Winning strategies for pseudo-telepathy games using single non-local box

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Using a single NL-box, a winning strategy is given for the impossible colouring pseudo-telepathy game for the set of vectors having Kochen-Specker property in four dimension. A sufficient condition to have a winning strategy for the impossible colouring pseudo-telepathy game for general d-dimension, with single use of NL-box, is then described. It is also shown that the magic square pseudo-telepathy game of any size can be won by using just two ebits of entanglement – for quantum strategy, and by a single NL-box – for non-local strategy.

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I. INTRODUCTION

By performing measurement on an entangled quantum system two separate observer can obtain correlations that are nonlocal, in the sense that no local hidden variable (LHV) model can reproduce it. This was first proved by Bell in 1964 in terms of Bell inequality [1]. Later on Clauser, Horne, Shimony and Holt gave an experimental proposition of Bell's inequality which is known as as CHSH inequality [2]. According to CHSH inequality all local hidden variable model must satisfy:

$$|\langle A_1B_1\rangle + \langle A_1B_2\rangle + \langle A_2B_1\rangle - \langle A_2B_2\rangle| \leq 2$$

where A_1, A_2 are observables of a spin-half particle in the possession of Alice and B_1, B_2 are observables of a spin-half particle in the possession of Bob. But local measurement carried out on entangled quantum system can reach the value $2\sqrt{2}$. Cirelson's showed [3] that this is the maximum value attainable by local measurement on entangled quantum system although the maximum nonlocal value of CHSH inequality can reach is 4.

Popescu and Rohrlich [4] asked a very interesting question: why quantum mechanics is not maximally nonlocal? Is there any stronger correlation than the quantum mechanical ones that do not allow signalling like quantum correlation? They have introduced a hypothetical non-local box (NL box for short) that does not allow signalling, yet violates CHSH inequality maximally. This NL-box has two input bits x and y, and yields two output bits a and b. The bits x and a are in Alice's hand, while y and b are in Bob's hand. The box is such that a and b are correlated according to simple relation:

$$x.y = a \oplus b,$$

where $\oplus$ is addition modulo 2. Afterwards many works have been done to characterize the NL-box in order to yield insights about the non-locality aspects of quantum mechanics [5 - 12].

Quantum pseudo-telepathy game [13] provides an intuitive way to understand quantum non-locality. Quantum pseudo-telepathy game is something which can not be won in the classical world without communication but can be won in the quantum world using entangled state without any use of classical communication. Thus, for an observer (ignorant about any sort of non-locality), the reason for winning of the game by the players would imply some *apriori* 'telepathic' connection between the players. Nevertheless, that sort of connection is impossible. Formally, according to ref. [13], a two-party [14] pseudo-telepathy game is given by a six-tuple $(X_A, X_B, Y_A, Y_B, P, W)$ where X_A and X_B are the input sets of parties Alice and Bob respectively, Y_A and Y_B are their respective output sets, P ($\subseteq X_A \times X_B$) is the set of all promises, and W ($\subseteq X_A \times X_B \times Y_A \times Y_B$) is the winning condition. Thus W is a relation between inputs and outputs that has to be satisfied by Alice and Bob whenever the promise is fulfilled. Once the respective inputs are supplied to Alice and Bob, they will no longer be allowed to communicate until the game is over. In each round of the game, Alice and Bob are supplied with the inputs $x \in X_A$ and $y \in X_B$ respectively. Their task is now to produce outputs $a \in Y_A$ and $b \in Y_B$ respectively. They will win the round if either $(x, y) \notin P$ or $(x, y, a, b) \in W$. They will win the game if they go on winning round after round. They will have a winning strategy for the game if they are mathematically certain to win the game as long as they have not exhausted all the classical information as well as quantum entanglement (if there is any) shared at the beginning of the game. Note that some observer of the game (other than Alice and Bob) can only have a statistical evidence towards making the hypothesis that Alice and Bob indeed have a winning strategy for the game, if Alice and Bob go on winning the game round after round. Quantum pseudo-telepathy games are proofs of non-locality. Moreover, they are stronger proofs than usual Bell theorems as well as Bell theorems without

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inequalities [15].

To understand the features of the NL-box it is necessary to understand its power in various quantum information processing protocols already discovered. There are entangled states (both bipartite as well as multipartite), the measurement correlations of which can be simulated by one or more than one NL-box [6, 7, 16]. But there are measurement correlations corresponding to some muti-partite entangled states which can not be simulated by NL-boxes. In this context it would be interesting to know whether all the pseudo-telepathy games, proposed so far, can be won with single use of the NL-box. Recently Broadbent and Méthot [17] showed that some of the pseudo-telepathy games can be won with single use of the NL-box where the quantum strategy requires more than a maximally entangled pair of qubits to succeed. It remained unsolved whether impossible colouring pseudo-telepathy game, constructed by using Kochen-Specker theorem, can be won with a single use of the NL-box. The problem, in general, will be extremely difficult as there are various sets of vectors satisfying Kochen-Specker property for Hilbert space of dimension three or more. On the other hand it is known that the magic square pseudo-telepathy game of *size* three (*i.e.*, where the size of the magic square matrix 3) can be won by using a single NL-box [17]. Whether this game, for any general size, can be won by a single NL-box is also an unresolved issue. There is a quantum winning strategy for the magic square pseudo-telepathy game of size three that uses two ebits of shared entanglement between the parties [13]. The corresponding situation for general size is unknown. As the magic square game of even size (see section VI for the definition of magic square game) does always have a classical solution (*i.e.*, the players are neither required to use classical communication, nor to share any entanglement, nor any NL-box), so we need to consider here only games each having odd size.

In this paper first we shall present a winning strategy of impossible colouring pseudo-telepathy game for the set of 18 vectors having Kochen- Specker property in four dimension with single use of NL-box. Then we discuss some sufficient condition for the winning strategy of impossible colouring pseudo-telepathy game for general d-dimension with single use of NL-box. We shall show here that the magic square pseudo-telepathy game of any odd size can also be won by using a single NL-box. Moreover, we shall describe a quantum winning strategy for this game (of any odd size) which requires only two ebits of shared entanglement between the parties.

In section II, we shall describe the Kochen-Specker theorem in four dimension that uses eighteen vectors from $\mathbb{R}^4$, and its corresponding impossible colouring pseudo-telepathy game is described in section III. A winning strategy for this game is described in section IV using only one NL-box. A winning strategy for the impossible colouring pseudo-telepathy games in d dimension, each of which satisfies a suitable sufficient condition, is described in section V where it uses a single NL-box. The magic square problem for general odd dimension is described in section VI, where it is then posed as a pseudo-telepathy game. A non-local winning strategy for this pseudo-telepathy game is described in section VII by using a single NL-box. For the sake of completeness, the quantum winning strategy for the magic square pseudo-telepathy game of size three is described briefly in section VIII which uses two ebits of shared entanglement between the players. This strategy is used in section IX to provide a quantum winning strategy for the magic square pseudo-telepathy game any general odd size by using again only two ebits of shared entanglement between the players. Section X draws the conclusion.

II. KOCHEN-SPECKER THEOREM

There exits an explicit, finite set of vectors in Hilbert space with dimension $d \geq 3$, that can not be assigned values $\{0, 1\}$ such that both of the conditions holds:

1. For every complete set of orthogonal basis vectors, only one vector will get value 1.

2. Value assignment of the vectors will be non-contextual.

We call such set of vectors a set with Kochen-Specker property.

Example:

The following set of 18 (unnormalized) vectors in $\mathbb{R}^4$ appearing in 9 sets of orthogonal basis has Kochen-Specker property [18]. If on the contrary, one assumes that this set satisfy both the conditions (1) and (2), one gets the following equations.

$$V(0,0,0,1)+V(0,0,1,0)+V(1,1,0,0)+V(1,-1,0,0) = 1$$

$$V(0,0,0,1)+V(0,1,0,0)+V(1,0,1,0)+V(1,0,-1,0) = 1$$

$$V(1,-1,1,-1)+V(1,-1,-1,1)+V(1,1,0,0)+V(0,0,1,1) = 1$$

$$V(1,-1,1,-1)+V(1,1,1,1)+V(1,0,-1,0)+V(0,1,0,-1) = 1$$

$$V(0,0,1,0)+V(0,1,0,0)+V(1,0,0,1)+V(1,0,0,-1) = 1$$

$$V(1,-1,-1,1)+V(1,1,1,1)+V(1,0,0,-1)+V(0,1,-1,0) = 1$$

$$V(1,1,-1,1)+V(1,1,1,-1)+V(1,-1,0,0)+V(0,0,1,1) = 1$$

$$V(1,1,-1,1)+V(-1,1,1,1)+V(1,0,1,0)+V(0,1,0,-1) = 1$$

$$V(1,1,1,-1)+V(-1,1,1,1)+V(1,0,0,1)+V(0,1,-1,0)=1$$

Here $V(0,0,0,1),\ldots,V(0,1,-1,0)$ denote the values taken from the set $\{0,1\}$ and are assigned to the respective vectors $(0,0,0,1),\ldots,(0,1,-1,0)$ (of $\mathbb{R}^4$). If one add these nine equations, the left hand side will be even as every vector has appeared twice and their value can be 1 or 0, while the right hand side is obviously odd. It proves that one can not assign values to all vectors satisfying both the conditions.

III. IMPOSSIBLE COLOURING PSEUDO-TELEPATHY GAME IN 4-DIMENSION

We now turn this Kochen-Specker theorem in to a pseudo-telepathy game as suggested by Brassard et al. [13]. Consider the nine complete orthogonal bases of real vectors in four dimension, described in the above-mentioned example. Denote them by $S^1, S^2, \ldots, S^9$, where each S^J contains the following four pairwise orthogonal vectors u_1^J, u_2^J, u_3^J, and u_4^J where, $u_1^1 = u_1^2 = (0,0,0,1)$, $u_2^1 = u_1^5 = (0,0,1,0)$, etc. Two players, say, Alice and Bob, are far apart from each other such that Alice is supplied with, at random, any one (S^k, say) of the nine bases mentioned above, while Bob is supplied with, at random, a vector (u_m^l, say) from the above-mentioned eighteen vectors. The promise of the game is that u_m^l must be a member of S^k. This round of the game will be won by Alice and Bob if the following conditions are satisfied:

Alice will have to assign value (0 or 1) to her four vectors u_1^k, u_2^k, u_3^k, u_4^k and Bob also will have to assign value (0 or 1) to his single vector u_m^k in such a way that

1. Exactly one of Alice's four vectors should receive the value 1.

2. Alice and Bob have to assign same value to their single common vector u_m^k.

with the condition that they will not be allowed to have any classical communication after the game starts and until the game is over. Thus they will win the game if they go on winning it for every round of the game. Interestingly, Brassard et al. [13] presented a quantum winning strategy for a general impossible colouring game in d dimension using $\log_2 d$ ebits of shared entanglement between Alice and Bob.

IV. WINING STRATEGY USING A SINGLE NL-BOX

Now we shall present a strategy to win this game by using a single NL-box. If one tries to satisfy all the above nine equations by assigning non-contextual values to the maximum possible no. of vectors, then one would see that seventeen vectors can be assigned non-contextual values and value assignment for the remaining one vector has to be contextual, *i.e.*, one vector out of eighteen has to take value 1 when it occurs in one basis and 0 when it occurs in another basis [19].

Let us now consider a contextual value assignment to the vector $(0,1,-1,0)$, which appeared in the above-mentioned nine equations twice – once in the basis S^6 and once in S^9. We call the following (contextual) value assignment strategy for this vector (together with the remaining seventeen vectors) as $A0$: The vector $(0,1,-1,0)$ takes value 1 when it occurs in S^6 and 0 when in S^9; and the values assigned to the remaining seventeen vectors are done non-contextually. Similarly we consider another contextual value assignment (call it as $A1$) where the vector $(0,1,-1,0)$ take value 1 when it appears in S^9 and 0 when in S^6, value assignments for the remaining vectors being non-contextual. Let $B0$ be the strategy where the eighteen vectors $u_1^1 = u_1^2 = (0,0,0,1)$, $u_2^1 = u_1^5 = (0,0,1,0)$, $u_3^1 = u_3^3 = (1,1,0,0)$, $u_4^1 = u_3^7 = (1,-1,0,0)$, $u_2^2 = u_2^5 = (0,1,0,0)$, $u_3^2 = u_3^8 = (1,0,1,0)$, $u_4^2 = u_4^3 = (1,0,-1,0)$, $u_1^3 = u_1^4 = (1,-1,1,-1)$, $u_2^3 = u_1^6 = (1,-1,-1,1)$, $u_4^3 = u_4^7 = (0,0,1,1)$, $u_2^4 = u_2^6 = (1,1,1,1)$, $u_4^4 = u_4^8 = (0,1,0,-1)$, $u_3^5 = u_3^9 = (1,0,0,1)$, $u_4^5 = u_3^6 = (1,0,0,-1)$, $u_4^6 = u_4^9 = (0,1,-1,0)$, $u_1^7 = u_1^8 = (1,1,-1,1)$, $u_2^7 = u_1^9 = (1,1,1,-1)$, $u_2^8 = u_2^9 = (-1,1,1,1)$, appeared above, are assigned the values 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0 respectively. Similarly, let $B1$ be the strategy where these eighteen vectors (in the same order as above) are assigned the values 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0 respectively. See the tables below for concise description of $A0$, $A1$, $B0$ and $B1$.

Let each of Alice and Bob adopts two strategies: $A0$ and $A1$ for Alice and $B0$ and $B1$ for Bob. One can now check that if, in any round of the game, Alice adopts the strategy $A0$ and Bob adopts the strategy $B0$, they will win that round of the game for all the cases except when Alice is supplied with the basis S^9 and Bob is supplied with the vector $u_4^6 = u_4^9 = (0\ 1\ -1\ 0)$. Same will hold good if, instead, Alice adopts the strategy $A1$ while Bob adopts $B1$. On the other hand both pairs of strategies $(A0, B1)$ and $(A1, B0)$ will give the winning condition of the game when Alice is supplied with the basis S^9 and Bob has given the vector $u_4^6 = u_4^9 = (0\ 1\ -1\ 0)$. The strategies $A0, B0, A1, B1$ are given in the following tabular form:

$A0$		$B0$		$A1$		$B1$	
set	value	set	value	set	value	set	value
S^1	1 0 0 0	S^1	1 0 0 0	S^1	1 0 0 0	S^1	1 0 0 0
S^2	1 0 0 0	S^2	1 0 0 0	S^2	1 0 0 0	S^2	1 0 0 0
S^3	1 0 0 0	S^3	1 0 0 0	S^3	1 0 0 0	S^3	1 0 0 0
S^4	1 0 0 0	S^4	1 0 0 0	S^4	1 0 0 0	S^4	1 0 0 0
S^5	0 0 1 0	S^5	0 0 1 0	S^5	0 0 0 1	S^5	0 0 0 1
S^6	0 0 0 1	S^6	0 0 0 1	S^6	0 0 1 0	S^6	0 0 1 0
S^7	1 0 0 0	S^7	1 0 0 0	S^7	1 0 0 0	S^7	1 0 0 0
S^8	1 0 0 0	S^8	1 0 0 0	S^8	1 0 0 0	S^8	1 0 0 0
S^9	0 0 1 0	S^9	0 0 1 1	S^9	0 0 0 1	S^9	0 0 0 0

Let us now assume that Alice and Bob share an NL-box. Alice and Bob use this NL-box to choose their strategies among those alternatives. The protocol is as follows: if Alice is supplied with one of the first eight bases, *i.e.*, S^1, S^2, ..., S^8, then she will provide 0 as input to the NL-box, otherwise she will choose 1 as input. On the other hand if Bob is given the vector $(0\ 1\ -1\ 0)$ he will provide 1 as input, otherwise he will choose 0 as input to the NL-box. They will now select their strategies according to the outputs of the NL-box, *i.e.*, if Alice gets 0 (1) as output, she will use the strategy $A0$ ($A1$). Similarly if Bob gets 0 (1) as output of the NL-box, then he will assign value to the vector given to him according to the strategy $B0$ ($B1$).

When Alice is told to assign values to the vectors of one of the bases S^1, S^2, ..., S^8 and Bob to any vector from that basis, the output of the NL-box will be either $0, 0$ or $1, 1$ to Alice and Bob respectively. Accordingly they will adopt either strategy $(A0, B0)$ or $(A1, B1)$. It is easy to verify from the table that, in each case under this scenario, Alice and Bob will assign same value to the vector given to Bob. When Alice's job is to assign values to the vectors in the basis S^9 and Bob to any vector except $(0, 1, -1, 0)$, the strategy will again be either $(A0, B0)$ or $(A1, B1)$ and again it will work, as described above. Only when Alice is asked to assign values to the vectors from the basis S^9 and Bob for vector $(0, 1, -1, 0)$, both will put the input 1 in the NL-box and get either $0, 1$ or $1, 0$ as their respective outputs. Here the strategy will be either $(A0, B1)$ or $(A1, B0)$. The vector $(0, 1, -1, 0)$ has same value for both the players. So the above-mentioned method produces a winning strategy for the impossible colouring pseudo-telepathy game.

V. WINNING STRATEGY FOR D-DIMENSION USING NL-BOX

Constructions of sets of vectors in general d dimensions (where $d \geq 3$), having Kochen-Specker property, have been done separately by using geometric method [20] and also by extending a construction in dimension d to dimension $d + 1$ [21]. Using each of these constructions, the above-mentioned impossible colouring pseudo-telepathy game can be generalized for any set of vectors having Kochen-Specker property for any dimension d for $d \geq 3$. Brassard et al. [13] have shown that if all the vectors are real, then there is always a quantum strategy to win this game, where Alice and Bob will have to share a maximally entangled state in $d \otimes d$ of the form

$$|\phi_{AB}\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i\rangle_A \otimes |i\rangle_B$$

Let in d-dimension, there are n number of vectors with which r number of orthogonal basis sets are formed with Kochen-Specker property. Now we give some sufficient condition on such sets for winning the pseudo-telepathy game constructed from these sets :
Let one start assigning values to the vectors appearing in the sets, ordered arbitrarily, in a non-contextual way to satisfy both the conditions of Kochen-Specker theorem (generalized to d dimension). After a certain steps, one finds that this non-contextual value assignments do not work for the remaining k (say) number of sets. Let us now assume that the following condition holds good:
Sufficient condition: Let $m(\leq k)$ be the number of different vectors to appear in those k sets such that no two or more than two of these m vectors appear in any one of these k sets.

We now consider a value assignment strategy (call it as $A0$) to the above-mentioned n vectors in d dimension in such a way that up to the first $(r - k)$ orthogonal bases, we non-contextually assign $\{0, 1\}$-values to all the $d(r - k)$ vectors, that appeared in these $(r - k)$ bases, maintaining both the conditions of Kochen-Specker theorem, while (i) each of the above-mentioned m vectors have to be assigned values (contextually) different from values already assigned to them when they appeared in first $(r - k)$ sets. By reversing the values of these m vectors in the last k sets, one can satisfy the condition (i) with non-contextual value assignment of the remaining $(n - m)$ vectors. We call this later strategy as $A1$. We also consider strategies $B0$ and $B1$: $B0$ is the strategy where the $\{0, 1\}$-value assignment to each of the above-mentioned n vectors will be same as those used in the strategy $A0$ except for the above-mentioned m vectors, for each of which, the value assignment will be same as in the strategy $A1$ for its (*i.e.* $A1$'s) assignment of values to these m vectors appeared in the last k sets (as described above). Similarly $B1$ is the strategy where the $\{0, 1\}$-value assignment to each of the above-mentioned n vectors will be same as those used in the strategy $A1$ except for the above-mentioned m vectors, for each of which, the value assignment will be same as in the strategy $A0$ for its (*i.e.*, $A0$'s) assignment of values to these m vectors appeared in the last k sets.

One can now find a strategy to win the game with a single NL- box. The protocol works as follows:

Alice can use either of the two strategies $A0$ and $A1$. Similarly Bob can use either of the two strategies $B0$ and $B1$. Let Alice and Bob are sharing a NL-box. They have fixed their protocol in this way: when Alice will get any one of the set given from those k sets, then she will give 1 as input to the NL-box, otherwise she will input 0. Similarly when Bob will get any one of those m vectors whose value has to be contextual, then he will give 1 as input to the NL-box and she will input 0 otherwise. They will use their strategies according to the outputs of NL-box, as described earlier. One can check that this is a winning strategy for Alice and Bob. Interestingly, the examples of non-colourable 37 vectors in 26 sets in $\mathbb{R}^3$ and 20 vectors in 11 sets in $\mathbb{R}^4$ [22] satisfy the sufficient condition given above.

Existence of a classical deterministic winning strategy for the impossible colouring game (*i.e.*, a strategy which does not use entanglement or NL-box or any communication but where Alice can assign $\{0,1\}$-values to the vectors of each supplied orthogonal basis (appeared in the associated Kochen-Specker theorem) x and Bob can also assign $\{0,1\}$-values to each supplied vector (which is a member of x) such that both the conditions in the Kochen-Specker theorem are satisfied) would amount to contradict the Kochen-Specker theorem itself. Hence such a strategy can not exist.

VI. GENERALIZATION OF THE MAGIC SQUARE PROBLEM

The magic square problem of size $n = 2d+1$, where d is any positive integer, is given as follows:

Provide an n by n square arrangement with entries from the set $\{0,1\}$ such that (i) the modulo 2 sum of all the elements in each row is 0 and (ii) the modulo 2 sum of all elements in each column is 1, when n is an arbitrary odd positive integer greater than 1.

Magic square problem as a pseudo-telepathy game:

Let there be two players Alice and Bob. Alice is supplied with an element $x^{(A)}$ from the set $\{1,2,\ldots,n\}$, and similarly, Bob is supplied with an element $x^{(B)}$ from the set $\{1,2,\ldots,n\}$. After receiving $x^{(A)}$, Alice will have to produce a *row* vector $\left(y^{(A)}_{x^{(A)}1}, y^{(A)}_{x^{(A)}2}, \ldots, y^{(A)}_{x^{(A)}n}\right) \in \{0,1\}^n$, and similarly, after receiving $x^{(B)}$, Bob will have to produce a *column* vector $\left(y^{(B)}_{1x^{(B)}}, y^{(B)}_{2x^{(B)}}, \ldots, y^{(B)}_{nx^{(B)}}\right)^T$ (where $\left(y^{(B)}_{1x^{(B)}}, y^{(B)}_{2x^{(B)}}, \ldots, y^{(B)}_{nx^{(B)}}\right) \in \{0,1\}^n$) such that the following conditions are simultaneously satisfied:

(1) modulo 2 sum of $y^{(A)}_{x^{(A)}1}$, $y^{(A)}_{x^{(A)}2}$, …, $y^{(A)}_{x^{(A)}n}$ is equal to 0,

(2) modulo 2 sum of $y^{(B)}_{1x^{(B)}}$, $y^{(B)}_{2x^{(B)}}$, …, $y^{(B)}_{nx^{(B)}}$ is equal to 1, and

(3) $y^{(A)}_{x^{(A)}x^{(B)}} = y^{(B)}_{x^{(A)}x^{(B)}}$

for every possible choice of $x^{(A)} \in \{1,2,\ldots,n\}$ and for every possible choice of $x^{(B)} \in \{1,2,\ldots,n\}$.

Note that here the question is not to produce a complete n by n square arrangement with 0's and 1's satisfying (i) and (ii) (which is, in fact, impossible), rather to provide a mathematical argument that would unquestionably establish the potentiality of the strategy to win the pseudo-telepathy game for every possible input pair. Classically there can't exist a winning strategy for this pseudo-telepathy game: A deterministic classical winning strategy will have to assign $\{0,1\}$-values to each of the n^2 entries of the magic square – which is impossible. And so, there can't be any probabilistic classical winning strategy either [23].

It is to be noted here that there is *no* restriction on the total number of the answers

$$\left(\left(y^{(A)}_{x^{(A)}1}, y^{(A)}_{x^{(A)}2}, \ldots, y^{(A)}_{x^{(A)}n}\right), \left(y^{(B)}_{1x^{(B)}}, y^{(B)}_{2x^{(B)}}, \ldots, y^{(B)}_{nx^{(B)}}\right)\right)$$

$$\in \{0,1\}^n \times \{0,1\}^n,$$

that Alice and Bob could give (if that is possible at all), corresponding each question $\left(x^{(A)}, x^{(B)}\right) \in \{1,2,\ldots,n\} \times \{1,2,\ldots,n\}$ in the magic square problem.

VII. A NON-LOCAL WINNING STRATEGY OF THE MAGIC SQUARE PSEUDO-TELEPATHY GAME USING A SINGLE NL-BOX

Let us consider the following row vectors from $\{0,1\}^n$:

$e_1 = (0,1,1,\ldots,1)$, *i.e.*, all the elements, starting from the second, are equal to 1, while the first element is equal to 0,

$e_2 = (1,0,1,\ldots,1)$, *i.e.*, all the elements, starting from the third, are equal to 1, while the first element is equal to 1 and the second element is equal to 0,

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$e_{n-1} = (1,1,\ldots,1,0,1)$, *i.e.*, all the elements up to $(n-2)$th position are equal to 1, while the $(n-1)$th element is equal to 0 and the nth element is equal to 1,

$e_n = (1,1,\ldots,1,0)$, *i.e.*, all the elements up to $(n-1)$th position are equal to 1, while the nth element is equal to 0;

$f_1 = (0,0,\ldots,0)$, *i.e.*, all the elements are equal to 0,

$f_2 = (0,0,\ldots,0,1,1)$, *i.e.*, all the elements up to $(n-2)$th position are equal to 0, while the $(n-1)$th as well as the nth elements are equal to 1;

$g_1 = (0,1,1,\ldots,1,0)$, *i.e.*, all the elements, starting from the second and up to the $(n-1)$th, are equal to 1, while the first and the nth elements are equal to 0,

$g_2 = (1,0,1,1,\ldots,1,0)$, *i.e.*, all the elements, starting from the third and up to the $(n-1)$th, are equal to 1, while the first element is equal to 1 and the second as well as the nth elements equal to 0,

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$g_{n-2} = (1,1,\ldots,1,0,1,0)$, *i.e.*, all the elements, starting from the first and up to the $(n-3)$th, are equal to 1, while the $(n-2)$th as well as the nth elements are equal to 0 and the $(n-1)$th element is equal to 1,

$g_{n-1} = (1,1,\ldots,1,0,0)$, *i.e.*, all the elements, starting from the first and up to the $(n-2)$th, are equal to 1, while the $(n-1)$th as well as the nth elements are equal to 0;

$h_1 = (1,1,\ldots,1)$, *i.e.*, all the elements are equal to 1.

Let us now consider the following two strategies (we call them as A0 and A1) to be adopted by Alice:

<u>Strategy A0:</u> If Alice adopts the strategy A0, then she will choose her row vectors according to the following rule:
$\left(y_{11}^{(A)}, y_{12}^{(A)}, \ldots, y_{1n}^{(A)}\right) = e_1$, $\left(y_{21}^{(A)}, y_{22}^{(A)}, \ldots, y_{2n}^{(A)}\right) = e_2$, … …, $\left(y_{(n-1)1}^{(A)}, y_{(n-1)2}^{(A)}, \ldots, y_{(n-1)n}^{(A)}\right) = e_{n-1}$, $\left(y_{n1}^{(A)}, y_{n2}^{(A)}, \ldots, y_{nn}^{(A)}\right) = f_1$.

<u>Strategy A1:</u> If Alice adopts the strategy A1, then she will choose her row vectors according to the following rule:
$\left(y_{11}^{(A)}, y_{12}^{(A)}, \ldots, y_{1n}^{(A)}\right) = e_1$, $\left(y_{21}^{(A)}, y_{22}^{(A)}, \ldots, y_{2n}^{(A)}\right) = e_2$, … …, $\left(y_{(n-2)1}^{(A)}, y_{(n-2)2}^{(A)}, \ldots, y_{(n-2)n}^{(A)}\right) = e_{n-2}$, $\left(y_{(n-1)1}^{(A)}, y_{(n-1)2}^{(A)}, \ldots, y_{(n-1)n}^{(A)}\right) = e_n$, $\left(y_{n1}^{(A)}, y_{n2}^{(A)}, \ldots, y_{nn}^{(A)}\right) = f_2$.

Similarly, let us consider the following two strategies (we call them as B0 and B1) to be adopted by Bob:
<u>Strategy B0:</u> If Bob adopts the strategy B0, then he will choose his column vectors according to the following rule:
$\left(y_{11}^{(B)}, y_{21}^{(B)}, \ldots, y_{n1}^{(B)}\right)^T = g_1^T$, $\left(y_{12}^{(B)}, y_{22}^{(B)}, \ldots, y_{n2}^{(B)}\right)^T = g_2^T$, … …, $\left(y_{1(n-1)}^{(B)}, y_{2(n-1)}^{(B)}, \ldots, y_{n(n-1)}^{(B)}\right)^T = g_{n-1}^T$, $\left(y_{1n}^{(B)}, y_{2n}^{(B)}, \ldots, y_{nn}^{(B)}\right)^T = h_1^T$.

<u>Strategy B1:</u> If Bob adopts the strategy B1, then he will choose his column vectors according to the following rule:
$\left(y_{11}^{(B)}, y_{21}^{(B)}, \ldots, y_{n1}^{(B)}\right)^T = g_1^T$, $\left(y_{12}^{(B)}, y_{22}^{(B)}, \ldots, y_{n2}^{(B)}\right)^T = g_2^T$, … …, $\left(y_{1(n-2)}^{(B)}, y_{2(n-2)}^{(B)}, \ldots, y_{n(n-2)}^{(B)}\right)^T = g_{n-2}^T$, $\left(y_{1(n-1)}^{(B)}, y_{2(n-1)}^{(B)}, \ldots, y_{n(n-1)}^{(B)}\right)^T = h_1^T$, $\left(y_{1n}^{(B)}, y_{2n}^{(B)}, \ldots, y_{nn}^{(B)}\right)^T = g_{n-1}^T$.

Note that if Alice adopts the strategy A0 and Bob adopts B0, all the three conditions (1), (2) and (3), mentioned above, are simultaneously satisfied for all $\left(x^{(A)}, x^{(B)}\right) \in \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$ except when $\left(x^{(A)}, x^{(B)}\right) = (n, n)$ (In this particular case, all these three conditions are not satisfied.).

If Alice adopts the strategy A1 and Bob adopts B1, all the three conditions (1), (2) and (3), mentioned above, are simultaneously satisfied for all $\left(x^{(A)}, x^{(B)}\right) \in \{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$ except when $\left(x^{(A)}, x^{(B)}\right) = (n, n)$ (In this particular case, all these three conditions are not satisfied.).

If Alice adopts the strategy A1 and Bob adopts B0, all the three conditions (1), (2) and (3), mentioned above, are simultaneously satisfied when $\left(x^{(A)}, x^{(B)}\right) = (n, n)$.

If Alice adopts the strategy A0 and Bob adopts B1, all the three conditions (1), (2) and (3), mentioned above, are simultaneously satisfied when $\left(x^{(A)}, x^{(B)}\right) = (n, n)$.

Thus we see that Alice and Bob can win the magic square game with *certainty* if they jointly adopt one of the two strategies (A0, B0) or (A1, B1) whenever $\left(x^{(A)}, x^{(B)}\right)$ is not equal to (n, n), and if they jointly adopt one of the two strategies (A0, B1) or (A1, B0) whenever $\left(x^{(A)}, x^{(B)}\right) = (n, n)$.

Let us now assume that Alice and Bob share a single NLB with Alice's input bit and Bob's input bit as $X^{(A)}$ and $X^{(B)}$ respectively, and with Alice's output bit and Bob's output bit as $Y^{(A)}$ and $Y^{(B)}$ respectively such that the modulo 2 sum of $Y^{(A)}$ and $Y^{(B)}$ is equal to the product of $X^{(A)}$ and $X^{(B)}$. Now given the input $x^{(A)} \in (\{1, 2, \ldots, n\} - \{n\})$ to Alice, she will supply the input $X^{(A)} = 0$ to the NLB, else she will supply the input $X^{(A)} = 1$ to the NLB. Similarly, given the input $x^{(B)} \in (\{1, 2, \ldots, n\} - \{n\})$ to Bob, he will supply the input $X^{(B)} = 0$ to the NLB, else he will supply the input $X^{(B)} = 1$ to the NLB. For these inputs to the NLB, Alice will then adopt the strategy A$Y^{(A)}$ and Bob will adopt the strategy B$Y^{(B)}$.

Thus we see that when $x^{(A)} \in (\{1, 2, \ldots, n\} - \{n\})$ and $x^{(B)} \in (\{1, 2, \ldots, n\} - \{n\})$, we have $X^{(A)} = X^{(B)} = 0$, and hence $\left(Y^{(A)}, Y^{(B)}\right) = (0, 0)$ or $(1, 1)$. So Alice and Bob can either adopt the strategies A0 and B0 respectively, or they can (equally well) adopt the strategies A1 and B1 respectively. And in both of these cases they can successfully win the magic square game.

When $x^{(A)} \in (\{1, 2, \ldots, n\} - \{n\})$ and $x^{(B)} = n$, we have $X^{(A)} = 0$, $X^{(B)} = 1$, and hence $\left(Y^{(A)}, Y^{(B)}\right) = (0, 0)$ or $(1, 1)$. So Alice and Bob can either adopt the strategies A0 and B0 respectively, or they can (equally well) adopt the strategies A1 and B1 respectively. And in both of these cases they can successfully win the magic square game.

When $x^{(A)} = n$ and $x^{(B)} \in (\{1, 2, \ldots, n\} - \{n\})$, we have $X^{(A)} = 1$, $X^{(B)} = 0$, and hence $\left(Y^{(A)}, Y^{(B)}\right) = (0, 0)$ or $(1, 1)$. So Alice and Bob can either adopt the strategies A0 and B0 respectively, or they can (equally well) adopt the strategies A1 and B1 respectively. And in both of these cases they can successfully win the magic square game.

When $x^{(A)} = n$ and $x^{(B)} = n$, we have $X^{(A)} = X^{(B)} = 1$, and hence $\left(Y^{(A)}, Y^{(B)}\right) = (0, 1)$ or $(1, 0)$. So Alice and Bob can either adopt the strategies A0 and B1 respectively, or they can (equally well) adopt the strategies A1 and B0 respectively. And in both of these cases they can successfully win the magic square game.

VIII. WINNING STRATEGY FOR THE MAGIC SQUARE GAME OF SIZE THREE WITH ENTANGLEMENT

Brassard et al. [13] have shown how to win the magic square game for $n = 3$ with certainty by sharing a two ebit entanglement between Alice and Bob. Let us describe that protocol below.

Alice and Bob are two far apart parties. Alice possess two qubits a and c while Bob possess another two qubits b and d. Let us now assume that Alice and Bob share the singlets $|\psi^-\rangle_{ab} = \frac{1}{\sqrt{2}}(|00\rangle_{ab} + |11\rangle_{ab})$ and $|\psi^-\rangle_{cd} = \frac{1}{\sqrt{2}}(|00\rangle_{cd} + |11\rangle_{cd})$. Thus we see that Alice and Bob share the following 2-ebit state:

$$|\Psi\rangle_{ac:bd} = \tfrac{1}{2}(|00\rangle_{ac} \otimes |11\rangle_{bd} - |01\rangle_{ac} \otimes |10\rangle_{bd} - |10\rangle_{ac} \otimes |01\rangle_{bd} + |11\rangle_{ac} \otimes |00\rangle_{bd}). \quad (1)$$

According to her input 1, or 2, or 3, Alice will first apply respectively the following 4×4 unitary operators on her two qubits:

$$U_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} i & 0 & 0 & 1 \\ 0 & -i & 1 & 0 \\ 0 & i & 1 & 0 \\ 1 & 0 & 0 & i \end{bmatrix}, \quad U_2 = \frac{1}{2}\begin{bmatrix} i & 1 & 1 & i \\ -i & 1 & -1 & i \\ i & 1 & -1 & -i \\ -i & 1 & 1 & -i \end{bmatrix}$$

$$U_3 = \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}. \quad (2)$$

Similarly, according to his input 1, or 2, or 3, Bob will first apply respectively the following 4×4 unitary operators on his two qubits:

$$V_1 = \frac{1}{2}\begin{bmatrix} i & -i & 1 & 1 \\ -i & -i & 1 & -1 \\ 1 & 1 & -i & i \\ -i & i & 1 & 1 \end{bmatrix}, \quad V_2 = \frac{1}{2}\begin{bmatrix} -1 & i & 1 & i \\ 1 & i & 1 & -i \\ 1 & -i & 1 & i \\ -1 & -i & 1 & -i \end{bmatrix}$$

$$V_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}. \quad (3)$$

After this, both Alice as well as Bob measure their respective two qubits in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. Let $|a_1a_2\rangle_{ac}$ and $|b_1b_2\rangle bd$ be the outputs of Alice and Bob, provided they would occur with some non-zero (joint) probability. Then Alice will supply the row vector $(a_1, a_2, a_1 \oplus a_2)$ and Bob will supply the column vector $(b_1, b_2, b_1 \oplus b_2 \oplus 1)^T$ as their respective outputs of the magic square pseudo-telepathy game.

One can check that, using this strategy, Alice and Bob will be able to win the game with certainty.

IX. WINNING STRATEGY FOR THE MAGIC SQUARE GAME OF ANY ODD SIZE WITH ENTANGLEMENT

Next we consider the question of winning the magic square pseudo-telepathy game of size $n = 2d + 1$ using entanglement where d is any positive integer greater than 1. So $n - 3 = 2(d - 1)$ will always be an even positive integer, greater than or equal to 2.

Let us again assume that two far apart parties Alice and Bob share the 2-ebit state $|\Psi\rangle_{ac:bd}$, as given in (1), where qubits a, c are in the possession of Alice and the qubits b, d are in the possession of Bob.

1. If the input $\left(x^{(A)}, x^{(B)}\right)$ for Alice and Bob belongs to the subset $\{1, 2, \ldots, n-3\} \times \{1, 2, \ldots, n-3\}$ of $\{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$, then they will provide their outputs as follows:

Upon receiving the inputs $\left(x^{(A)}, x^{(B)}\right)$, Alice and Bob first performs the unitary operators U_1 (given in equation (2)) and V_1 (given in equation (3)) on their respective two qubits. And then they perform measurements in the computational basis on their respective two qubits. If $|a_1a_2\rangle_{ac}$ and $|b_1b_2\rangle_{bd}$ be the outcomes of Alice and Bob respectively in the above measurements (so, the probability of their joint occurrence must be positive), then Alice will provide the output row vector according to the rule $\left(y^{(A)}_{11}, y^{(A)}_{12}, \ldots, y^{(A)}_{1n}\right) = (1, 1, \ldots, 1, 0, 0, 0)$ (*i.e.*, all the elements, except the last three, are equal to 1), $\left(y^{(A)}_{21}, y^{(A)}_{22}, \ldots, y^{(A)}_{2n}\right) = (0, 0, \ldots, 0)$ (*i.e.*, all the elements are equal to 0), $\left(y^{(A)}_{31}, y^{(A)}_{32}, \ldots, y^{(A)}_{3n}\right) = (0, 0, \ldots, 0)$, … …, $\left(y^{(A)}_{(n-3)1}, y^{(A)}_{(n-3)2}, \ldots, y^{(A)}_{(n-3)n}\right) = (0, 0, \ldots, 0)$;

$\left(y^{(B)}_{11}, y^{(B)}_{21}, \ldots, y^{(B)}_{n1}\right)^T = (1, 0, 0, \ldots, 0)^T$ (*i.e.*, all the elements, except the first one, are equal to 0), $\left(y^{(B)}_{12}, y^{(B)}_{22}, \ldots, y^{(B)}_{n2}\right)^T = (1, 0, 0, \ldots, 0)^T$, $\left(y^{(B)}_{13}, y^{(B)}_{23}, \ldots, y^{(B)}_{n3}\right)^T = (1, 0, 0, \ldots, 0)^T$, … …, $\left(y^{(B)}_{1(n-3)}, y^{(B)}_{2(n-3)}, \ldots, y^{(B)}_{n(n-3)}\right)^T = (1, 0, 0, \ldots, 0)^T$.

Thus we see that the outputs of Alice and Bob, in this case, does not depend on the choice of the unitary operators and measurement outcomes.

2. If the input $\left(x^{(A)}, x^{(B)}\right)$ for Alice and Bob belongs to the subset $\{1, 2, \ldots, n-3\} \times \{n-2, n-1, n\}$, then they will provide their outputs as follows:

Upon receiving the inputs $\left(x^{(A)}, x^{(B)}\right)$, Alice and Bob first performs the unitary operators U_1 and $V_{x^{(B)}-n+3}$ on their respective two qubits. And then they perform measurements in the computational basis on their respective two qubits. If $|a_1a_2\rangle_{ac}$ and $|b_1b_2\rangle_{bd}$ be the outcomes of Alice and Bob respectively in the above measurements (so, the probability of their joint occurrence must be positive), then Alice will provide the output row vector according to the rule

$\left(y_{11}^{(A)}, y_{12}^{(A)}, \ldots, y_{1n}^{(A)}\right) = (1,1,\ldots,1,0,0,0)$, $\left(y_{21}^{(A)}, y_{22}^{(A)}, \ldots, y_{2n}^{(A)}\right) = (0,0,\ldots,0)$, $\left(y_{31}^{(A)}, y_{32}^{(A)}, \ldots, y_{3n}^{(A)}\right) = (0,0,\ldots,0)$, … …, $\left(y_{(n-3)1}^{(A)}, y_{(n-3)2}^{(A)}, \ldots, y_{(n-3)n}^{(A)}\right) = (0,0,\ldots,0)$,

while Bob will provide the output column vector according to the rule

$\left(y_{1x^{(B)}}^{(B)}, y_{2x^{(B)}}^{(B)}, \ldots, y_{nx^{(B)}}^{(B)}\right)^T = (0,0,\ldots,0,b_1,b_2,b_1 \oplus b_2 \oplus 1)^T$ (*i.e.*, all the elements, except the last three, are equal to 0).

3. If the input $\left(x^{(A)}, x^{(B)}\right)$ for Alice and Bob belongs to the subset $\{n-2, n-1, n\} \times \{1,2,\ldots,n-3\}$, then they will provide their outputs as follows:

Upon receiving the inputs $\left(x^{(A)}, x^{(B)}\right)$, Alice and Bob first performs the unitary operators $U_{x^{(A)}-n+3}$ and V_1 on their respective two qubits. And then they perform measurements in the computational basis on their respective two qubits. If $|a_1a_2\rangle_{ac}$ and $|b_1b_2\rangle_{bd}$ be the outcomes of Alice and Bob respectively in the above measurements (so, the probability of their joint occurrence must be positive), then Alice will provide the output row vector according to the rule

$\left(y_{x^{(A)}1}^{(A)}, y_{x^{(B)}2}^{(A)}, \ldots, y_{x^{(A)}n}^{(A)}\right) = (0,0,\ldots,0,a_1,a_2,a_1 \oplus a_2)$ (*i.e.*, all the elements, except the last three, are equal to 0),

while Bob will provide the output column vector according to the rule

$\left(y_{11}^{(B)}, y_{21}^{(B)}, \ldots, y_{n1}^{(B)}\right)^T = (1,0,0,\ldots,0)^T$, $\left(y_{12}^{(B)}, y_{22}^{(B)}, \ldots, y_{n2}^{(B)}\right)^T = (1,0,0,\ldots,0)^T$, $\left(y_{13}^{(B)}, y_{23}^{(B)}, \ldots, y_{n3}^{(B)}\right)^T = (1,0,0,\ldots,0)^T$, … …, $\left(y_{1(n-3)}^{(B)}, y_{2(n-3)}^{(B)}, \ldots, y_{n(n-3)}^{(B)}\right)^T = (1,0,0,\ldots,0)^T$.

4. If the input $\left(x^{(A)}, x^{(B)}\right)$ for Alice and Bob belongs to the subset $\{n-2, n-1, n\} \times \{n-2, n-1, n\}$, then they will provide their outputs as follows:

Upon receiving the inputs $\left(x^{(A)}, x^{(B)}\right)$, Alice and Bob first performs the unitary operators $U_{x^{(A)}-n+3}$ and $V_{x^{(B)}-n+3}$ on their respective two qubits. And then they perform measurements in the computational basis on their respective two qubits. If $|a_1a_2\rangle_{ac}$ and $|b_1b_2\rangle_{bd}$ be the outcomes of Alice and Bob respectively in the above measurements (so, the probability of their joint occurrence must be positive), then Alice will provide the output row vector according to the rule

$\left(y_{x^{(A)}1}^{(A)}, y_{x^{(B)}2}^{(A)}, \ldots, y_{x^{(A)}n}^{(A)}\right) = (0,0,\ldots,0,a_1,a_2,a_1 \oplus a_2)$,

while Bob will provide the output column vector according to the rule

$\left(y_{1x^{(B)}}^{(B)}, y_{2x^{(B)}}^{(B)}, \ldots, y_{nx^{(B)}}^{(B)}\right)^T = (0,0,\ldots,0,b_1,b_2,b_1 \oplus b_2 \oplus 1)^T$.

Using this strategy, one can check (which is simple but tedious) that Alice and Bob will be able to win the game with certainty.

Note that instead of applying the unitary operator U_1, Alice could also have applied any one (but fixed) of U_2, U_3 whenever she receives her input $x^{(A)}$ from the subset $\{1,2,\ldots,n-3\}$. Similarly, Bob also could have applied any one (but fixed) of V_1, V_2, V_3 whenever he receives her input $x^{(B)}$ from the subset $\{1,2,\ldots,n-3\}$.

X. CONCLUSION

One should note that in the discussion of the sufficient conditions for having non-local winning strategy for the impossible colouring pseudo-telepathy game in d dimension, we have not mentioned that vectors have to be real. That condition is necessary for quantum protocol [13] but has no relevance for protocol using NL-box. This difference may be important to understand particular features of quantum entanglement in the context of general non-local theory with no signalling. As we go on increasing the size of the inputs in a pseudo-telepathy game, we might expect of using more resources both for quantum as well as for non-local winning strategies. But our result proves it to be not true for the magic square pseudo-telepathy game.

In order to characterize properties of the non-local correlation associated to the NL-box, it is important to classify all possible non-local correlations (including quantum one), each of which can be simulated by one or more than one NL-box (without allowing any communication). For example, the EPR correlation, for von Neumann measurements, can be simulated by a single NL-box [6]. The quantum correlations, arising out from the quantum winning strategy of the magic square pseudo-telepathy game of size three using two EPR pairs, can be simulated by a single NL-box by using the method for having a non-local winning strategy for the above game [17]. It would be similarly interesting to see whether both the quantum correlations – one in impossible colouring pseudo-telepathy game in d dimension and another in general magic square pseudo-telepathy game – can be simulated by their corresponding non-local winning strategies, each of which uses only a single NL-box.

XI. ACKNOWLEDGEMENT

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[1] J.S. Bell, *Physics* **1**, 195 (1964).
[2] F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, *Phys. Rev. Lett.* **23**, 880 (1969).
[3] B. S. Cirelson, *Lett. Math. Phys.* **4**, 93 (1980).
[4] S. Popescu and D. Rohrlich, *Found. Phys.* **24**, 379 (1994).
[5] J. Barrett, N. Linden, S. Massar, S. Pironio, S. Popescu, D. Roberts *Phys. Rev. A* **71**, 022101 (2005).
[6] N. J. Cerf, N. Gisin, S. Massar, S. Popescu, *Phys. Rev. Lett.* **94**, 220403 (2005).
[7] N. Brunner, N. Gisin, V. Scarani, *New J. Phys.* **7**, 88 (2005).
[8] W. V. Dam *quant-ph/0501159*.
[9] T. Short, N. Gisin, S. Popescu *quant-ph/0504134*.
[10] H. Buhrman, M. Christandl, F. Unger, S. Wehner, A. Winter, *quant-ph/0504133*.
[11] T. Short, S. Popescu, N. Gisin *quant-ph/0508120*.
[12] G. Brassard, H. Buhrman, N. Linden, A. A. Méthot, A. Tapp, F. Unger, *quant-ph/0508042*.
[13] G. Brassard, A. Broadbent and A. Tapp, *quant-ph/0407221*.
[14] We only consider here two-party quantum pseudo-telepathy games.
[15] A. A. Méthot, *quant-ph/0507149*.
[16] J. Barrett and S. Pironio, *Phys. Rev. Lett.* **95**, 140401 (2005).
[17] A. Broadbent and A. A. Méthot, *quant-ph/0504136*.
[18] A. Cabello, J.M. Estebaranz and G. Garcia-Alcaine *Phys. Lett. A* **212** , 183 (1996).
[19] G. Kar, *quant-ph/0504191*.
[20] R. D. Gill and M. S. Keane, *J. Phys. A: Math. Gen.* **29**, L289 (1996).
[21] A Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, Dordrecht, 1993).
[22] J. Bub, *Interpreting the quantum world* (Cambridge University press, 1997) p. 82-100.
[23] A probabilistic classical winning strategy for the magic square pseudo-telepathy game requires implementation of some strategy $(A1, B1)$ for Alice and Bob respectively with non-zero probability p_1 (where this strategy can be implemented by Alice and Bob by sharing some random variables and then tossing local coins to get values of the local random variables, which, in turn, will fix the strategy), such that the strategy can produce correct outputs for all possible inputs with probability at least p where $p > 0$. Note that p has to be 0 here as any non-zero p would amount to successfull implementation of a classical deterministic winning strategy for the magic square pseudo-telepathy game (see [13] for details).