STUDIES IN EDUCATIONAL TESTS NO. 4. A PRELIMINARY STUDY OF THE INTELLIGENCE QUOTIENT OF BENGALI SCHOOL CHILDREN

By P. C. MAHALANOBIS.

Introduction.

A group-test of intelligence conducted through the medium of the Bengali language was fully described in a previous paper in this Journal (Vol. 1, Part 1, June, 1933, pp. 25-49). The test had been intentionally divided into two parts I and II in order that the reliability of the test could be measured in terms of the correlation between scores in the two parts. The Pearsonian (product-moment) coefficient was high and of the order of +0·88 for a sample of 1212 Bengali school children. The variation of raw scores with age was discussed in another issue (Vol. 1, Parts 2 and 3, July, 1934, pp. 231-244). The regression of scores on age was non-linear but graduation by a parabolic regression of the second degree was found adequate. The graduated scores for different age-groups, determined from these parabolic regressions are reproduced in Table 1 for reference. (Appendix)

If we wish to compare the intelligence of two students belonging to two different age groups, it is clearly necessary to make suitable allowances for the change of scores with age. The most extensively used device for this purpose is the I.Q. (Intelligence Quotient) which is defined below.

Let x_{pq} be the actual observed score of a particular (say the qth) boy in a particular (say the pth) age group. The chronological age of the student is thus t_p. Now corresponding to each age group we have a graduated score given in Table 1. From this Table 1 we can also find for each score a corresponding age-group[1]. Let t_{pq} be the age corresponding to a score x_{pq}. If t_{pq} is less than t_p, then the student is clearly older than the boys whose average score is the same as his. On the other hand if t_{pq} is greater than t_p, the student is ahead of boys of his own age. It is usual to call t_{pq} the mental (or intelligence) age of the student. The Intelligence Quotient (I.Q.) is defined as the percentage ratio of the mental age to the chronological age, or

$$\text{I.Q.} = 100.\ t_{pq}/t_p = u_{pq} \qquad \ldots\ldots(1)$$

The I.Q. of each of the 1212 students was calculated separately for scores in Part I, and Part II, and also for the combined scores, with the help of the graduated scores given in Table 1.

Reliability in terms of I.Q.

We thus have two different estimates of the I.Q. of the same student based on scores in the two parts. We may, therefore, measure the reliability of the test by finding the

[1] It may be noted in passing that an alternative statistical procedure is also open. Instead of using the regression of scores on age, we can determine the age corresponding to a given score from the regression of age on scores. This procedure however does not appear to have been adopted in practice.

correlation between the two I.Q.'s based on Parts I and II respectively. Let u represent the observed I. Q. based on scores in Part I, v the observed I. Q. based on scores in Part II, and w the I. Q. based on the combined scores. The correlation chart for u and v is given in Table 2.

TABLE 2. CORRELATION CHART FOR u (I. Q. FOR PART I SCORES) AND v (I. Q. FOR PART II SCORES)

u (I. Q. for Part I Scores) \ v (I. Q. for Part II Scores)	25-34	35-44	45-54	55-64	65-74	75-84	85-94	95-104	105-114	115-124	125-134	135-144	145-154	155-164	165-174	175-184	Total
25— 34	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35— 44	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	4
45— 54	—	9	10	15	5	1	—	—	—	—	—	—	—	—	—	—	40
55— 64	—	1	16	34	27	12	6	2	—	—	—	—	—	—	—	—	98
65— 74	—	—	3	30	28	26	9	3	2	—	—	—	—	—	—	—	101
75— 84	—	—	2	15	18	29	21	15	3	1	1	—	—	—	—	—	105
85— 94	—	—	1	6	16	24	44	22	8	4	2	1	—	—	—	—	128
95—104	—	—	1	1	15	18	26	107	11	10	2	—	1	—	—	—	192
105—114	—	—	—	—	2	11	16	24	84	9	15	2	2	—	—	—	165
115—124	—	—	—	1	2	2	8	16	17	95	3	8	—	—	—	—	152
125—134	—	—	—	—	1	—	4	16	5	4	76	—	3	—	—	—	109
135—144	—	—	—	—	—	—	2	4	5	8	3	39	1	—	—	—	62
145—154	—	—	—	—	—	—	—	1	3	5	6	1	25	—	—	—	41
155—164	—	—	—	—	—	—	—	—	—	—	3	1	—	7	—	—	11
165—174	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
175—184	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	2
185—194	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
195—204	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Total	1	12	34	103	114	123	136	210	138	136	111	53	32	7	1	1	1212

The statistical constants are given in Table 3. The two mean values (98·86 and 97·74) are both slightly less than 100, but do not differ much while the two standard deviations (26·65 and 26·15) are practically equal. The coefficient of correlation is $+0{\cdot}8481 \pm 0{\cdot}0054$ which is only slightly less than the correlation between the raw scores ($+0{\cdot}8781 \pm 0{\cdot}0044$) but is still quite high. The values of η, the Pearsonian correlation ratio, are 0·8493 and 0·8504, and are very nearly equal to r showing that the regression is practically linear.

TABLE 3. CONSTANTS FOR u AND v

Statistics	u (I.Q. for Part I Scores)	v (I.Q. for Part II Scores)
Mean	98·8647 ± 0·5150	97·7426 ± 0·5069
S.D.	26·6503 ± 0·3692	26·1524 ± 0·3585
r	+0·8481 ± 0·0054	...
η	0·8493	0·8 04
$\eta^2 - r^2$	0.0019	0·0037

The analysis of variance for the regression of v (Part II) on u (Part I) is given in Table 4.

TABLE 4 ANALYSIS OF VARIANCE FOR REGRESSION OF v (I.Q. FOR PART II SCORES) ON u (I.Q. FOR PART I SCORES)

Factor of Variation	Degrees of Freedom	Sum of Squares	Variance	Ratio of Variances	
				Observed	1% Expected
Deviations from Linear Regression ...	14	15·5	1·11	0·58	2·078
Linear Regression	1	5963·1	5963·1	3086·5	6·657
Between u-Arrays	15	5978·6	398·57	206·3	2·092
Within u-Arrays	1196	2311·0	1·932	1·00	—
TOTAL ..	1211	8289·6	6·845	—	—

The variance for deviations from linear regression is 1·11 while the variance within arrays is 1·93. A linear regression is therefore fully adequate.

The analysis of variance for the regression of u (Part I) on v (Part II) is given in Table 5.

TABLE 5. ANALYSIS OF VARIANCE FOR REGRESSION OF u (I.Q. FOR PART I SCORES) ON v (I.Q. FOR PART II SCORES)

Factor of Variation	Degrees of Freedom	Sum of Squares	Variance	Ratio of Variances	
				Observed	1% Expected
Deviations from Linear Regression ...	14	32·3	2·31	1·16	2·078
Linear Regression	1	6192·3	6192·3	3107·0	6·657
Between v-Arrays	15	6224·6	414·97	208·2	2·092
Within v-Arrays	1196	2383·5	1·993	1·00	—
TOTAL ...	1211	8608·1	7·108	—	—

The linear regression is again adequate. The two regression equations are given below.

$$u = 0{\cdot}8643 \,.\, (v) + 14{\cdot}386 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$v = 0{\cdot}8323 \,.\, (u) + 15{\cdot}458 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The observed and calculated mean values for different ranges are given in Table 6 and are shown graphically in Chart 1.

CHART I. REGRESSION LINES OF 'u' AND 'v'

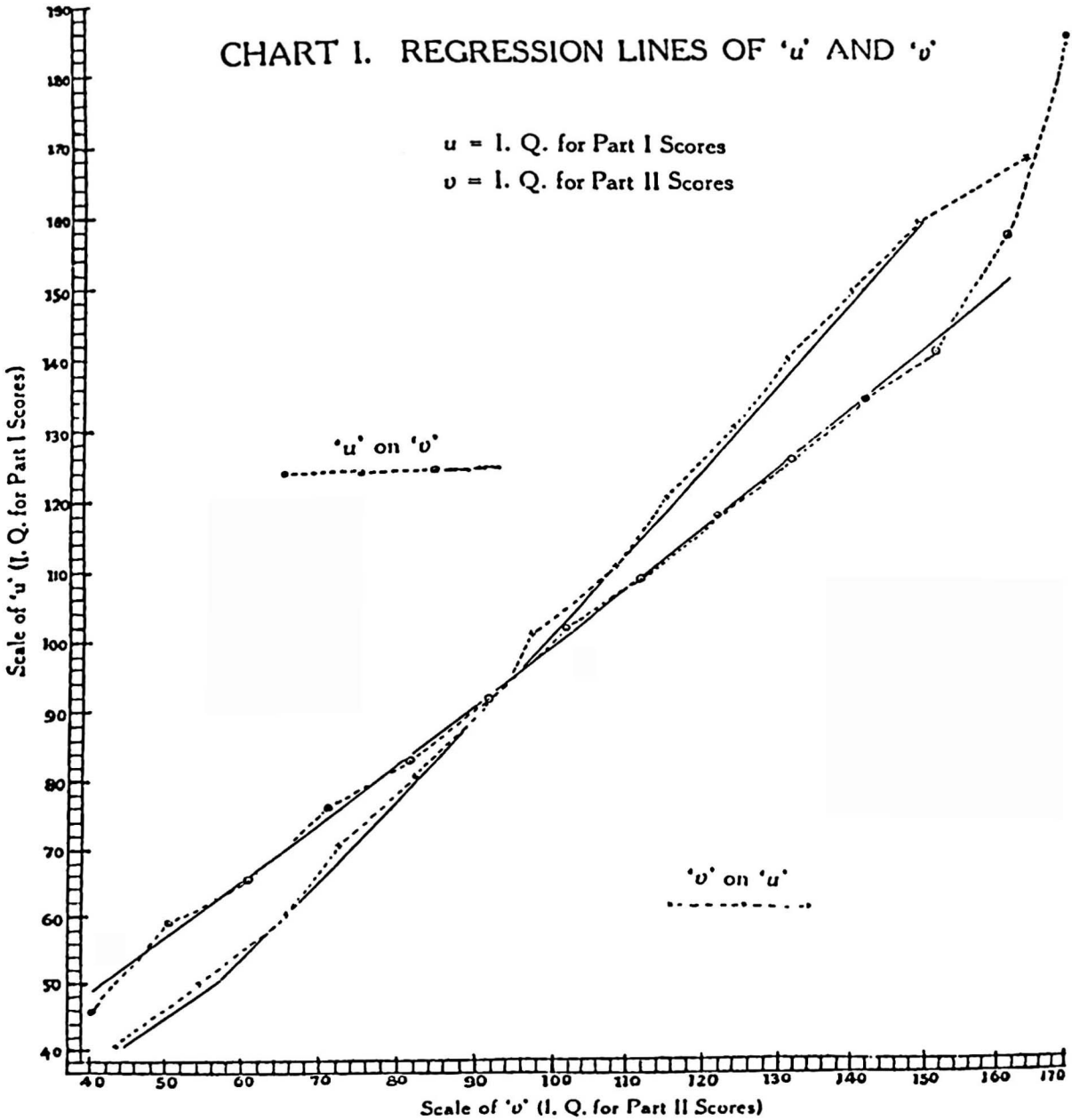


TABLE 6 OBSERVED AND CALCULATED VALUES OF v (I.Q. FOR PART II SCORES) AND u (I.Q. FOR PART I SCORES).

Range of u: I.Q. for Part I Scores	Number	Mean Value of v: I.Q. for Part II Scores		Range of v: I.Q. for Part II Scores	Number	Mean Value of u: I.Q. for Part I Scores	
		Observed	Calculated			Observed	Calculated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25—34	1	45·92	48·53	25—34	1	48·50	44·08
35—44	12			35—44	4		
45—54	84	58·82	57·17	45—54	40	54·25	56·57
55—64	103	65·12	65·81	55—64	98	65·52	64·89
65—74	114	75·42	74·46	65—74	101	71·97	73·21
75—84	128	82·13	83·10	75—84	105	80·93	81·54
85—94	136	91·98	91·74	85—94	128	88·33	89·86
95—104	210	100·90	100·38	95—104	192	95·65	98·18
105—114	138	108·96	109·03	105—114	165	106·79	106·51
115—124	136	117·50	117·67	115—124	152	113·87	114·83
125—134	111	125·71	126·31	125—134	109	122·84	123·35
135—144	53	134·46	134·95	135—144	62	129·98	131·47
145—154	32	141·98	143·60	145—154	41	138·61	139·80
155—164	7	158·23	152·24	155—164	11	149·50	148·12
165—174	1	188·23	...	165—174	...		
175—184	1		..	175—184	2	162·83	...
				185—194	...		
				195—204	1		
TOTAL ...	1212	...	...	TOTAL ...	1212	...	...

CHANGE OF I. Q. WITH AGE.

It will be remembered that the chief purpose of using the I. Q. is to eliminate the effect of the variation of scores with age in making comparison. It is therefore necessary to investigate how far this has been achieved in practice, that is, to see whether I. Q. depends in any way on the age of the candidates.

TABLE 7. CORRELATION CHART FOR AGE-DISTRIBUTION OF u (I. Q. FOR PART I SCORES)

Values of u (I. Q. for Part I Scores) \ Age in months	100–105	106–111	112–117	118–123	124–129	130–135	136–141	142–147	148–153	154–159	160–165	166–171	172–177	178–183	184–189	190–195	196–201	202–207	208–213	214–219	220–225	226–231	232–237	238–243	244–249	250–255	256–261	262–267	268–273	Total
25–34	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1
35–44	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	2	–	–	1	–	–	–	–	4
45–54	–	–	–	–	–	–	–	–	4	–	4	8	5	4	6	4	2	1	2	–	–	–	–	–	–	–	–	–	–	40
55–64	–	–	–	–	4	5	22	16	9	6	6	9	3	3	1	2	3	1	4	1	1	1	1	–	–	–	–	–	–	98
65–74	–	–	3	5	8	10	10	7	4	7	8	4	5	8	1	7	3	2	3	1	1	1	1	–	–	–	–	–	–	101
75–84	2	4	7	3	3	3	5	5	3	4	7	11	5	9	2	5	4	5	10	2	3	1	1	–	–	–	1	–	–	105
85–94	–	1	1	4	5	6	8	6	10	6	9	7	10	11	9	4	5	4	3	1	11	3	–	2	1	–	–	–	1	128
95–104	–	–	2	1	4	5	2	8	7	6	6	11	11	8	3	7	37	38	19	17	–	–	–	–	–	–	–	–	–	192
105–114	2	–	1	2	2	3	8	3	10	2	23	7	4	3	54	41	–	–	–	–	–	–	–	–	–	–	–	–	–	165
115–124	2	–	2	2	6	11	3	5	8	7	–	–	40	66	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	152
125–134	2	1	1	2	1	2	5	6	–	–	41	48	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	109
135–144	1	–	1	3	2	2	–	–	33	20	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	62
145–154	1	–	3	3	1	2	14	17	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	41
155–164	–	–	–	–	5	6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	11
165–174	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
175–184	–	–	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
185–194	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
195–204	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
Total	10	6	23	26	41	55	77	73	88	58	104	105	83	112	76	70	54	51	44	22	16	8	3	2	2	–	1	–	2	1212

The distribution of u (I. Q. for Part I scores) with age is shown in the correlation Table 7. The analysis of variance is given in Table 8.

TABLE 8. ANALYSIS OF VARIANCE FOR REGRESSION OF u (I.Q. FOR PART I SCORES) ON AGE

Factor of Variation	Degrees of Freedom	Sum of Squares	Variance	Ratio of Variances	
				Observed	1% Expected
Deviations from Parabolic Regression	25	265·0	10·60	1·584	1·765
Parabolic Regression (2nd Degree) ...	1	124·4	124·4	18·58	6·657
Deviations from Linear Regression ...	26	389·4	14·98	2·237	1·676
Linear Regression	1	290·0	290·0	43·31	6·657
Between Age-groups	27	679·4	25·16	3·758	1 696
Within Age-groups	1184	7928·7	6·696	1·00	—
TOTAL ...	1211	8608·1	7·108	-	—

The variance within age-groups is 6.696 against a variance between age-groups of 25·16. The ratio of variances is 3·758, while the expected ratio[2] is 1·696 on the basis of 1% (one per cent.) probability. The observed ratio is highly improbable, and we must conclude that the value of u (I. Q. for Part I scores) depends appreciably on the age of the candidate. This is disappointing but cannot be helped. The following values of the correlation co-efficient and the correlation ratio confirm the above results.

$$\eta_{u|t} = 0{\cdot}2809 \qquad \eta^2_{u|t} = 0.0789$$

$$r_{ut} = -0{\cdot}1836 \pm 0{\cdot}0187 \qquad r^2_{u|t} = 0.0337$$

$$\eta^2 - r^2 = 0.0452$$

The mean value of $(\eta^2)_0$ for no association is given by $(n-1)/(N-1) = 27/1211 = 0{\cdot}0233$, while the observed value is 0·0789. The standard deviation of $(\eta^2)_0$ is 0·0059, so that the observed correlation ratio must be considered significant.

Proceeding with the analysis of variance we find that naturally enough linear regression is definitely significant. Deviations from linear regression yield a ratio of variances of 2·237 which also is significant on the one per cent. level.

As the regression is definitely non-linear, we may next try Pearson's Non-linear Regression of the second degree. The relevant formulæ (taken from Karl Pearson's fundamental memoir of 1905) have been already given in the first paper of the present series in Note No. 9 (this Journal, Vol. 1, Part 1, June 1933, pp. 46-47).

[2] P. C. Mahalanobis: "Auxiliary Tables for Fisher's Z-test for use in the Analysis of Variance" (*Ind. Jour. Agri. Sc.*, Vol. II, Part VI, Dec., 1932).

TABLE 9. CORRELATION CHART FOR AGE-DISTRIBUTION OF v (I. Q. FOR PART II SCORES)

Age in months

v (I. Q. for Part II Scores)

Range	100–105	106–111	112–117	118–123	124–129	130–135	136–141	142–147	148–153	154–159	160–165	166–171	172–177	178–183	184–189	190–195	196–201	202–207	208–213	214–219	220–225	226–231	232–237	238–243	244–249	250–255	256–261	262–267	268–273	Total
25–34	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1
35–44	–	–	–	–	–	–	–	–	–	–	–	3	2	–	3	–	2	–	–	–	–	2	–	–	–	–	–	–	–	12
45–54	–	–	–	–	–	–	5	1	4	3	5	2	3	2	3	2	1	–	2	–	–	–	–	–	1	–	–	–	–	34
55–64	–	–	1	5	3	9	7	10	8	1	13	12	4	5	1	3	4	3	7	2	1	1	2	–	1	–	–	–	–	103
65–74	–	2	3	2	5	5	14	7	5	9	8	8	7	10	6	7	3	3	2	3	1	1	1	–	–	–	–	–	–	114
75–84	1	2	2	1	6	7	9	12	5	7	6	10	8	13	5	9	–	5	8	3	2	1	–	–	–	–	–	–	4	123
85–94	1	1	5	–	7	4	9	8	11	6	7	8	9	10	11	9	6	3	3	–	12	3	–	2	–	–	1	–	–	136
95–104	2	1	5	5	8	7	5	8	4	7	14	10	7	9	5	4	38	37	22	12	–	–	–	–	–	–	–	–	–	210
105–114	1	–	1	6	2	9	6	5	9	5	7	3	5	1	42	36	–	–	–	–	–	–	–	–	–	–	–	–	–	138
115–124	2	–	3	3	3	5	4	2	7	4	1	2	38	62	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	136
125–134	1	–	–	2	3	3	5	3	2	2	43	47	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	111
135–144	1	–	1	–	–	3	1	–	33	14	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	53
145–154	1	–	1	1	–	–	12	17	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	32
155–164	–	–	–	–	4	3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	7
165–174	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
175–184	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
Total ...	10	6	23	26	41	55	77	73	88	58	104	105	83	112	76	70	34	31	44	22	16	8	3	2	2	–	1	–	2	1212

The actual parabolic equation is given below.

$$u = 38{\cdot}445 + 10{\cdot}9758\ (t) - 0{\cdot}4634\ (t^2) \quad \ldots \quad (8)$$

where u = I.Q. for scores in Part I

and t = age in years.

The graduated and observed values are given in Table 13, columns 5 and 4, and are shown plotted in Chart 2.

Proceeding with the analysis of variance in Table 8, we find that the parabolic regression is significant, but deviations from the parabolic regression give a ratio of variance of 1·584 which is no longer statistically significant. We conclude that a parabolic regression of the second degree gives adequate graduation.

In the same way we can investigate the dependence of v (I. Q. for Part II scores) on age. The correlation chart is given in Table 9, and the analysis of variance in Table 10.

TABLE 10. ANALYSIS OF VARIANCE FOR REGRESSION OF v (I. Q. FOR PART II SCORES) ON AGE.

Factor of Variation	Degrees of Freedom	Sum of Squares	Variance	Ratio of Variances	
				Observed	1% Expected
Deviations from Parabolic Regression	25	147·1	5·88	0·896	1·765
Parabolic Regression (2nd degree) ...	1	164·2	164·2	25·00	6·657
Deviations from Linear Regression ...	26	311·3	11·97	1·828	1·676
Linear Regression	1	201·5	201·5	30·68	6·657
Between Age-groups	27	512·8	18·98	2·890	1·696
Within Age-groups	1184	7776·8	6·568	1·00	...
TOTAL ..	1211	8289·6	6·845	...	...

The variance between age-groups is significantly greater than the variance within age-groups (since the observed ratio of variances is 3·487 against an one per cent. expected value of 1·696).

The values of the correlation coefficient and the correlation ratio, which are given below also show that v (I. Q. for Part II scores) clearly depends upon age.

$$\eta_{vt} = 0{\cdot}2409 \qquad \eta^2_{vt} = 0{\cdot}0619$$

$$r_{vt} = 0{\cdot}155 \pm 0{\cdot}019 \qquad r^2_{vt} = 0{\cdot}0243$$
$$\eta^2 - r^2 = 0{\cdot}0376$$

The analysis of variance also shows that deviations from linear regression give a ratio of variances of 1·823 against an 1% expected value of 1·676. The linear regression

TABLE 11. CORRELATION CHART FOR AGE-DISTRIBUTION OF w (I. Q. FOR COMBINED SCORES.)

Range of w (I. Q. for Combined Scores)	Age in months																													Total
	100–105	106–111	112–117	118–123	124–129	130–135	136–141	142–147	148–153	154–159	160–165	166–171	172–177	178–183	184–189	190–195	196–201	202–207	208–213	214–219	220–225	226–231	232–237	238–243	244–249	250–255	256–261	262–267	268–273	
25–34	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	1	2
35–44	–	–	–	–	–	–	–	–	–	–	–	2	2	1	3	1	2	–	–	–	–	1	–	–	1	–	–	–	–	13
45–54	–	–	–	–	–	–	7	4	6	1	6	6	2	1	2	3	1	–	3	–	–	–	–	–	–	–	–	–	–	42
55–64	–	–	1	5	5	11	12	10	6	4	5	8	2	5	3	2	4	3	2	1	–	1	2	–	–	–	–	–	–	92
65–74	1	3	2	3	5	3	9	8	4	7	9	5	10	7	–	6	–	3	7	4	3	1	1	–	1	–	–	–	1	103
75–84	1	2	6	–	4	4	8	6	6	6	10	14	7	14	5	7	3	3	10	3	2	2	–	2	–	–	–	–	–	127
85–94	–	–	3	–	6	8	8	8	8	7	3	11	6	12	14	11	6	6	5	1	11	2	–	–	–	–	1	–	–	137
95–104	1	–	1	4	5	4	4	9	9	5	20	5	11	7	5	9	38	36	17	11	–	–	–	–	–	–	–	–	–	201
105–114	1	–	3	5	8	11	6	5	10	8	8	6	7	5	44	31	–	–	–	–	–	–	–	–	–	–	–	–	–	158
115–124	4	1	3	2	2	4	8	4	7	3	5	2	36	60	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	141
125–134	–	–	1	4	1	3	4	2	–	1	38	46	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	100
135–144	1	–	2	2	1	4	1	2	32	16	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	61
145–154	1	–	–	–	–	–	10	15	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	26
155–164	–	–	–	–	4	3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	7
165–174	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
175–184	–	–	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
TOTAL ...	10	6	21	26	41	55	77	73	88	58	104	105	83	112	76	70	54	51	44	22	16	8	3	2	2	–	1	–	2	1212

cannot thus be considered entirely adequate. Using a second degree regression we obtain the following equation :—

$$v = 23{\cdot}342 + 12{\cdot}7214\ (t) - 0{\cdot}5128\ (t^2) \quad \ldots\ldots(4)$$

where v = I.Q. for Part II scores
and t = age in years.

It will be noticed that the deviations from the above parabolic regression are statistically negligible so that the graduation is fully adequate. The graduated and observed values are given in columns 7 and 6 of Table 13, and are shown graphically in Chart 3.

The I.Q. based on combined scores naturally show similar features. The correlation chart is given in Table 11, and the analysis of variance in Table 12.

TABLE 12. ANALYSIS OF VARIANCE FOR REGRESSION OF w (I. Q. FOR COMBINED SCORES) ON AGE.

Factor of Variation	Degrees of Freedom	Sum of Squares	Variance	Ratio of Variances	
				Observed	1% Expected
Deviations from Parabolic Regression	25	206·1	8·244	1·276	1·765
Parabolic Regression	1	198·9	198·9	30·80	6·657
Deviations from Linear Regression ...	26	405·0	15·58	2·412	1·676
Linear Regression	1	203·3	203·3	31·48	6·657
Between Age-groups	27	608·3	22·52	3·487	1·697
Within Age-groups	1184	7647·4	6·459	1·00	...
TOTAL ...	1211	8255·7	6·818	...	..

The correlation coefficients and the correlation ratio are given below.

η_{wt} 0·2714 $\eta^2_{wt} = 0{\cdot}037$

r_{wt} 0·1569 ± ·0189 $r^2_{wt} = 0{\cdot}0246$, $\eta^2 - r^2 = 0{\cdot}0491$

The above values as well as the analysis of variance given in Table 12 show the appreciable dependence of w (I.Q. for combined scores) on age. Deviations from linear regression are definitely significant ; and we obtain the following parabolic regression :—

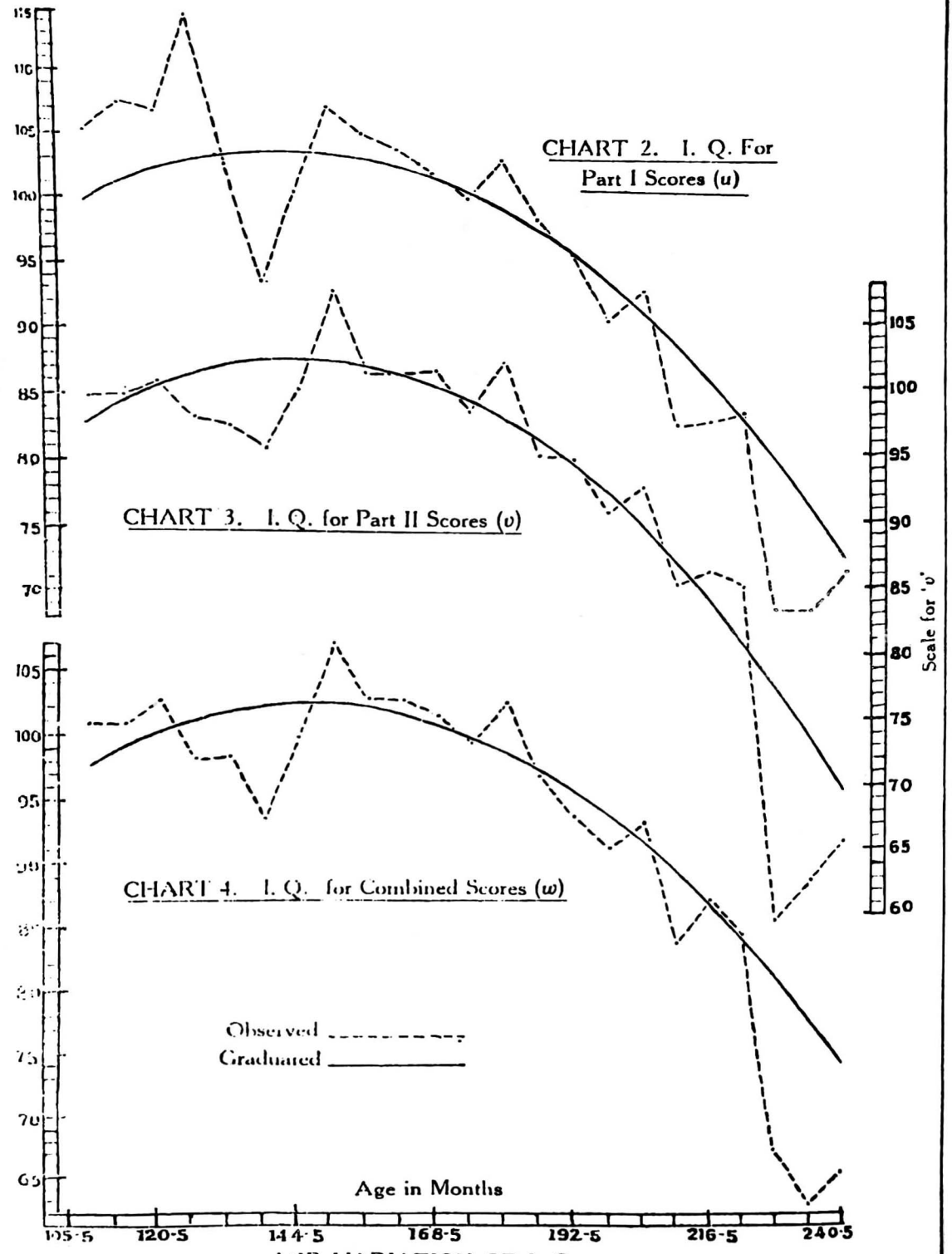
$$w = 33{\cdot}736 + 11{\cdot}230\ (t) - 0{\cdot}4594\ (t^2) \quad \ldots\ldots(5)$$

where w = I.Q. for combined scores
and t = age in years.

This graduation is adequate since deviations from the parabolic regression are not statistically significant. The graduated and observed values will be found in columns No. 9 and 8 of Table 13 and are plotted in Chart 4.

TABLE 13. GRADUATED AND OBSERVED VALUES OF I. Q.'S (VARIATION WITH AGE)

Age-Groups		Number	Mean u: I. Q. for Part I Scores		Mean v: I. Q. for Part II Scores		Mean w: I. Q. for Combined Scores	
Range (months)	Mid. point (months)		Observed	Graduated	Observed	Graduated	Observed	Graduated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
100—105	102·5	10	105·11	99·71	99·75	96·04	100·75	97·73
106—111	108·5	6						
112—117	114·5	23	107·86	100·99	99·94	97·63	100·81	99·07
118—123	120·5	26	106·69	101·94	100·65	98·97	102·58	100·19
124—129	126·5	41	114·47	102·73	98·28	100·05	98·04	101·08
130—135	132·5	55	102·41	103·14	97·50	100·88	98·23	101·74
136—141	138·5	77	93·43	103·40	95·73	101·45	93·53	102·17
142—147	144·5	73	100·29	103·43	100·19	101·77	99·73	102·36
148—153	150·5	88	106·64	103·22	107·80	101·82	107·00	102·33
154—159	156·5	58	101·61	102·78	101·05	101·62	102·95	102·07
160—165	162·5	104	103·52	101·41	101·23	101·17	102·58	101·58
166—171	168·5	105	101·47	101·11	101·41	100·46	101·41	100·86
172—177	174·5	83	99·68	100·07	98·54	99·49	99·38	99·91
178—183	180·5	112	102·69	98·77	102·09	99·26	102·45	98·73
184—189	186·5	76	97·78	97·11	95·03	96·80	96·87	97·33
190—195	192·5	70	95·09	95·28	94·64	94·27	93·64	95·69
196—201	198·5	54	90·08	93·21	90·61	92·10	91·17	93·82
202—207	204·5	51	92·54	90·92	92·83	89·65	93·03	91·72
208—213	210·5	44	82·32	88·40	85·18	86·96	83·82	89·39
214—219	216·5	22	82·78	85·64	86·32	84·01	87·23	86·84
220—225	222·5	16	88·23	82·65	85·13	80·80	84·50	84·05
226—231	228·5	8	68·23	79·43	59·50	77·83	67·20	81·03
232—237	234·5	8	68·23	75·98	62·88	78·61	62·83	77·78
238—243	240·5	2	71·10	72·29	65·99	69·64	65·22	74·81
Beyond	243	5	—	—	—	—	—	—



AGE VARIATION OF I. Q. s

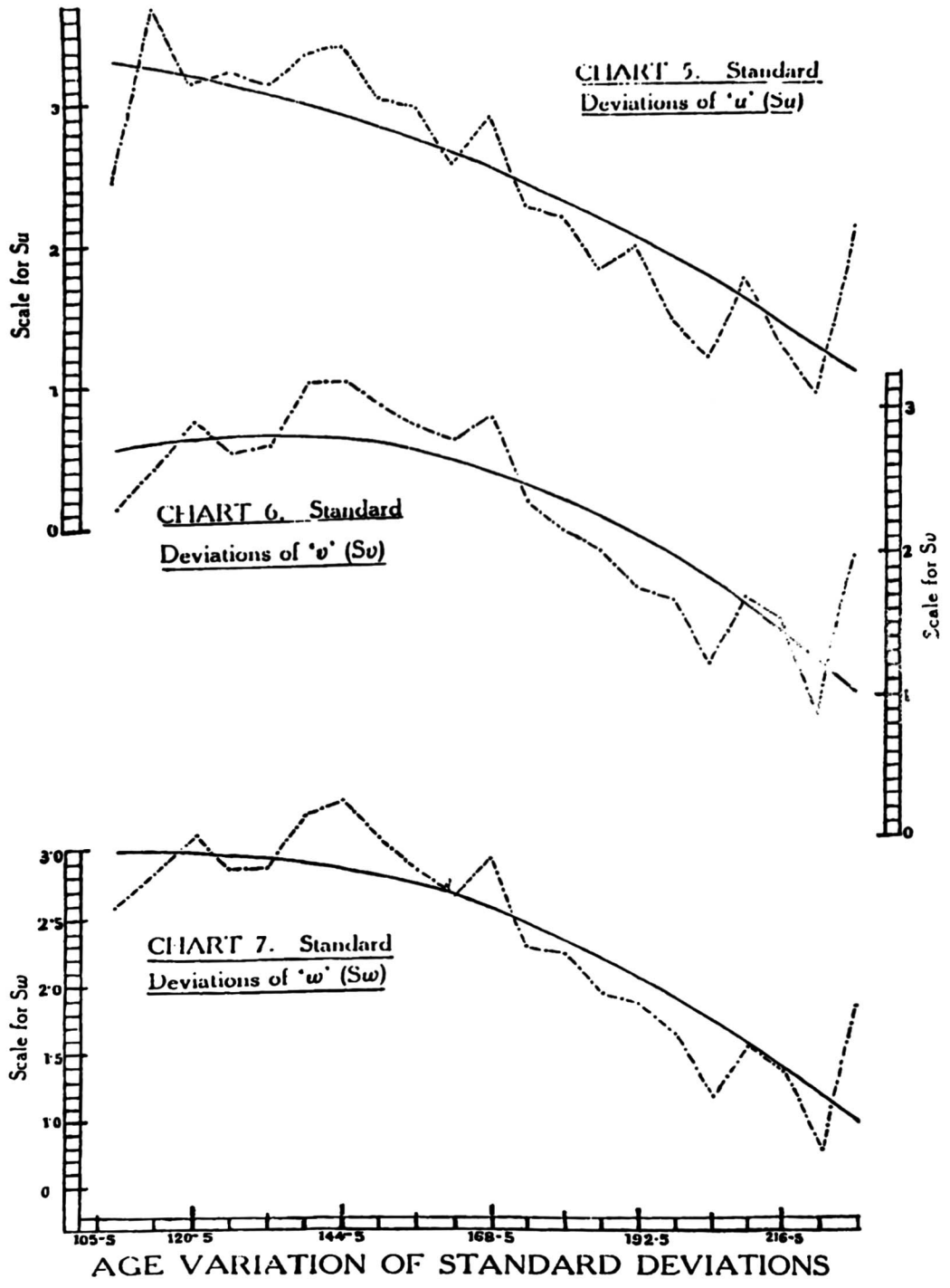


CHART 5. Standard Deviations of 'u' (Su)
Scale for Su
0
1
2
3
CHART 6. Standard Deviations of 'v' (Sv)
Scale for Sv
0
1
2
3
CHART 7. Standard Deviations of 'w' (Sw)
Scale for Sw
0
1·0
1·5
2·0
2·5
3·0
105-5
120-5
144-5
168-5
192-5
216-5
AGE VARIATION OF STANDARD DEVIATIONS

It can be seen from Charts 2, 3, and 4 that *u*, *v* and *w* show practically the same kind of variations with age. The I.Q.'s all slightly increase at first up to the age of about 12 or 13 years and then decrease with increasing age. The rate of decrease itself is at first small, but increases rapidly at higher ages.

So far as the present material is concerned the I.Q. evidently fails to eliminate completely the effect of age variations. It cannot therefore serve as a fully satisfactory tool for comparing the intelligence of candidates of different ages.

TABLE 14. OBSERVED AND GRADUATED VALUES OF STANDARD DEVIATIONS OF I. Q.'s

Age-Group in Months		Number	S_u=S.D. of *u* (I.Q. for Part I Scores)		S_v=S.D. of *v* (I.Q. for Part II Scores)		S_w=S.D. of *w* (I.Q. for Combined Scores)	
Range	Mid-point		Observed	Graduated	Observed	Graduated	Observed	Graduated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
100—105	102·5	16	2·442	3·318	2·377	2·767	2·585	2·966
106—111	108·5							
112—117	114·5	23	3·670	3·269	2·629	2·813	2·802	2·975
118—123	120·5	26	3·152	3·218	2·990	2·846	3·098	2 978
124—129	126·5	41	3·212	3·149	2·744	2·864	2·842	2·960
130—135	132·5	55	3·135	3·079	2·709	2·868	2·870	2·934
136—141	138·5	77	3·356	3·002	3.239	2·859	3·220	2·898
142—147	144·5	73	3·400	2·919	3·245	2·835	3·310	2·850
148—153	150·5	88	3·034	2·828	3·065	2·818	3·042	2·790
154—159	156·5	58	2·938	2·730	2·929	2·745	2·802	2·719
160—165	162·5	104	2·534	2·626	2·827	2 679	2·620	2·637
166—171	168·5	103	2·860	2·514	2·992	2·599	2 905	2·543
172—177	174·5	83	2·223	2·396	2·383	2·504	2·273	2·437
178—183	180·5	112	2·137	2·271	2·162	2·896	2·218	2·320
184—189	186·5	76	1·798	2·139	2·022	2·294	1·915	2·192
190—195	192·5	70	1·901	2·000	1·795	2·186	1·863	2·052
196—201	198·5	54	1·428	1·854	1·674	1·985	1·619	1·900
202—207	204·5	51	1·142	1·702	1·215	1·820	1·185	1·737
208—213	210·5	44	1·607	1·542	1·684	1·641	1·581	1·568
214—219	216·5	22	1·225	1·376	1·519	1·448	1·346	1·377
220—225	222·5	16	0·866	1·208	0·864	1·240	0·791	1·180
226—231	228·5	8	2·032	1·022	1·977	1·019	1·919	0·971

Variation of Standard Deviations with Age.

We may now enquire whether the standard deviations of the I. Q.'s are influenced by age or not. A glance at Table 14 will show that in the case of all three I.Q.'s (u, v and w for Part I, Part II, and combined scores respectively), the standard deviation changes systematically. There is just a slight increase at first, and then a steady decrease with growing age. For u (I.Q. for Part I scores) for example it drops from a value higher than 3 for lower ages to a value of about 1 for higher age-groups. The variation in the values of the standard deviation is thus proportionately much greater than the variation of the I.Q.'s themselves.

Graduation by weighted parabolas lead to the following equations :—

$$s_u = 2{\cdot}514 - 0{\cdot}2296\ (t - 13{\cdot}987) - 0{\cdot}01376\ (t - 13{\cdot}987)^2 \quad \ldots\ldots(6)$$

$$s_v = 2{\cdot}598 - 0{\cdot}1748\ (t - 13{\cdot}987) - 0{\cdot}02824\ (t - 13{\cdot}987)^2 \quad \ldots\ldots(7)$$

$$s_w = 2{\cdot}543 - 0{\cdot}1950\ (t - 13{\cdot}987) - 0{\cdot}02296\ (t - 13{\cdot}987)^2 \quad \ldots\ldots(8)$$

The observed and graduated values of the Standard Deviation of I.Q.'s are given in Table 14, and shown graphically in Chart 5, 6, and 7.

The decrease in the Standard Deviations of I.Q.'s makes it difficult to compare the I.Q.'s of candidates of different ages. For example consider u (I.Q. for Part I scores) for the age-group of 190-195 months or say 16 years. The S.D. is about 2·0 ; so that a difference of 4·0 units in the I.Q. will be definitely significant. Now consider the age-group of 118-123 months. The S.D. is 3·2, so that the difference must now be at least 6·5 to attain the same standard of significance.

The dependence of the I.Q.'s as well as their standard deviations on age thus take away a good deal from the usefulness of the I.Q. in comparing the intelligence of candidates of different ages.

Frequency Distribution of I.Q.'s.

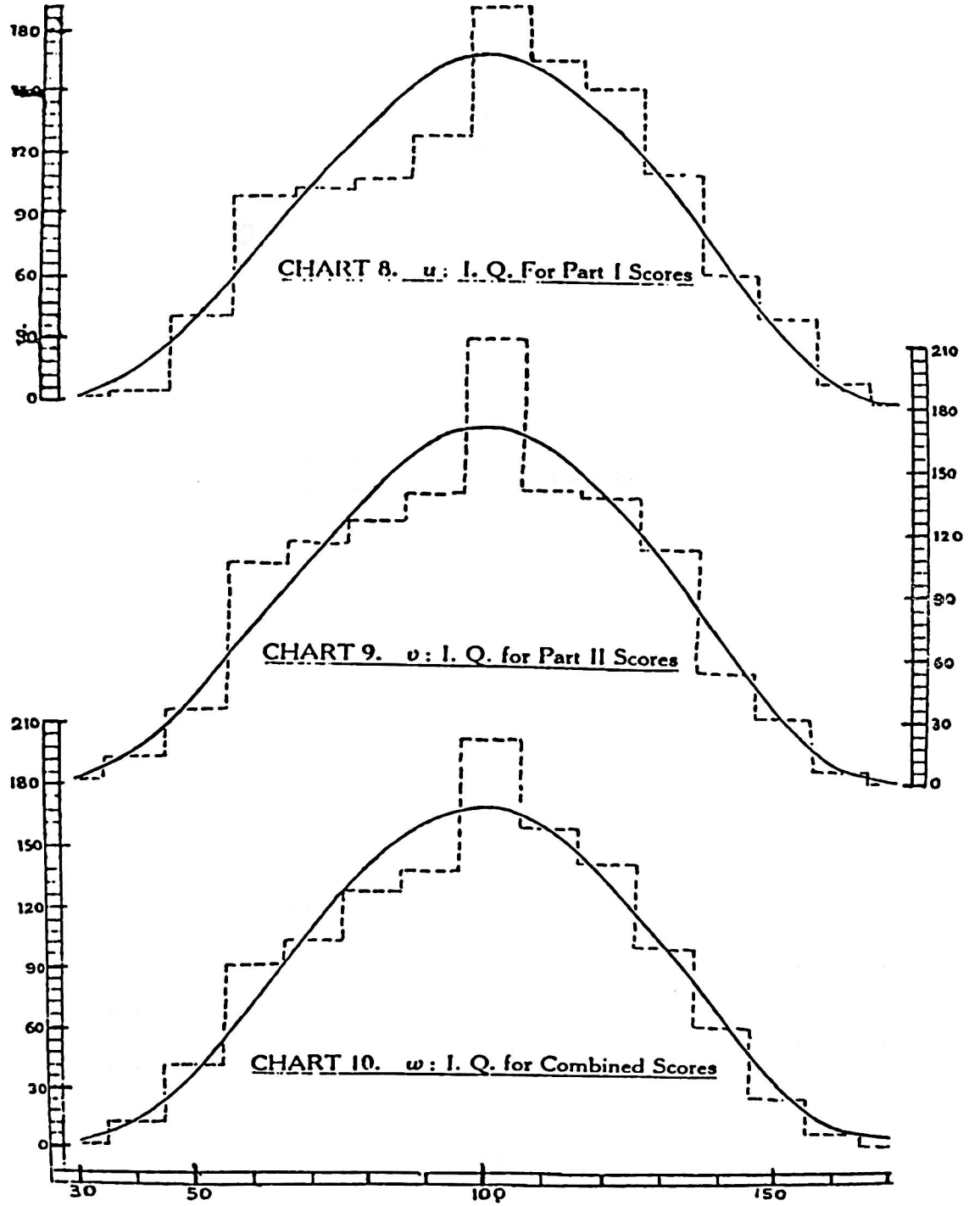
We may now consider the actual frequency distributions of the three I.Q.'s. The relevant statistical constants are given in Table 15.

Table 15. Statistical Constants for the I. Q.'s ($N = 1212$)

Statistics	u : I. Q. for Part I Scores	v : I. Q. for Part II Scores	w : I.Q. for Combined Scores
Mean	98·865 ± 0·5183	97·748 ± 0·5036	97·867 ± 0·5026
Standard Deviation	26·6503 ± 0·3692	26·1524 ± 0·3585	25·980 ± 0·3354
β_1	0·0000	0·0003	0·0052
β_2	2·4879 ± 0·0478	2·3855 ± 0·0422	2·4723 ± 0·0165
Type of Curve	Type II	Type II	Type II
m	3·858	2·382	3·185
a^2	68·195	52·454	63·046
b	167·98	167·35	171·00

All three distributions are practically symmetrical, but the values of β_2 are all significantly less than 3 showing that the distributions are definitely non-normal.

They can therefore be all graduated by the Pearsonian Type II curve whose formulæ are given below.



FREQUENCY DISTRIBUTIONS OF I. Q. s

$$y = y_0 \left(1 - \frac{x^2}{a^2}\right)^m \quad \ldots \quad \ldots \quad (9.0)$$

$$m = \frac{5\beta_2 - 9}{2(3-\beta_2)} \quad \ldots \quad \ldots \quad \ldots \quad (9{\cdot}1)$$

where

$$a^2 = \frac{2\beta_2 \cdot \mu_2}{3-\beta_2} \quad \ldots \quad \ldots \quad \ldots \quad (9{\cdot}2)$$

$$y_0 = \frac{N}{a} \cdot \frac{\Gamma(m+3/2)}{\Gamma(m+1)} \quad \ldots \quad \ldots \quad \ldots \quad (9{\cdot}3)$$

The values of m, a^2 and y_0 for the three curves are shown in Table 15 and the equations are given below.

For u: $$y = 167{\cdot}98\left(1 - \frac{x^2}{68.195}\right)^{3{\cdot}358} \quad \ldots \quad (10)$$

For v: $$y = 167{\cdot}35\left(1 - \frac{x^2}{52.454}\right)^{2{\cdot}382} \quad \ldots \quad (11)$$

For w: $$y = 171{\cdot}00\left(1 - \frac{x^2}{63.346}\right)^{3{\cdot}185} \quad \ldots \quad (12)$$

The observed and graduated values are shown in Table 16. The fit as judged by the (χ^2, P) test cannot be considered satisfactory. We conclude that although the distributions are non-normal, they cannot be conveniently graduated by the Pearsonian family of curves using terms up to fourth moments.

TABLE 16. OBSERVED AND GRADUATED FREQUENCY DISTRIBUTIONS OF I. Q.'s.

Range of I. Q.	u: I. Q. for Part I. Scores			v: I.Q. for Part II Scores			w: I.Q. for Combined Scores		
	Observed (f')	Expected (f)	$(f'-f)^2/f$	Observed (f')	Expected (f)	$(f'-f)^2/f$	Observed (f')	Expected (f)	$(f'-f)^2/f$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
25— 34	1	2·0	0·50	1	2·0	0·50	2	3·0	0·88
— 44	4	18·5	6·69	12	15·0	0·60	13	13·0	0
— 54	40	36·0	0·44	34	43·0	1·88	42	37·5	0·54
— 64	98	64·5	17·40	103	77·0	8·77	92	72·0	5·58
— 74	101	101·0	0	114	109·5	0·19	103	109·0	0·33
— 84	105	134·5	6·47	123	138·0	1·63	127	141·0	1·39
— 94	128	156·0	5·03	136	162·0	4·17	137	161·0	3·56
—104	192	165·0	4·42	210	167·5	10·78	201	168·0	6·48
—114	165	160·5	0·13	138	155·0	1·80	158	157·5	0
—124	152	141·0	0·86	136	133·0	1·06	141	132·0	0·61
—134	109	108·0	0·01	111	103·5	0·54	100	101·0	·01
—144	62	71·0	0·14	53	64·5	2·05	61	69·5	1·04
—154	41	43·0	0·01	32	31·0	0·03	26	31·5	0·96
—164	11	14·0	0·64	7	8·0	0·12	7	10·0	·09
Beyond 164	8	2·0	0·50	2	3·0	0·33	2	6·0	2·67
TOTAL	1212	1212·0	44·24	1212	1212·0	33·45	1212	1212·0	24·40
	c= 5,	n′=11,	P<0·00002	c=15,	n′=11,	P<0·0009	c=14,	n′=10,	P=0·0097

SUMMARY

The present paper gives an analysis of the I.Q.'s for 1212 Bengali school children based on a Group Test of Intelligence conducted through the medium of the Bengali language. The test was divided into two parts I and II, and three I.Q.'s: u for scores in Part I, v for scores in Part II, and w for combined scores were calculated separately. Chief results are summarized below.

(1) The reliability of the test as measured by the association between u (I.Q. for scores in Part I) and v (I. Q. for scores in Part II) is high. The actual value of the coefficient of correlation between u and v ($+ 0{\cdot}8481 \pm 0{\cdot}0054$) is only slightly less than that between raw scores ($+0{\cdot}8781 \pm 0{\cdot}0044$).

(2) The relation between u and v is linear, so that gains in u and v are practically proportional.

(3) The I.Q.'s are not independent of age. The change with age is not strictly proportional, that is, is not linear, but a parabolic regression of the second degree is found adequate in every case.

(4) All three I.Q.'s slightly increase at first up to the age of 12 or 13 years and then decrease with increasing age. The rate of decrease is slow at first but rapidly increases at higher age-groups.

The I.Q.'s thus fail to eliminate completely the effect of age variations of the scores, and cannot be used with safety for purposes of comparisons.

(5) The variability of the I.Q.'s as measured by the Standard Deviation also changes with age. All three Standard Deviations increase slightly at first and then steadily decrease.

(6) The frequency distributions of all three I.Q.'s are practically symmetrical, so that the number of children who are in advance of their age is more or less balanced by the number of retarded children.

(7) All three distributions are however definitely non-normal, and belong to Type II of the Pearsonian Family of Curves but the goodness of fit is not satisfactory. It is worth noting in this connection that there is a great preponderance of observed I.Q.'s between 95 and 104.

The general conclusion is that the I.Q.'s have high reliability, and are *symmetrically (but not normally) distributed*, with a preponderance of mediocre values. Further, the *I.Q.'s as well as their standard deviations show significant changes with age*, so that they are not completely independent of the age of the candidate. The I.Q.'s, at least so far as the present material is concerned, cannot serve as fully satisfactory tools for comparing the intelligence of children of different ages.*

*Communicated to the Indian Science Congress, 1933.

APPENDIX:—TABLE 1. GRADUATED AND OBSERVED SCORES.

Age-group	Number n_p	Part I		Part II		Combined	
		Graduated	Observed	Graduated	Observed	Graduated	Observed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
100 - 102	2	5·80	11·50	4·51	11·50	10·29	23·00
-105	8	6·13	7·87	5·08	8·12	11·17	16·00
-108	3	6·48	4·33	5·67	4·00	12·13	8·33
-111	3	6·85	2·00	6·28	0·33	13·13	2·33
-114	6	7·24	8·83	6·92	8·83	14·18	17·67
-117	17	7·65	7·82	7·58	8·71	15·26	16·53
-120	15	8·08	8·47	8·25	8·00	16·38	16·47
-123	11	8·52	9·64	8·92	10·60	17·53	19·54
-126	8	8·97	13·50	9·59	13·38	18·70	26·87
-129	33	9·43	8·67	10·29	9·09	19·90	17·76
-132	26	9·90	9·35	10·99	9·77	21·12	19·12
-135	29	10·38	10·17	11·69	10·17	22·34	20·34
-138	25	10·86	8·92	12·40	11·24	23·57	20·16
-141	52	11·35	10·15	13·11	2·69	24·81	22·85
-144	45	11·83	12·15	13·81	11·02	25·65	26·18
-147	28	12·32	9·89	14·51	11·79	27·27	21·68
-150	33	12·80	13·91	15·20	17·15	28·49	31·36
-153	55	13·27	11·87	15·88	19·13	29·70	31·00
-156	35	13·74	12·88	26·55	14·83	30·53	27·71
-159	23	14·19	15·74	17·20	19·08	32·61	34·85
-162	44	14·65	15·18	17·83	17·41	33·17	32·58
-165	60	15·08	16·15	18·15	20·02	34·27	36·17
-168	61	15·50	15·00	19·01	18·87	35·23	33·87
-171	44	15·91	15·82	19·61	20·52	36·34	36·34
-174	27	16·29	16·15	20·15	19·48	37·31	35·63
-177	56	16·65	16·73	20·66	21·41	38·23	38·14
-180	60	16·99	16·80	21·14	21·90	39·10	38·70
-183	52	17·31	18·19	21·58	23·58	39·90	41·77
-186	33	17·60	18·76	21·99	22·88	40·64	41·64
-189	43	17·86	18·81	22·35	23·37	41·30	41·95
-192	44	18·09	17·40	22·68	22·23	41·90	39·64
-195	26	18·29	17·58	22·95	23·27	42·42	40·85
-198	18	18·45	18·78	23·18	22·00	42·85	40·78
-201	36	18·58	18·86	23·36	24·80	43·19	43·67
-204	21	18·66	19·09	23·49	25·00	43·45	44·10
-207	30	18·72	19·77	23·56	25·73	43·60	45·50
-210	20	18·72	16·60	23·58	21·50	43·66	38·10
-213	24	18·68	16·67	23·53	21·96	43·61	38·63
-216	12	18·60	18·67	23·42	24·00	43·45	42·66
-219	10	18·46	19·80	23·25	22·80	43·17	42·10
-222	9	18·29	20·11	23·01	28·33	42·78	48·44
-225	7	18·05	18·00	22·60	24·00	42·26	42·00
-228	8	17·76	14·00	22·12	18·00	41·61	32·00
-231	0	17·42	..	21·56		40·83	..
-234	1	17·02	17·00	20·98	14·00	39·92	31·60
-237	2	16·56	12·00	20·21	15·00	38·86	27·00
-240	0	16·03	...	19·42		37·65	..
-243	2	15·44	21·50	18·54	32·00	36·30	53·50
-246	2	14·79	12·00	17·57	10·00	34·79	22·50
-249	0	14·07	...	16·01	...	33·11	
-252	0	13·28	...	14·86	...	31·28	
-255	0	12·42	..	13·61	...	29·27	
-258	1	11·48	21·00	12·27	20·00	27·05	50·00
-261	0	10·47	...	10·83	...	24·74	
-264	0	9·39	..	9·28	..	22·19	..
-267	0	8·22	...	7·63	..	19·47	
-270	2	6·97	9·50	5·88	12·00	16·55	21·50