LETTERS TO THE EDITOR

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A Method of Estimating Variance of Sample Grand-mean and Zone Variances in unequal nested Sampling

In nested sampling the fundamental problem is estimating the error of the sample mean. This error is an expression involving all the zone-variances which are unknown. The mathematical expectations of the known variances from Analysis of Variance table are linear functions of these unknown ones. These linear functions have been found and the problem solved for most generalized case with unequal sampling in each zone.

Any sample in a w-fold nesting is represented by $Xi_1i_2......i_\omega = A + {}^1Bi_1 + {}^2Bi_1i_2 + ... {}^\omega Bi_1i_2 ...... i_\omega$ where A is a constant and for $1 \leq K \leq \omega$

(1) i_k goes from 1 to ${}^kNi_1i_2 \ldots \ldots i\omega$

(2) $E\{{}^kBi_1i_2 \ldots \ldots i_k\} = 0$

(3) $E\{{}^kBi_1i_2 \ldots \ldots i_k\}^2 = \sigma_k{}^2$

(4) $\sum\limits_{i_p} \sum\limits_{i_{p+1}} \ldots \sum\limits_{i_{k-1}} {}^kNi_1i_2 \ldots i_{k-1} = {}^kNi_1i_2 \ldots i_{p-1}$

$(1 \leq p \leq k-1)$ and ${}^kNi_0 = {}^kN$ say.

Typical term in analysis of variance will be $S(Xi_1i_2 \,..\, i_k \ldots - Xi_1i_2 \ldots i_{k-1} \ldots)^2$ which is "sum of squares between Z_k-zones within Z_{k-1}-zones" and its corresponding degrees of freedom and variance being f_k and V_k respectively.

Then, for $1 \leq k \leq \omega - 1$ and $0 \leq l \leq \omega - k - 1$

(1) The variance of the sample grand-mean will be

$$\sum_{k=1}^{\omega-1} \left[\frac{\sigma_p{}^2}{{}^\omega N^2} \sum_{i_1} \sum_{i_2} \ldots \sum_{i_k} ({}^\omega Ni_1i_2 \ldots i_k)^2 \right] + \frac{\sigma_\omega{}^2}{{}^\omega N}$$

(2) Mathematical expectation of V_kf_k will be

$$\sum_{l=0}^{\omega-k-1} \left[\sum_{i_1} \sum_{i_2} \ldots \sum_{i_{k+l}} ({}^\omega Ni_1i_2 \ldots i_{k+l})^2 \left\{ \frac{1}{{}^\omega Ni_1i_2 \ldots i_k} - \frac{1}{{}^\omega Ni_1i_2 \ldots i_{k-1}} \right\} \sigma^2{}_{k+l} \right] + \sigma^2{}_\omega$$

Further work on estimation based on maximum likelihood method is proceeding.

The problem has previously been tackled by Messrs S. N. Roy and K. Banerjee[1] where they gave the results in case of equal sampling within each fold, and by Mr W. G. Cochran[2] where he published the result in case of unequal sampling in a 2-fold nesting.

The problem tackled here concerns the case of unequal sampling within each fold. I had taken up the above study in connection with work on nested sampling under the direction of Prof. P. C. Mahalanobis in 1940 while I was working in Statistical Laboratory.

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[1] SCIENCE AND CULTURE, *6*, 189, 1940-41.
[2] *Journ. Am. St. Ass.*, *34*, No. 206, 1939.