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Economies of Scale in Household Consumption: An Application of Indirect Addilog Engel Model

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1. *Introduction*

In most of the earlier work done on Engel curve analysis, per capita household expenditure on a specific item has been related to per capital household total expenditure (used as a proxy for income). As pointed out by Prais and Houthakker (1971), this kind of formulation tacitly assumes that there are no economies or diseconomies of scale in household consumption in the sense that an equi-proportionate change in household total expenditure (or household income) and household size would lead to the proportionate change in household expenditure on specific items to the same extent. There have been many studies[1] in the past which have brought to light the limitations of this assumption. They have considered explicitly the effect of household size in addition to that of household income in the estimation of Engel functions and examined the possibility of economies (or diseconomies) of scale in household consumption. In almost all these studies the well-known Cobb-Douglas type (or log-linear form) of Engel function has been employed which, however, has two limitations. First it implies constant partial elasticities with respect to household income and household size which means the economies scale factor remains constant irrespective of the level of household income and household size, and secondly, it does not possess the property of aggregation across items of expenditure.

The present study considers a system of additive Engel functions [see Jain (1978) also] which is free from both the limitations of the log-linear specification. We use this system, referred as the modified addilog model (MAM for short), to examine the possibility of economies (diseconomies) of scale in household consumption. The modified addilog model and the method of estimating its parameters are described briefly in Section 2. Here we also present the expressions for partial elasticities with respect to household income and household size along with their properties and

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[1]See Tobin (1950, Houthakker (1957), Crocket (1960), Liviaton (1964). Prais and Houthakker (1971) and Iyengar, Jain and Srinivasan (1975).

discuss how far it is possible to examine the existence of economies (diseconomics) of scale in household consumption based on the modified addilog model. The modified model is estimated by employing cross-section data on consumer expenditure with individual households as units of observation (Section 3.1). Discussion of the results relating overall and specific economies (diseconomies) of scale is taken up in Section 3.2. Conclusions and the limitations of the present study are indicated in Section 4.

II. *Modified Indirect Addilog System*

2.1. Indirect addilog model with household size effect. The conventional per capita indirect addilog model, developed originally by Leser (1941) and then by Houthakker (1960), is based on the classical demand theory with individual consumers as the relevant decision units. Following Barten[2] (1964), Muellbaue (1974) incorporated the effect of household composition in the indirect addilog model by taking the indirect utility function for a household as

$$V[\, E/m_1 p_1, \ldots, E/m_K p_K \,] = \sum_{i=1}^{K} (\bar{a}_i/b_i)\,(E/m_i p_i)^{b_i} \tag{2.1.1}$$

and obtained a new system of demand functions represented by

$$q_i = (\bar{a}_i E^{1+b_i} m_i^{-b_i} p_i^{-b_i-1}) \Big/ \Big(\sum_{j=1}^{K} \bar{a}_j E^{b_j} m_j^{-b_j} p_j^{-b_j}\Big) \text{ for } i = 1, \ldots, K \tag{2.1.2}$$

where p_i and q_i are the price and the quantity consumed of commodity i, E[3] the household total outlay, and m_i the total number of equivalent standard consumers with respect to any commodity 'i'. In order to ensure $q_i > 0$ and compliance with the second-order conditions, it is required that $\bar{a}_i \geq 0$ and $b_i > -1$ for $i = 1, \ldots, K$, respectively [see Blokland and Somermayer (1970)]. As for any commodity z, m_i is a function of the number of household members per age-sex class, it involves at least G specific scale coefficient parameters if there are G age-sex classes. Therefore, (2.1.2) will contain K times (G+2) parameters and this renders the estimation of (2.1.2) too much complicated. Moreover, such a model is not

[2]Bertan regarded the joint family to be a more relevent decision-making unit for spending the family total income on various items of consumption than (any of) its members separately.

[3]Hereafter named 'household income' for the sake of brevity.

identified in a single cross-section of families as pointed out by Muellbauer (1975).

In view of these difficulties, a simplified version of (2.1.2) is suggested in the present study which follows from (2.1.2) on replacing $m_i^{-b_i}$ by N^{c_i}, N being the household size. This version in a cross-sectional situation where prices of the different commodities can be treated as constant, is

$$(2.1.3)\quad e_i = a_i E^{1+b_i} N^{c_i} \Big/ \sum_{j=i}^{K} a_j E^{b_j} N^{c_j} \quad (i = 1, \ldots, K)$$

We shall call (2.1.3) a 'modified system of indirect addilog Engel Functions' and refer to it, for the sake of brevity, the 'modified addilog model' in our later discussions where $a_i = \bar{a}_i p_i^{-b_i}$ and e_i is the household expenditure on the i-th item. The parameters c_i's may be negative, zero and positive.

It may be worth noting that when as a particular case c_i is equal to $-b_i$ for all $i = 1, \ldots, K$, then model (2.1.3) corresponds to the conventional per capita indirect addilog model which also follows from (2.1.2) by taking $m_i = N$ for all $i = 1, \ldots, K$. As parameters c_i's in (2.1.3) can assume any real values and which in turn allows the effect of household consumption to vary across different commodities, the model (2.1.3) is an improved version of the conventional per capita indirect addilog model. An interesting feature of this model relevant to the present study is that it gives rise to the variable economies scale factor which varies with the level of household income and household size, whereas earlier studies (mentioned in footnote 1) have assumed a constancy of the economies scale factor.

The model (2.1.3) can also be obtained from the log-linear Engel functions model $e_i = a_i' E^{b_i'} N^{c_i'} (i = 1, \ldots, K)$ by making it add up across items of expenditure [see Houthakker (1960)].

2.2. Incomes and Household Size Elasticities. The income and household size elasticities implied by (2.1.3) are:

$$(2.2.1)\qquad \eta_E^{e_i} = (\partial \log e_i / \partial \log E) = 1 + b_i - \bar{b} \text{ or}$$

$$(2.2.2)\qquad \eta_N^{e_i} = (\partial \log e_i / \partial \log N) = c_i - \bar{c} \text{ or}$$

where $\bar{b} = \sum_{j=1}^{K} w_j b_j$ and $\bar{c} = \sum_{j=1}^{K} w_j c_j$ are the means of b_j's and c_j's, respectively, weighted by the budget shares $w_j = e_j/E$. Evidently, both elasticities are variable ones, being functions of both the variables E and N. Furthermore,

$$\frac{\partial \eta_N^{e_i}}{\partial E} = -E^{-1} \sum_{j=1}^{K} w_j (b_j - \bar{b})^2 < 0$$

$$\frac{\partial \eta_N^{e_i}}{\partial N} = -N^{-1} \sum_{j=1}^{K} w_j (c_j - \bar{c})^2 < 0$$

It means that both partial elasticities are monotonically decreasing functions of the level of E or N (as the case may be). The pace of decrease in both elasticities with increase of the corresponding variables (for fixed E and N) is same for all items. As there is no such general theoretical presumption regarding the behaviour of the two partial elasticities with respect to the corresponding variables, the modified addilog model is, therefore, restrictive one.

Relation (2.2.1) suggests that an item i is more a "necessity" or more a "luxury" according as b_i is lower (especially if negative and nearer to —1) and higher (especially if strongly positive), respectively.

Relation (2.2.2) shows that the modified addilog model satisfies the relation

$$\sum_{i=1}^{K} w_i \eta_N^{e_i} = 0$$

which means that the total effect of a change in household size N on expenditures on all the specific items must be zero [see Houthakker (1957)].

Finally,

$$\eta_E^{e_i} - \eta_E^{e_j} = b_i - b_j$$

and $$\eta_N^{e_i} = \eta_N^{e_j} = c_i - c_j$$

which implies that for any two item difference in income elasticities or in family size elasticities is invariant to the level of family income and family size, respectively.

2.3. Estimation of the Parameters.[4] We make the modified addilog model stochastic by incorporating into it disturbance term u_i ($i = 1, \ldots, K$). As a sesult, we get

$$(2.3.1) \qquad e_i = a_i E^{1+b_i} N^{c_i} e^{u_i} \Big/ \sum_{j=1}^{K} a_j E^{b_j} N^{c_j} e^{u_j} \qquad \text{for } i = 1, \ldots, K.$$

This specification has the advantage that also the randomized specific expenditures satisfy the budget constraint. This may be rewritten as

$$(2.3.2) \qquad e_i = a_{ik} E^{1+b_{ik}} N^{c_{ik}} e^{u_{ik}} \Big/ \sum_{j=1}^{K} a_{jk} E^{b_{jk}} N^{c_{jk}} e^{u_{jk}}, \qquad i = 1, \ldots, K$$

where $a_{ik} = a_i/a_k$, $b_{ik} = b_i - b_k$, $c_{ik} = c_i - c_k$ and $u_{ik} = u_i - u_k$.

For estimating the parameters of (2.3.2) we derive by division and logarithmic transformation.

$$(2.3.3) \qquad \log (e_i/e_r) = \log a_{ir} + b_{ir} \log E + c_{ir} \log N + u_{ir}, \qquad (i, r = 1, \ldots, K; i \neq r)$$

We observe that (2.3.3) consists of $\frac{1}{2}K(K-1)$ equations, but only $K-1$ of these are mutually independent which in turn are obtained by setting $r = 1$ and varying i. Somermeyer and Langhout (1972), and also Huyser and Somermeyer (1971) applied the method of "ordinary least squares" (O.L.S) separately to the equations of system similar to the system (2.3.3). This yields best linear unbiased and consistent estimates of the parameters under the assumption that the disturbances u_{ik}'s over the individual households are homoscedastic. The same estimation procedure is followed in the present study. The O.L.S. estimates of the parameters $\log a_{i1}$, b_{i1} and c_{i1} ($i = 2, \ldots, K$) alongwith their standard errors are presented in Appendix Table A. 1.

It may be easily seen that the above estimation remains uneffected by setting r equal to any number and varying i in (2.3.3).

[4]As the present study employs cross-section data on individual households described in the later section 3.1, this estimation procedure refers to models applied to cross-section ungrouped data.

At a given level of E and N, estimates of income and housecholod-size elasticities may be obtained as

$$\hat{\eta}_E^{e_i}(E, N) = 1 + \hat{b}_{i1} - \sum_{j=2}^{K} \hat{w}_j \hat{b}_{j1}$$

and

$$\hat{\eta}_N^{e_i}(E, N) = \hat{c}_{i1} - \sum_{j=2}^{K} \hat{w}_j \hat{c}_{j1}$$

These, however, are not BLU-estimators as $\hat{w}_j$ $(= \hat{e}_j/E)_i$ are non-linear functions of the estimates $\hat{a}_{j1}, \hat{b}_{j1}$ and $\hat{c}_{j1}$. However, by linearization of $\hat{w}_j$'s one may construct an approximate standard errors for the two elasticities estimates for the purpose of statistical inference [see Jain and Tendulkar (1973)].

II.4. *Economies of Scale in the Context of the Modified Addilog Model.* The simplest hypothesis allowing for the effect of variation in household size is that consumption per person depends only on the level of income per person. It corresponds to the homogeneity hypothesis that there are constant returns to scale. While this assumption no doubt is easy to handle and may explain the broad phenomena of consumption patterns it can be easily shown that it will not be exactly true in general.[5] For it is generally observed that if the incomes per person are the same, larger households are able to enjoy a higher standard of living than smaller households. As aptly pointed out by Prais and Houthakker (1971) also, scale economies may arise in purchasing, storage and the cost of carriage to the house. There may be price or/and quality advantage on the bulk purchase as the packing cost, and overhead trade and transport margins borne by large size packets are proportionally low. It may be that a commodity is indivisible one, e.g. durables or it is only sold in multiple of certain minimum quantities such as a packet or a tin. This certainly is to the disadvantage of a smaller household compared to a large household. This suggests the existence of economies of scale in household consumption.

As the modified model does not assume the homogeneity hypothesis, it is expected to exhibit non-constant returns-to-scale phenomena in household consumption. At the point (E, N), a local measure of returns-to-scale

[5]A discussion of this point may be found in Stone (1951), pp. 10-11.

for an item 'i' is defined as $\eta_E^{e_i} + \eta_N^{e_i} - 1$ for an equal proportional change in each household income E and household size N. For the modified model, as

$$\eta_E^{\theta_i} + \eta_N^{\theta} - 1 = \sum_{j=1}^{K} (b_{ij} + c_{ij})\, w_j$$

where $w_j = e_j / E = 1 / \sum_{k=1}^{K} a_{kj}\, E^{b_{kj}} N^{c_{kj}}$ one may therefore say that in respect of an item 'i' there exist

(i) diseconomies of scale, if $\sum_{j=1}^{K} (b_{ij} + c_{ij})\, w_j > 0$

(ii) scale neutrality, if $\sum_{j=1}^{K} (b_{ij} + c_{ij})\, w_j = 0$

(iii) economies of scale, if $\sum_{j=1}^{K} (b_{ij} + c_{ij})\, w_j < 0$.

As $\sum_{j=1}^{K} (b_{ij} + c_{ij})\, w_j = 0$ is equivalent to $b_{ij} + c_{ij} = 0$ for all $j \neq i$, the case of scale neutrality in any item of consumption may, alternatively, be referred by the condition $b_1 + c_1 = b_2 + c_2 = \dots = b_k + c_k$.

It may further be easily seen that both

$$\text{(2.4.1)} \quad \frac{\partial}{\partial E}[n_E^{e_i} + n_N^{e_i} - 1] = - E^{-1} \sum_{j=1}^{K} (b_j + c_j)(b_j - \bar{b})\, w_j$$

and

$$\text{(2.4.2)} \quad \frac{\partial}{\partial N}[\eta_E^{e_i} + \eta_N^{e_i} - 1] = - N^{-1} \sum_{j=1}^{K} (b_j + c_j)(c_j - \bar{c})\, w_j$$

are constant across all items at a given level of E and N. As the sign of $\eta_E^{e_i} + \eta_N^{e_i} - 1$ determines whether there are economies or diseconomies of scale, the above result implies that if all goods do not exhibit economies (diseconomies) of scale for all E and N, then *no* good can exhibit economies (diseconomies) of scale for all E and N. Thus, the best test for economies (diseconomies) of scale in a *particular* good is the test for no economies (diseconomies) of scale in *any* good, i.e. hypothesis

$$H_o : b_1 + c_1 = b_2 + c_2 = \ldots = b_k + c_k$$

tested simultaneously. As the per capita model is nested within the modified model and the latter under H_o reduces to the former, the test for H_o is the same as the test for overall (dis) economies of scale. Therefore, to test H_o one may use the test-statistic

$$F(\nu_1 - \nu_o, \nu_o) = \frac{S_1 - S_o}{\nu_1 - \nu_o} \Big| \frac{S_o}{\nu_o}$$

where S_o and S_1 are the residual sums of squares obtained on applying O.L.S. method to the modified model (2.3.3) and (2.3.3) under H_o with the corresponding degrees of feedom $\nu_o = (k - 1)(n - 3)$ and $\nu_1 = (K - 1)(n - 1)$, respectively.

The returns to scale factor $F_i(E, N) = \eta_E^{e_i} + \eta_N^{e_i} - 1$ for the modified model happens to be a function of the joint variables E and N which move in the same direction. Furthermore, it declines[6] with an equi-proportionate[7] increase in both E and N (for proof, see Appendix B). Therefore, if there are (dis) economies of scale in a commodity at all, the scale factor F_i may behave in either of the two different ways; the first, when the sign of F_i remains same at all the level of E and N exhibiting economies or diseconomies for every household and the second, F_i changes sign from positive to negative at certain levels of E and N, thus exhibiting diseconomies of scale below it and economies of scale above it. This level

[6]In the theory of production, it is assumed that the return to scale decreases with the scale of production [see Frisch (1965)]. There have been some empirical attempts to verify whether this is true or not. Nerlove (1963) and Ringstad (1974) exmined this hypothesis by considering such forms of production function which did not presume a decreasing return to scale. Their results yielded convincing support for the theory.

[7]In general, when E and N change at unequal proportionate rates, it does not seem possible to prove that the scale factor F_i is a declining function of E and N without imposing complicated and unwielding constraints on the parameters of the modified model. However, these possibilities are not relevant in the persent context.

of E and N, which divides the entire ranges of E and N into two parts behaving differently on the economies scale, is a solution of the non-linear equation F_i (E, N)=0. In order to solve this equation one may work out a broad picture of the scale factor by computing values of F_i at observed mean levels of E and N for some broad groups of households formed in respect of E, and then use a trial and error method.

III. *Application*

III.*1*. *The Data Used*. The present study is based on the consumption expenditures data for individual households from the 17th round of National Sample Survey of India covering the period September 1961 to July 1962. In this round the sample was selected in the form of three independent sub-samples. However, because of limited resources, the ready availability of data and the exploratory nature of this exercise, only one sub-sample for rural and urban areas in the state of 'Uttar Pradesh' is considered. Number of households in the rural and the urban samples in 316 and 200. But the entire sample could not be used as the present model has the limitation that it can be applied to data containing only those sample households which report non-zero expenditures on all the items. Large number of sample households especially with low income are known to be reporting zero expenditure on relatively luxury items, viz., clothing and milk and milk products. To meet this limitation the following five broad[8] items-groups of consumption adding upto total expenditure (income) could be considered:

1. Cereals and cereal substitutes (or total cereals)
2. Pulses
3. Other food items (except 1 and 2)
4. Fuel and light
5. Other non-food items (excluding 4).

This provides us with the samples size as 305 and 189, reducing the original sample by 3.5 per cent and 5.5 per cent, for rural and urban areas, respectively.

Based on this data, two complete modified addilog models, one for rural and other for urban areas, have been estimated.

To examine the behaviour of scale economies at various broad levels of E and N, as suggested in Section 2.4, the entire sample of households is classified into four somewhat homogenous groups with respect to household income. The distribution of the sample households according to the four income-groups alongwith the average household income and the average household size for each income-group is displayed in Table 1.

[8]Broad grouping is also dictated by the additive nature of the utility function implied by the addilog model.

TABLE 1

DISTRIBUTION OF SAMPLE HOUSEHOLDS, AVERAGE HOUSEHOLD INCOME AND HOUSEHOLD SIZE BY MONTHLY HOUSEHOLD INCOME—CLASSES IN RURAL AND URBAN AREAS OF UTTAR PRADESH 1961-62

Monthly household income-class (Rs)	*Number of sample households*	*Average household income (Rs)*	*Average household size*
		Rural	
0—55	71	35.79	3.34
55—90	82	72.71	5.73
90—135	77	109.92	6.38
⩾135	75	211.49	8.58
All classes	305	107.64	6.06
		Urban	
0—55	37	40.79	2.51
55—90	62	74.08	5.00
90—135	47	108.04	5.89
≥135	43	264.16	7.53
All classes	189	119.25	5.30

III.2. *Results Relating to Economies of Scale.* As pointed out in Section 2.4, test for overall (dis) economies of scale consists in testing the null hypothesis

$$H_0\colon\ b_1+c_1 = b_2+c_2 = \dots = b_k+c_k$$
$$\text{or}:\ b_{21}+c_{21} = b_{31}+b_{31} = \dots = b_{k1}+c_{k1} = 0$$

simultaneously. On using the data of Section 3.1, the values of the test-statistic F turn out as 21.54 and 28.85 for the rural and the urban samples, respectively, indicating that H_0 is rejected even at 1% level of singificance in both the areas. This provides evidence that there exists overall economies of scale in household consumption, both in rural and urban areas. This also proves that the modified addilog model performs better than the per capita addilog model in both the areas when comparison of the two models is based on the entire household budget.

We may now proceed to examine specific economies (diseconomies) of scale in household consumption. This consists in testing the null hypothesis H_0: F_i (E, N)=0 at various levels of E and N, which obviously is a daunting prospect. In view of this difficulty we test H_0 at the observed mean levels of E and N for four broad income groups given in Table 1.

In Table 2 we present estimates of the economies scale factor F_i alongwith their approximate standard errors[9] at the mean levels of E and N

TABLE 2

ITEM-WISE ESTIMATES OF THE ECONOMIES SCALE FACTOR AND THEIR APPROXIMATE STANDARD ERRORS FOR FOUR HOUSEHOLD INCOME GROUPS, RURAL AND URBAN AREAS OF UTTAR PRADESH: 1961-62

Monthly household income class (Rs)	*Total cereals*	*Pulses*	*Other food items*	*Fuel and light*	*Other non-food items*
			Rural		
0—55	—0.013	—0.151	0.138	—0.379**	0.561**
	(0.046)	(0.120)	(0.128)	(0.082)	(0.202)
55—90	—0.056	—0.193	0.096	—0.421**	0.518**
	(0.055)	(0.123)	(0.123)	(0.086)	(0.195)
90—135	—0.105	—0.243*	0.046	—0.471**	0.469**
	(0.069)	(0.126)	(0.119)	(0.090)	(0.185)
⩾135	—0.193**	—0.330**	—0.041	—0.559**	0.381**
	(0.094)	(0.138)	(0.120)	(0.103)	(0.164)
All classes	—0.107	—0.244*	0.045	—0.473**	0.467**
	(0.070)	(0.126)	(0.119)	(0.090)	(0.185)
			Urban		
0—55	—0.202**	—0.310**	0.224*	—0.183**	0.532**
	(0.058)	(0.124)	(0.116)	(0.090)	(0.157)
55—90	—0.242**	—0.350**	0.184*	—0.223**	0.492**
	(0.066)	(0.127)	(0.112)	(0.090)	(0.153)
90—135	—0.296**	—0.404**	0.130	—0.277**	0.438**
	(0.075)	(0.131)	(0.107)	(0.091)	(0.147)
⩾135	—0.444**	—0.552**	—0.018	—0.424**	0.290**
	(0.102)	(0.143)	(0.101)	(0.097)	(0.123)
All classes	—0.326**	—0.434**	0.100	—0.307**	0.408**
	(0.081)	(0.133)	(0.105)	(0.091)	(0.143)

Notes: 1. Figures marked as '*' and '**' are significant at 5 and 1 per cent levels, respectively.
2. Figures given within brackets refer to the approximate standard errors of the estimates.

[9]Approximate standard error of estimated F_i at a given level of E and N is obtained by linearizing F_i. For details, see Appendix C.

for various income-groups. As pointed out in Section 2.4 also that there may be straightfoward discounts for making bulk purchases of necessities like total cereals, pulses and fuel and light, it may be reasonable to expect the presence of economies of scale in their consumption. However, in case of other item-groups it may not be possible to formulate any such hypothesis a priori. We, therefore, adopt the one sided test H_0: $F_i=0$ against the alternative H_1: $F_i<0$ for 'necessities'[10] and the two-sided test H_0: $F_i=0$ against H_1: $F_i \neq 0$ for the remaining item-groups. Results of Table 2 conform to the theoretical property of the modified model that the scale factor for each item of consumption monotonically decreases with the level of household income and household size. Occurrence of same sign for the scale factor, with singificant values in general, over the entire ranges of E and N seems to suggest that the consumption expenditures on necessary item—'cereals', 'pulses' and 'fuel and light' are subject to "economies of scale" relative to expenditures on 'other foods' and 'other non-food items'. The possible reason of this perhaps is that expenditures on items like health, milk and milk products, and durables go up for large households especially in urban areas (i. e., strong diseconomies) and staple foods and fuel must take the brunt showing up as economies of scale.

It may be interesting to note that in both the sectors estimated F_i for 'other food items' carries positive sign over ranges of E and N covered by bottom three income-groups and negative sign (although statistically insignificant) over the rest. This seems to lend support to the possibility indicated earlier in Section 2.4 that when one moves from lower to higher values of E (or N) one may come across successive ranges of E and N characterised respectively by scale diseconomies, scale neutrality and scale economies. As the middle ranges of E and N is comprised of nonsignificant values of $\hat{F}_i$, it corresponds to the interval $[\mp 1.645 \text{ s. e. } (\hat{F}_i)]$. To determine this, we present the variation of F_1 against each individual variable E and N in Figures[11] 1. R and 1. U of Appendix D, employing the results of Table 2. In each figure two horizontal lines corresponding to the end points $\mp$ 1.645 s.e. $(\hat{F}_i)$[12] are drawn so as to yield approximately

[10]It may be noted that the application of the both sided test for necessities leads to the same conclusion as the one based on the one sided test.

[11]It must be noted that in Figures 1.R and 1.U, while plotting $\hat{F}_i$ against E (or N), N (or E) is not held constant.

[12]In view of the standard errors of estimated values F_i for 'other food items at various levels of household income, given in Table 2, values of s. e. $(\hat{F}_i)$ chosen in the rural and the urban areas are 0.130 and 0.112, respectively, which are on the higher side.

the corresponding levels of E and N. Around these approximate levels several joint values of E and N are considered with a view to solve the equations $\hat{F}_i = \mp 1.645$ s. e. $(\hat{F}_i)$. The solutions indicate that in the rural areas, below the joint level of E = Rs 550 and N = 10.4, which corresponds to per capita income level of Rs 53, there appears scale-neutrality and above it economies of scale. In the urban areas, however the entire ranges of E and N gets trisected by the two point (E = 75, N = 5.0) and (E = 925, N = 8.4) such that the bottom section comprised of poor households with per capita income less than Rs 15 exhibits diseconomies, the top rich section with per capita income more than Rs 110 exhibits economies, and the middle section scale-neutrality.

The reasons for the top rich section of the population experiencing economies in respect of expenditure on 'other food item' seems to lie in the facts that (i) 'other food' is regarded as necessity by this top section as its income elasticity is less than unity and (ii) other non-food has income elasticity more than unity and shows diseconomies of scale. The explanation of this interesting finding may be that the rich families usually purchase other food, e. g., processed ones—biscuits, fruits jam, butter and so on, more than their requirement. When the size of such a family increases the expenditures on 'non-food other than fuel' go up because of diseconomies and its brunt falls on the necessary items—other food items.

As 'other food items' is a broad group of items, results relating it perhaps can not be taken as definitive.

IV. *Concluding Remarks*

This study attempted to investigate the existence of economies (diseconomies) of scale in household consumption at different levels of income by employing a modified model of indirect addilog Engel functions wherein the effect of household size is explicity introduced. The economies scale factor related to the modified model happens to be a variable one which declines with the equiproportionate increase in the levels of household income (E) and household size (N). This suggests the possibility that the different ranges of the joint variables E and N exhibit different types of economies of scale in respect to household consumption.

This study provided some evidence based on the modified addilog model regarding the existence of over-all economies of scale in household consumption. It also indicated that the modified addilog model gives better agreement to cross-sectional household consumer expenditure data than the conventional indirect addilog model in per capita terms. Consequently it confirmed the earlier impression of Iyengar, Jain and Srinivasan (1975) that the Engel curves formulation in per capita terms may lead to specification errors in the estimation of income elasticities. This, in turn, may result in errors in the projection of aggregate consumption expenditures on specific items if elasticity estimates based on models in per capita terms are used. In general, economies of scale were exhibited in case of necessity items-groups; i. e., cereals, pulses, and fuel and light, and diseconomies for luxury items viz., other food and other non-food. In respect of food items group excluding cereals and pulses different types of returns-to-scale were noted in different segments of the entire joint ranges of E and N. In the rural areas, the entire joint range got bisected such that the lower section exhibited scale—neutrality and the upper one economies of scale. However, in the urban areas the entire joint range got classified into three different ranges, exhibiting scale diseconomies in the lower range, scale neutrality in the middle one and scale-economies in the upper range.

Certain limitations of the present approach may finally be noted. First, the present formulation has ignored the variations of the socio-economic factors like, age, sex, education and occupation etc. Ignoring the use of equivalence weights may cause serious problem while testing the economies of scale. For instance, large households usually have a high proportion of children and economies of scale in case of necessities such as 'food' may simply reflect the different consumption requirements of children. However, nothing could be done to meet this shortcoming in view of the fact that simultaneous estimation of equivalence weights and economies scale factors is not possible from the single cross-section data as it involves identification problem. Secondly, the present model is able to investigate only commodity specific economies or diseconomies. This is because over all goods, weighting by budget shares, the scale factor $\eta_E^{\theta_i} + \eta_N^{\theta_i} - 1$ averages to zero. The question of the size of *overall* economies of scale is, therefore, left untouched. Lastly, the present study could consider only five broad items groups which form the entire houehold budget, whereas a more detailed break-down of household total expenditure into several specific expenditures would have been much more meaningful.

APPENDIX A

TABLE A-1

LEAST SQUARES ESTIMATES OF THE PARAMETERS OF THE MODIFIED ADDILOG MODEL, RURAL AND URBAN AREAS OF UTTAR PRADESH: 1961-62

Name of the item	$\hat{\log a_{i1}}$	$\hat{b}_{i1}$	$\hat{c}_{i1}$
		Rural	
Total cereals	0.0000 (0.0000)	0.0000 (0.0000)	00.000 (0.0000)
Pulses	—2.4098 (0.2636)	0.2458 (0.0734)	—0.3833 (0.0819)
Other food items	—3.5944 (0.3049)	0.9048 (0.0849)	—0.7533 (0.0948)
Fuel and light	—1.2514 (0.2027)	—0.0028 (0.0564)	—0.3631 (0.0630)
Other non-food items	—7.1824 (0.4160)	1.8227 (0.1158)	—1.2488 (0.1293)
		Urban	
Total cereals	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
Pulses	—2.4246 (0.3082)	0.1777 (0.0770)	—0.2853 (0.0722)
Other food items	—3.6214 (0.3619)	0.9624 (0.0905)	—0.5361 (0.0848)
Fuel and light	—2.4254 (0.2670)	0.3379 (0.0667)	—0.3184 (0.0625)
Other non-food items	—6.5495 (0.4177)	.6642 (0.1044)	—0.9299 (0.0978)

NOTE: The figures within brackets refer to the standard errors of the estimated parameters.

APPENDIX B

Here, we prove that for the modified model, the economies scale factor for an item of household consumption is a declining function for an equiproportionate change in both variables, household income and household size.

The economies scale factors for an item 'i' is given by

$$F_i\,(E,\ N) = \eta^{e_i}_{E} + \eta^{e_i}_{N} - 1$$

$$= b_i + c_i - \sum_{j=1}^{K} (b_j + c_j)\, w_j \qquad \ldots (1)$$

where $w_j = a_j E^{b_j} N^{c_j} \Big/ \sum_{r=1}^{K} a_r E^{b_r} N^{c_r}$, E and N being household income and household size variables. Suppose both E and N increase at equiproportionate rate α, i.e. $dE/E = dN/N = \alpha$. Totally differentiating (1), we have

$$dF_i = \frac{\partial F_i}{\partial F} dE + \frac{\partial F_i}{\partial N} dN$$

$$= -\alpha \left(E \frac{\partial F_i}{\partial E} + N \frac{\partial F_i}{\partial N} \right)$$

$$= -\alpha \sum_j (b_j + c_j)(b_j - \bar{b})\, w_j - \alpha \sum_j (b_j + c_j)(c_j - \bar{c})\, w_j$$

[(from (2.4.1) and (2.4.2) }

$$= -\alpha \,[\sum_j (b_j + c_j)^2 w_j - (\bar{b} + \bar{c})^2\,]$$

$$= -\alpha \,[\sum_j \{ (b_j + c_j) - (\bar{b} + \bar{c}) \}^2 w_j$$

$$< 0$$

Thus it proves that the economies scale factor for every item of consumption is a declining function for an equal proportional increase in both E and N.

APPENDIX C

Derivation of Approximate Standard Error of Economies Scale Factor

As noted in Section 2.4, the economies scale factor related to the modified model is given by

$$F_l\,(E,N) = \eta_E^{e_l} + \eta_N^{e_l} - 1$$

$$= b_{l1} + c_{l1} - \bar{b}_1 - \bar{c}_1$$

where $\bar{b}_1 = \sum_{j=1}^{K} b_{j1}\, w_j$, and $w_j = e^{\log a_{j1}}\, E^{b_{j1}}\, N^{c_{j1}} \Big/ \sum_{j=1}^{K} e^{\log\ a_{r1}}\, Eb_{r1}\, N^{c_{r1}}$

are non-linear functions of the parameters of the modified model. Therefore, at a given level of E and N, we may write F_l in the parametric functional form $F_l = f_l\,(A,B,C)$, where A, B and C denote the following row vectors:

$$A = [\log a_{21}, \ldots, \log a_{k1}],\ B = [b_{21}, \ldots, b_{k1}],\ C = [c_{21}, \ldots, c_{k1}].$$

The estimate of the scale factor F_l at a given E and N is, therefore, given by $\hat{F}_l = f_l\ (\hat{A},\hat{B},\hat{C})$, where $\hat{A},\hat{B}$ and $\hat{C}$ are the least squares estimates as indicated in Section 2.3. One may now linearise the function f_l [now onward f_l refers to $f_l\ (\hat{A},\hat{B},\hat{C})$] on applying to it Taylor's series expansion so as to obtain the following formula for the approximate standard error of $\hat{F}_l$:

$$\text{s.e.}^2\,(\hat{F}_l) \simeq \sum_{k=2}^{K} \sum_{j=2}^{K} \hat{\text{cov}}\ (\log \hat{a}_{j1}, \log \hat{a}_{k1}) \frac{\partial f_l}{\partial \log \hat{a}_{j1}} \frac{\partial f_l}{\partial \log \hat{a}_{k1}}$$

$$+ \hat{\text{cov}}\,(\hat{b}_{j1}, \hat{b}_{k1}) \frac{\partial f_l}{\partial \hat{b}_{j1}} \frac{\partial f_l}{\partial \hat{b}_{k1}} + \hat{\text{Cov}}\ (\hat{c}_{j1}, \hat{c}_{k1}) \frac{\partial f_l}{\partial \hat{c}_{j1}} \frac{\partial F_l}{\partial \hat{c}_{k1}}$$

$$+ 2\,\hat{\text{Cov}}\,(\log \hat{a}_{j_1}, \hat{b}_{k_1}) \frac{\partial f_i}{\partial \log \hat{a}_{j_1}} \frac{\partial f_i}{\partial \hat{b}_{k_1}} + 2\,\hat{\text{Cov}}\,(\hat{b}_{j_1}, \hat{c}_{k_1}) \frac{\partial f_i}{\partial \hat{b}_{j_1}} \frac{\partial f_i}{\partial \hat{c}_{k_1}}$$

$$+ 2\hat{\text{Cov}}\,(\log \hat{a}_{j_1}, \hat{c}_{k_1}) \frac{\partial f_i}{\partial \log \hat{a}_{j_1}} \frac{\partial f_i}{\partial \hat{c}_{k_1}},$$

where

$$\hat{\text{Cov}}\,(\log \hat{a}_{j_1}, \log \hat{a}_{j_1}) = \hat{\sigma}_{jk} \left(\frac{1}{n} + \bar{x}_1^2 s^{11} + 2\bar{x}_1 \bar{x}_2 s^{12} + \bar{x}_2^2 s^{22} \right),$$

$$\hat{\text{Cov}}\,(\log \hat{a}_{j_1}, \hat{b}_{k_1}) = -\hat{\sigma}_{jk} (\bar{x}_1 s^{11} + \bar{x}_2 s^{12}),$$

$$\hat{\text{Cov}}\,(\log \hat{a}_{j_1}, \hat{c}_{k_1}) = -\hat{\sigma}_{jk} (\bar{x}_1 s^{12} + \bar{x}_2\, {}^{22})$$

$$\hat{\text{Cov}}\,(\hat{b}_{j_1}, \hat{b}_{k_1}) = \hat{\sigma}_{jk} s^{11}, (\hat{\text{Cov}}\,(\hat{b}_{j_1}, \hat{c}_{k_1}) = \hat{\sigma}_{jk} s^{12},$$

$$\hat{\text{Cov}}\,(\hat{c}_{j_1}, \hat{c}_{k_1}) = \hat{\sigma}_{jk} s^{22}, \partial f_i / \partial \log \hat{a}_{j_1} = -\hat{w}_j \hat{F}_j$$

$$\partial f_i / \partial \hat{b}_{j_i} = \begin{cases} -\hat{w}\,(1 + \hat{b}_{j_1} \hat{F}_j / E), \text{ if } j \neq i \\ -\hat{w}_j\,(1/\hat{w}_j + 1 + \hat{b}_{j_1} \hat{F}_j / E), \text{ if } j = i \end{cases}$$

$$\partial f_i\, \partial \hat{c}_{j_1} = \begin{cases} -\hat{w}\,(1 + \hat{c}_{j_1} \hat{F}_j / N), \text{ if } j \neq i \\ -\hat{w}_j\,(-1/\hat{w}_j + 1 + \hat{c}_{j_1} \hat{F}/N), \text{ if } j = i \end{cases}$$

$$\hat{\sigma}_{jk} = (s_{y_j y_k} - \hat{b}_{j_1} s_{x_1 y_k} - \hat{c}_{j_1} s_{x_2 y_k}) \,/\, (n-3), ((s^{ij})) = ((s_{x_i x_j}))^{-1}$$

$$s_{x_i x_j} = \sum_{t=i}^{n} x_{it} x_{jt} - n\bar{x}_i \bar{x}_j, \quad s_{x_i y_k} = \sum_{t=1}^{n} x_{it} y_{kt} - n\bar{x}_i \bar{x}_k, \text{ and}$$

$$y_j = \log (e_j / e_1), \bar{x}_1 = \log E \text{ and } x_2 = \log N.$$

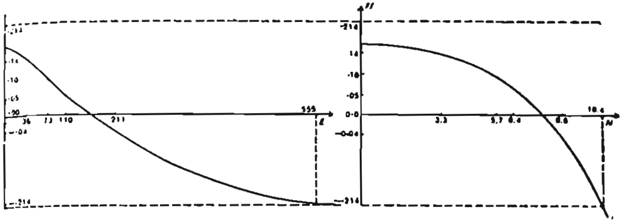
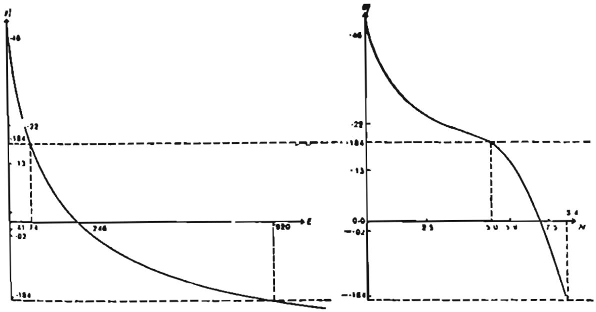


FIGURE 1R. Free hand plotting of economies scale factor F for 'other food items group' separately against household Income E and household size N, rural areas of Uttar Pradesh: 1961-62.



FIGRE 1U. Free hand plotting of economies scale factor F for 'other food items groups' separately against household income E and household size N, urban areas of Uttar Pradesh: 1961-62.

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