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TEMPORARY COMPETITIVE EQUILIBRIUM IN A MONETARY ECONOMY WITH UNCERTAIN TECHNOLOGY AND MANY PLANNING PERIODS

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1. Introduction

The results of Grandmont (1974) and Sondermann (1974) in the context of temporary equilibrium analysis are well known. Our object is to extend their results in certain directions.

First, both authors deal with a two-period problem. Sondermann interprets the two periods as the 'present' and the 'future' and hence allows for the possibility of the second period being considerably longer than the first. Both Grandmont (1974) and Sondermann (1974), however, assume the existence of traders who expect prices in the second period to belong to a compact set with a high probability irrespective of the level of current prices. The above interpretation of two periods would require traders to have 'tight expectations' over a very long future. Moreover, in the case of Sondermann, all production decisions are taken in period 1, which implies that over a long future firms do not consider any new production decisions.

We avoid such complications by considering a many-period model, in which traders have tight expectations regarding at least one future period (which could vary from trader to trader). Also, we allow production and investment plans for future periods.

Secondly, we extend Sondermann's work by assuming technological uncertainty.

Both Grandmont and Sondermann use dynamic programming to solve the problem. Such an approach leads to difficulties when a two-period horizon is replaced by a three-period one. Dynamic programming methods give rise to a third-period optimal policy which is dependent upon the distribution of prices in the third period, conditional on both the first- and second-period prices. In

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the absence of independence, decisions for the third period based on the conditional distribution of third-period prices, given second-period prices, will, in general, lead to a higher expected utility than those which do not depend on the second-period prices. But prices in the second period being unknown, the distribution of third-period prices is not known with certainty.

In this paper, we solve the problem by viewing it as one of programming in a space of functions and probability measures.

2. A decision model for temporary equilibrium analysis when traders plan for many periods

Sondermann (1974) has presented a general decision model, which provides the framework for a broad class of short-run equilibrium models. In this section, we consider a special case of that model suitable for temporary equilibrium analysis when traders plan for many periods.

Let T, J, L be positive integers. In what follows L and J would also be interpreted as the sets $\{1, 2, \dots, L\}$, $\{1, 2, \dots, J\}$ and T as the set $\{2, 3, \dots, T\}$. For $t = 1, 2, \dots, T$, let $P_t \subset R^L$ and $R_t \subset R^J_+ \times \{1\}$. For $t \in T$, Q'_t is a polish space and let $S_t = P_t \times R_t$, $Q_t = S_t \times Q'_t$, and $\Omega = \prod_{t=2}^{T} Q_t$. The elements of Ω describe the states of the world at time periods $t \in T$. The elements of the Borel σ-field, $\mathscr{F} = \mathscr{B}(\Omega)$ induced by the product topology on Ω are called the events. Denote by $\mathscr{F}_i$ the σ-field generated by all rectangles of the form $\prod_{t \in T} F_t$, with $F_t \in \mathscr{B}(Q_t)$ and $F_t = Q_t$ for $t > i, i \in T$.

The space of consequences is denoted by $X = \prod_{t=1}^{T} (X_t \times M_t)$, where X_t is a compact, convex subset of R^L and $M_1 = \overline{M}_1 \times [0,\infty)$, $M_t = \overline{M}_t \times [0,\infty]$, $t \in T$, where $\overline{M}_t$ is a non-empty, compact, convex subset of R^J containing zero. The set $[0,\infty]$ is the Alexandroff one-point compactification of $[0,\infty)$ with the usual topology. For $t \in T$, let G_t be a set of maps from $(\Omega, \mathscr{F}_t)$ into $((X_t \times M_t), \mathscr{B}(X_t \times M_t))$. Any action the decision-maker may take at time t is an element of G_t. Define

$$G := \{f = ((x_1, l_1), f_2, \dots, f_T) : (x_1, l_1) \in X_1 \times M_1, f_t \in G_t, t \in T\}.$$

The sets G_t and G are endowed with the product topologies. Assume that G_t is compact convex and first countable.

Define the projection of $f(\cdot)$ on $\prod_{t=1}^{T} X_t$ by $\text{proj} \cdot f(\cdot)$. Similarly, the projection of $(f_i(\cdot), f_{i+1}(\cdot), \dots, f_j(\cdot))$ on $\prod_{t=i}^{j} X_t$ will be denoted by $\text{proj} \cdot (f_i(\cdot), f_{i+1}(\cdot), \dots, f_j(\cdot))$. The projection of $f_t(\cdot)$ on M_t, $t \in T$, will be denoted by $l_t(\cdot)$. The last component of $l_t(\cdot)$ will be denoted by $m_t(\cdot)$ and that of l_1 by m_1. Define $f_{i,\varepsilon} = f_i + (0, \dots, 0, \varepsilon)$. We assume that given any $f_i \in G_i$ and $\eta > 0$, $\exists\, 0 < \varepsilon \leqq \eta$ such that $f_{i,\varepsilon} \in G_i$.

Let $\mathscr{M}$ be the set of probability measures on $\prod_{t\in T} S_t$ endowed with the topology of weak convergence. Define the map

$$\psi : S_1 \times \mathscr{B}\left(\prod_{t\in T} S_t\right) \to [0,1],$$

such that $\psi(s_1,.) \in \mathscr{M}$. Assume:

(P. 1) $\psi(\cdot, E)$ is a continuous function on $\prod_{t\in T} S_t$ for all closed events E.

Assume also that σ is a fixed probability measure on

$$\left(\prod_{t\in T} Q'_t, \mathscr{B}\left(\prod_{t\in T} Q'_t\right)\right).$$

For $s_1 \in S_1$, $(f, e) \in G \times G$, we assume that

$$\operatorname*{ess\,sup}_{\substack{\omega\in\Omega \\ t\in T}} e_{t,L+J+1}(\omega) < \infty,$$

with respect to the measure $(\psi(s_1, \cdot) \times \sigma)$,

$$(e_{t,L+1}, \ldots, e_{t,L+J}) \geqq 0, \qquad t \in \{1, T\},$$

$$s_1((x_1, l_1) - e_1) \leqq 0,$$

$$s_t f_t(\omega) - r_t l_{t-1}(\omega) = s_t e_t(\omega) \leqq 0, \qquad t \in T, \quad s_t \in S_t, \quad r_t \in R_t, \tag{2.1}$$

a.s. $(\psi(s_1, \cdot) \times \sigma)$.

The set of feasible actions is given by the correspondence β of $G \times S_1$ into G with

$$\beta(e, s_1) := \{f \in G : (f, e, s_1) \text{ satisfies (2.1)}\}. \tag{2.2}$$

The preference on the set of actions is given by the map $V: G \times \mathscr{M} \to R$, where

$$V(f, \lambda) := \int_\Omega u(\text{proj}\cdot f(\cdot))\, d(\lambda \times \sigma)(\cdot), \qquad f \in G, \quad \lambda \in \mathscr{M}, \tag{2.3}$$

$$u : \prod_{t=1}^{T} X_t \to [0, 1], \quad \text{continuous.}$$

The decision problem is to choose, given $e \in G$ and $s_1 \in S_1$, an

$$f^* \in \beta(e, s_1) \quad \text{s.t.} \quad V(f^*, \lambda) \geqq V(f, \lambda), \qquad \forall f \in \beta(e, s_1),$$

with $\lambda = \psi(s_1, \cdot)$.

3. Continuity properties[1]

In this section, we study the continuity properties of the correspondence β and the function V.

Lemma 3.1. The correspondence $\beta: G \times S_1 \to G$ defined by (2.2) *is u.h.c., compact valued and continuous in s_1.*

Proof. Let $f^n \in \beta(e, s_1)$ and $f^n \to f$. We then have, for all $\omega \in \Omega$, $s_1((x_1^n, l_1^n) - e_1) \leqq 0$, and for $t \in T$,

$$s_t f_t^n(\omega) - r_t\, l_{t-1}^n(\omega) - s_t\, e_t(\omega) \leqq 0.$$

The pointwise convergence of f_t^n to f_t implies that

$$l_t^n(\omega) \to l_t(\omega),$$

$$s_1((\omega, l) - e_1) \leqq 0,$$

and

$$s_t f_t(\omega) - r_t\, l_{t-1}(\omega) - s_t\, e_t(\omega) \leqq 0.$$

This proves that $\beta(e, s_1)$ is a closed subset of the compact set G and hence compact.

Let $(e^n, s_1^n) \in G \times S_1$ and $f^n \in \beta(e^n, s_1^n)$ be such that $(e^n, s_1^n, f^n) \to (e, s_1, f)$. Again, we have,

$$s_1^n(x_1^n, l_1^n) - s_1^n\, e_1^n \leqq 0,$$

and

$$s_t^n f_t^n(\omega) - r_t^n\, l_{t-1}^n(\omega) - s_t^n\, e_t^n(\omega) \leqq 0,$$

for $t \in T$ and $\omega \in \Omega$. Letting $n \to \infty$, we have

$$s_1((x_1, l_1) - e_1) \leqq 0,$$

and

$$s_t f_t(\omega) - r_t\, l_{t-1}(\omega) - s_t e_t(\omega) \leqq 0,$$

which shows that (e, s_1, f) belongs to the graph of β. Thus β has a closed graph. Since G is compact, the u.h.c. of β follows.

[1]Jordan (1975) has proved the continuity of similar decision functions, using the weak convergence topology. Since we would like the budget constraint to be satisfied for each state, we prefer to use the pointwise convergence topology.

To prove the l.h.c. of β in s_1, we will show that the strong inverse image of any closed set $F \subset G$ is closed. If $\beta^{-1}(F) = \emptyset$ or a finite set, there is nothing to prove. Let $\beta^{-1}(F)$ be an infinite set and s_1^* be an accumulation point of $\beta^{-1}(F)$. We will show that $\beta(e, s_1^*) \subset F$.

Let

$$f^* = ((x_1^*, l_1^*), f_2^*, f_3^*, \ldots, f_T^*) \in \beta(e, s_1^*).$$

For $\varepsilon > 0$, the open neighbourhood of f^* is given by

$$\begin{aligned} N_\varepsilon = \{f \in G : \text{for } k > \infty, \| ((x_1^*, l_1^*), f_2^*(\omega_i), \ldots, f_T^*(\omega_i)) \\ -((x_1, l_1), f_2(\omega_i), \ldots, f_T(\omega_i)) \| < \varepsilon, \\ \text{for } (x_1, l_1) \in X_1 \times M_1, \omega_i \in \Omega, i = 1,2, \ldots, k\}, \end{aligned}$$

where $\|\cdot\|$ is a metric for the product topology on G. For $\eta > 0$, let 0_η be an η-neighbourhood of s_1^*. Since s_1^* is an accumulation point of $\beta^{-1}(F)$,

$$\beta^{-1}(F) \cap 0_\eta \neq \emptyset \quad \text{for all} \quad \eta > 0.$$

Choose $h := ((x_1', l_1'), h_2, \ldots, h_T) \in G$, such that

$$(x_1', l_1') = (e_{11}, \ldots, e_{1L}, 0, \ldots, 0, m_1/2),$$

and for each $\omega \in \Omega$,

$$s_t h_t(\omega) - r_t(h_{t-1,\, L+1}(\omega), \ldots, h_{t-1,\, L+J+1}(\omega)) - s_t e_t \leqq 0, \tag{3.1}$$

for $t \in T$.

For $0 < \lambda < 1$, define

$$f_\lambda^* = \{(x_1, l_1)_\lambda, f_{2,\lambda}^*, \ldots, f_{T,\lambda}^*\},$$

with

$$(x_1, l_1)_\lambda = \lambda(x_1', l_1') + (1-\lambda)(x_1^*, l_1^*),$$

and

$$f_{t,\lambda}^* = \lambda h_t + (1-\lambda) f_t^*.$$

Choose λ such that $f_\lambda^* \in N_\varepsilon$.

Since $f^* \in \beta(e, s_1^*)$, it follows that

$$s_1^*((x_1, l_1) - e_1) < 0,$$

and

$$s_t f_{t,\lambda}^*(\omega) - r_t l_{t-1,\lambda}^*(\omega) - s_t e_t(\omega) \leqq 0, \qquad \forall\, t \in T, \quad w \in \Omega. \tag{3.2}$$

Choose λ so small that, for some $\bar{s} \in 0_\eta \cap \beta^{-1}(F)$, we have

$$\bar{s}((x_1, l_1) - e_1) < 0.$$

This implies that $f_\lambda^* \in \beta(e, \bar{s}) \subset F$. This proves that f^* is an accumulation point of the closed set F and hence $f^* \in F$. Q.E.D.

Let $\bar{R}$ be the one point compactification of R and $Z = (\bar{R}^L)^T$. For $Z' \subset Z, s_1 \in S_1$ and $e^* \in G$, define $\beta^{Z'}(e^*, s_1)$ as in $\beta(e, s_1)$, with X replaced by Z'. Let Z_n be an increasing sequence of closed subsets of Z converging in Hausdorff distance topology of Z. Define $X_n = X \cap Z_n$. We then have the following:

Corollary. For every $s_1^n, s_1^* \in S_1, s_1^n \to s_1^*$ *and* $\| e(\omega) \| < K < \infty\ \forall \omega \in \Omega$, $\beta^X(e^*, s_1^*) \subset Li\beta^{X_n}(e^*, s_1^n) = F$.

Proof. Let $h^* \in \beta^X(e^*, s_1^*)$. Since F is closed, it is enough if we prove that h^* is an accumulation point of F. Let $\varepsilon > 0$. Consider an ε-neighbourhood of h^*, N_ε, defined by

$$N_\varepsilon = \{h \in \beta^X(e^*, s_1^*) : \text{for some } k < \infty, \| ((x_1, l_1), h_2(\omega_i) \dots h_T(\omega_i))$$
$$((x_1^*, l_1^*), h_2^*(\omega_i), \dots, h_T^*(\omega_i)) \| < \varepsilon,$$
$$\text{for } (x_1, l_1) \in X_1 \times M_1, \omega_i \in \Omega, i = 1,2, \dots, k\}.^2$$

Let $\hat{h} = (\hat{e}_1, e_2^*, \dots, e_T^*)$ with $\hat{e}_1 = (e_{11}^*, e_{12}^*, 0, \dots, 0, m_1/2)$.

For $0 < \lambda < 1$, define $\bar{h} = \lambda \hat{h} + (1-\lambda) h^*$.

For n large, $e^* \in X_n$. Consider the case, where $\| h^*(w) \| < K' < \infty$ in the usual norm. Choose λ so small that $\bar{h} \in N_\varepsilon$. For large enough n, $s_1^n((\bar{x}_1, \bar{l}_1) - e_1^*) < 0$, since $s_1^*((\bar{x}_1, \bar{l}_1) - e_1^*) < 0$. Also $\bar{h}(\omega) \in X_n, \forall \omega \in \Omega$. Hence $\bar{h} \in \beta^X n(e^*, s_1^*)$ for large n. This proves that $h^* \in F$.

[2] $\| \cdot \|$ will be used for the metric on the compactified space also.

Consider the case where h^* is unbounded. This means that h_t^* is unbounded for at least one $t \in T$. By the construction of the Alexandroff one-point compactification, we can choose an h_t^{**} such that h_t^{**} belongs to an $\varepsilon/2$-neighbourhood of h_t^* and $h_t^{**}(\omega) \in X_n$ for n large. Now the previous arguments may be applied to h^{**}. This completes the proof.

Lemma 3.2. The mapping $V: G \times \mathcal{M} \to R$ defined in (2.3) *is continuous.*

Proof. Follows immediately from Delbaen (1974, lemma 8, remark 3).

4. Consumers

There is a finite set of consumers denoted by A. An individual consumer will be indexed by $a \in A$. Till the beginning of Proposition 4.5 we shall be concerned with an individual consumer's decision problem and as such the index 'a' will be dropped. Indexing of consumers will be necessary only in Proposition 4.5 and the notation used there will be self-explanatory.

The individual consumer's decision problem may be described as follows. In period t, he is restricted to choose his commodity bundle from a non-empty, compact and convex subset of R^L, which we denote by X_t. Commodities are assumed to be non-durable and have to be consumed within the period in question. Similarly, his choice of durable, non-monetary assets, viz. shares in the J firms, is restricted to a non-empty, compact and convex subset of R^J_+, viz. $\overline{M}_t$. As in Sondermann, we shall take $\overline{M}_t := [0,1]^J$. The consumer's demand for money in period 1 is assumed to come from the set $[0,\infty) \subset R_+$ and for periods $2, \ldots, T$, from the sets $[0, \infty]$, where $[0, \infty]$ is the Alexandroff one-point compactification of R_+ with the usual topology.

The sets P_t and R_t of section 2 will be used to denote commodity and asset prices in period $t \in \{1, T\}$. Similarly, Q_t' will denote the set of states of the world other than prices (say, weather). For any $r_t \in R_t$, $r_{t,J+1} = 1$, which means that the price of money is identically 1.

The consumer's endowments of goods and financial assets in periods $t \in T$ are uncertain. Let $e_t'(\cdot)$ denote his endowments for period $t \in T$. We assume $e_t' \in G_t'$, where G_t' is defined in exactly the same manner as G_t. For period 1, the consumer has a fixed endowment of goods and assets, which we denote by $e_1' \in X_1 \times M_1$.

Given a price vector of commodities and assets in the first period, the consumer has to specify a vector of demand for consumption goods and assets, say (x_1^*, l_1^*) for the first period along with a set of optimal policy functions, say f_t^*, $t \in T$, for the remaining periods. It will be assumed that $(x_1^*, l_1^*) \in X_1 \times M_1$ and $f_t^* \in G_t'$, $t \in T$. Thus, f_t^* describes the demand for commodities and assets for period $t \in T$ as a function of prices and other states of nature in periods 2 through t.

Any action $f: = \{(x_1, l_1), f_2, \ldots, f_T\}$ of the consumer has to satisfy the budget constraint. Given $s_1 \in S_1$, this is expressed by the requirement that $f \in \varphi(e', s_1)$, where

$$e'(\cdot): = \{e'_1, e'_2(\cdot), \ldots, e'_T(\cdot)\},$$

and $\varphi(\cdot,\cdot)$ is identically the same as $\beta(\cdot,\cdot)$ of section 2.

The consumer is endowed with a continuous and convex preference ordering $\succsim$ over $\prod_{t=1}^{T} X_t$. Instead of working directly with $\succsim$, we shall be dealing with a bounded utility function u representing this ordering. It is assumed that

$$u': \prod_{t=1}^{T} X_t \to [0,1]$$

is a continuous function.

The consumer's expectations about future prices is incorporated in the stochastic kernel $\psi(s_1, \cdot)$ describing the joint distribution of prices in $\prod_{t \in T} S_t$. $\psi(\cdot,\cdot)$ is assumed to satisfy (P.1) of section 2. Moreover, the consumer has a fixed expectation pattern about Q', which is given by the distribution σ.

The consumer's expected utility index is given by

$$V': G' \times \mathscr{M} \to R,$$

where

$$V'(f, \psi(s_1, \cdot)): = \int_{\Omega} u'(\text{proj.} f(\cdot))\, d(\psi(s_1, \cdot) \times \sigma(\cdot)). \tag{4.1}$$

The consumer's choice problem is now exactly the same as that posed at the end of section 2.

The following three propositions can be immediately derived:

Proposition 4.1. The correspondence $\varphi: G' \times S_1$ into G' is compact valued and continuous in s_1.

Proof. Follows immediately from Lemma 3.1.

Proposition 4.2. The expected utility functional $V': G' \times \mathscr{M} \to R$ is continuous.

Proof. Follows immediately from Lemma 3.2.

Define

$$\gamma(s_1): = \{f^* = ((x_1^*, l_1^*), f_2^*, \ldots, f_T^*) \in \varphi(e', s_1):$$
$$V'(f^*, \psi(s_1, \cdot) \geqq V'(f, \psi(s_1, \cdot) \;\forall f \in \varphi(e', s_1)\}.$$

The consumer's demand relation for period 1 is then given by

$$\xi(s_1):=\{(x_1^*, l_1^*):((x_1^*, l_1^*), f_2^*, \ldots, f_T^*) \in \gamma(s_1)\}. \tag{4.2}$$

Proposition 4.3. The excess demand relation $\xi(s_1)-e_1'$ is non-empty, convex, compact-valued and u.h.c.

Proof. Compactness and u.h.c. follow from Proposition 4.1, 4.2 and a result of Hildenbrand (1974, Corollary (b) of theorem 3, p. 30). Convexity is obvious.

The remaining part of this section is devoted to derive certain propositions about the individual consumer's demand for money.

Let

$$Q_{T-2}:=Q_2 \times Q_3 \times \ldots \times Q_{i-1} \times Q_i' \times Q_{i+1} \times \ldots \times Q_T.$$

Let $\psi s_1(\cdot | q_{T-2})$ denote the distribution of s_i given $s_1 \in S_1$ and $q_{T-2} \in Q_{T-2}$. For any agent, in addition to (P.1) of section 2, we assume

(P.2) $\forall \varepsilon > 0$, $\exists K_i \subset S_i$ for *some* $i \in T$, K_i compact, such that $\forall s_1 \in S_1$ and

$$q_{T-2} \in Q_{T-2},\ \psi s_1(K_i | q_{T-2}) > 1-\varepsilon.$$

Assumption (P.2) means that every consumer expects prices to be in a compact set in at least one future period with very high probability. Given

$$f:=((x_1, l_1), f_2, \ldots, f_{i-1}, f_i, f_{i+1}, \ldots, f_T) \in \varphi(e', s_1),$$

define

$$f_{T-1}:=((x_1, l_1), f_2, \ldots, f_{i-1}, f_{i+1}, \ldots, f_T).$$

For $g_i \in G_i'$, define

$$(f_{T-1}, g_i):=((x_1, l_1), f_2, \ldots, f_{i-1}, g_i, f_{i+1}, \ldots, f_T).$$

Also, for $q_{T-2} \in Q_{T-2}$, $s_i \in S_i$, let

$$\begin{aligned}(q_{T-2}, s_i) &:= (q_1, \ldots, q_{i-1}, (s_i, q_i'), q_{i+1}, \ldots, q_T)\\ &\equiv (q_1, \ldots, q_{i-1}, q_i, q_{i+1}, \ldots, q_T) = \omega \in \Omega.\end{aligned}$$

With q_{T-2} specified, f determines $(i-1)$ vectors, viz. (x_t, l_t), $1 \leqq t \leqq i-1$ and $(T-i+1)$ functions of s_i, viz. $f_i(q_{T-2},\cdot), \ldots, f_T(q_{T-2},\cdot)$. In other words,

$$f(q_{T-2},\cdot) := ((x_1, l_1), f_2(q_{T-2},\cdot), \ldots, f_{i-1}(q_{T-2},\cdot), f_i(q_{T-2},\cdot), f_{i+1}(q_{T-2},\cdot), \ldots, f_T(q_{T-2},\cdot))$$

$$= ((x_1, l_1), (x_2, l_2), \ldots, (x_{i-1}, l_{i-1}), f_i(q_{T-2},\cdot), f_{i+1}(q_{T-2},\cdot), \ldots, f_T(q_{T-2},\cdot)).$$

For $\varepsilon > 0$, define

$$\begin{aligned} X_\varepsilon^f(s_i) := \{x_i \in X_i : \forall q_{T-2} \in Q_{T-2}, & \, u'(x_1, x_2, \ldots, x_{i-1}, x_i, \operatorname{proj}(f_{i+1}(q_{T-2}, s_i), \ldots, f_T(q_{T-2}, s_i))) \\ & \geqq u'(x_1, x_2, \ldots, x_{i-1}, \operatorname{proj}(f_i(q_{T-2}, s_i), f_{i+1}(q_{T-2}, s_i), \ldots, f_T(q_{T-2}, s_i))) + \delta_\varepsilon, \\ & s_i(x_i, l_i(q_{T-2}, s_i)) - s_i e_i'(q_{T-2}, s_i) - r_i l_{i-1}(q_{T-2}, s_i) \leqq \varepsilon\}, \end{aligned}$$

where $\delta_\varepsilon > 0$ and independent of f.

Finally, define

$$S_f^\varepsilon := \{s_i \in S_i : X_\varepsilon^f(s_i) \neq \emptyset\}.$$

Thus, the set S_f^ε consists of prices in period i, for which the consumer is not able to improve upon f due to lack of funds.

We assume the following:

(P.3) For any $f \in G'$ and any $(s_1, q_{T-2}) \in S_1 \times Q_{T-2}$,

$$\psi s_1(S_f^\varepsilon | q_{T-2}) \geqq \delta_{\bar{\varepsilon}},$$

and

$$\exists\, h_i : \Omega \to X_i,$$

such that

$$(h_i, l_i) \in G_i',$$

$$s_i(\operatorname{proj}\cdot(h_i(q_{T-2}, s_i), l_i(q_{T-2}, s_i))) \leqq s_i e_i'(q_{T-2}, s_i) + r_i\, l_{i-1}(q_{T-2}, s_i) + \varepsilon,$$

$$h_i(q_{T-2}, s_i) \in X_\varepsilon^f(s_i),\ s_i \in S_f^\varepsilon,$$

and

$$u(\text{proj}\cdot(f_{T-1}(q_{T-2}, s_i)), h_i(q_{T-2}, s_i))$$
$$\geqq u(\text{proj}\cdot f(q_{T-2}, s_i)), s_i \in S_i \backslash S_f^\varepsilon.$$

Assumption (P.3) means that for every $f \in \varphi(e', s_1)$ there is always a set of prices $s_i \in S_i$ with positive (conditional) measure (given any $(s_1, q_{T-2}) \in S_1 \times Q_{T-2}$) for which there are points in the consumption set which are preferred to proj$\cdot f(q_{T-2}, s_i)$ but not satisfying the budget constraints at these prices. In other words, the consumer is never sure of achieving a maximal element in X_i.

Let $\psi_i \in \mathscr{M}(S_i)$ satisfy (P.3). For $f \in \varphi(e'_a, s_1)$ and $q_{T-2} \in Q_{T-2}$, define

$$V'_i(f, \psi_i) := \int_{S_i} u'(\text{proj}\cdot f(q_{T-2}, \cdot)\, d\psi_i(\cdot).$$

As an immediate consequence of (P.3), we have:

Proposition 4.4. $\exists\, g_i \in G'_i$ *such that*

$$s_i(\text{proj}\cdot g_i(q_{T-2}, s_i), l_i(q_{T-2}, s_i))$$
$$\leqq s_i e'_i(q_{T-2}, s_i) + r_i\, l_{i-1}(q_{T-2}, s_i) + \varepsilon,$$

and (4.3)

$$V'_i(f_{T-1}(q_{T-2}, \cdot), g_i(q_{T-2}, \cdot), \psi_i)$$
$$\geqq V'_i(f(q_{T-2}, \cdot), \psi_i) + \delta_\varepsilon^2, \qquad \forall\, q_{T-2} \in Q_{T-2}.$$

Proof. Obvious.

Define

$$H_i(f, \psi_i) := \{g_i : (4.3) \text{ are satisfied}\}.$$

It may be noted that H_i is u.h.c. in f and ψ_i.

Corollary. For every $s_1 \in S_1$ *and each consumer a, the value of the excess demand correspondence at any period is zero,* i.e., $f^* \in \gamma(s_1)$ *implies* $s_1((x_1^*, l_1^*) - e'_1) = 0$ *and* $s_i f_i^*(\cdot) - r_i l_{i-1}^*(\cdot) - s_i e'_i(\cdot) = 0$, $\forall\, s_i \in S_i$, $i \in T$.

Consider the total excess demand correspondence for the first period, viz. $\sum_{a \in A} [\xi_a(s_1) - e'_{a1}]$.

Proposition 4.5.[3] *Assume that*

$$\sum_{a \in A} e'_{a1}, \in \operatorname{int} \sum_{a \in A} X_{a1} \times M_{a1}.$$

Then, if $\{s_1^k\}$ *is a sequence of prices in* S_1 *with* $\lim_k \|s_1^k\| = \infty$, *for every sequence* $\{z^k\}$ *with* $z^k \in \sum_{a \in A} [\xi_a(s_1) - e'_{a1}]$, *one has* $\lim_k z^k_{L+J+1} = \infty$, *i.e., the total demand for money in the first period tends to infinity.*

Proof. Consider the sequence of normalized prices π^k with $\pi^k = s_1^k / \|s_1^k\|$. Since $\|\pi^k\| = 1$ for all k, $\exists$ a convergent subsequence, still denoted by $\{\pi^k\}$, converging to a π^0.

Suppose now, that the proposition is false. Then, without loss of generality, $z^k \to z^0$ (say). By assumption,

$$\sum_{a \in A} \pi^0 e'_{a1} > \inf \sum_{a \in A} \pi^0 (X_{a1} \times M_{a1})$$

$$= \sum_{a \in A} \inf \pi^0 (X_{a1} \times M_{a1}).$$

Thus, $\exists$ at least one consumer $\bar{a} \in A$ with

$$\pi^0 e'_{\bar{a}1} > \inf \pi^0 (X_{\bar{a}1} \times M_{\bar{a}1}). \tag{4.4}$$

Let

$$z^k = \sum_{a \in A} (x^k_{a1}, l^k_{a1}) - \sum_{a \in A} e'_{a1},$$

where

$$f_a^k := \{(x^k_{a1}, l^k_{a1}), f^k_{a1}, \ldots, f^k_{aT}\} \in \gamma_a(s_1^k).$$

Since $z^k \to z^0$, it follows that $(x^k_{a1}, l^k_{a1}) \to (x^0_{a1}, l^0_{a1}), \forall a \in A$, where $z^0 = \sum_{a \in A} (x^0_{a1}, l^0_{a1}) - \sum_{a \in A} e'_{a1}$.

From now on we shall restrict our attention to $\bar{a} \in A$ and once again drop the index $\bar{a}$ for convenience. We have

$$\pi^0 (x_1^0; l_1^0) \leqq \pi^0 e'_1.$$

Also, from (4.4), $\exists (\bar{x}_1, \bar{l}_1) \in X_1 \times M_1$ with

$$\pi^0 (\bar{x}_1, \bar{l}_1) < \pi^0 e'_1.$$

[3]In proving Proposition 4.5, we use (P.2). Professor Grandmont pointed out to us that a similar proposition can be proved under the weaker assumption that with a *positive probability* the price of money is expected to be 1. [See Grandmont–Hildenbrand (1974).]

Define

$$(x_1^\lambda, l_1^\lambda) := ([\lambda(x_1^0, l_{11}^0, \ldots, l_{1J}^0) + (1-\lambda)(\bar{x}_1, \bar{l}_{11}, \ldots, \bar{l}_{1J})], m), \qquad m - m_1^0 = \varepsilon > 0.$$

Clearly,

$$\pi^0(x_1^\lambda, l_1^\lambda) < \pi^0 e_1'. \tag{4.5}$$

Since $m_1^k \to m_1^0$, $\exists\, N$ s.t. $\forall\, k \geqq N$, $m_1^k < m$.

Given $\psi_i \in \mathscr{M}(S_i)$, $\exists\, g_i^k \in G_i'$, by Proposition 4.4, such that

$$s_i(\text{proj}\cdot g_i^k(\cdot), l_i^k(\cdot)) \leqq s_i e_i' + r_i l_{i-1}^k(\cdot) + \varepsilon,$$

and

$$V_i'((f_{T-1}^k, g_i^k), \psi_i) > V_i'(f^k, \psi_i), \qquad \forall\, q_{T-2} \in Q_{T-2}.$$

Define the following:

1. $(x_1^{\lambda,k}, k, l_1^{\lambda,k}) := ([\lambda(x_1^k, l_{11}^k, \ldots, l_{1J}^k) + (1-\lambda)(\bar{x}_1, \bar{l}_{11}, \ldots, \bar{l}_{1J})], m)$.
2. $f_{i,\varepsilon}^k(\cdot) := f_i^k(\cdot) + (0, 0, \ldots, 0, \varepsilon), \qquad \varepsilon = m - m_1^0$.

   That is, $f_{i,\varepsilon}^k(\cdot)$ adds $\varepsilon > 0$ to the money component of $f_i^k(\cdot)$.
3. $g^{\lambda,k} := \{(x_1^{\lambda,k}, l_1^{\lambda,k}), f_{2,\varepsilon}^k, \ldots, f_{i-1,\varepsilon}^k, \lambda g_i^k + (1-\lambda) f_i^k, f_{i+1}^k, \ldots, f_T^k\}$.
4. $g^k := \{((x_1^k, l_{11}^k, \ldots, l_{1J}^k), m), f_{2,\varepsilon}^k, \ldots, f_{-i1,\varepsilon}^k, g_i^k, f_{i+1}^k, \ldots, f_T^k\}$.

For k large enough, say $k \geqq N' \geqq N$,

$$g^{\lambda,k} \in \varphi(e', s_1^k).$$

We claim $\exists\, q_{T-2}^k \in Q_{T-2}$, such that $\mu(q_{T-2}^k, \cdot) \neq 0$ and

$$V_i^k(f^k, \psi_{s_1 k}(\cdot \mid q_{T-2}^k)) \geqq V_i'(g^{\lambda,k}, \psi_{s_1 k}(\cdot \mid q_{T-2}^k)). \tag{4.6}$$

If not, $\forall\, q_{T-2} \in Q_{T-2}$ s.t. $\mu(q_{T-2}^k, \cdot) \neq 0$ and $k \geqq N'$,

$$V_i'(f^k, \psi_{s_1 k}(\cdot \mid q_{T-2})) < V_i'(g^{\lambda,k}, \psi_{s_1 k}(\cdot \mid q_{T-2})),$$

or

$$\int_{S_i} u'(\text{proj}\cdot f^k(q_{T-2}, \cdot))\, d\psi_{s_1 k}(\cdot \mid q_{T-2}) < \int_{S_i} u'(\text{proj}\cdot g^{\lambda,k}(q_{T-2}, \cdot))\, d\psi_{s_1 k}(\cdot \mid q_{T-2}),$$

or

$$\int_{Q_{T-2}} \int_{S_i} u'(\text{proj}\cdot f^k(q_{T-2},\cdot)d\,\psi_{s_1k}(\cdot|q_{T-2})d\psi^k_{T-2}(\cdot)d\sigma$$
$$< \int_{Q_{T-2}} \int_{S_i} u'(\text{proj}\cdot g^{\lambda,k}(q_{T-2},\cdot)d\psi_{s_1k}(\cdot|q_{T-2})d\psi^k_{T-2}(\cdot)d\sigma, \quad (4.7)$$

where, $\psi^k_{T-2}(\cdot)$ denotes the marginal distribution of $\psi(s^k_1,\cdot)$ on Q_{T-2}. However, (4.7) contradicts the optimality of f^k. Hence (4.6) is satisfied. Define

1. $h^k : S_i \to \prod_{t=1}^{T} (X_t \times M_t)$ as $h^k(s_i) := f^k(q^k_{T-2}, s_i)$.

2. $h^k_\varepsilon : S_i \to \prod_{t=1}^{T} (X_t \times M_t)$ as $h^k_\varepsilon(s_i) := g^k(q^k_{T-2}, s_i)$.

3. $h^{\lambda,k}_\varepsilon : S_i \to \prod_{t=1}^{T} (X_t \times M_t)$ as $h^{\lambda,k}_\varepsilon(s_i) := g^{\lambda,k}(q^k_{T-2}, s_i)$.

Since $\prod_{t=2}^{T} (X_t \times M_t)$ is compact and (x^k_1, l^k_1) is a bounded sequence, we may choose h^k and $h^{\lambda k}_\varepsilon$ to converge pointwise (on S_i) to some h and h^λ_ε. Also $\psi_{s_1k}(\cdot|q^k_{T-2})$ may be assumed to converge weakly to some $\psi^0 \in \mathcal{M}(S_i)$, by virtue of (P.2). Hence

$$V^t_i(h,\psi^0) \leqq V^t_i(h^\lambda_\varepsilon,\psi^0),$$

since $V^t_i(\cdot,\cdot)$ is continuous by Lemma 3.2. Letting $\lambda \to 1$,

$$V^t_i(h,\lambda^0) \geqq V^t_i(h_\varepsilon,\lambda^0),$$

where h_ε is the pointwise limit (on S_i) of h^k_ε. Since ψ^0 is the weak limit of a sequence of measures satisfying (P.3), ψ^0 satisfies (P.3) also. Also, g^k_i depends in a u.h.c. fashion on $(f^k, \psi_{s_1k}(\cdot|q^k_{T-2}))$. But this is a contradiction. Hence, the total demand for money in the first period goes to infinity. Q.E.D.

5. Producers

There is a finite set of producers, denoted by J. An individual producer will be denoted by $j \in J$. As in the previous section, most of the ensuing discussion will be related to the behaviour of the individual producer and hence the index 'j' will be dropped.

The production possibilities in period i for the producer are given by the correspondence

$$Y_i : \left(\Omega \times R^L_+, \prod_{t\in T} S_t \times \mathcal{B}\left(\prod_{t=2}^{i} Q'_t\right)\right) \to R^L_+. \quad (5.1)$$

The set $Y_i(\omega, x)$, $(\omega, x) \in \Omega \times R_+^L$ is to be interpreted as the set of outputs available at time i, when the input at time $i-1$ was x and the state of the world is ω. Moreover, (5.1) says that outpot in period i will depend solely upon the states of the world (other than prices) occurring up to period i.

We assume that the probability distribution of output for any given input is completely known to the producers. We further assume:

(T.1) a. Y_t is closed-valued.

b. For any bounded set $F \subset R_+^L$, the set $\{y \in Y_t(\Omega, x) : x \in F\}$ is bounded.

(T.2) For each $\omega \in \Omega$, the graph of Y_t is closed, convex and contains (0, 0). Note that this implies that the correspondence Y_t is continuous for each ω.

Production is financed through the issue of shares and the market value of each firm belongs to its shareholders. The total number of shares of each firm is normalized to unity. Producers may also carry a portfolio over time and they are allowed to trade assets and money on the capital market.

At the beginning of period 1, the producer has to choose a list of inputs and a portfolio he wants to carry over to the next period, for every given price vector $s_1 \in S_1$. But the choice of the portfolio depends upon what he wants to produce in the next period and at what price he is able to sell his output. Both the future price and the output are uncertain. Thus he has to make his plans for production and portfolio for the future periods conditionally on the future states of the world. His decision functions for $t = 1, 2, \ldots, T-1$ are formally described by the following maps:

$$\begin{aligned}
&\alpha_1 : S_1 \to R_+^L, &&\alpha_t : (\Omega, \mathscr{F}_t) \to R_+^L, &&t = 2, \ldots, T-1,\\
&\beta_1 : S_1 \to R_+^L, &&\beta_{t+1} : (\Omega, \mathscr{F}_t) \to R_+^L, &&t = 2, \ldots, T-1,\\
&g_1 : S_1 \to [0, 1]^J, &&g_t : (\Omega, \mathscr{F}_t) \to [0, 1]^J, &&t = 2, \ldots, T-1.
\end{aligned} \tag{5.2}$$

The maps are restricted as in the case of G_t and G (see section 2). A firm's endowments of goods and money in the different periods are represented, as in the case of the consumer, by $e'' := (e_1'', e_2''(\cdot), \ldots, e_T''(\cdot)) \in G'' \equiv G$. We will call $(\alpha, \beta, g) := (\alpha_t, \beta_{t+1}, g_t)_{t=1}^{T-1}$ a feasible programme if for $(e'', s_1) \in G'' \times S_1$,

$$\beta_{t+1}(\omega) \in Y_{t+1}(\omega, \alpha_t(\omega)), \qquad \omega \in \Omega, \tag{5.3}$$

and

$$s_1((\alpha_1(\omega), g_1(\omega)) - e_1'') \leqq 0, \tag{5.4}$$

$$s_t((\alpha_t(\omega), g_t(\omega)) - (\beta_t(\omega), g_{t-1}(\omega)) - e_t''(\omega)) \leqq 0, \qquad t = 2, \ldots, T-1.$$

Let $D := \{(\alpha, \beta, g) : (5.2), (5.3)$ and (5.4) are satisfied$\}$. The set D is endowed with the topology of pointwise convergence. The consequence of the plan (α, β, g) will be a certain market value of the firm for each period. For each $\omega \in \Omega$, this is given by the map

$$C_i : D \times \Omega \to R \quad \text{with} \quad C_i(\alpha, \beta, g, \omega) = \sum_{t=2}^{i} [p_t \beta_t(\omega) + r_t\, g_{t-1}(\omega)]. \tag{5.5}$$

The set of consequences C is given by $\{C_2(d), \ldots, C_T(d), d \in D\}$.

Like the consumers, each producer has expectations about future prices and endowments, which is given by a Markoff Kernel $\psi'(\cdot, \cdot)$ satisfying (P.1). The producer's utility function is given by $u'' : R^{T-1} \to [0, 1]$, where u'' is continuous, concave and increasing. The producer's expected utility index is then given by

$$V''(d, s_1) := \int_\Omega u''(C_2(\cdot), \ldots, C_T(\cdot)) d(\psi'(s_1, \cdot) \times \sigma(\cdot)), \quad \forall\, (d, s_1) \in D \times S_1. \tag{5.6}$$

The set of feasible production and financial plans for a producer at price $s_1 \in S_1$ and $e'' \in G''$ is given by the correspondence

$$\theta : G'' \times S_1 \to D. \tag{5.7}$$

Each producer chooses an element from his feasible set so as to maximize his expected utility index. We assume that each producer has an endowment of a positive amount of money at the beginning of period 1. Moreover,

$$\sup_{\substack{\omega \in \Omega \\ t \in \{1, T\}}} e''_{t, L+J+1}(\omega) < \infty,$$

i.e., the initial endowments of money are in a bounded set.

The following propositions are easily established by standard methods:

Proposition 5.1. The correspondence $\theta : G'' \times S_1 \to D$ is compact-valued and continuous in $s_1 \in S_1$.

Proposition 5.2. The functional V'' *is continuous.*

For $s_1 \in S_{1'}$, define

$$\delta(s_1) := \{d^* \in \theta(e'', s_1) : V''(d^*, s_1) \geqq V''(d, s_1), \forall\, d \in \theta(e'', s_1)\}.$$

The demand relation for period 1 is given by

$$\eta(s_1) := \{(\alpha_1^*, g_1^*) \equiv (x_1^*, l_1^*) : (\alpha_t^*, \beta_{t+1}^*(\cdot), g_t^*) \in \delta(s_t)\}. \tag{5.8}$$

Let $T = \prod_{t=2}^{T} T_t$, where T_t is a convex and compact subset of R_+^L. Restrict $\beta = \{\beta_2, \ldots, \beta_T\}$ to take values in $\prod_{t=2}^{T} T_t \times Y_t(\Omega, R_+^L)$.

For such (α, β, g), we have:

Proposition 5.3. The excess demand relation $\eta(s_1) - e_1''$ is non-empty, compact-valued, convex-valued and u.h.c.

Finally, we have:

Proposition 5.4. $\forall\, s_1 \in S_1$ *and* $z \in (\eta(s_1) - e_1'')$, $s_1 z = 0$.

6. Market equilibrium

In this section, we prove the existence of a temporary equilibrium in period 1. First, we consider the case of compact consumption sets. Then we extend the results to the case of unbounded sets when there is free disposal.

The economy is completely described by

$$\mathscr{E} := \{(X_a, u_a', \psi_a, e_a'), (Y_{j2}, \ldots, Y_{jT}, u_j'', \psi_j', e_j'')\}, \qquad a \in A, \quad j \in J.$$

The economy is called regular, if all elements of $\mathscr{E}$ satisfy the assumptions made in sections 4 and 5.

Definition 6.1. A price vector $s_1^* \in S_1$ and an $(A+J)$-tuple $(f_{a1}^*(s_1^*), (\alpha_{j1}^*(s_1^*), g_{j1}^*(s_1^*))$ is an equilibrium in period 1 *if*

(i) $f_{a1}^*(s_1^*) \in \xi_a(s_1^*), \quad \forall\, a \in A,$

(ii) $(\alpha_{j1}^*(s_1^*), g_{j1}^*(s_1^*)) \in \eta_j(s_1^*), \quad \forall\, j \in J.$

(iii) $\sum_{a \in A} f_{a1}^*(s_1^*) + \sum_{j \in J} (\alpha_{j1}^*(s_1^*), g_{j1}^*(s_1^*)) = \sum_{a \in A} e_{a1}' + \sum_{j \in J} e_{j1}''.$

Theorem 6.1. If a regular economy satisfies the additional condition

$$\sum_{a \in A} e_{a1}' \in \operatorname{int} \sum_{a \in A} [X_{a1} \times M_{a1}],$$

then there exists an equilibrium in period 1.

Proof. In view of the results of sections 4 and 5 and, in particular, Proposition 4.5, the proof of the theorem follows exactly as in Sondermann (1974).

So far our analysis has been based on the assumption that the consumption and technology sets are compact. In the traditional static models of general equilibrium without money it is customary to assume that consumption sets are unbounded. The consumption set in such models cannot be assumed to be compact for otherwise the twin assumptions of continuity of preferences and local non-satiation would become inconsistent. Thus for a compact consumption set there is always a satiation point. However, in a monetary economy, Sondermann (1974) got around this difficulty by assuming that the agent's expectations of future prices are such that there is always a positive probability of not attaining the satiation point. This is roughly the content of our Assumption (P.3) also.

In traditional analysis it is assumed that consumption sets are bounded below, the idea being that there are physical limitations on the amounts an agent can supply (e.g., only 24 hours of labour). For exactly the same reason one should insist on physical limitations on the amounts of any good a person can consume. This will, however, imply that the consumption set is bounded. In that case there would exist a consumption bundle x^0 such that $x \not\succ x^0$ for any $x \geqq x^0$. Under such circumstances the assumption of local nonsatiation leads to a contradiction.

Thus there are conceptual difficulties involved in assuming that individuals have non-compact consumption sets. However, in the context of our model, one might consider a planning problem by introducing a government or a social planner. For example, this is done in Chetty and Nayak (1976) by introducing an agent whose consumption set includes all individual consumption sets. Such considerations lead us to extend our results to the case of unbounded consumption and technology sets.

Before going into the details of this case, let us examine the usual steps involved in the proof of Theorem 6.1 and see where the assumption of compactness is crucial. First, one deals wuth truncated economies, in which case compactness of the entire consumption or technology sets plays no role. Hence, we can find a sequence of equilibrium aggregate excess demands and prices. Also, Proposition 4.5 can be modified in this case so that $\|z^k(s_1^k)\| \to \infty$ instead of $z_{L+J+1}^k \to \infty$, whenever $\|s_1^k\| \to \infty$. In view of this a convergent subsequence of equilibrium excess demand prices can be extracted. Now to claim that the limiting allocation is an equilibrium allocation for the economy in the limit, i.e., the given economy, we need the u.h.c. of the demand correspondence for the consumption and technology sets in the limit, and hence non-emptiness of the demand correspondence. Since an agent can *imagine* an unbounded budget set in a future period, his demand correspondence may be empty! This type of difficulty does not arise with respect to the first period, for he will soon find out the current prices of scarce goods are not zero and hence, with limited endowments, budget set is compact. Thus, if we could compactify the feasible sets in future periods, the agent's demand correspondence would not be empty, though the future period's policy functions will now be required to provide plans for impossible prices (for the economy in the future period, but feasible in the imaginary economy of some consumers) like zero for some essential goods. The existence of a temporary equilibrium in period one will then be possible, it we could find a suitable compactification ot the commodity space.

Consider an economy in which there is free disposal of goods, i.e.,

(D.1) if $y \in Y_{jt}(\omega, x)$, $y' \leqq y$ and $x' \geqq x$, then $y' \in Y_{jt}(\omega, x')$, $\forall \omega \in \Omega$, $\forall j \in J$.

In this case there are no noxious goods in the economy. Hence we could assume that each consumer's preference preordering is (weakly) monotone, i.e., if $x \geqq y$, then $u(x) \geqq u(y)$. Any consumer has strictly monotone preferences with respect to a subset of commodities and weakly monotone preferences with respect to others.

Let the economy $\mathscr{E}^*$ be such that it satisfies (D.1) and all the assumptions of sections 4 and 5, with the commodity space R^L replaced by $\bar{R}^L$, where $\bar{R}$ is the Alexandroff one point compactification of R.

We have then the following:

Theorem 6.2. If the economy $\mathscr{E}^*$ satisfies the additional assumption that almost all consumers have weakly monotone preference, then there exists an equilibrium in period 1.

Proof. Consider a consumer $a \in A$. Let his preference preordering on X_a be given by the continuous utility function $u_a: X_a \to [0, 1]$. Note that u_a is increasing in commodities. Now extend the function u to $\bar{X}$, the closure of X in $\bar{R}^L$. For $\bar{x} \in \bar{X}$, define $u_a(\bar{x}) = \lim_n u_a(x_n)$, $x_n \in X$, $x_n \uparrow \bar{x}$. It is easily checked that u_a is well-defined in $\bar{X}$ and is continuous. Also, define $0 \cdot \infty = 0$, so that the points $\bar{x} \in \bar{X} \setminus X$ are feasible only if the intial endowment of money is ∞ or if the prices of the commodities for which the components of $\bar{x}$ are ∞ are zero.

The utility functions of producers have already been assumed to be monotone and hence would extend in the same manner. Their feasible sets are also similarly defined.

By assumption, the set of feasible decision functions G' and G'' are compact. It follows from Lemma 3.1 and its corollary that the correspondence φ and θ are continuous. Hence the demand correspondences of consumers and producers are not empty. For every convergent sequence of elements from the first-period demand correspondence of consumer 'a', we can choose a convergent sequence of decision functions for each future period. It is possible for any trader, that the future decision functions may be unbounded and hence may converge to ∞ at some points. When the markets meet during the second period, these policy functions are irrelevant, since they will be planning using the second period's prices, which will then be known. From this point on the standard arguments go through. Note that the compactification points will not be chosen in period 1 in equilibrium by any trader. Q.E.D.

We may also remark that in the presence of noxious commodities, such Alexandroff's one point compactification is not useful, since we cannot assume a monotone preference and hence we cannot extend any continuous function to $\bar{R}^L$. One can use Stone–Čech compactification, but then it is not possible to demonstrate the u.h.c. of the budget set correspondence for subsets of the compactified space. In the absence of free disposal, compactness of consumption and technology sets is indispensable, since at any fixed price, the budget set could be unbounded (in the usual norm).[4]

[4] In an earlier version of this paper, the authors have proved all the results for a 'continuum economy'.

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