# Quantum ion acoustic shock waves in planar and nonplanar geometry

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The effects of unbounded planar geometry and bounded nonplanar geometry on quantum ion acoustic shock waves (QIASWs) in unmagnetized plasmas, where plasma kinematic viscosities are taken into account, are investigated. By the reductive perturbation method, deformed Korteweg–de Vries Burger (dKdVB), cylindrical, and spherical dKdVB equations are obtained for quantum ion acoustic shock waves in an unmagnetized two-species quantum plasma system, comprising electrons and ions. The properties of quantum ion acoustic shock waves are studied taking into account the quantum-mechanical effects in planar and nonplanar geometry. It is shown that quantum ion acoustic shock waves in nonplanar geometry differ from planar geometry. We have studied the change of QIASW structure due to the effect of the geometry, quantum parameter H, and ion kinematic viscosities by numerical calculations of the planar dKdVB, cylindrical, and spherical dKdVB equations.

## I. INTRODUCTION

Recently, considerable interest has been shown in quantum effects[1] in plasma. This is mainly due to the fact that these effects are of considerable importance in many aspects of plasma, such as quantum plasma echo,[2] dense plasma (particularly in astrophysical and cosmological studies),[3–6] quantum plasma instabilities in Fermi gases,[7] quantum Landau damping,[8] among others. Among the prevalent models to study quantum effects in plasma, the quantum hydrodynamic (QHD)[7,9–13] model has become popular because it extends the usual fluid model to one incorporating the quantum effect. The QHD model is similar to the classical fluid model as it is comprised of a set of equations describing charge, momentum, and energy transport. The deviation from the classical model occurs because of the presence of a term, the so called Bohm potential. This term contains Planck's constant $\hbar$, an indication of quantum effect. Another significant quantum plasma theory is of the Wigner-Poisson system,[14–16] which involves the integrodifferential system. Haas *et al.*[17] used the QHD model to study quantum ion acoustic waves in the weakly nonlinearized theory and obtained a deformed Korteweg–de Vries (dKdV) equation, which involves the parameter H, proportional to Planck's constant $\hbar$. It has already been shown that the linear quantum ion acoustic waves are described by a dispersion relation that tends to the classical dispersion relation as the quantum effect tends to zero. Opher *et al.*[4] studied the effects of highly damped models in the energy and reaction rates in plasma and discussed the implication of introducing highly damped models, taking into account the quantum effects, in the nuclear reaction rates in a plasma. More recently, Haas[18] used a magnetohydrodynamical quantum model to extend their study in magnetized plasma. Garcia *et al.*[19] used the hydrodynamical model to study the modified Zakharov equation in a plasma with a quantum correction. Ali and Shukla[20] studied dust acoustic solitary waves in quantum plasma. Misra and Roychowdhury[21] used the one-dimensional QHD model to study modulation of dust acoustic waves. However, most of the studies mentioned above were in either one-dimensional or planar geometry, except the one by Haas,[18] who used cylindrical geometry to study the magnetostatic equilibrium. Recently, Sahu and Roychoudhury[22] derived cylindrical and spherical dKdV equations for quantum ion acoustic waves in an unmagnetized plasma. In this paper, we have studied planar and cylindrical and spherical deformed Korteweg–de Vries Burger's (dKdVB) equations to study shock-wave-like solutions. The Burger term in the nonlinear wave equation arises when one takes into account the kinematic viscosities of the plasma constituents. We have considered a two-species plasma comprising electrons and ions in both planar and nonplanar (cylindrical and spherical) geometry. The plan of the paper is as follows. The planar dKdVB equation is derived in Sec. II for the one-dimensional case. Derivation of dKdVB in nonplanar geometry is shown in Sec. III. In Sec. IV the numerical solutions are discussed, while Sec. V is contains the conclusion.

## II. BASIC EQUATIONS AND DERIVATION OF DEFORMED KdVB EQUATIONS IN PLANAR GEOMETRY

We consider a two-species quantum plasma system comprising electrons and ions in a planar geometry and study the nonlinear propagation of ion acoustic shock waves. The one-dimensional quantum hydrodynamic mode consists of the continuity and momentum balance equations for both electrons and ions together with Poisson's equation for the self-consistent potential. The nonlinear dynamics of the ion acoustic waves in quantum plasma system in planar geometries are governed by

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$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0, \tag{1}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{2}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{e}{m_e}\frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right) + \mu_e \frac{\partial^2 u_e}{\partial x^2}, \tag{3}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} + \frac{\hbar^2}{2m_i^2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_i}/\partial x^2}{\sqrt{n_i}}\right) + \mu_i \frac{\partial^2 u_i}{\partial x^2}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - n_i), \tag{5}$$

where $n_e$, $u_e$, $m_e$, $-e$ ($n_i$, $u_i$, $m_i$, e) are the electron (ion) density field, velocity field, mass, and charge, respectively, and $\varepsilon_0$ and $\hbar$ are the dielectric and Planck constant divided by $2\pi$, respectively. $\phi$ is the electrostatic wave potential, $p_e$ is the pressure effects for electrons, $\mu_e$ and $\mu_i$ are the electron and ion kinematic viscosity, respectively. Pressure effects for ions are neglected for simplicity. We assume that the electrons obey the equation of state pertaining to a one-dimensional zero-temperature Fermi gas,[7]

$$p_e = \frac{m_e v_{Fe}^2}{3n_0^2} n_e^3, \tag{6}$$

where $n_0$ is the equilibrium density for both electrons and ions, $v_{Fe}$ is the electronic Fermi velocity connected to the Fermi temperature $T_{Fe}$ by $m_e v_{Fe}^2/2 = k_B T_{Fe}$, and $k_B$ is Boltzmann's constant. Now we introduce the following normalization:

$$\bar{x} = \omega_{pi} x/t, \quad \bar{t} = \omega_{pi} t, \quad \bar{n}_e = n_e/n_0, \quad \bar{n}_i = n_i/n_0,$$
$$\bar{u}_e = u_e/c_s, \quad \bar{u}_i = u_i/c_s, \quad \bar{\phi} = e\phi/(2k_B T_{Fe}), \tag{7}$$

where $\omega_{pe}$ and $\omega_{pi}$ are the corresponding electron and ion plasma frequencies,

$$\omega_{pe} = \left(\frac{n_0 e^2}{m_e \varepsilon_0}\right)^{1/2}, \quad \omega_{pi} = \left(\frac{n_0 e^2}{m_i \varepsilon_0}\right)^{1/2},$$

and $c_s$ is the quantum ion acoustic velocity given by

$$c_s = \left(\frac{2k_B T_{Fe}}{m_i}\right)^{1/2}.$$

We have denoted nondimensional quantum parameter

$$H = \frac{\hbar \omega_{pe}}{2k_B T_{Fe}} (>0).$$

Using the above normalization, we obtain from Eqs. (3) and (4) (dropping bars)

$$\frac{m_e}{m_i}\left(\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x}\right) = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right) + \eta_e \frac{m_e}{m_i}\frac{\partial^2 u_e}{\partial x^2}, \tag{8}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} + \frac{m_e}{m_i}\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_i}/\partial x^2}{\sqrt{n_i}}\right) + \eta_i \frac{\partial^2 u_i}{\partial x^2}, \tag{9}$$

where $\eta_e = \mu_e \omega_{Pi}/c_s^2$ and $\eta_i = \mu_i \omega_{Pi}/c_s^2$. As $m_e/m_i \ll 1$, after integrating Eq. (8) once and assuming the boundary conditions $n_e = 1$, $\phi = 0$ at infinity, we get

$$\phi = -\frac{1}{2} + \frac{n_e^2}{2} - \frac{H^2}{2\sqrt{n_e}}\frac{\partial^2}{\partial x^2}\sqrt{n_e}. \tag{10}$$

This equation gives the electrostatic potential in terms of electron density and its derivatives. In the momentum equation (9), the quantum diffraction term may be neglected due to $m_e/m_i \ll 1$.

Now the continuity equation (2), momentum equation (9), and Poisson's equations become

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{11}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} + \eta_i \frac{\partial^2 u_i}{\partial x^2}, \tag{12}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i. \tag{13}$$

Equations (11)–(13) and Eq. (10) are the four basic equations with four unknown quantities $n_i$, $u_i$, $n_e$, and $\phi$. The only remaining free parameter is H, which measures the effect of quantum diffraction. Physically, H is the ratio between the electron plasmon energy and the electron Fermi energy. The electron fluid velocity can be found from the electron continuity equation.

We now introduce the stretched coordinates,

$$\xi = \epsilon^{1/2}(x - t), \quad \tau = \epsilon^{3/2} t \tag{14}$$

and expand $n_i$, $u_i$, and $n_e$ in a power series of $\epsilon$ as

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{15}$$

$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{16}$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots. \tag{17}$$

In many experimental situations, the value of $\eta_i$ is small, so we may set $\eta_i = \epsilon^{1/2}\eta_{i0}$. $\eta_{i0}$ is $O(1)$. Due to the above expansion of $n_e$, the expansion for $\phi$ [Eq. (10)] becomes

$$\phi = \epsilon\left(n_e^{(1)} - \frac{H^2}{4}\frac{\partial^2 n_e^{(1)}}{\partial x^2}\right) + \frac{\epsilon^2}{2}\left[n_e^{(1)2} + 2n_e^{(2)} + \frac{H^2}{2}\left\{n_e^{(1)}\frac{\partial^2 n_e^{(1)}}{\partial x^2} - \frac{\partial^2 n_e^{(2)}}{\partial x^2} + \left(\frac{1}{2}\frac{\partial n_e^{(1)}}{\partial x}\right)^2\right\}\right] + \cdots . \tag{18}$$

Now we develop Eqs. (11)–(13) in the form of a power series of $\epsilon$. Then the system of equations can be written as with the help of Eq. (18),

$$\frac{\partial}{\partial \xi}(u_i^{(1)} - n_i^{(1)}) + \epsilon\left\{\frac{\partial n_i^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi}(u_i^{(2)} - n_i^{(2)} + n_i^{(1)}u_i^{(1)})\right\} = O(\epsilon^2), \tag{19}$$

$$\frac{\partial}{\partial \xi}(n_e^{(1)} - u_i^{(1)}) + \epsilon\left\{\frac{\partial u_i^{(1)}}{\partial \tau} - \frac{\partial u_i^{(2)}}{\partial \xi} + u_i^{(1)}\frac{\partial u_i^{(1)}}{\partial \xi} - \frac{H^2}{4}\frac{\partial^3 n_e^{(1)}}{\partial \xi^3}\right\} + \epsilon\left\{\frac{1}{2}\frac{\partial}{\partial \xi}(n_e^{(1)2} + 2n_e^{(2)}) - \eta_{i0}\frac{\partial^2 u_i^{(1)}}{\partial \xi^2}\right\} = O(\epsilon^2), \tag{20}$$

$$n_i^{(1)} - n_e^{(1)} + \epsilon\left\{n_i^{(2)} - n_e^{(2)} + \frac{\partial^2 n_e^{(1)}}{\partial \xi^2}\right\} = O(\epsilon^2). \tag{21}$$

The zeroth-order terms of the above equations together with the assumption that the $u_i^{(1)}$ and $n_i^{(1)}$ vanish as $\xi \to 0$ yield

$$n_e^{(1)} = n_i^{(1)} = u_i^{(1)} \equiv U(\xi,\tau) \tag{22}$$

defining a new function $U(\xi,\tau)$.

From Eqs. (19)–(21), considering the first-order terms using Eq. (22), we have

$$\frac{\partial U}{\partial \tau} + \frac{\partial}{\partial \xi}(u_i^{(2)} - n_i^{(2)} + U^2) = 0, \tag{23}$$

$$\frac{\partial U}{\partial \tau} + \frac{\partial}{\partial \xi}\left(n_e^{(2)} - u_i^{(2)} + U^2 - \frac{H^2}{4}\frac{\partial^2 U}{\partial \xi^2}\right) - \eta_{i0}\frac{\partial^2 U}{\partial \xi^2} = 0, \tag{24}$$

$$\frac{\partial^2 U}{\partial \xi^2} = n_e^{(2)} - n_i^{(2)}. \tag{25}$$

Combining Eqs. (23)–(25), we deduce a modified deformed KdVB equation for quantum ion acoustic waves,

$$\frac{\partial U}{\partial \tau} + 2U\frac{\partial U}{\partial \xi} + \frac{1}{2}\left(1 - \frac{H^2}{4}\right)\frac{\partial^3 U}{\partial \xi^3} - \frac{\eta_{i0}}{2}\frac{\partial^2 U}{\partial \xi^2} = 0. \tag{26}$$

## III. DERIVATION FOR DEFORMED KdVB EQUATIONS IN NONPLANAR (CYLINDRICAL AND SPHERICAL) GEOMETRY

In a nonplanar cylindrical or spherical geometry, the nonlinear dynamics of the ion acoustic waves in a quantum plasma system is governed by

$$\frac{\partial n_e}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_e u_e) = 0, \tag{27}$$

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_i u_i) = 0, \tag{28}$$

$$\frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial r} = \frac{e}{m_e}\frac{\partial \phi}{\partial r} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial r} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial r}\left\{\frac{\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial\sqrt{n_e}}{\partial r}\right)}{\sqrt{n_e}}\right\} + \mu_e\left[\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial u_e}{\partial r}\right) - \frac{\nu u_e}{r^2}\right], \tag{29}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial r} = -\frac{e}{m_i}\frac{\partial \phi}{\partial r} + \frac{\hbar^2}{2m_i^2}\frac{\partial}{\partial r}\left\{\frac{\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial\sqrt{n_i}}{\partial r}\right)}{\sqrt{n_i}}\right\} + \mu_i\left[\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial u_i}{\partial r}\right) - \frac{\nu u_i}{r^2}\right], \tag{30}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial \phi}{\partial r}\right) = \frac{e}{\varepsilon_0}(n_e - n_i), \tag{31}$$

where $\nu=0$ for one-dimensional geometry and $\nu=1$ and 2 for cylindrical and spherical geometry, respectively.

Now we introduce the following normalization:

$$\bar{r} = \omega_{pi}r/t, \quad \bar{t} = \omega_{pi}t, \quad \bar{n}_e = n_e/n_0, \quad \bar{n}_i = n_i/n_0,$$
$$\bar{u}_e = u_e/c_s, \quad \bar{u}_i = u_i/c_s, \quad \bar{\phi} = e\phi/(2k_BT_{Fe}). \tag{32}$$

Using the above normalization, we obtain from Eqs. (29) and (30) (dropping bars)

$$\frac{m_e}{m_i}\left(\frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial r}\right) = \frac{\partial \phi}{\partial r} - n_e\frac{\partial n_e}{\partial r} + \frac{H^2}{2}\frac{\partial}{\partial r}\left\{\frac{\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial\sqrt{n_e}}{\partial r}\right)}{\sqrt{n_e}}\right\} + \eta_e\frac{m_e}{m_i}\left[\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial u_e}{\partial r}\right) - \frac{\nu u_e}{r^2}\right], \tag{33}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial r} = -\frac{\partial \phi}{\partial r} + \frac{m_e}{m_i}\frac{H^2}{2}\frac{\partial}{\partial r}\left\{\frac{\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial\sqrt{n_i}}{\partial r}\right)}{\sqrt{n_i}}\right\} + \eta_i\left[\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial u_i}{\partial r}\right) - \frac{\nu u_i}{r^2}\right]. \tag{34}$$

As $m_e/m_i \ll 1$, after integrating Eq. (33) once and assuming the boundary conditions $n_e=1$, $\phi=0$ at infinity, we get

$$\phi = -\frac{1}{2} + \frac{n_e^2}{2} - \frac{H^2}{2\sqrt{n_e}}\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial\sqrt{n_e}}{\partial r}\right). \tag{35}$$

This equation gives the electrostatic potential in terms of electron density and its derivatives. In the momentum equation (34), the quantum diffraction term may be neglected due to $m_e/m_i \ll 1$.

Now the continuity equation (28), momentum equation (34), and Poisson's equations become

$$\frac{\partial n_i}{\partial t}+\frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_i u_i)=0, \tag{36}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial r}=-\frac{\partial \phi}{\partial r}, \tag{37}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu\frac{\partial \phi}{\partial r}\right)=n_e-n_i. \tag{38}$$

The electron fluid velocity can be found from the continuity equation.

We now introduce the stretched coordinates $\xi=\epsilon^{1/2}(r-t)$, $\tau=\epsilon^{3/2}t$ and expand $n_i$, $u_i$, and $n_e$ in a power series of $\epsilon$ as given by (15)–(17).

Due to the above expansion of $n_e$, the expansion for $\phi$ [Eq. (35)] becomes

$$\phi=\epsilon\left(n_e^{(1)}-\frac{H^2}{4}\frac{\partial^2 n_e^{(1)}}{\partial r^2}-\frac{H^2}{4}\frac{\nu}{r}\frac{\partial n_e^{(1)}}{\partial r}\right)+\frac{\epsilon^2}{2}\Bigg[n_e^{(1)2}+2n_e^{(2)} +\frac{H^2}{2}\Bigg\{\frac{n_e^{(1)}}{4}\left(\frac{\partial^2 n_e^{(1)}}{\partial r^2}+\frac{\nu}{r}\frac{\partial n_e^{(1)}}{\partial r}\right)-\frac{\partial^2 n_e^{(2)}}{\partial r^2}+\frac{1}{8}\frac{\partial^2 n_e^{(1)2}}{\partial r^2} +\frac{\nu}{r}\left(\frac{\partial n_e^{(2)}}{\partial r}+\frac{1}{4}\frac{\partial n_e^{(1)2}}{\partial r}\right)\Bigg\}\Bigg]+\cdots. \tag{39}$$

Now we develop Eqs. (36)–(38) in the form of a power series of $\epsilon$. Then the system of equations can be written, with the help of Eq. (39), as

$$\frac{\partial}{\partial \xi}(u_i^{(1)}-n_i^{(1)})+\epsilon\left\{\frac{\partial n_i^{(1)}}{\partial \tau}+\frac{\partial u_i^{(2)}}{\partial \xi}-\frac{\partial n_i^{(2)}}{\partial \xi}+\frac{\partial}{\partial \xi}(n_i^{(1)}u_i^{(1)}) +\frac{\nu}{\tau}u_i^{(1)}\right\}=O(\epsilon^2), \tag{40}$$

$$\frac{\partial}{\partial \xi}(n_e^{(1)}-u_i^{(1)})+\epsilon\left\{\frac{\partial u_i^{(1)}}{\partial \tau}-\frac{\partial u_i^{(2)}}{\partial \xi}+u_i^{(1)}\frac{\partial u_i^{(1)}}{\partial \xi}-\frac{H^2}{4}\frac{\partial^3 n_e^{(1)}}{\partial \xi^3}\right\} +\epsilon\left\{\frac{1}{2}\frac{\partial}{\partial \xi}(n_e^{(1)2}+2n_e^{(2)})-\eta_{i0}\frac{\partial^2 u_i^{(1)}}{\partial \xi^2}\right\}=O(\epsilon^2), \tag{41}$$

$$n_i^{(1)}-n_e^{(1)}+\epsilon\left\{n_i^{(2)}-n_e^{(2)}+\frac{\partial^2 n_e^{(1)}}{\partial \xi^2}\right\}=O(\epsilon^2). \tag{42}$$

The zeroth-order terms of the above equations together with the assumption that the $u_i^{(1)}$ and $n_i^{(1)}$ vanish as $\xi\to 0$ yield

$$n_e^{(1)}=n_i^{(1)}=u_i^{(1)}\equiv U(\xi,\tau) \tag{43}$$

defining a new function $U(\xi,\tau)$.

From (40)–(42), considering the first-order terms using Eq. (43), we have

$$\frac{\partial U}{\partial \tau}+\frac{\partial}{\partial \xi}(u_i^{(2)}-n_i^{(2)}+U^2)+\frac{\nu}{\tau}U=0, \tag{44}$$

$$\frac{\partial U}{\partial \tau}+\frac{\partial}{\partial \xi}\left(n_e^{(2)}-u_i^{(2)}+U^2-\frac{H^2}{4}\frac{\partial^2 U}{\partial \xi^2}\right)-\eta_{i0}\frac{\partial^2 U}{\partial \xi^2}=0, \tag{45}$$

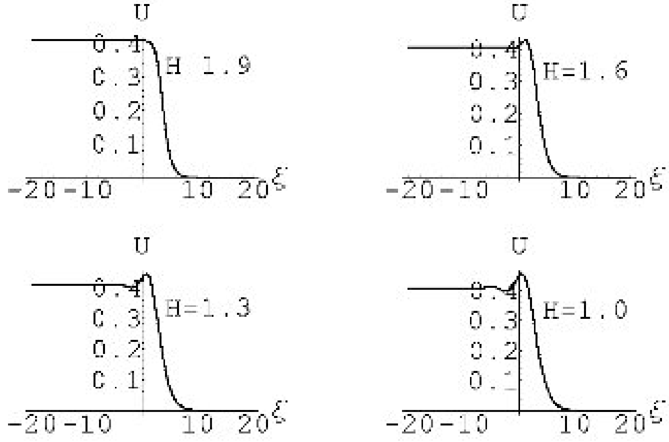


FIG. 1. Numerical solution for Eq. (26), for different values of H, where $\eta_{i0}=0.5$, $V=1$, and $\tau=-3$.

$$\frac{\partial^2 U}{\partial \xi^2}=n_e^{(2)}-n_i^{(2)}. \tag{46}$$

Combining Eqs. (44)–(46), we deduce a modified deformed KdVB equation for quantum ion acoustic waves,

$$\frac{\partial U}{\partial \tau}+\frac{\nu}{2\tau}U+2U\frac{\partial U}{\partial \xi}+\frac{1}{2}\left(1-\frac{H^2}{4}\right)\frac{\partial^3 U}{\partial \xi^3}-\frac{\eta_{i0}}{2}\frac{\partial^2 U}{\partial \xi^2}=0. \tag{47}$$

## IV. NUMERICAL SOLUTIONS

The traveling solution of Eq. (26) can be obtained by the so called "tanh method."[23] The solution of the dKdVB equation turns out to be

$$\phi(\xi,\tau)=a_0+a_1\tanh\{\alpha(\xi-V\tau)\}+a_2\tanh^2\{\alpha(\xi-V\tau)\}, \tag{48}$$

where

$$a_0=\frac{1}{2}(V+12A\alpha^2),\quad a_1=-\frac{6B\alpha}{5},\quad a_2=-6A\alpha^2,$$

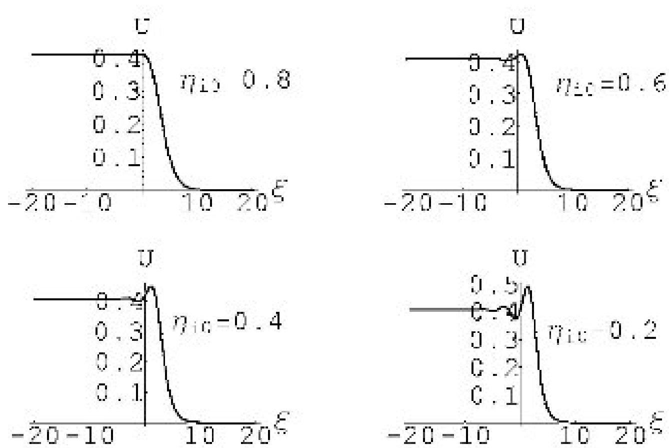


FIG. 2. Numerical solution for Eq. (26), for different values of $\eta_{i0}$, where $H=1.5$, $V=1$, and $\tau=-3$.

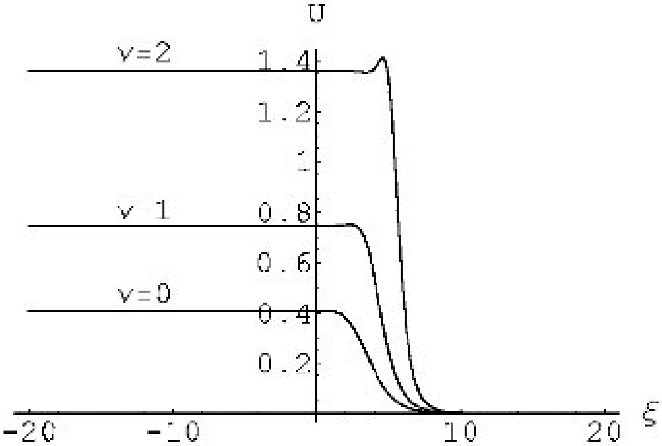


FIG. 3. Plot of the numerical solution for Eq. (47) for different values of $\nu$, where $H=1.8$, $V=1$, $\eta_{i0}=0.6$, $\tau=-3$, and $\alpha=0.3$.

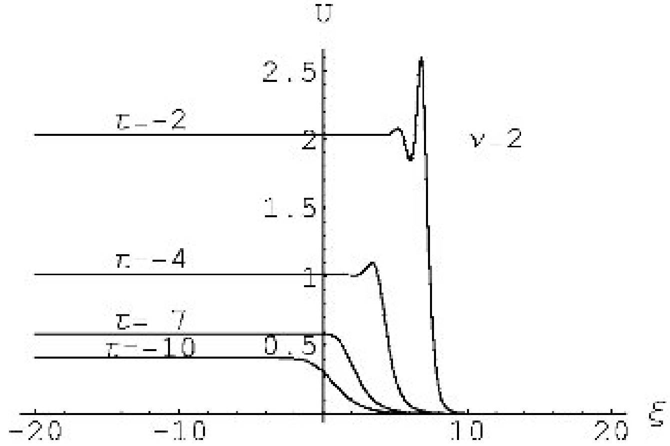


FIG. 5. Plot of the numerical solution for Eq. (47) for different values of $\tau$ for $\nu=2$, where $H=1.8$, $V=1$, $\eta_{i0}=0.4$, and $\alpha=0.21$.

$$\alpha = \pm \frac{B}{10A}, \quad A = \frac{1}{2}\left(1 - \frac{H^2}{4}\right), \quad B = \frac{\eta_{i0}}{2},$$

where $V$ is the shock wave velocity. For $H=2$, Eq. (26) reduces to purely Burger's equation, and Eq. (47) reduces to the purely cylindrical or spherical Burger's equation. The initial profile that we have used in all our numerical results is the stationary solution (48). Here we have seen that the shock height and steepness increases with an increase of H and $\eta_{i0}$. In Figs. 1 and 2, we plot the numerical solution of Eq. (26) for different values of H and $\eta_{i0}$, respectively. It is seen that the shape of the developed shock wave changes appreciably with a decrease of H and $\eta_{i0}$. The effects of geometry on the shock wave due to quantum diffraction are also studied. Figure 3 shows the shock wave structure evolved at $\tau=-3$ in different geometries. It is clear that the shock height and shape change remarkably in different geometries. In Figs. 4 and 5, we plot the solutions of Eq. (47) for several values of $\tau$ in cylindrical ($\nu=1$) and spherical ($\nu=2$) geometry, respectively. We can see that as the value of $|\tau|$ increases, the solution looks like those for one-dimensional KdVB solutions. This is because for large values of $|\tau|$ the nonplanar geometrical effect is no longer dominant. As the value of $|\tau|$ decreases, the nonplanar geometrical effects represented by $(\nu/2\tau)U$ will become effective, and shock waves differs from each other in cylindrical or spherical geometry. In Fig. 6, we have plotted the solutions of (47) for several values of H for the cylindrical case. Here we have also seen that the shape of the shock waves changes for different H. As expected for values of H near 2, the KdV-type kink disappears, whereas for values of H away from 2, the kink is quite prominent. This is because KdV behavior is prominent when $H-2$ is large numerically.

## V. CONCLUSION

We have derived the dKdVB equation and cylindrical and spherical dKdVB equations for quantum ion acoustic waves in an unmagnetized two-species quantum plasma system, comprising electrons and ions taking into account the viscosities of the plasma constituents. The standard reductive perturbation method is employed to derive the dKdVB equation and the cylindrical and spherical dKdVB equations. We have found that the propagation of quantum ion acoustic shock waves in nonplanar geometry differs from that in one-dimensional planar geometry, and the quantum effect plays a significant role in the nature of shock-wave-type solutions. It

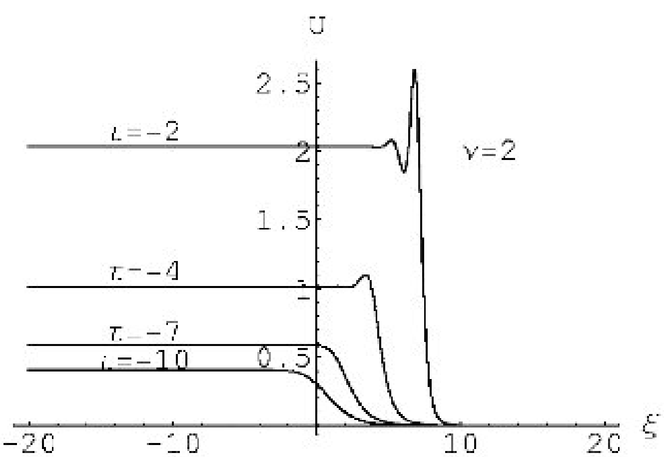


FIG. 4. Plot of the numerical solution for Eq. (47) for different values of $\tau$ for $\nu=1$, where $H=1.8$, $V=1$, $\eta_{i0}=0.4$, and $\alpha=0.21$.

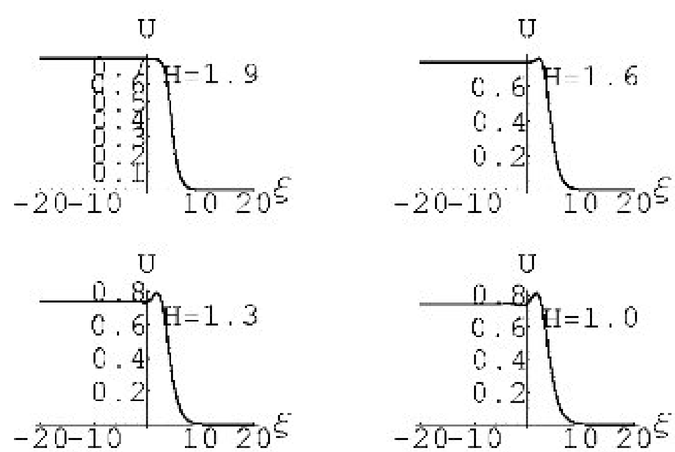


FIG. 6. Plot of the numerical solution for Eq. (47) for different values of H for $\nu=1$, where $\eta_{i0}=0.7$, $V=1$, and $\tau=-3$.

should be noted that for small values of $|\tau|$, both the KdV soliton behavior and Burger's shock wave behavior are present, while for large $|\tau|$ only the shock-like structure is dominant as in the one-dimensional KdVB solution. It is also seen that the shape of the shock structure changes substantially depending on the geometry and the value of H, the quantum effect parameter. In fact, the value of H determines whether the KdV effect will be more prominent than the Burger effect.

## ACKNOWLEDGMENTS

The authors are grateful to the referee for helpful suggestions.

One of us (R.R.) is grateful to the Council of Scientific and Industrial Research (CSIR) for a grant [Project No. 21/(0659)/06/EMR-II].

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