On the Distribution of Certain Symmetric Functions of-pStatistics on the Null hypothesis

In the research note entitled "The Distribution of the p-statistics on the non-null hypothesis" sent for publication in "Science and Culture" in January last and being published in the present issue of the Journal the distribution of k_i's defined in (1) is obtained in the form (2) on the null hypothesis *i.e.*, when the populations sampled have the same dispersion matrix $\| \prec_{ij} \|$ and is obtained in the form (5) on the non-null hypothesis *i.e.*, when the populations sampled have different dispersion matrices, say, $\| \prec_{ij} \|$ and $\| \prec'_{ij} \|$. All the numbers refer to the foregoing note. If we consider the distribution (2) of that note and define a new set of quantities $u_1, u_2, \ldots\ldots u_p$ which are the symmetric functions of the roots k_i^2s of the determinantal equation (1) of the foregoing note, and which are given by

$$\left.\begin{aligned} u_1 &= \frac{1}{p_{c_1}} \sum_{i=1}^{p} k_i^2, \qquad (u_2)^2 = \frac{1}{p_{c_2}} \sum_{i,j=1}^{p} k_i^2 k_j^2 \quad (i \neq j), \ldots \\ (u_p)^p &= \frac{1}{p_{c_p}} \, k_1^2 k_2^2 \ldots k_p^2 \end{aligned}\right\} \quad (1)$$

then the joint distribution of u_i's $(i=1, 2, \ldots p)$ has been found out in the form

$$\text{Const.} \; u_2 \, (u_3)^2 \, (u_4)^3 \ldots \; (u_{p-1})^{p-2} \; u_p^{\frac{p(n-p)-2}{2}} \prod_{i=1}^{p} du_i \quad (2)$$
$$\overline{(1+\lambda^2 n_{c_1} u_1 + \lambda^4 n_{c_2} (u_2^*)^2 + \ldots \lambda^{2p} n_{c_p} (u_p)^p)^{\frac{n+n'-2}{2}}}$$

where $\lambda^2 = n/n'$.

Just as in (2) of the foregoing paper k varies from 0 to k_{p-1} k_{p-1} varies from 0 to k_{p-2} and so on, and finally k_2 varies from 0 to k_1 and k_1 from 0 to ∞, so here it has been found that u_p varies from 0 to u_{p-1}, u_{p-1} from 0 to u_{p-2} and so on, and finally u_2 from 0 to u_1 and u_1 from 0 to ∞. The distribution of the u_i's on the non-null hypothesis is being worked out and will be published in due course. The set of k_i's defined in (1) of the foregoing note may be useful for purposes of testing one category of hypotheses and the set u_i's defined in (1) of this note may be useful for testing another category of hypotheses and other functions of the k_i's may be useful for other purposes. Detailed investigations into these aspects of the problem are now being carried on by the author and the results will be announced later on.

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