Int J Theor Phys (2007) 46: 2976–2982
DOI 10.1007/s10773-007-9412-4

Double Coil Resonance Experiment and Partial Reduction of Wave Packet in Quantum Mechanics

Ratan Dasgupta · Sisir Roy

Published online: 15 May 2007

Abstract Interference and measurement aspects for the double coil resonance experiment are reanalyzed. The resulting situation is analogous to partial reduction of wave packet in quantum mechanics. Using convergence results of relative frequencies, magnitudes of the intensity are calculated when prior probabilities are assigned to the coefficients associated with the states.

Neutron interferometry has become a very powerful tool in testing the physical basis of quantum theory. In a series of experiments, Badurek et al. [1, 2] confirmed once again the predictions of quantum theory. It may be considered as powerful success of quantum formalism. However, Dewdney et al. [3] reported the inadequacy of Copenhagen interpretation. Apart from the interpretational problem, the aspect which is becoming more and more focused, concerns the theory of measurement without the collapse of the state vector [4]. Regarding this experiment, Rauch himself expressed doubt whether the energy exchange with the resonance frequency coils should be considered as real measuring process. In this paper, we reanalyze the whole situation within the conventional framework of quantum mechanics. Instead of considering the resonance frequency coils as constituting the measuring process, we consider them as the devices to producing two orthogonal states in the two paths in double slit experiment. Taking this intermediate splitting in the wave functions, we study the interference phenomena. Moreover, this experiment confirms that we can get interference pattern even for a single neutron and, as a consequence, the whole experiment

R. Dasgupta (✉)
Indian Statistical Institute, Kolkata 700108, India
e-mail: rdgupta@isical.ac.in

S. Roy
Center for Earth Observing and Space Research, School of Computational Sciences, George Mason University, Fairfax, VA 22030, USA
e-mail: sisir@isical.ac.in

S. Roy
e-mail: sroy@scs.gmu.edu

can be thought of as independently repeating the experiment N-times for N nearly identical neutrons, in the sense that these are prepared within certain (narrow) velocity and (narrow) angular range. In a real experiment, neutrons cannot be prepared identically. They can only be prepared within a certain narrow velocity band or/and within a certain narrow angular range. The limits of the preparations are given by the uncertainty relation.

An attempt is made to analyze this situation in the light of computing relative frequency. It has raised a possibility to consider a situation analogous to partial reduction of wave packet.

In double coil experiment, the wave functions $\phi = a\phi_{\mathrm{I}} + b\phi_{\mathrm{II}}$ are a linear superposition of the wave functions belonging to two different paths I and II of two coherent sub-beams within the interferometer. For an ideal empty interferometer, the relation $\phi_{\mathrm{I}} = \phi_{\mathrm{II}}$ holds.

In the real experiment with double coil, as performed by Badurek et al. [1, 2], there will be spin flip due to passage of neutron through resonance frequency coils and the energy exchanges are measured by resonance condition of both the coils. Moreover, an interference pattern is also observed simultaneously. At this stage, doubt has been expressed whether this energy exchange should be considered as constituting the measuring process. Without going into this controversy instantly, let us suppose that the two separated coherent beams ϕ_{I} and ϕ_{II} are prepared to be in mutually orthogonal spin eigenstates, say $|{+Z}\rangle$ and $|{-Z}\rangle$ by inserting spin flippers. The initial states ϕ_{I} and ϕ_{II} may be considered as superposition of mutually orthogonal states:

$$|\phi_{\mathrm{I}}\rangle = \langle\phi_{1/2}|\phi_{\mathrm{I}}\rangle|\phi_{1/2}\rangle + \langle\phi_{-1/2}|\phi_{\mathrm{I}}\rangle|\phi_{-1/2}\rangle, \tag{1}$$

$$|\phi_{\mathrm{II}}\rangle = \langle\phi_{1/2}|\phi_{\mathrm{II}}\rangle|\phi_{1/2}\rangle + \langle\phi_{-1/2}|\phi_{\mathrm{II}}\rangle|\phi_{-1/2}\rangle. \tag{2}$$

Here, we have considered a coherent superposition of $|{+Z}\rangle$ and $|{-Z}\rangle$ in both the beams. This situation has been realized recently in the experiment by Hasegawa et al. [5, 6]. It is to be mentioned that Badurek et al. [7] considered a $|{+Z}\rangle$ in one beam and $|{-Z}\rangle$ in the other beam. Now, let

$$\alpha = \langle\phi_{1/2}|\phi_{\mathrm{I}}\rangle, \qquad \beta = \langle\phi_{-1/2}|\phi_{\mathrm{I}}\rangle.$$

Also, write

$$\begin{aligned} &\gamma = \langle\phi_{1/2}|\phi_{\mathrm{II}}\rangle, \qquad \delta = \langle\phi_{-1/2}|\phi_{\mathrm{II}}\rangle, \\ &\alpha^2 + \beta^2 = 1, \qquad \gamma^2 + \delta^2 = 1, \quad \alpha, \beta, \gamma, \delta > 0. \end{aligned}$$

Again, we know

$$\phi = a\phi_{\mathrm{I}} + b\phi_{\mathrm{II}} \tag{3}$$

or,

$$\phi = a\alpha\phi_{1/2} + a\beta\phi_{-1/2} + b\gamma\phi_{1/2} + b\delta\phi_{-1/2} = (a\alpha + b\gamma)\phi_{1/2} + (a\beta + b\delta)\phi_{-1/2} \tag{4}$$

or,

$$\begin{aligned} |\phi|^2 &= |a\alpha + b\gamma|^2|\phi_{1/2}|^2 + |a\beta + b\delta|^2|\phi_{-1/2}|^2 \\ &= (a^2\alpha^2 + b^2\gamma^2)|\phi_{1/2}|^2 + (a^2\beta^2 + b^2\delta^2)|\phi_{-1/2}|^2 \\ &\quad + 2a\alpha b\gamma|\phi_{1/2}|^2 + 2a\beta b\delta|\phi_{-1/2}|^2. \end{aligned} \tag{5}$$

In the above, we consider $|\phi_{\pm 1/2}|^2$ as inner product. Next,

$$a^2\alpha^2 + b^2\gamma^2 + a^2\beta^2 + b^2\delta^2 = a^2 + b^2.$$

Then the terms in (5) can be rearranged as

$$|\phi|^2 = (a^2\alpha^2 + b^2\gamma^2)|\phi_{1/2}|^2 + (a^2\beta^2 + b^2\delta^2)|\phi_{-1/2}|^2 + ab(\phi_{\mathrm{I}}^*\phi_{\mathrm{II}} + \phi_{\mathrm{I}}\phi_{\mathrm{II}}^*) \tag{6}$$

where,

$$\phi_{\mathrm{I}}^*\phi_{\mathrm{II}} + \phi_{\mathrm{I}}\phi_{\mathrm{II}}^* = 2\{\alpha\gamma|\phi_{1/2}|^2 + \beta\delta|\phi_{-1/2}|^2\} = 2(\alpha\gamma + \beta\delta) > 0.$$

Therefore, the interference term does not vanish. One may rewrite (5)/(6) as

$$|\phi|^2 = (a^2\alpha^2 + b^2\gamma^2 + 2ab\alpha\gamma)|\phi_{1/2}|^2 + (a^2\beta^2 + b^2\delta^2 + 2ab\beta\delta)|\phi_{-1/2}|^2 \tag{7}$$

with,

$$a^2 + b^2 + 2ab(\alpha\gamma + \beta\delta) = 1. \tag{8}$$

At a first glance, it appears that there is no interference term present in (7), but note that the coefficient of ab is non zero. In this case if we want to calculate the probability of the particle in $\phi_{1/2}$ state (which is $a^2\alpha^2 + b^2\gamma^2 + 2ab\alpha\gamma$), it requires the knowledge of the coefficients b, γ which is connected to ϕ_{II}. In other words, if we want to detect the particle to remain in any one of the states, it is necessary to have the information from the two slits simultaneously. This situation has already been revealed by the double coil experiment. But the question arises whether the detection of energy exchange due to the spin flips should be considered as measurement. Equation (7) indicates that we have reached to a mixed state as $|\phi|^2$ contains the mixture of $|\phi_{1/2}|^2$ and $|\phi_{-1/2}|^2$ only. In this situation, we should say that the wave packet has been reduced and it satisfies the condition of measurement. However, the coefficient of $|\phi_{1/2}|^2$ or $|\phi_{-1/2}|^2$ depends on the coefficients of both the states. This is contrary to the usual calculation of the probabilities of the particle to remain in either of the states. Therefore, if we can detect the particle in either of the states by using the spin flippers, we can get some information regarding both the paths.

Badurek et al. [1, 2] claimed the inability of measurement of energy exchange for every neutron separately, because of the neutron-number-phase uncertainty relation. Namiki et al. [8] has shown that it is possible to circumvent the phase-number uncertainty relation. Scully and Walther [9] also proposed a simple experiment to show that the loss of coherence occasioned by 'which path' information i.e., by the presence of a 'Welcher Weg' detector is due to the establishing of quantum correlations and it is not associated with large random-phase factors as in Einstein's recoiling slits or Heisenberg's microscope. Rauch and Vigier [10, 11] reanalyzed the whole situation of neutron interferometry and proposed experiment bypassing the phase number uncertainty relation.

It is true that the quantum limit of the photon number and phase measurements are $\Delta n \simeq \sqrt{\langle n\rangle}$ and $\Delta\phi \simeq \frac{1}{(2\sqrt{\langle n\rangle})}$. This standard quantum limit is attributable to the quantum noise of a coherent state. However, some quantum states of electromagnetic field can reduce quantum noise for one observable [12], say, photon number and preserve the uncertainty relation by a increase quantum noise for the conjugate variable, say phase ϕ. In real experiments, the energy exchange of the neutron with one of the resonance frequency coils can be detected from the resonance condition of both the coils. It means that though we can detect energy exchange through one coil, still it requires the knowledge of the whole system. Now, if it is possible to detect the energy exchange without using the resonance condition of both the coils (i.e., the probability of the particle to be in $\phi_{1/2}$, given that it was in the first path is independent of the particle to be in $\phi_{1/2}$ given that it was in the second path) the above analysis within the conventional formalism of quantum mechanics may not be able to explain the new situation.

The situation of mutually orthogonal spin eigenstates prepared by the flippers particularly exists if the flippers are operated 'off-resonance', where the flipping probability becomes considerably smaller than 1. This experimental situation has been tested by Summhammer et al. [13]. Using polarized neutron interferometer, the photon exchange probabilities can be resolved below 1%. However, the situations depicted in (6) and (7) are peculiar even within the present formalism of quantum theory. By retaining the $(\phi_{\mathrm{I}}^*\phi_{\mathrm{II}} + \phi_{\mathrm{I}}\phi_{\mathrm{II}}^*)$ term in (6) separately i.e., with full interference term, we can not say that no measurement is being done due to the spin flips since ultimately we are getting sort of mixed states as $|\phi|^2$ contains terms involving only $|\phi_{1/2}|^2$ and $|\phi_{-1/2}|^2$. We want to emphasize that, it is necessary to reformulate the theory of measurement so as to accommodate this sort of intermediate situations. It is to be mentioned that Vigier [14] explained the above situation saying that:

"Particle and wave properties exist simultaneously and particle will pass through either of the slits but the wave will pass through both the slits. The wave will carry all the information for both the slits. There will be no collapse of the state vector at all. Hence one can detect the existence of the particles through the spin flippers even by maintaining the interference pattern but may not be able to say by which particular path the particle passed through."

This is exactly depicted by (6), within the present formalism of quantum theory. But this might be an inconsistent description if it is possible to detect the exchange of energy through one of the coils only. In that case, it will not be described by (6) or (7). The relation (8) imposes a restriction on the magnitude of the interference. The maximum value $(\alpha\gamma + \beta\delta)$ may take is 1, since this term of interference can be expressed as a *cosine* of an angle. In that case, taking $a = b$ which maximizes the interference term, we obtain $2a^2 = 1 - 2a^2$ i.e., $a^2 = 1/4$, with the maximum value of interference as $2ab(\alpha\gamma + \beta\delta) = 2ab = 2a^2 = 1/2$. Another interesting aspect of this experiment is that, it is possible to send one neutron at a time and interference is observed for a single neutron. So, we may consider this whole experiment as N-independent and identical experiments, being repeated for N (nearly) identical neutrons, within certain (narrow) velocity and (narrow) angular range to get the interference pattern. Each time the two coils are being placed at the two paths to flip the spin of the neutron. This picture is consistent with the frequency interpretation proposed and developed by several authors [15] for Born's probability postulate. Here relative frequency is treated as an observable and is represented by Hermitean operator.

Let ϕ_{I} and ϕ_{II} be the two coherent sub-beams passing through the two slits as depicted in the above experiment. Then,

$$\phi = w_1^{1/2}\phi_{\mathrm{I}} + w_2^{1/2}\phi_{\mathrm{II}}. \tag{9}$$

If w_1 and w_2 be taken as random weights with uniform distribution, then we have $\int_0^1 w_1\rho(w_1)dw_1 = 1/2$ and $\int_0^1 w_2\rho(w_2)dw_2 = 1/2$, as usual in quantum mechanics. This implies both slits are equally likely, $\rho(w_1)$ and $\rho(w_2)$ are taken as uniform densities.

Now, if we consider that the double coils placed along the two paths are producing the orthogonal states for individual particles in a random manner, then $\alpha, \beta, \gamma, \delta$ should be considered as random weights. Therefore the desired intensity is obtained by averaging:

$$\int_0^1 |\phi|^2 \rho(\alpha)\rho(\gamma) d\alpha d\gamma = \int_0^1 (a^2\alpha^2 + b^2\gamma^2)|\phi_{1/2}|^2 \rho(\alpha)\rho(\gamma) d\alpha d\gamma + \int_0^1 \{a^2(1-\alpha^2) + b^2(1-\gamma^2)\}|\phi_{-1/2}|^2 \rho(\alpha)\rho(\gamma) d\alpha d\gamma + 2ab \int_0^1 (\alpha\gamma + \beta\delta)\rho(\alpha)\rho(\gamma) d\alpha d\gamma. \tag{10}$$

The densities of α^2 and γ^2 may be taken as uniform, assuming equally likely possibilities of all the probability amplitudes for spin eigenstates $|+Z\rangle$ and $|-Z\rangle$; and this should hold for each path I and II. This is a sort of homogeneity assumption. Due to the interaction with the apparatus, the resulting densities may not remain as uniform. On the measuring apparatus the value of $(\alpha\gamma + \beta\delta)$ can be varied, but in principle *it can not be reduced to zero*.

So, the interference term of (10) can be reduced, but it is a non vanishing term provided the choice of $\alpha, \beta, \gamma, \delta$ are positive.

It should be mentioned that, there is no mathematical restriction on $\alpha, \beta, \gamma, \delta$ to assume only positive values, either from the Schrödinger equation or from the interpretation or quantum probability amplitude. Therefore, if one relaxes such a restriction, it is possible to have $(\alpha\gamma + \beta\delta) = 0$ for some values of $\alpha, \beta, \gamma, \delta$ for which interference is zero.

In a series of papers, the authors [16, 17] studied the different aspects of Bose–Einstein statistics as it is a compound distribution of multinomial distribution with Dirichlet distribution. Let $W = (W_1, W_2, \ldots, W_{k-1})$ be a random vector *uniformly distributed* in the region

$$\Delta = \{W_1, W_2, \ldots, W_{k-1} : W_i \geq 0\}$$

such that,

$$\sum_{i=1}^{k-1} W_i \leq 1. \tag{11}$$

Therefore the density of W is

$$f_W(w) = (k-1)! \quad \text{on } \Delta.$$

Let $N = (N_1, N_2, \ldots, N_k)$ be a non-negative integral valued random vector such that given $W = w, N$ has multinomial distribution with parameters $n, w_1, w_2, \ldots, w_k$, where $w_k = 1 - \sum_{i=1}^{k-1} w_i$.

Thus,

$$P\left(N_1 = n_1, \ldots, N_k = n_k | W = w, \sum_{i=1}^{k} n_i = n\right) = \frac{n!}{n_1! n_2! \ldots n_k!} w_1^{n_1} w_2^{n_2} \ldots w_k^{n_k}. \tag{12}$$

This multinomial distribution when integrated over Δ by the uniform density of W, results in Bose–Einstein statistic.

In this formulation, it is not that the distinguishable particles suddenly decide to behave in an indistinguishable manner, but the fact is that their original characteristic of distinguishability is totally lost, after averaging. From Bayesian point of view, one may say that the particles loose their distinguishability, after being averaged by uniform prior on Δ. A prior distribution other than uniform may also lead to an intermediate situation where the particles are not be fully distinguishable as in the case of BE statistic. Description of partly distinguishable particles may arise, even if the perturbation to the uniform prior distribution on Δ is negligible. This intermediate situation may occur, if one assumes that occupation of a state by a particle, has an influence on the occupation of other states in a particular way [16, 17]. Therefore, these kind of distributions are useful in the study of the interaction of the particle and the apparatus which gives rise to measurement.

Now, let us assume uniform densities for α^2 and γ^2. Consider the following special cases:

(1) $\alpha = \gamma, \beta = \delta$ then, $\langle(\alpha\gamma + \beta\delta)\rangle = \langle(\alpha^2 + \beta^2)\rangle = 1$.
(2) α and γ are independent variables, then $\langle(\alpha\gamma + \beta\delta)\rangle = \langle u^{1/2}v^{1/2} + (1-u)^{1/2}(1-v)^{1/2}\rangle$ where, $u = \alpha^2$, $v = \gamma^2$ are independent and uniform over (0, 1).

Next, assume independence of a, b with $(\alpha, \beta, \gamma, \delta)$. Also let a^2 be uniform on (0, 1), which implies both the slits are equally likely [see (9)] with $a = w_1^{1/2}$. Then,

For (1), assuming uniform density of a^2 on (0, 1), the interference is

$$\langle 2ab\rangle = \langle 2a(1-a)\rangle = \int_0^1 (1 - u^{1/2})\,du = 1/3,$$

where, we use $1 = a^2 + b^2 + 2ab = (a+b)^2$, from (8) and u is uniform on (0, 1).

For (2), averaging (8) over $\alpha, \beta, \gamma, \delta$ we get,

$$a^2 + b^2 + \frac{8}{9}ab = 1, \text{ since } \langle(\alpha\gamma + \beta\delta)\rangle = 4/9.$$

Solving this quadratic we get the approximate solution,

$$b \simeq \frac{7a^2}{18} - \frac{49a^4}{648}.$$

Therefore, the magnitude of interference, with uniform distribution of a^2 on (0, 1) is

$$\left\langle \frac{8}{9}ab \right\rangle = 0.0752,$$

approximately. Finally, consider the situation

$$\alpha = \delta, \qquad \gamma = \beta.$$

Then,

$$\langle \alpha\gamma + \beta\delta)\rangle = 2\langle\alpha\beta\rangle = 1/3,$$

assuming α^2 to be uniform on (0, 1). So the interference term is

$$\langle 2ab(\alpha\gamma + \beta\delta)\rangle = \langle 2ab\rangle\langle(\alpha\gamma + \beta\delta)\rangle = 1/9.$$

As the probability weights are considered here is random with uniform density attached to each state, it is quite natural to have an interpretation [18, 19] within the traditional probability theory.

The average value of the relative frequency operator converges to the square of the coefficient associated with that state, under appropriate assumptions. This relative frequency of a state is computable for N independent and identical experiments. Hence, an interpretation of the coefficients $(\alpha, \beta, \gamma, \delta)$ is possible in terms of the relative frequencies, which are observable quantities. The appropriate relative frequencies converges to the square of the coefficients i.e., $(\alpha^2, \beta^2, \gamma^2, \delta^2)$.

In this way the predictions of quantum mechanics will be recovered from the assertions about individual systems.

However, this is not the case in the conventional formulation of quantum mechanics because, the probability interpretation of the wave function is a fundamental assumption, the predictions of the theory for ensembles of identical systems are not deduced from its predictions for the individual systems. Now, in the picture depicted in our analysis, the wave function of a single neutron or its absolute square can be determined by measurements of an ensemble of neutrons provided these have been nearly identically prepared. In the absence of such a knowledge, the wave function for a single particle can not be determined from the frequencies of the various measurement results.

From recent experiments, Hasegawa et al. [5, 6] demonstrated the violation of Bell-like inequality. They showed that the entanglement is not merely limited to different particles like neutrons, but even applicable to different degrees of freedom in single particle. Here, they considered single particle in neutron interferometer using static spin flippers. However, there is a difference between our proposal and the set up by Hasegawa et al. In our view, if this experiment is repeated with *resonant spin flippers* instead of static spin flippers, then it might be possible to verify our results.

Our analysis of double resonance coil experiment clearly indicates that, the non-vanishing interference term as well as the appearance of the orthogonal states in $|\phi|^2$, raise some difficulties regarding our usual understanding of the measurement. An intermediate situation i.e., partial reduction of wave packet is possible within our framework. A formulation of quantum mechanics is also possible by postulating assertions for individual systems which is in complete agreement with result of double coil experiment.

Acknowledgements We thank the referees for constructive suggestions. One of the authors(SR) is greatly indebted to CEOSR (Center for Earth Observing and Space Research), George Mason University, USA for the support where part of the work has been completed.

References

1. Badurek, G., Rauch, H., Tuppinger, D.: Ann. New York Acad. Sci. **480**, 133 (1986)
2. Badurek, G., Rauch, H., Tuppinger, D.: Phys. Rev. A **34**, 2600 (1986)
3. Dewedney, C., Kyprianidis, A., Vigier, J.P.: Phys. Lett. A **102**, 291 (1984)
4. Namiki, M.: Preprint, Waseda University (1988)
5. Hasegawa, Y., et al.: Nature **425**, 45 (2003)
6. Hasegawa, Y., et al.: Nucl. Instrum. Methods A **529**, 182 (2004)
7. Badurek, G., et al.: Phys. Rev. Lett. **51**, 1015 (1983)
8. Namiki, M., et al.: Prog. Theor. Phys. **77**, 508 (1987)
9. Scully, M.O., Walther, H.: Phys. Rev. A **39**, 5229 (1989)
10. Rauch, H., Vigier, J.P.: Phys. Lett. A **151**, 269 (1990)
11. Rauch, H., Vigier, J.P.: Phys. Lett. A **157**, 311 (1991)
12. Hartle, J.P.: Am. J. Phys. **38**(8), 704 (1968)
13. Summhamer, J., et al.: Phys. Rev. Lett. **75**, 3206 (1995)
14. Vigier, J.P.: In: Proceedings of New York Academy of Sciences, USA (1986)
15. Yamamoto, Y., Haus, A.H.: Rev. Mod. Phys. **58**(4), 1001 (1986)
16. Dasgupta, R., Roy, S.: Phys. Lett. A **149**, 63 (1990)
17. Kunte, S.: Sankhya A **39**, 305 (1977)
18. Dewitt, B.: Battle Recentres. Benjamin Elmsford (1968). The results were obtained by R.N. Graham independently and compiled in his doctoral dissertation, University of North Carolina (1967)
19. Everett, H., Wheeller, J., et al.: The Many World Interpretation of Quantum Mechanics. Princeton University Press, Princeton (1973)