STUDIES IN EDUCATIONAL TESTS No. 2. THE AGE VARIATION OF SCORES IN A GROUP TEST OF INTELLIGENCE IN BENGALI

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INTRODUCTION.

In a previous paper (this Journal, June, 1933) I have described in detail a Group Test of Intelligence conducted through the medium of the Bengali language. The test was intentionally divided into two portions, Part I and Part II, with 23 and 37 as the respective full marks. The reliability of the test was investigated by correlating the marks in the two parts for 1212 Bengali school children of various ages from about 8 years to over 22 years. The coefficient of correlation was of the order of +0·88 for the whole data. It remained high for even small age-groups, the weighted average value for one year age-groups being +0·87, and for six months age-groups +0·86. The regression was significantly non-linear, but a parabolic equation of the second degree was found adequate. The Pearsonian correlation ratio was of the order of $\eta = 0{\cdot}90$ in the case of both the regressions. The reliability of the test as measured by the correspondence in marks in the two parts was, therefore, very high.

ANALYSIS OF VARIANCE DUE TO AGE.

I shall now consider how the average score increases with the age of the candidate. I have adopted for this purpose age-groups of 3 months. With this unit of grouping, the number of individuals in certain age-groups becomes so small as 2 or 3 ; it is therefore the smallest unit of grouping which can be used for the present material.

The analysis of variance due to age for marks in Part I is given below.

TABLE 1.—ANALYSIS OF VARIANCE FOR PART I SCORES.

Factor of Variation	D. F.	Sum of Squares	Variance	Ratio of Variances: Observed	Ratio of Variances: 1% Expected
Deviations from Linear Regression ...	47	40 99·14	87·22	2·27	1·87
Linear Regression	1	1 09 51·15	109 51·15	285·14	6·66
Between Age-groups	48	1 50 50·29	8 13·55	8·16	1·32
Within Age-groups	11 68	4 46 68·30	38·41	1·00	...
Total ...	12 11	5 97 18·59	49·32	...	...

The variation between age-groups is definitely significant, which shows that test-scores depend to a great extent on the age.

The correlation ratio $\eta = 0{\cdot}5020$, while the correlation coefficient $r = 0{\cdot}4788$, showing that the regression is non-linear. Calculating the sum of squares of deviations from a linear regression we find that the corresponding variance is 87·22 (with $n_1 = 47$), against a residual (within age-groups) variance of 38·41 (with $n_2 = 1163$). The observed ratio is 2·27 while the one per cent. expected value is 1·37. We conclude, therefore, that the regression of scores on age is definitely non-linear.

Similar results are obtained for scores in Part II. The analysis of variance is shown below.

TABLE 2.—ANALYSIS OF VARIANCE FOR PART II SCORES.

Factor of Variation	D. F.	Sum of Squares	Variance	Ratio of Variances: Observed	Ratio of Variances: 1% Expected
Deviations from Linear Regression	47	83 63·34	1 77·94	2·10	1·37
Linear Regression ..	1	2 44 22·84	2 44 22·84	288·48	6·65
Between Age-groups	48	3 27 86·18	6 83·05	8·07	1·32
Within Age-groups	11 63	9 84 44·66	84·65	1·00	...
Total	12 11	13 12 30·84	1 08·37	...	..

Deviations from the linear regression are again statistically significant. The regression in this case also is, therefore, non-linear.

The analysis of variance for combined scores given below naturally shows the same features.

TABLE 3.—ANALYSIS OF VARIANCE FOR COMBINED SCORES.

Factor of Variation	D. F.	Sum of Squares	Variance	Ratio of Variances: Observed	Ratio of Variances: 1% Expected
Deviations from Linear Regression ..	47	2 33 39·33	4 96·58	2·32	1·37
Linear Regression	1	6 81 84·41	6 81 84·41	321·32	6·66
Between Age-groups	48	9 15 23·74	19 06·74	9·12	1·32
Within Age-groups	11 63	25 48 95·85	2 19·17	1·00	
Total	12 11	34 64 19·59	2 86·06		...

The regression of total scores on age is thus definitely non-linear in character.

I shall now try to graduate the scores according to age. Pearson's Non-Linear Regression equations[1] would be undoubtedly the most suitable expressions for this purpose. I give below the relevant formulæ for convenience of reference.

[1] Karl Pearson, Skew Correlation and Non-Linear Regression, 1905, p. 28.

Let Y_p represent the graduated mean score for the pth age-group, $\bar{y}$ and $\bar{T}$ the general average score and the mean age, and s_y, s_t the corresponding standard deviations. Then as usual, we write :—

$$Y_p = (y_p - \bar{y})/s_y, \quad \text{and} \quad t_p = (T_p - \bar{T})/s_t.$$

$$N.p_{qq'} = S[(T - \bar{T})^q.(y - \bar{y})^{q'}]$$

$$s_t = \sqrt{p_{20}}, \qquad s_y = \sqrt{p_{02}},$$

$$\beta_1 = p_{30}^2/p_{20}^3 \qquad \beta_2 = p_{40}/p_{20}^2$$

$$\beta_3 = p_{50}p_{30}/p_{20}^4 \qquad \beta_4 = p_{60}/p_{20}^3$$

$$f_2 = \beta_2 - \beta_1 - 1, \qquad f_3 = (\beta_3 - \beta_2 \cdot \beta_1 - \beta_1)/\sqrt{\beta_1}$$

$$f_4 = \beta_4 - \beta_2^2 - \beta_1, \qquad \text{and } g = f_2 f_4 - f_3^2$$

In these equations we adopt the convention that the sign of $\sqrt{\beta_1}$ will be the same as that of p_{30}.

Further, the coefficient of correlation is given by

$$r = p_{11}/\sqrt{p_{20} \cdot p_{02}},$$

and the correlation ratio by

$$\eta^2 = \frac{\text{Sum of squares of deviations between age-groups}}{\text{Total sum of squares of deviations}}$$

We also put

$$d_2 = (p_{21} \cdot p_{20} - p_{11} \cdot p_{30})/s_t^4 \cdot s_y$$

$$d_3 = (p_{31} \cdot p_{20} - p_{11} \cdot p_{40})/s_t^5 \cdot s_y$$

We can then write the cubic regression equation in the following form :—

$$Y_p = c_0 + c_1(t_p) + c_2(t_p^2) + c_3(t_p^3)$$

where

$$c_3 = (d_3 f_2 - d_2 f_3)/g$$

$$c_2 = (d_2 f_4 - d_3 f_3)/g$$

$$c_1 = +(r - c_2\sqrt{\beta_1} - c_3\beta_2)$$

$$c_0 = -(c_2 + c_3\sqrt{\beta_1})$$

A slight simplification is possible when the cubic curve is really adequate. A necessary condition for this enables us to write :—

$$c_2' = (d_2 - c_3 f_3)/f_2$$

$$c_3' = \pm\sqrt{[f_2(\eta^2 - r^2) - d_2^2]/g}$$

with the same values for c_1 and c_0 as before. The sign of c_3 must be determined by inspection, but its use avoids the trouble of calculating p_{31}.

The work is perfectly straightforward. Adopting a grouping unit of 3 months we find the following constants for the distribution of age.

Grouping unit	= 3 months,		$N = 1212$	
Mean age	=	$\overline{T}$	= 168·07 months	
Standard Deviation	=	s_t	= 27·98 ,,	
p_{20}	=	21·744845	$p_{40} =$	3226·6925
$\sqrt{p_{20}}$	=	4·663	$p_{50} =$	+ 1273·7279
p_{30}	=	+ 10·3519	$p_{60} =$	129544·0745
β_1	=	0·010422	$\beta_3 =$	0·149366
β_2	=	2·693792	$\beta_4 =$	12·59936
$\sqrt{\beta_1}$	=	+ 0·102067		
f_2	=	+ 1·683378	$f_3 =$	1·089890
f_4	=	+ 5·332320	$g =$	7·787450

These same values will be used for all three graduations.

I give below the constants for Part I, Part II and the combined score separately.

TABLE 5.—REGRESSION CONSTANTS

Element.	Part I	Part II	Combined.
Mean	14 ·91 67	18 ·48 85	33 ·40 52
S. D.	7 ·01 95	10 ·40 55	16 ·90 64
p_{11}	+14 ·03 45	+20 ·96 55	+35 ·00 00
p_{21}	−27 ·27 15	−38 ·85 01	−66 ·12 16
p_{31}	+5 96 ·92 17	+9 26 ·45 87	+15 23 ·38 04
r	+0 ·42 88	+0 ·43 21	+0 ·44 40
η	0 ·50 20	0 ·49 96	0 ·51 40
d_2	−0 ·22 24 33	−0 ·21 58 22	−0 ·22 51 89
d_3	−0 ·31 63 06	−0 ·28 58 44	−0 ·30 72 91
c_3	−0 ·03 73 03	−0 ·03 21 46	−0 ·03 49 72
c_2	−0 ·10 80 63	−0 ·10 75 09	−0 ·11 12 04
c_1	+0 ·54 02 91	+0 ·53 05 15	+0 ·54 95 12
c_0	+0 ·10 42 56	+0 ·10 37 02	+0 ·11 47 73

It will be noticed that the regression constants c_0, c_1, c_2 and c_3 are practically identical for both Part I and Part II, and hence naturally also for the combined score. This agreement is satisfactory, since it shows definitely that the entities measured by the scores in Part I and Part II exhibit the same kind of change with age.

TABLE 6.—GRADUATED AND OBSERVED SCORES

Age-group	Number n_p	Part I		Part II		Combined	
		Graduated	Observed	Graduated	Observed	Graduated	Observed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
100–102	2	5·80	11·50	4·51	11·50	10·29	23·00
–105	8	6·18	7·87	5·08	8·12	11·17	16·00
–108	3	6·48	4·33	5·67	4·00	12·13	8·33
–111	3	6·85	2·00	6·28	0·33	13·13	2·33
–114	6	7·24	8·83	6·92	8·83	14·18	17·67
–117	17	7·65	7·82	7·58	8·71	15·26	16·53
–120	15	8·08	8·47	8·25	8·00	16·38	16·47
–123	11	8·52	9·64	8·92	10·00	17·53	19·54
–126	8	8·97	13·50	9·59	13·38	18·70	26·87
–129	33	9·43	8·67	10·29	9·09	19·90	17·76
–132	26	9·90	9·85	10·99	9·77	21·12	19·12
–135	29	10·38	10·17	11·69	10·17	22·34	20·34
–138	25	10·86	8·92	12·40	11·24	23·57	20·16
–141	52	11·35	10·15	13·11	12·69	24·81	22·85
–144	45	11·83	12·15	13·81	14·02	26·05	26·18
–147	28	12·32	9·89	14·51	11·79	27·27	21·68
–150	33	12·80	13·91	15·20	17·45	28·49	31·36
–153	55	13·27	14·87	15·88	19·13	29·70	34·00
–156	35	13·74	12·88	16·55	14·83	30·88	27·71
–159	23	14·19	15·74	17·20	19·08	32·04	34·83
–162	44	14·65	15·18	17·83	17·41	33·17	32·59
–165	60	15·08	16·15	18·45	20·02	34·27	36·17
–168	61	15·50	15·00	19·04	18·87	35·33	33·87
–171	44	15·91	15·82	19·61	20·52	36·34	36·34
–174	27	16·29	16·15	20·15	19·48	37·31	35·63
–177	56	16·65	16·73	20·66	21·41	38·23	38·14
–180	60	16·99	16·80	21·14	21·90	39·10	38·70
–183	52	17·31	18·19	21·58	23·58	39·90	41·77
–186	33	17·60	18·76	21·99	22·88	40·64	41·64
–189	43	17·86	18·81	22·35	23·37	41·30	41·95
–192	44	18·09	17·40	22·68	22·23	41·90	39·64
–195	26	18·29	17·58	22·95	23·27	42·42	40·85
–198	18	18·45	18·78	23·18	22·00	42·85	40·78
–201	36	18·58	18·86	23·36	24·80	43·19	43·67
–204	21	18·66	19·09	23·49	25·00	43·45	44·10
–207	30	18·72	19·77	23·56	25·73	43·60	45·50
–210	20	18·72	16·60	23·58	21·50	43·66	38·10
–213	24	18·68	16·67	23·53	21·96	43·61	38·63
–216	12	18·60	18·67	23·42	24·00	43·45	42·66
–219	10	18·46	19·80	23·25	22·30	43·17	42·10
–222	9	18·29	20·11	23·01	28·33	42·78	48·44
–225	7	18·05	18·00	22·60	24·00	42·26	42·00
–228	8	17·76	14·00	22·12	18·00	41·61	32·00
–231	0	17·42	...	21·56	...	40·83	...
–234	1	17·02	17·00	20·93	14·00	39·92	31·60
–237	2	16·56	12·00	20·21	15·00	38·86	27·00
–240	0	16·08	...	19·42	...	37·65	...
–243	2	15·44	21·50	18·54	32·00	36·30	53·50
–246	2	14·79	12·00	17·57	10·00	34·79	22·50
–249	0	14·07	...	16·01	...	33·11	...
–252	0	13·28	...	14·86	...	31·28	...
–255	0	12·42	...	13·61	...	29·27	...
–258	1	11·48	21·00	12·27	29·00	27·09	50·00
–261	0	10·47	...	10·88	...	24·74	...
–264	0	9·39	...	9·28	...	22·19	...
–267	0	8·22	...	7·63	...	19·47	...
–270	2	6·97	9·50	5·88	12·00	16·55	21·50

Substituting for t_p and using corresponding values of c_0, c_1, c_2 and c_3 given above, and writing X_p, Y_p, Z_p for graduated scores in Part I, Part II, and combined respectively, we have the following regression equations :—

$$\frac{X_p-14{\cdot}92}{7{\cdot}02}=+0{\cdot}1043+0{\cdot}5403\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)-0{\cdot}1081\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)^2-0{\cdot}0373\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)^3$$

$$\frac{Y_p-18{\cdot}49}{10{\cdot}41}=+0{\cdot}1037+0{\cdot}5305\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)-0{\cdot}1075\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)^2-0{\cdot}0321\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)^3$$

$$\frac{Z_p-33{\cdot}41}{16{\cdot}19}=+0{\cdot}1148+0{\cdot}5495\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)-0{\cdot}1112\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)^2-0{\cdot}0350\left(\frac{T_p-168{\cdot}07}{27{\cdot}98}\right)^3$$

These may be written more simply in the form :—

$$X_p = 22{\cdot}24 - 0{\cdot}5516\ (T_p) + 0{\cdot}005061\ (T_p^2) - 0{\cdot}000012\ (T_p^3)$$
$$Y_p = 18{\cdot}51 - 0{\cdot}6153\ (T_p) + 0{\cdot}0063\ (T_p^2) - 0{\cdot}000015\ (T_p^3)$$
$$Z_p = 38{\cdot}97 - 1{\cdot}1503\ (T_p) + 0{\cdot}0112\ (T_p^2) - 0{\cdot}000027\ (T_p^3)$$

The graduated and observed values of the mean scores for each age-group are given in Table 6. The corresponding graphs are shown in figures 1, 2, and 3.

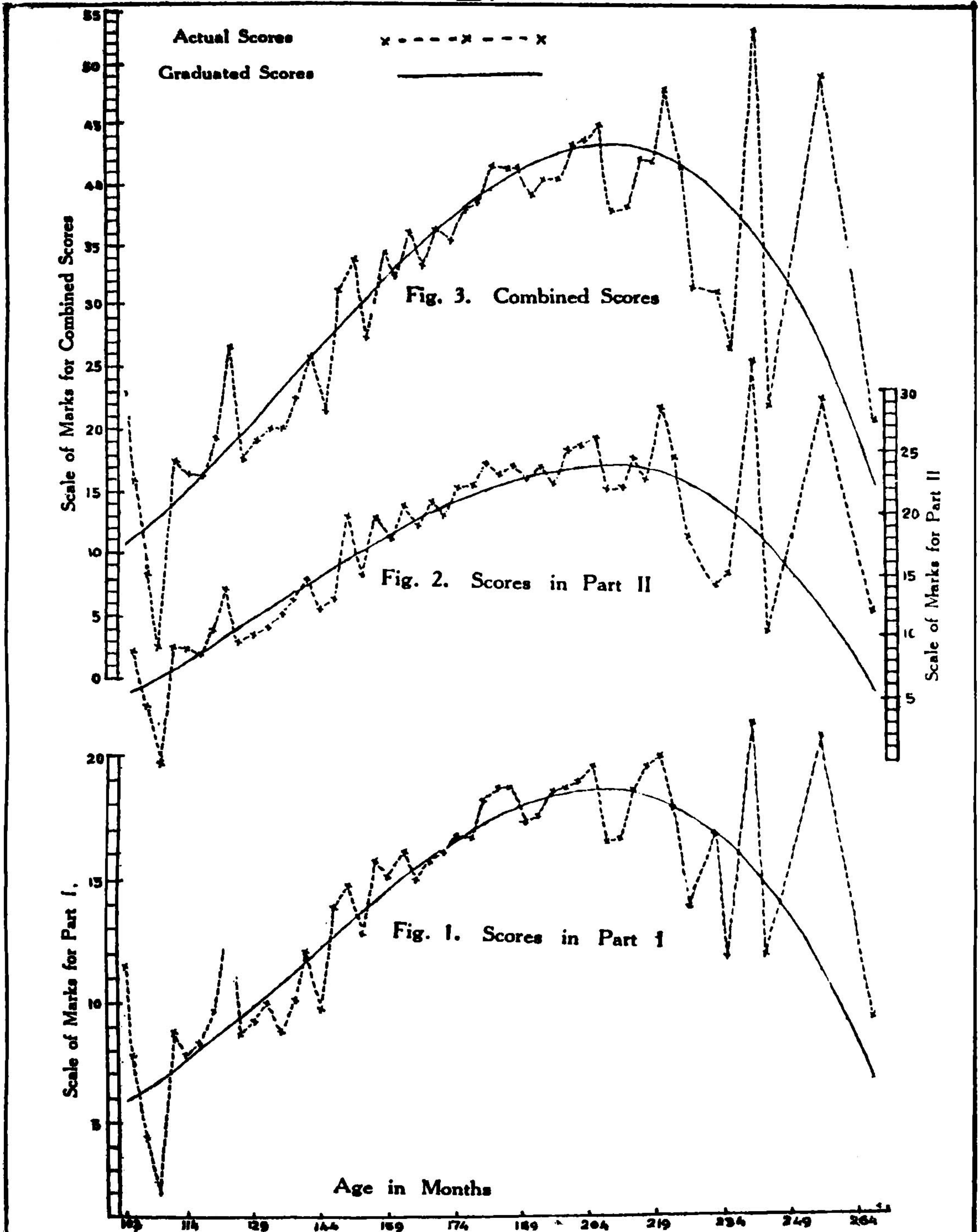
We may now test the adequacy of fit of the cubic equations by calculating the sum of the squares of deviations from the cubic graduation. This is obtained by squaring the difference between columns (3) and (4), (5) and (6), (7) and (8) in Table 6 for Part I, Part II, and the combined scores respectively, multiplying by the corresponding values of n_p given in column 2 and adding.

We then find for Part I :

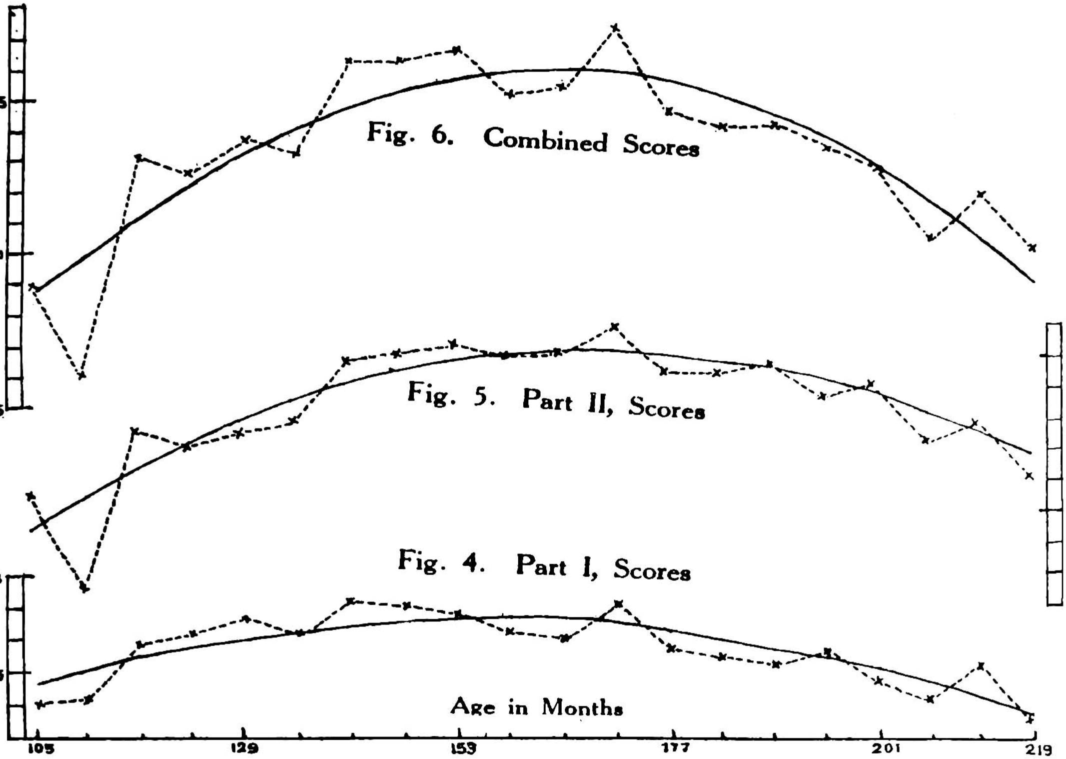
TABLE 7.—ANALYSIS OF VARIANCE FOR PART I SCORES (CUBIC REGRESSION)

Factor of Variation	D. F.	Sum of Squares	Variance	Ratio of Variances: Observed	Ratio of Variances: 1% Expected
Deviations from Cubic Regression	45	1878·72	41·75	1·09	1·34
,, ,, Linear Regression ...	47	4099·14	87·22	2·27	1·37
Within Age-groups	1163	44668·50	38·41	1·00	..

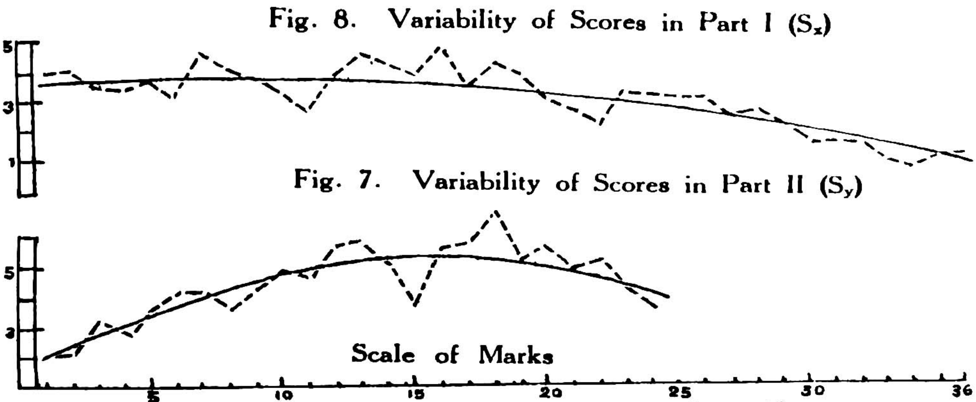
The number of degrees of freedom for deviations from the cubic equation is 45, since the cubic equation contains three arbitrary constants. The variance due to deviations from cubic regression is 41·75, against a residual (within age-groups) variance of 38·41. The observed ratio of variances is 1·09, while the one per cent. expected value is 1·34 (for $n_1=45$, $n_2=1163$). The observed ratio cannot, therefore, be considered significant, and we conclude that the graduation by the cubic curve is adequate.



Age Variation of Marks



Variability of Scores with Ages



Variability of Scores in Parts I & II

Similarly for Part II, I find :—

TABLE 8.—ANALYSIS OF VARIANCE FOR PART II SCORES (CUBIC REGRESSION)

Factor of Variation	D. F.	Sum of Squares	Variance	RATIO OF VARIANCE	
				Observed	1% Expected
Deviations from Cubic Regression	45	2455·70	54·57	0·64	1·84
,, ,, Linear Regression ...	47	8363·84	177·94	2·10	1·37
Within Age-groups	1163	98444 00	84·65	1·00	...

In this case the deviations from the cubic equation are actually lower (proportionately to the respective numbers of degrees of freedom) than the deviations within age-groups. Evidently we cannot hope to better the fit by taking a higher order curve.

Finally for the combined scores, I get :—

TABLE 9,—ANALYSIS OF VARIANCE FOR COMBINED SCORES (CUBIC REGRESSION)

Factor of Variation	D. F.	Sum of Squares	Variance	RATIO OF VIARANCES	
				Observed	1% Expected
Deviations from Cubic Regression ..	45	10396·04	231·02	1·05	1·34
,, ,, Linear Regression ...	47	23339·33	496·58	2·27	1·37
Within Age-groups	1163	254895·85	219·17	1·00	...

For the cubic regression the ratio of variances is 1·05 against an one per cent. expected value of 1·34 (with $n_1 = 45$, $n_2 = 1163$). The fit is therefore extremely satisfactory.

The above analysis shows definitely that in each case (Part I, Part II, and the combined results) the graduation by a cubic regression equation is fully adequate, and is a distinct improvement on the graduation by a linear regression line.

THE AGE FOR MAXIMUM SCORES.

A glance at the graphs shows that the maximum score is reached in each case at about the age of 209 months or roughly at about 17½ years. The actual maximum point can be easily calculated from the cubic curves by solving the equation,

$$\frac{dy}{dt} = c_1 + 2c_2(t) + 3c_3(t^2) = 0$$

$$\text{or } (t)_m = \pm \left[\sqrt{(c_2^2 - 3c_1c_3)} - c_2 \right] / 3c_3$$

Substituting the value of c_3, c_2 and c_1 we first obtain the values of t_m, and then transfer to months with the help of the equation :—

$$T_m = 168{\cdot}07 + 27{\cdot}98 \ t_m$$

We obtain the following results:—

TABLE 10.—AGE OF MAXIMUM SCORES.

Scores	t_m	Months
Part I ...	+ 1·4345	203·20 Months
Part II ...	+ 1·4821	209·53 ,,
Combined ...	+ 1·4621	208·98 ,,

The decline in test scores sets in therefore slightly before the age of 17½ years. The explanation I think is obvious The more intelligent students pass the Matriculation Examination at about this age, and go away from schools. It will be remembered that according to the Regulations of the Calcutta University the minimum age for matriculation was 16 years. The *average age* of matriculates who are neither in advance nor behind their proper age in ability will be 16½ years (or 198 months) approximately. The fact that maximum scores are obtained at the age of 208 or 209 months, that is about 10 or 11 months later indicates I think that, on an average, our school boys are retarded by about 10 or 11 months.

The rapid decline in test scores after the age of 17½ years merely implies, therefore, that those who still remain in schools beyond the age of 17½ years are distinctly below the matriculates in intelligence. This result is interesting inasmuch as it indicates that intelligence (or at least whatever entity is being measured by the present test) is a factor of importance in passing the Matriculation Examination of the Calcutta University.

RATE OF GROWTH OF SCORES WITH AGE.

Looking at the graph we notice that up to about the age of 17½ years the relation between test-scores and age appears to be almost linear. Let us consider this point in greater detail.

Choosing 1082 pupils of age from 100 to 204 months, I find the following analysis of variance for scores in Part I.

The variance for deviations from linear regression is 44·90 against a residual (within age-groups) variance of 39·88. The observed ratio of variances is thus 1·13, while the five

TABLE 11.—ANALYSIS OF VARIANCE FOR PART I SCORES (*Selected*).

Factor of Variation	D. F.	Sum of Squares	Variance	Ratio of Variances: Observed	Ratio of Variances: 1% Expected
Deviation from Linear Regression ...	33	14 81·59	44·90	1·13	1·55
Linear Regression	1	1 16 58·93	116 58·93	292·38	6·66
Between Age-groups	34	1 31 40·52	3 86·49	9·69	1·53
Within Age-groups	1047	4 17 50·64	39·88	1·00	...
Total	1081	5 48 91·16	50·78	...	

per cent. expected value is 1·55 (for $n=33$, $n=1047$). The deviations from linear regression are, therefore, clearly negligible, and we conclude that a linear regression gives an adequate fit up to the age of 17 years.

We also have $\eta = 0{\cdot}4893$, $r = +0{\cdot}4788$, with $(\eta^2 - r^2) = 0{\cdot}010143$ against a value of $(\eta^2 - r^2) = 0{\cdot}068665$ for the total sample of 1212. It will be noticed that there is substantial reduction in the value of $(\eta^2 - r^2)$ showing a better approach to linearity.

Similarly we obtain the following analysis for scores in Part II.

TABLE 12.—ANALYSIS OF VARIANCE FOR PART II SCORES (*Selected*)

Factor of Variation	D. F.	Sum of Squares	Variance	Ratio of Veriances	
				Observed	1 % Expected
Deviation from Linear Regression ...	33	27 21·57	82·47	·94	1·55
Linear Regression	1	2 53 74·32	2 53 74·32	2 90·47	6·66
Between Age-groups	34	2 80 95·89	8 26·35	9·48	1·53
Within Age-groups	1047	9 14 52·23	87·35	1·00	...
Total	1081	11 95 48·12	110·59	...	

The ratio of variances (for deviation from linear regression) is again negligible. Also $\eta = 0{\cdot}4869$, $r = +0{\cdot}4609$ and $(\eta^2 - r^2) = 0{\cdot}02\ 29\ 39$, while the value of $(\eta^2 - r^2) = {\cdot}06\ 62\ 39$ for the whole sample. Thus in this case also a linear regression gives an adequate fit up to the age of 17 years.

It is not necessary to repeat the analysis for the combined scores as we are bound to get similar results. We conclude therefore that up to the age of 17 years, the growth in test scores is practically proportional to the growth in age.

CHANGES IN THE VARIABILITY OF SCORES WITH AGE.

It is not merely the mean score that changes with age. The variability of scores within different age-groups also shows characteristic changes with age.
respectively.

The observed values of the Standard Deviation for 6 months age-groups are shown in Table 13, columns (3), (5), and (7) for scores in Part I, Part II, and combined scores

I have graduated these observed scores by simple parabolic curves whose equations are given below. Putting $t = (T - 162)/6$ we have :—

$$s_x = 6{\cdot}6922 - 0{\cdot}0742(t) - 0{\cdot}0277(t^2)$$
$$s_y = 9{\cdot}9533 + 0{\cdot}0917(t) - 0{\cdot}0490(t^2)$$
$$s_z = 16{\cdot}1743 - 0{\cdot}0152(t) - 0{\cdot}0807(t^2)$$

The graduated values are given in Table 13, columns 3, 5, and 7. The corresponding graphs are shown in Figures 4, 5 and 6. In each case the variability increases at first, attains a maximum value[2], and then slowly declines.

It will be noticed from Table 13 that for scores in Part I, the maximum value of the graduated S.D., 6·74, is attained at the mean age of 156·5. The variability remains fairly steady and greater than 6·0 between mean ages 126·5 and 180·5.

[2] The theoretical values of the points of maximum variability deduced from the graduation equations by solving for dy/dt, are 154·5, 168·1, and 162·0 for Part I, Part II, and combined scores respectively.

For scores in Part II, graduated variability reaches its maximum value of 10·0 at the mean age of 170·5 months, and remains stationary (over 9·0) from 144·5 to 192·5 months The maximum variability in scores for Part II occurs distinctly, nearly 14 months, after the maximum variability is reached for Part I.

For the combined scores the highest graduated variability (16·17) occurs at the mean age of 162·5, which is about mid-way between the points of maximum variability for Part I and Part II. The graduated variability remains greater than 14 from 132·5 to 192·5 months.

TABLE—18 GRADUATED AND OBSERVED VALUES OF STANDARD DEVIATIONS

Age-group	Number n_p	Part I (s_x)		Part II (s_y)		Combined (s_z)	
		Graduated	Observed	Graduated	Observed	Graduated	Observed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
100–105	10	4·66	4·01	4·14	5·29	8·26	8·96
–111	6	5·11	4·22	5·16	2·40	9·78	6·15
–117	23	5·51	5·99	6·08	7·47	11·13	13·05
–123	26	5·85	6·32	6·91	6·96	12·33	12·65
–129	41	6·14	6·81	7·64	7·38	13·36	13·73
–135	55	6·37	6·35	8·27	7·59	14·23	13·35
–141	77	6·55	7·28	8·80	9·54	14·94	16·41
–147	78	6·67	7·15	9·24	9·76	15·49	16·37
–153	88	6 78	6·94	9·57	10·18	15·88	16·69
–159	58	6·74	6·84	9·81	9·72	16·11	15·34
–165	104	6·69	6·24	9·95	9·98	16·17	15·54
–171	105	6·59	7·27	10·00	10·77	16·08	17·53
–177	83	6·43	5·91	9·94	9·36	15·82	14·79
–183	112	6·22	5·58	9·79	9·32	15·40	14·38
–189	76	5·95	5·37	9·54	9·57	14·82	14 39
–195	70	5·63	5 66	9·19	8·61	14·08	13·65
–201	54	5·25	4 92	8·74	8·75	13·18	13·03
–207	51	4·81	4·20	8·20	7·11	12·11	10·72
–213	44	4·32	5·26	7·55	7·73	10·89	12·20
–219	22	3·88	3·74	6·81	6·08	9·50	10·60

The increase in the variability of the score in the earlier years is I believe a real peculiarity of the growing child, for an exactly similar change is known to occur in a large number of physical characters.

The decline in variability observed in later years, on the other hand, must be traced I think to another cause. It will be remembered that the maximum score possible in Part I is only 23. The graduated mean score at the age of 157 months (when the variability attains its maximum value) is 14·19. The graduated S.D. at this age is 6·74, which places the limit of the score which may be attained by a pupil of normal ability at 20·93 (which limit is obtained by adding 6·74 to 14·19). At a still higher age, say, at the mean age of 165, the graduated mean score is 16·65. At the upper end, this leaves a range of variation of only 6·35 (*i.e.* 23—16·65), which is actually *less* than the maximum variability of 6·74. It is obvious therefore that owing to the curtailment of the score-scale at 23, and to the fact that at higher ages the mean score more and more approaches this limit, the possible range of variation is progressively reduced. In other words, the observed variability is progressively reduced owing to congestion in the score-scale at the upper end.

For Part II the progressive decrease in variability occurs about 14 months later at about the age of 170·5 months which shows that the test-score begins to get congested at this point. The combined score merely shows the joint effect of both scales. The congestion begins roughly mid-way between the critical points for the two scales. We notice however that the decline in variability is not serious up to the mean age of about 15 years.

The reality of the congestion at the upper end of the test-scores is clearly shown by the change of s_x (variability of scores in Part I) with the increase in the values of Y (mean scores in Part II), and also by the change of s_y with X. The observed values of s_x and s_y are given in Table 14, and the corresponding graphs* in figures 7 and 8.

Table 14—Standard Deviations of Scores

Scores in Part I	Number	Standard Deviation of Scores in Part II	Scores in Part II	Number	Standard Deviation of Scores in Part I	Scores in Part II	Number	Standard Deviation of Scores in Part I
X	n	s_y	Y	n'	s_x	Y	n	s_x
(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
0	37	2·00	0	38	2·04	24	32	3·19
1	31	2·15	1	22	4·03	25	32	3·14
2	20	3·24	2	27	3·53	26	47	2·58
3	36	2·90	3	34	3·45	27	43	2·64
4	28	3·54	4	27	3·70	28	57	2·43
5	21	4·19	5	33	3·07	29	46	1·67
6	28	4·19	6	36	4·61	30	54	1·68
7	29	3·52	7	21	4·29	31	40	1·61
8	28	4·32	8	33	3·91	32	39	1·08
9	38	4·97	9	38	3·40	33	35	0·97
10	37	4·76	10	32	2·87	34	26	1·20
11	39	5·73	11	29	3·83	35	22	1·33
12	38	5·89	12	31	4·56	36	5	...
13	44	5·12	13	31	4·41	37	1	...
14	47	3·69	14	33	4·01	...	...	...
15	38	5·70	15	26	4·97	...	...	...
16	37	5·77	16	32	3·64	...	...	...
17	58	6·80	17	26	4·36	...	...	...
18	60	5·30	18	36	4·00	...	...	...
19	69	5·67	19	29	3·25	...	...	...
20	88	5·08	20	31	2·88	...	...	...
21	112	5·20	21	30	2·30	...	...	...
22	132	4·80	22	33	3·35	...	...	...
23	122	3·57	23	25	3·33	...	...	...
Total ...	1,212	...	...	...	...	1,212	...	...

It will be noticed that the variability of scores in Part II (s_y) becomes restricted beyond the mean score of about 20 in Part I. The decline in the variability of scores in Part I (s_x) is more pronounced, and occurs comparatively earlier, at about the score of 16 or 17 for Part II. This is of course quite natural, since pupils securing scores of 16 or more in Part II are almost sure to have obtained high scores in Part I, and hence must show diminished variability with regard to Part I scores.

We conclude therefore that the test scale is seriously congested at the upper end, and does not offer full scope to the older subjects. But, on the whole the balance of evidence in favour of the view that the congestion is not serious up to the age of about 15 or 15½.

*The smooth curve has been drawn by hand.

SUMMARY

The results of an Intelligence Test conducted through the medium of the Bengali language for 1212 Bengali school children of various ages from about 8 years to over 22 years have been analysed in this paper. The test was arranged in two parts I and II, and it had been shown previously (this Journal, 1(1), June, 1933) that the reliability of the Test, as measured by the correlation in the two parts, was very high, with $r = +0.88$, and the Pearsonian correlation ratio $\eta = 0.90$ approximately. The chief results of the present paper are summarised below.

(1) Scores in Part I and in Part II (and hence naturally the combined scores) show significant variation with age.

(2) Upto the age of about 204 months, that is, from about 8 years to about 17 years, the increase in the score is practically proportional to the growth in age, and a linear regression is found fully adequate in all three cases (Part I. Part II, and combined).

(3) Beyond the age of 17 or 17½ years there is a pronounced drop in the scores which is statiscally significant. The minimum age of Matriculation in the Calcutta University being 16, it is likely that all school students who are able to pass the Matriculation Examination begin to pass out of the School at the age of 16 or 17 years. Students of older age who are still left in the School will thus represent the retarded students who find it difficult or impossible to pass the Matriculation examination. It is therefore only to be expected that the higher age groups would show lower scores in the Intelligence Test. This result also indicates indirectly that intelligence is a factor of importance in passing the Calcutta Matriculation examination.

(4) For students of all ages, the regression of scores on age is definitely non-linear.

(5) A cubic regression is found fully adequate for describing the dependence of scores on age in all cases.

(6) Apart from mean scores, the variability of scores (as measured by the Standard Deviation within each age-group) also changes appreciably with age.

(7) The variability increases at first, remains fairly steady for medium ages, and then diminishes.

(8) The increase in variability of intelligence in earlier years is probably a real characteristic of growing children.

(9) The decline in variability in higher age-groups on the other hand is caused by the progressive curtailment in the scale of scores at the upper end.

(10) On the whole the balance of evidence is in favour of the view that the congestion in the scale is not serious up to 15 or 15½ years, so that the test may be administered with safety up to this age, and with slightly diminished efficiency up to the age of 17 or 17½ years.

In subsequent papers we intend to make a comparative study of the Intelligence Quotient (I.Q.) and other alternative measures of intelligence.*

*The present paper was communicated to the Indian Science Congress, 1932.