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A GENERALISATION OF SINGLE SAMPLING MULTIATTRIBUTE PLANS

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SUMMARY. Under the assumption of independence of defect occurrences of different types, several acceptance criteria were compared with respect to cost and discrimination power by the author (Majumdar 1990) in an earlier paper. A linear cost model for the discrete prior distribution of process average was also developed. The present paper generalises these results to the situation where defect occurrences are mutually exclusive.

1. **Introduction**

The problem of acceptance sampling by attributes involving more than one quality characteristic is well known. The general practice of dealing in such situations is to consider each of the characteristics separately. Military standard - 105 - D/ISO 2859 suggests grouping of the characteristics in order of seriousness viz. critical, major and minor groups and choosing AQL accordingly. Bray *et al.* (1973) have considered simultaneously three mutually exclusive classes viz. good, marginal and bad for developing sampling schemes. Majumdar (1990) considered this problem under the assumption that defect occurrence involving any one of the characteristics is independent of those involving others.

We will now make an attempt to arrive at a general scheme applicable to both these situations. We suppose that there are r attribute characteristics inspectable separately for any product. For example, we may have a number of visual characteristics as well as functional characteristics. Suppose all the inspection results are expressed as attributes. If we take a sample of size n from a lot of size N, all the r characteristics should be observable in any order. The sampling scheme we are interested in should be applicable when lots are submitted in a more or less continuous manner. Let us consider the type B acceptance probability of such schemes. We recall that for a single characteristic

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Hald (1980) has shown that if the lot quality follows binomial for any specified process average, p, the probability of obtaining x defectives in a sample of size n taken from lots, without replacement, will follow binomial with parameters n, p. When the defect occurrences are independent for products involving r characteristics the probability of any $(X_1, X_2, \ldots X_r)$, X_i denoting the number of defectives in a lot is

$$Pr(X_1, X_2, \ldots X_r) = \prod_{1}^{r} Pr(X_i). \qquad \ldots(1)$$

However the probability of obtaining x_i defectives corresponding to characteristic i from a sample of size n is

$$Pr(x_i|X_i) = \binom{n}{x_i}\binom{N-n}{X_i - x_i} \Big/ \binom{N}{X_i}. \qquad \ldots(2)$$

If now, each X_i is assumed to follow binomial with parameters N and p_i and each is independent of the others then the unconditional probability of obtaining $(x_1, x_2, \ldots, x_r)$ defectives in a sample of size n can be easily shown as

$$Pr(x_1, x_2, \ldots, x_r | p_1, p_2, \ldots, p_r) = \prod_{i=1}^{r} b(x_1, n, p_i) \qquad \ldots(3)$$

where

$$b(x_i, n, p_i) = \binom{n}{x_i} p_i^{x_i}(1-p_i)^{n-x_i} \qquad \ldots 3(a)$$

When the defect occurrences are mutually exclusive the expression for the probability of observing $(x_1, x_2, \ldots x_r)$ defectives in a sample of size n from a lot of size N, containing $(x_1, x_2, \ldots, x_r)$ defectives corresponding to the different characteristics will be multivariate hypergeometric given by

$$Pr(x_1, x_2, \ldots, x_r | X_1, X_2, \ldots, X_r)$$
$$= \binom{X_1}{x_1}\binom{X_2}{x_2}\cdots\binom{X_r}{x_r}\binom{N - X_1 - X_2 - \ldots X_r}{n - x_1 - x_2 - \ldots x_r} \Big/ \binom{N}{n} \qquad \ldots(4)$$

At any process average the joint probability distribution of $(X_1, X_2, \ldots X_r)$ can be assumed to be multinominal with parameters $N, (p_1, p_2, \ldots, p_r)$ as

$$Pr(X_1, X_2, \ldots X_r | p_1, p_2, \ldots p_r)$$
$$= \binom{N}{X_1}\binom{N - X_{(1)}}{X_2}\cdots\binom{N - X_{(r-1)}}{X_r} p_1^{X_1} \ldots p_r^{X_r}(1 - p_{(r)})^{N - X_{(r)}} \qquad \ldots(5)$$

where

$$X_{(i)} = X_1 + \ldots X_i, \text{and} p_{(i)} = p_1 + \ldots + p_i. \qquad \ldots 5(a)$$

From (4) and (5) it follows that the unconditional probability is

$$Pr(x_1, x_2, \dots x_r | p_1 \dots p_r)$$
$$= \binom{n}{x_1}\binom{n-x_1}{x_2}\cdots\binom{n-x_{(r-1)}}{x_r} p_1^{x_1}\cdots p_r^{x_r}(1-p_{(r)})^{n-x_{(r)}}. \qquad \dots (6)$$

Thus the average probability follows multinominal distribution with the parameters $n, p_1, p_2, \dots p_r$.

2. Approximation Under Poisson Condition

The phrase 'under poisson conditions' was used by Hald (1980) when poisson probability can be used as an approximation to the binomial for small p's. More precisely if $p_i \to 0$ and $n \to \infty$ such that $np_i \to m_i$ the binomial probability $b(x_i, n, p_i)$ tends to poisson probability $g(x_i, np_i)$ where

$$b(x_i, n, p_i) = \binom{n}{x_i} p_i^{x_i}(1-p_i)^{n-x_i} \qquad \dots (7)$$

and

$$g(x_i, np_i) = e^{-np_i}(np_i)^{x_i}/(x_i)!. \qquad \dots (8)$$

Under this condition the R.H.S. of equation (3) can be approximated by the product of corresponding poisson terms. It is interesting to note that the multinomial expression of R.H.S. of equation (6) can also be approximated by the same product expression under poisson approximation. Thus under both the situations the average probability can be approximated by poisson probability.

The additional assumption needed is $\sum_{i=1}^{r} p_i \to 0$. Then,

$$Pr(x_1, x_2, \dots x_n | p_1, \dots p_r) \simeq \prod_{i=1}^{r} g(x_i, m_i). \qquad \dots (9)$$

For a general acceptance criteria described as : accept if $(x_1, x_2, \dots, x_r) \in A$ and reject if $(x_1, x_2, \dots, x_r) \in \overline{A}$, A and $\overline{A}$ being two complementary sets, the type B OC function under poisson condition can be written as

$$P(\underset{\sim}{p}) = \sum_{x \epsilon A} \prod_{i=1}^{r} g(x_i, m_i). \qquad \dots (10)$$

This is exactly same as in the independent case.

3. Remarks

Majumdar (1990) proposed several aceptance criteria viz. Scheme A., Scheme B, Scheme C and Scheme D. The discrimination prower of these schemes was evaluated and compared the 'independent' case. Results proved as theorem 1 and theorem 2 for independent case will also be valid for the mutually exclusive case under poisson condition.

The cost model for the 'independent' case; has been developed by Majumdar (1990). Under the assumption of multinomial distribution of lot quality, similar cost model for the mutually exclusive case can be arrived at. For a d point prior distribution of process average the over all average cost or regret function can be easily shown as

$$K(N,n) = \sum_{j=1}^{d} k(N,n,\underset{\sim}{p}^{(j)} w_j') \qquad \ldots (11)$$

where $\underset{\sim}{p}^{(j)} = (p_1^{(j)}, p_2^{(j)} \ldots p_r^{(j)})$, with probability w_j such that $\sum_{j=1}^{d} w_j = 1$.

The expression for $k(N,n,p^{(j)})$ is the same as that in the independent case excepting that probability of acceptance $P(\underset{\sim}{p})$ has to be replaced by the corresponding expression involving multinomial probability of acceptance. For a two point prior distribution the expression can be conveniently written as

$$K(N,n) = n + (N-n)[v_1(Q(\underset{\sim}{p}^{(1)}) + v_2 P(\underset{\sim}{p}^{(2)})] \qquad \ldots (12)$$

Where v_j denotes the cost parameters as defined in equation (27) by Majumdar (1990). $P(\underset{\sim}{p})$ and $Q(\underset{\sim}{p})$ are the probability of acceptance and probability of rejection respectivly.

Under Poisson condition the cost functions in (11) and (12) are the same as the corresponding expressions in the independent case. Obviously the results relating to comparison of cost obtained by Majumdar (1990) for the independent case shall become valid for the mutually exclusive case.

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