LETTERS TO THE EDITOR

SOME ARITHMETICAL PROPERTIES OF THE FOURIER COEFFICIENTS OF THE MODULAR INVARIANT $j(\tau)$

THE arithmetical properties of the Fourier coefficients $c(n)$ of $j(\tau)$, the classical modular invariant of Klein, have been the subject of investigation by several authors. Some of the more important results obtained by Lehner, Newman, and Kolberg have been collected together in a recently published book by Lehner.[1] The present writer has obtained a number of new results, some of which are given below.

If p stands for any of the primes 2, 3, 5 and 7, then

$$c(pn) \equiv -p^u n^v \sigma_k(n) \pmod{p^w},$$

where p, u, v, w, k stand for any of the following four sets of values:

(i) 2, 11, 3, 15, 1; (ii) 3, 5, 2, 7, 1;

(iii) 5, 2, 1, 3, 1; and (iv) 7, 1, 1, 2, 3;

$\sigma_k(n)$ being, as usual, the sum of the k-*th* powers of the divisors of n. [This result can be generalised to cover $c(p^\lambda n)$, λ being a positive integer.]

Another result is that, if a, b, c, d are non-negative integers and

$$n \equiv 0 \pmod{2^a 3^b 5^c 7^d},\ n > 0,$$

then

$$c(n) \equiv 0 \pmod{2^{3a+12}\, 3^{2b+5}\, 5^{c+2}\, 7^{d+1}} \quad (1)$$

for "almost all" such values of n. It is interesting to compare this result with the one established by Lehner, *viz.*,

$$c(n) \equiv 0 \pmod{2^{3a+8}\, 3^{2b+3}\, 5^{c+1}\, 7^d} \quad (2)$$

for all such n's. [It is understood that if one or more of the primes 2, 3, 5, 7 are not divisors of n then the corresponding prime powers are to be removed from the moduli in the congruences (1) and (2) for $c(n)$].

As illustration of another type of result we have the following The least (positive) residue of $c(7^{6d+4})$ to the modulus 7^{6d+5} is 7^{6d+4}. There are similar results for other exponents, and also for the primes 2, 3 and 5.

Again, there are results of the type

$$c[2^a(4n+3)] \equiv 0 \pmod{2^{3a+10}},\ a > 0.$$

All these and other results will be published elsewhere with necessary proofs.

Dept. of National Sample Survey, Indian Statistical Institute, Calcutta-35, *February* 8, 1965. D. B. LAHIRI.

1. Lehner, J., *Discontinuous Groups and Automorphic Functions*, American Mathematical Society, Mathematical Survey No VIII, 1964, p 306.