THE ONE-TENTH PER CENT LEVEL OF THE RATIO OF VARIANCES

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Fisher defines $z = \log_e (s_1/s_2)$,

where s_1 and s_2 are the estimated standard deviations of two samples drawn from the same normal population. For a test of significance, therefore, one has to calculate the Naperian logarithm of the ratio of the two standard deviations (or what is the same thing, the difference between the Naperian logarithms of the two standard deviations). Finding the Naperian logarithms however takes time, and to avoid this, Mahalanobis published in 1932, 5 per cent and 1 per cent tables of s_1^2/s_2^2 (and also of s_1/s_2) and similar tables have also been given in G. W. Snedecor's book on the Analysis of Variance and Covariance[2]. Now that C. G. Colcord and L. S. Deming are publishing a one-tenth per cent table of z, it will be convenient to have a table of one-tenth per cent values of (s_1^2/s_2^2) and (s_1/s_2).

The present Table has been derived directly from Colcord and Deming's* "One-tenth per cent Level of z" in the following way :—

$$z = \tfrac{1}{2} \log_e (s_1^2/s_2^2) = \tfrac{1}{2} \log_e (x) = 1{\cdot}1513 \log_{10} (x)$$

Thus $$x = \text{Antilog}_{10} (z/1{\cdot}1513) = \text{Antilog}_{10}\, 0{\cdot}8686\,(z)$$

The following example will illustrate the use of the Table. In a randomised block experiment the following analysis of variance was obtained.

	Mean Square	Ratio of Variance
Block ... 5	23·146	—
Treatment ... 4	19·708	—
Error ... 20	2·725	7·23

* Since the table has been constructed from the corresponding table by Colcord and Deming, given correct to 4 places of decimals, the present table is correct within one unit in the fifth figure; but this will not in any way affect the usual test of significance of two observed variances.

$$z = \tfrac{1}{2}\log_e (19{\cdot}708/2{\cdot}725)$$

$$= 1{\cdot}1513\,(\log_{10} 19{\cdot}708 - \log_{10} 2{\cdot}725)$$

$$= 1{\cdot}1513 \times 0{\cdot}8593$$

$$= 0{\cdot}9893$$

To test the significance, we have the following table :—

$n_1 = 4$ and $n_2 = 20$

	Values of z	Values of x
5·0 per cent	0·5265 (Fisher)	2·866 (Mahalanobis)
1·0 per cent	·7443 (,,)	4·431 (,,)
0·1 per cent	·9798 (Colcord & Deming)	7·102 (Present Tables)

Since the observed value of z ($=0{\cdot}9893$) exceeds not only the 1 per cent but also the 0·1 per cent value of z given in Colcord and Deming's Table, the treatment is significant on the 0·1 per cent level.

The same result is reached more easily by calculating the ratio of variances and using the present Table. We have, in the present example,

$$x = \text{the ratio of variances} = \frac{19{\cdot}708}{2{\cdot}725} = 7{\cdot}23$$

But 7·23 exceeds the 0·1 per cent value of the ratio of variances (7·102) given in the table, so that we reach the same conclusion almost immediately.

REFERENCES

1. Mahalanobis P. C. (1932) :—Auxiliary Tables for Fisher's z-test in Analysis of Variance (*Ind. Jour. Ag. Sc.* Vol. II (6).

2. Snedecor, G. W. (1934) :—Calculation and Interpretation of Analysis of Variance and Covariance (*Collegiate Press, Ames, Iowa.*)

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ONE-TENTH PERCENT VALUES OF THE RATIO OF VARIANCES (s_1^2/s_2^2)

Values of n_2	Values of n_1									
	1	2	3	4	5	6	7	8	9	10
1	406491'	499976'	536657'	562182'	576375'	585906'	592960'	598100'	602181'	605563'
2	998·4	999·0	999·2	999·2	999·2	999·2	999·2	999·4	999·4	999·4
3	167·46	148·5	141·11	137·08	134·58	132·81	131·57	130·6	129·87	129·29
4	74·126	61·238	56·181	53·436	51·706	50·520	49·658	48·994	48·471	48·047
5	47·039	36·612	33·201	31·087	29·748	28·835	28·168	27·634	27·237	26·918
6	35·509	26·998	23·702	21·902	20·803	20·029	19·464	19·029	18·686	18·412
7	29·218	21·688	18·772	17·188	16·206	15·521	15·020	14·634	14·323	14·083
8	25·416	18·493	15·826	14·392	13·485	12·858	12·404	12·054	11·768	11·540
9	22·855	16·387	13·901	12·561	11·714	11·127	10·697	10·369	10·109	9·893
10	21·039	14·906	12·553	11·282	10·481	9·921	9·518	9·201	8·953	8·753
11	19·687	13·813	11·560	10·346	9·577	9·047	8·655	8·354	8·118	7·922
12	18·643	12·972	10·803	9·633	8·892	8·378	8·001	7·711	7·480	7·292
13	17·814	12·312	10·208	9·072	8·354	7·855	7·489	7·206	6·982	6·799
14	17·143	11·780	9·730	8·623	7·922	7·435	7·078	6·802	6·582	6·405
15	16·587	11·338	9·335	8·253	7·567	7·092	6·741	6·470	6·256	6·081
16	16·110	10·970	9·005	7·948	7·272	6·804	6·460	6·195	5·983	5·811
17	15·721	10·650	8·727	7·683	7·022	6·563	6·228	5·962	5·753	5·584
18	15·379	10·380	8·487	7·459	6·807	6·355	6·021	5·763	5·558	5·390
19	15·080	10·157	8·280	7·264	6·609	6·176	5·845	5·590	5·387	5·222
20	14·820	9·952	8·098	7·102	6·461	6·018	5·692	5·440	5·239	5·075
21	14·588	9·773	7·937	6·946	6·318	5·880	5·557	5·308	5·109	4·946
22	14·379	9·612	7·796	6·814	6·192	5·758	5·438	5·190	4·993	4·832
23	14·194	9·468	7·669	6·695	6·079	5·648	5·330	5·086	4·890	4·729
24	14·027	9·339	7·555	6·589	5·976	5·550	5·233	4·991	4·797	4·638
25	13·875	9·222	7·450	6·493	5·885	5·462	5·148	4·907	4·713	4·555
26	13·738	9·116	7·356	6·406	5·802	5·382	5·069	4·829	4·638	4·480
27	13·612	9·020	7·272	6·326	5·726	5·308	4·999	4·759	4·568	4·411
28	13·498	8·930	7·194	6·253	5·656	5·240	4·932	4·694	4·503	4·349
29	13·391	8·852	7·121	6·187	5·592	5·179	4·876	4·645	4·447	4·291
30	13·292	8·774	7·054	6·124	5·533	5·122	4·817	4·581	4·393	4·238
40	12·614	8·251	6·600	5·698	5·128	4·731	4·436	4·207	4·024	3·874
60	11·972	7·765	6·172	5·307	4·757	4·373	4·088	3·865	3·688	3·542
120	11·262	7·312	5·703	4·947	4·415	4·041	3·765	3·546	3·373	3·229
∞	10·826	6·908	5.423	4·616	4·103	3·743	3·474	3·265	3·098	2·959

ONE-TENTH PERCENT VALUES OF THE RATIO OF VARIANCES (s_1^2/s_2^2)—*Contd.*

Values of n_2	Values of n_1: 12	15	20	24	30	40	60	120	∞
1	610549.	615699.	620893.	623381.	626506.	628610.	631286.	633942.	636483.
2	999.6	999.4	999.4	999.4	999.4	999.4	999.4	999.4	999.4
3	128.30	127.35	126.41	125.94	125.48	124.96	124.46	124.01	123.49
4	47.407	46.758	46.098	45.768	45.430	45.090	44.745	44.388	44.052
5	26.416	25.909	25.396	25.143	24.883	24.650	24.332	24.061	23.783
6	17.989	17.559	17.119	16.891	16.663	16.444	16.216	15.987	15.746
7	13.708	13.324	12.933	12.733	12.531	12.322	12.119	11.912	11.695
8	11.194	10.842	10.483	10.302	10.113	9.922	9.728	9.535	9.335
9	9.570	9.239	8.898	8.723	8.547	8.369	8.180	8.004	7.813
10	8.445	8.129	7.802	7.637	7.460	7.298	7.123	6.943	6.762
11	7.625	7.321	7.008	6.847	6.684	6.518	6.348	6.174	5.998
12	7.005	6.709	6.404	6.248	6.090	5.927	5.763	5.593	5.419
13	6.519	6.231	5.934	5.782	5.626	5.467	5.305	5.139	4.967
14	6.130	5.848	5.557	5.408	5.254	5.099	4.939	4.775	4.604
15	5.812	5.536	5.248	5.101	4.950	4.795	4.638	4.475	4.307
16	5.548	5.274	4.992	4.846	4.697	4.545	4.388	4.228	4.059
17	5.324	5.054	4.777	4.631	4.484	4.332	4.177	4.017	3.850
18	5.132	4.867	4.590	4.448	4.301	4.155	3.996	3.837	3.671
19	4.967	4.704	4.427	4.286	4.143	3.994	3.840	3.681	3.515
20	4.823	4.562	4.290	4.150	4.005	3.856	3.703	3.546	3.378
21	4.697	4.437	4.167	4.026	3.884	3.736	3.584	3.425	3.257
22	4.583	4.326	4.058	3.918	3.774	3.629	3.475	3.312	3.151
23	4.482	4.227	3.961	3.822	3.679	3.534	3.380	3.223	3.054
24	4.393	4.139	3.873	3.735	3.594	3.447	3.294	3.137	2.968
25	4.311	4.058	3.794	3.657	3.516	3.363	3.216	3.057	2.890
26	4.238	3.986	3.722	3.586	3.444	3.299	3.147	2.990	2.820
27	4.170	3.920	3.658	3.521	3.380	3.233	3.082	2.925	2.754
28	4.109	3.860	3.598	3.462	3.319	3.176	3.023	2.861	2.695
29	4.053	3.804	3.543	3.407	3.268	3.122	2.971	2.812	2.640
30	4.000	3.753	3.492	3.358	3.217	3.072	2.919	2.758	2.589
40	3.642	3.400	3.145	3.012	2.874	2.727	2.571	2.408	2.233
60	3.315	3.079	2.828	2.694	2.555	2.408	2.252	2.085	1.896
120	3.016	2.767	2.522	2.396	2.261	2.112	1.952	1.782	1.561
∞	2.742	2.513	2.266	2.132	1.990	1.835	1.660	1.448	1.000