X^1_{ijt}}{X^1_{jt}}$ in diagram 1. The curve is read off at X^o_j and let the ordinate CD measure upto b^1_{ij}. In the next central calculations, b^o_{ij} is replaced by b^1_{ij} yielding a revised approximation to the output level, X^1_j. This is now handed over to the sectoral planners who now prepare a second optimal project π_j^2 ; and thus define the curves OR and OS in Diagram 2 representing X^2_{jt} and X^2_{ijt}. With the help of these two curves we draw in diagram 1 the curve EF representing $b^2_{ijt} = \frac{X^2_{ijt}}{X^2_{jt}}$. The curve is read-off at X^1_j (the ordinate RS) yielding the estimate b^2_{ij} to be made use of in the third cycle of operations.

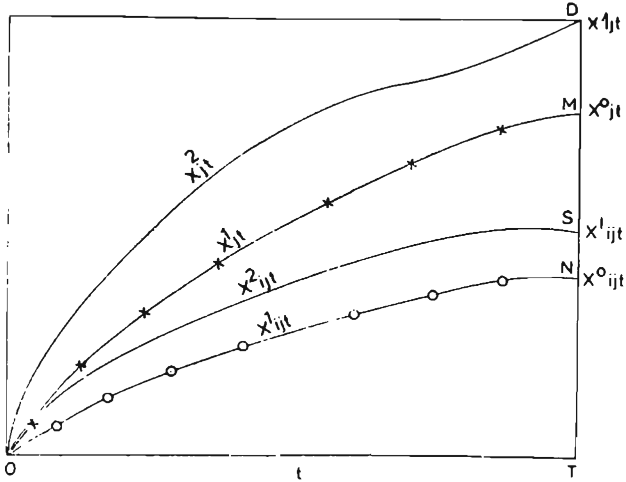

Diagram 2

CONCLUSION

What we have achieved in the course of the paper is a mathematical rationalisation of the decentralised process of planning as could be practised in India, given the organisational structure of planning in India. We have used the mathematical language to bring out the inherent logical and quantitative interdependences and sketched possible sequences of computational programmes. We have, however, not considered the more essentially mathematical problems of the existence of solutions to equations and the convergence of decision sequences. Important as these problems are, they do not constitute the central problem we set to ourselves. We may, however, state that we have been considering a situation where the production functions holding for a sector are not necessarily linear, are not known and are not such as can be generated by combinations of finite number of independent or basic activities. As such results already obtained by other writers in the field of decentralised linear and non-linear programming do not seem to be directly applicable to our problem. The mathematical theorems and proofs required to give completion to the frame presented in this prper are currently receiving the attention of the writer.

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Notes

1. There is of course the growing literature on decentralised decision making within the purview of central calculations. The early works of Lange, Taylor, Dickinson and the more recent works of Kantorovitch, Dantzig and Wolfe, Malinvaud, and Arrow and Hurwicz belong to this field. The present paper has not been written after fully taking into account all the results arrived at by these different workers. It is the presumption of the present writer that the problem of decentralised planning has not been posed by any one of these authors in the generality in which it has been posed here, and none of the results already published are of direct relevance to the problem as presented in this paper.

2. Thus the convergence of the decentralised computational technique presented by Dantzig and Wolfe (1) is based on the assumption of there being for each sector a finite number of basic technologies each representable by a vector of fixed co-efficients.

3. Thus Stone and Brown (2) in their computable growth model assume technological coefficients to follow known functions of time.

4. Three prices are distinguished for each commodity : an import price; a domestic cost price or ex-factory price so determined as to ensure payment of wages at certain specified money rates and a certain rate of return to capital; and a market price so determined as to ensure the equality of demand and supply of consumer goods. The difference between the two prices is maintained by a system of indirect taxes and subsidies, the rates of which are such as to satisfy the needs of balancing receipts and disbursements of the government sector.

5. The quantities X^o_{jt}, X^o_{ijt} x_{ijt} and L^o_{ijt} would correspond to full capacity utilisation of the stock of capital in sector j surviving from before the beginning of the plan. The inequality $X^o_{jt} \leqslant X^o_{jo}$ follows from the assumption that this stock is not replaced. These quantities are non-variables for any given t as the maximum scale of output and the technology are both non-variables for the surviving stock of capital. It is, however, not necessary to assume full capacity utilization of the surviving stock of capital. We can treat x^o_{ijt} etc as non-variables and corresponding to full capacity utilisation of the surviving stock of capital by interpreting X_{jt}, X_{ijt}, x_{ijt}, L_{ijt} etc. as the outputs and inputs of the new units *net* of the adjustments due to less than full capacity utilization of the old units.

6. The distinctions between c^o_j and c_j, s^o_j and s_j, d^o_j and d_j etc are made on the implicit assumption that the propensities to save and consume would be different as between the old and new units of the same sector due to differences in the distribution of the income flows y^o_j and y_j. The average saving rates v^o_j and v_j are also distinguished for the same reason. The distribution of the income flows from the new units being dependent on technological choice s_j, c_j, d_j, and v_j are to be treated as variables whereas s^o_j, c^o_j, d^o_j and v^o_j would either be non-variables (if full capacity utilization is assumed) or at least may be treated so (as explained in note 5 for X^o_{jt}, X^o_{ijt}, etc.)

7. $b_{ij}(X_j)$, $a_{ij}(X_j)$, $l_{ij}(X_j)$ and $k_{ij}(X_j)$ are independent of time t and are functions only of the scale of output, X_j. They have been derived, it is true, from observations $b_{ijt}(X_{jt})$, $a_{ijt}(X_{jt})$, $l_{ijt}(X_{jt})$ and $k_{ijt}(X_{jt})$ over time, but time only gives rise to scale change and not

technological change. Technology for the new units is chosen once for all, so that there is no change in technology over time within the planning horizon.

8. It is to be doubted whether a strict balance between production and demand for each individual year would be consistent with optimality and a long period.

9. It is to be noted that the inequality objective is stated in terms of the distribution of consumption expenditure (net of indirect taxes) rather than that of income. The emphasis thus is on the reduction of inequality in the standards of living.

10. The log-normal law is found to give good fit to National Sample Survey data on the distribution of per capita expenditure.

11. It is to be noted that the price equation 4.7 refers only to the new units for which capital stock is evaluated by taking the market prices of the commodities going into capital formation. This is a means deliberately adopted for by-passing the problem of evaluating the stock of capital existing from before the plan period. However, if the rate of return to profit thus determined is considered to be applicable to the old stock of capital as well then a mode of evaluation of the old stock of capital is achieved as a by-product. Thus, we may write :

$$\sum_{i=1}^{(N-1)} x^o_{ij} \; p^o_i(d) = \sum_{i=1}^{n} X^o_{ij} \, p_i + \sum_{s=1}^{u} L^o_{sj} w_s + K^o_j \, \pi$$

and substitute for the prices and the profit rate their solution values and treat that value of K^o_j which equalises the two sides as the value of the stock of capital surviving from before the beginning of the plan.

12. We are ignoring, for the sake of simplicity, any surplus of the government current account going to investment.

13. By this means we avoid the bringing in of the pattern of growth after the plan period to determine the structure of capital formation during the end year of the plan.

14. Take a sector like agriculture. The proportions of output to such inputs as fertilizers, seeds, labour, equipment, etc. may be made to vary continuously and it is not at all certain that all the possibilities can be generated by taking combinations of a few basic activities.

Appendix : 1

Glossary of Symbols

1.	T	:	length of the plan period ; also the end year of the plan.
2.	P	:	Population in end year of plan.
3.	N	:	number of sectors.
	n	:	number of commodities.
	n_o	:	number of essential consumer commodities.
	u	:	number of categories of labour.
4.	X_j	:	level of activity in sector j in new units.
5.	X^o_j	:	level of activity in sector j in old units.
6.	x_{ij}	:	output of commodity i in sector j in new units.
7.	x^o_{ij}	:	output of commodity i in sector j in old units.
8.	X_{ij}	:	input of commodity i in sector j in new units.
9.	X^o_{ij}	:	input of commodity i in sector j in old units.
10.	K_{ij}	:	quantity of commodity i in the capital stock. formed in sector j during the plan period.
11.	h_i	:	quantity of commodity i going to household consumption.
12.	g_i	:	quantity of commodity i going to government consumption.
13.	z_i	:	quantity of commodity i going to capital formation.
14.	e_i	:	quantity of commodity i going to exports.
15.	m_i	:	quantity of commodity i going to imports.
16.	f_i	:	quantity of commodity i going to stock formation.
17.	p_i (d)	:	ex-factory price of commodity i.
18.	p_i	:	market price of commodity i.
19.	p_i (m)	:	import price of commodity i.
20.	τ_i	:	indirect tax on commodity i.
21.	τ_i (m)	:	import tax on commodity i.
22.	w_s	:	wage rate for the sth category of labour.
23.	π	:	rate of return to capital.
24.	L^o_{sj}	:	quantity of labour of sth category employed in seetor j in the old units.
25.	L_{sj}	:	quantity of labour of sth category employed in sector j in the new units.

26. y^o_j : income generated in sector j in the old units.

27. y_j : income generated in sector j in the new units.

28. c^o_j : part of y^o_j consumed by persons.

29. c_j : part of y_j consumed by person.

30. s^o_j : part of y^o_j saved.

31. s_j : part of y_j saved.

32. d^o_j : direct tax paid out of y^o_j.

33. d_j : direct tax paid out of y_j.

34. X : set the elements of which are (4) to (16).

35. Λ : set the elements of which are (17) to (23).

36. Δ : set the elements of which are (24) to (31).

37. π_j : project for the realisation of the target X_j for sector j.

38. $b_{ij}(X_j)$: production of commodity i per unit of output in sector j, at activity ievel X_j.

39. $a_{ij}(X_j)$: input of commodity i per unit of output in sector j at activity level X_j.

40. $l_{sj}(X_j)$: input of labour of category s per unit of output in sector j at activity level X_j.

41. $k_{ij}(X_j)$: input of commodity i into capital formation per unit of output in sector j at activity level X_j.

42. $\Omega : X_j$: array the elements of which are (38) to (41) for all i and a given j, representing technology for sector j.

43. $\Omega : X$: matrix arrangement of (38) to (41) for all i and all j representing technology for the entire economy.

44. $b_{ij}, a_{ij}, l_{sj}, k_{ij}$: values of fuuctions $b_{ij}(X_j)$, $a_{ij}(X_j)$, $l_{sj}(X_j)$ and $k_{ij}(X_j)$ when X_j takes a particular given value.

45. Ω_j : array the elements of which are b_{ij}, a_{ij}, l_{sj} and k_{ij} for all values of i and a given value of j.

46. Ω : matrix arrangements of the element of b_{ij}, a_{ij}, l_{sj} and k_{ij} for all i and all j.

47. r^o_j : growth coefficient representing ratio of cumulative output in sector j from old units to end year output from the same.

48. r_j : the same for new units for sector j.

49. θ : Lorentz ratio for the distribution of persons according to consumption expenditure during the end year of the plan.

50. θ^o : Lorentz ratio for the distribution of persons according to consumption expenditure during the base year.

51. ψ : ratio of gross fixed capital formation to gross national product.

52. C	:	total consumption expenditure net of indirect tax for a person.
53. $C\alpha$	:	average consumption expenditure of lowest $\alpha\%$ of the population.
54. $F(C, \rho_1, \rho_2)$	:	log normal distribution of C with mean ρ_1 and variance ρ_2.
55. m_o	:	payments to be made in foreign exchange over and above those for imports.
56. β_i	:	proportion of commodity i produced in the plan year to be added to stocks.
57. e_i^o	:	export projection for commodity i.
58. z	:	value of gross fixed capital formation in end year of the plan.
59. $D_i (C, p_1, p_2 \ldots\ldots p_n)$	:	demand function for commodity i in per capita total expenditure and the market prices.
60. q_i	:	minimum guaranteed per capita consumption of essential commodity i.
61. K_j	:	value of the capital stock added to that of sector j during the plan period.
62. μ_i^o	:	proportion representing the share of commodity i in capital formation in the end year of plan.
63. λ	:	criterion function in the elements of x, Λ and Δ for central calculations representing gap in the requirement and availability of capital resources.
64. λ_j	:	criterion function in the elements of the array Ω for use by the sectoral planners for making technological choice.
65. y	:	gross national product at current prices.
66. v_i	:	the rate of savings of the income of sector i net of taxes valid for new units.
67. v_i^*	:	the same valid for old units.

Appendix : 2

Use of Criterion λ_j : A Numerical Example

We shall consider a numerical illustrative example of the use of the criterion λ_j with the help of a two sector model. Let the targets set by the central planners for the two sectors be 307.7 and 284.6 corresponding to the solution of the following programming problem.

$$\text{(A.2.1)}\quad \begin{aligned} X_1 - a_{12}X_2 + m_1 &= 100 \\ -X_1a_{21} + X_2 + m_2 &= 100 \\ m_1 + m_2 &= 20 \end{aligned}$$

$$\lambda = X_1\gamma_1 + X_2\gamma_2 = X_1[k_1-(1-a_{21})r_1v_1] + X_2[k_2-(1-a_{12})r_2v_2] = \min$$

The final demands of 100 each and the availability of foreign exchange, 20, are functions of the price system determined by the central calculation; we are however not concerned here with the price calculations. Let the preliminary estimates of the parameters of the model made use of by the central planners be :

	sector 1	sector 2
input coefficients	$a_{21} = 0.6$	$a_{12} = 0.8$
capital coefficients	$k_1 = 2.40$	$k_2 = 1.60$
saving rates	$v_1 = 0.20$	$v_2 = 0.10$
growth coefficients	$r_1 = 4$	$r_2 = 7$
weight in objective function	$\gamma_1 = 2.08$	$\gamma_2 = 1.46$

γ_1 being greater than γ_2, the solution will correspond to $m_2=0$ and λ will take the minimal value of 1055. The expression for λ in terms of the above coefficients, as long as $\gamma_1 > \gamma_2$; is

$$\text{(A. 2.2)}\quad \lambda = \frac{80+100a_{12}}{1-a_{12}.a_{21}}\Big[k_1-(1-a_{21})v_1r_1\Big] + \frac{100+80\,a_{21}}{1-a_{12}\,a_{21}}\Big[k_2-(1-a_{12})\,v_2r_2\Big]$$

The criterion for sector 1 is therefore :

$$\text{(A. 2.3)}\quad \lambda = \frac{160\,[k-(1-a_{21})\,v_1\,r_1]+(100+80\,a_{21})\,1.46}{1-0.8\,a_{21}}$$

and that for sector 2 :

$$\text{(A. 2.4)}\quad \lambda = \frac{148\,[\,k_2-(1-a_{12})\,v_2.r_2\,] + (80+100\,a_{12})\,2.08}{1-0.6\,a_{12}}$$

We shall assume that the planners of sector 1 start out to achieve the output target of 307.7 by adopting minimum cost techniques.

Let the quantitative specification of the first project they prepare be as follows :

output	X_1	=	307.7
input	X^1_{21}	=	170
fixed capital	K^1_1	=	900
cumulated output over the plan period	$\sum_t X^1_{1t}$	=	1000

so that they imply the following norms :

input coefficient	a^1_{21} (307.7)	=	0.55

capital coefficient	k^1_1 (307.7)	=	2.92
growth coefficient	r^1_1	=	3.2

Let the project also determine the following rate of savings applicable o income generated in sector 1.

rate of savings	v^1_1	=	0.20
so that the weight will be	γ^1_1	=	2.63

The value of λ_1, corresponding to this project thus will

$$\lambda^1_1 = 1126.9$$

Finding that by choosing the minimum cost techniques one has actually enlarged the capital resources gap, the sectoral planners may now try out the other extreme alternative, namely the combination of techniques that would minimise fixed capital requirements. Let the second project corresponding to this alternative have the following specifications :

input	X^2_{21}	=	231
fixed capital	K^2_1	=	675
cumulated output over the plan period	$\Sigma_t X^2_{1t}$	=	1350
input coefficient	a^2_{21} (307.7)	=	0.75
capital coefficient	k^2_1 (307.7)	=	2.19
growth coefficients	r^2_1	=	4.4
rate of savings	v^2_1	=	0.20
	γ^2_1	=	1.97

The criterion λ_1 will however take an even higher value :

$$\lambda^2_1 = 1372$$

The sectoral planners will have now to search for technological combinations intermediate between the extremes of minimum current cost techniques and the minimum fixed capital techniques.

Appendix : 3

An Alternative Approach : The Shortest Time Path

A.3.1 We have, all through the paper assumed a given fixed internal T for the plan period and treated the gap in the demand and supply of capital resources as a variable to be minimised. However, this gap may be assumed as variable only if net foreign assistance is treated as variable. If however net foreign assistance is either nil or equal to a given quantity then the system of equations as presented in section 4 will be altered in that there will be an additional constraint $\lambda = \lambda_0$ where λ is as in (5.4) and λ_0 is a number representing net foreign assistance. This additional constraint however makes the problem uninteresting as long as T is kept fixed. That is because, for a given T, the minimum value of λ will be either greater than or less than λ_o. In the former case the equality $\lambda = \lambda_o$ cannot be realised which would mean that the objectives set for the plan just cannot be achieved within the period T. If on the other hand the minimum value of λ is less than λ_o, it would mean that the objectives set for the plan would be realised with less of capital resources than what can be generated during the period. This may be interpreted as indicating that one could have set out to achieve more ambitious objectives.

A. 3.2. The objective may be revised upwards in one of two ways, either one keeps the length of the plan period the same and changes the state of things aimed at for the end of the period, or one tries to shorten the time path to reach a desired state of things. Given the nature of our objectives, namely the transition of the economy from a given lower stage of growth to a next higher stage of growth, it seems to us more appropriate to accept this second operative interpretation of the minimal value of λ being smaller than λ_o.

A.3.3. An alternative formulation of the criterion of efficiency for a longterm plan may then be the minimisation of the time path to reach a state of the economy wherein the five objectives stated in section 3 are realised. Stated mathematically, the problem is of so solving equation (4.1) — (4.10) together with constraint $\lambda = \lambda_o$ as to minimise T. T enters the equation system explicitly in the expression (5.4) for λ and implicitly in equation (4.2) and subsidiary equation (4.12), g^o_i and P being functions of T.

A. 3.4. Considering the way T occurs in the equation system, it is difficult to think of any mathematical technique for the solution of the time minimisation problem except, of course, that of trial and error. Apractical course would be as follows. One would start with a trial value T^1 and prepare a long term plan by minimising λ. This minimum value, say λ^1, will be either greater than or less than λ_o. If $\lambda^1 < \lambda_o$ then a trial will be made with a $T^2 < T^1$. If on the other hand $\lambda^1 > \lambda_o$ one will try $T^2 > T^1$. Proceeding in this manner, it will be possible to generate a sequence T^1, T^2, T^3............such that the corresponding sequence of minimal values λ^1, λ^2, λ^3,............would converge to λ_0.