Intellectual property rights protection and unemployment in a North South model: A theoretical analysis ☆

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Abstract

In this paper, we analyse a North South dynamic general equilibrium model of the international product cycle to study the effects of strengthening the Intellectual Property Rights (IPR) protection in the South on the rate of innovation in the North and on the level of unemployment in the South. Introducing efficiency wage hypothesis into the standard Grossman–Helpman [Grossman, G. and E. Helpman, 1991b. Endogenous product cycles. The Economic Journal, 101, 1214–1229.] product variety framework we explain unemployment equilibrium in the unskilled labour market in the South. We show that the strengthening of Intellectual Property Rights (IPR) protection has a negative effect on the steady state equilibrium rate of innovation. However, its effect on the level of unemployment in the South depends on the relative wage gap between these two regions. In the wide gap equilibrium case, stronger IPR protection lowers the level of unemployment. However, in the narrow gap case, this raises the level of unemployment. We also analyse the effects of changes in labour endowments in both the countries.

JEL classification: O31; O34; O40

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1. Introduction

The purpose of this paper is to analyse the effect of strengthening the Intellectual Property Rights (IPR) protection on the rate of economic growth in developed countries and on the level of unemployment in less developed countries when different countries of the world are involved in free and unrestricted trade. We use a North South product cycle model as developed by Grossman and Helpman (hereafter called GH) (1991b). Endogenous growth is driven by the

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introduction of the new differentiated products in the North which are later imitated in the South; and the unemployment in the unskilled labour market in the South is explained by the efficiency wage hypothesis.

There are two branches of the existing literature to which our paper is related. The first one focuses on the relationship between the long run rate of economic growth and the level of unemployment in a closed economy framework.[1] A subset of these works considers unemployment resulting from efficiency wage.[2] However, these papers do not consider the North South product transfer through imitation in the South; and hence do not analyse the role of IPR protection in the South. The other branch of the literature deals with the effects of strengthening IPR protection in the less developed countries on the rate of economic growth and on the welfare in the different trading countries of the world. However, these papers use North South models based on GH (1991a,b) framework and assume full employment of labour in all the countries.[3] The model of Arnold (2002) is the only exception in the North South literature because it deals with the unemployment problem in the North. However, Arnold (2002) does not consider unemployment in the South caused by the efficiency wage hypothesis. In reality, less developed countries suffer from severe unemployment and underemployment problems of unskilled (uneducated) labour in agricultural sectors and in urban informal sectors.

In this paper, we extend the GH (1991b) model introducing unemployment in the South caused by the efficiency wage hypothesis. We consider a South with two types of labour — skilled and unskilled; and introduce efficiency wage hypothesis in the unskilled labour market. Mirrlees (1976), Stiglitz (1976), Dasgupta and Ray (1986) and many others explain unemployment of unskilled labour in less developed countries using the efficiency wage hypothesis in static models. However, the North has only skilled labour. In the Western developed countries, the illiteracy rate is negligible and the percentage of unskilled (uneducated) workers is very low. So we do not consider unskilled labour in the North. However, illiteracy is a serious problem in poor countries of South Asia and Africa. Agricultural sectors and Urban informal sectors in less developed countries are mainly dependent on unskilled (uneducated) workers. Size of the formal sector in a less developed country is far lower than the size of the unorganised (agriculture and urban informal) sector. Efficiency wage hypothesis is generally valid for those workers who are underpaid because the improvement in their income raises their levels of consumption and working abilities. Rodgers (1975), Bliss and Stern (1978) etc. provide empirical evidences in favour of this hypothesis. Unskilled workers of agricultural and informal sectors earn substantially less than skilled workers of the formal sectors.

This framework allows us to analyse the effects of strengthening IPR protection on the rate of economic growth and on the level of unemployment in the steady state equilibrium of the world economy. We show that the movements of growth rate and unemployment level due to strengthening of IPR protection may not be unidirectional; and the nature of their movements depends on the North South wage gap. In both the wide gap and the narrow gap cases, stronger IPR protection in the South lowers the balanced rate of growth of both the regions and raises the North South relative wage in the new steady state equilibrium. However, the level of unemployment in the South is increased in the narrow gap case and is decreased in the wide gap case.

We also analyse the effects of changes in factor endowments of both the regions. The expansion of the Southern skilled (Northern) labour endowment raises the rate of innovation (growth) and raises (lowers) the rate of imitation. A similar result is also obtained from the GH (1991b) model. The level of unemployment in the South is increased (decreased) due to an expansion of the Northern (Southern skilled) labour endowment in the narrow gap case. However, in the wide gap case, the expansion of the Northern labour endowment does not affect the Southern unemployment at all. So the effect of the change in the Northern labour endowment on the level of Southern unemployment crucially depends on the North South wage gap. The expansion of the unskilled labour endowment in the South does not affect the innovation rate and the imitation rate and only raises the level of unemployment there. Like GH (1991b), the North South relative wage varies directly (inversely) with the size of the Northern (Southern skilled) labour endowment in this model. However, the change in the unskilled labour endowment in the South does not affect the North South relative wage at all.

This paper is organised as follows. The model of the international product cycle with unemployment in the South is presented in Section 2. Section 3 presents the reduced form steady state equilibrium conditions and analyses the effects of strengthening IPR protection and of changes in labour endowments on the unemployment level and on the growth

[1] See, for example, Bean and Pissarides (1993), Aghion and Howitt (1994), Palokangas (1996), Van Schaik and De Groot (1998), Stadler (1999) etc.

[2] See, for example, De Groot (1998), Van Schaik and De Groot (1998), Stadler (1999) etc.

[3] See, for example, Grossman and Helpman (1991b), Helpman (1993), Lai (1998), Yang and Maskus (2001), Glass and Saggi (2002) etc.

rate. In Subsection 3.1, we analyse these effects in the wide gap case; and, in Subsection 3.2, we do the same in the narrow gap case. Concluding remarks are made in Section 4.

2. The model

There are two countries in the world — the North and the South. They are linked by free trade in differentiated products which are invented in the North and imitated in the South. A representative Northern firm incurs an upfront innovation cost to invent a new product and then earns a stream of monopoly profits from that product until it gets imitated by a potential Southern firm. Patents are perfectly protected in the North but are imperfectly protected in the South which leads to imitation there. Due to lower labour cost, a successful imitator from the South earns an infinite stream of positive profit which it balances against the positive imitation cost. The structure of this international product cycle model is adapted from GH (1991b). However, unlike GH (1991b), we introduce two types of labour – skilled and unskilled – in the labour market of the South; and assume that the efficiency of the unskilled worker varies positively with the relative wage of the unskilled labourer to that of the skilled worker.[4] Thus the level of endowment of the Southern unskilled labour expressed in efficiency unit is endogenous to this model. The introduction of the efficiency wage function leads to an unemployment equilibrium in the unskilled labour market of the South.[5] The level of endowment of Southern skilled labour is exogenously given and is fully employed. The skilled labour is used in imitation as well as in production. However, the unskilled labour is used only in the production sector. The North has only skilled labour and it is used in production as well as in R&D. Its level of endowment is given and it is fully employed.

2.1. The demand for goods

We consider a world where all households are identical in terms of preferences irrespective of their origin. The representative household maximises the intertemporal utility function given by

$$W = \int_t^{\infty} e^{-\theta(\tau - t)} \log[U(\tau)] \mathrm{d}\tau$$

subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r(\tau - t)} E(\tau) \mathrm{d}\tau = \int_t^{\infty} e^{-r(\tau - t)} I(\tau) \mathrm{d}\tau + A(t) \quad \text{for all } t.$$

Here $E(\tau)$, $I(\tau)$, $U(\tau)$ and $A(\tau)$ stand for the level of instantaneous expenditure, level of instantaneous income, level of instantaneous utility and current stock of assets at time τ. θ and r stand for the rate of time preference and the nominal interest rate respectively. The instantaneous utility function is assumed to have the following form.

$$U(\tau) = \left[\int_0^n x(z)^{\alpha} \mathrm{d}z \right]^{\frac{1}{\alpha}} \quad \text{with } 0<\alpha<1.$$

Here n and $x(z)$ stand for the number of varieties and the level of consumption of the zth variety. It is assumed that the proportions of unemployed members are same for all the households in the South; and thus we ignore the income distributional aspect of unemployment.

Solving the optimisation problem we obtain the following demand function for the zth variety.

$$x(z) = \frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon} \mathrm{d}u} E. \tag{1}$$

[4] Many models of efficiency wage hypothesis assume worker's efficiency to be a function of relative wage. See, for example, Summers (1988), Agell and Lundborg (1992, 1995), Akerlof and Yellen (1990) etc.

[5] The idea that efficiency wage hypothesis leads to an unemployment equilibrium is wellknown in the literature on the theory of unemployment.

This is true for all $z \in [0, n]$. Here $\varepsilon = \frac{1}{1-\alpha} > 1$ is the constant price elasticity of demand. We also obtain the following optimal time path of expenditure given by

$$\frac{\dot{E}}{E} = r - \theta. \tag{2}$$

Its derivation is given in the Appendix A. In the Appendix A (see Eq. 1E.6) we have shown that

$$\frac{\dot{U}}{U} = \frac{\dot{E}}{E} - \frac{\dot{P}}{P}$$

where

$$P^{(1-\varepsilon)} = \int_0^n p(u)^{(1-\varepsilon)} du.$$

Here the subscripts N and S stand for the North and the South respectively. Also we have

$$n = n_N + n_S \tag{3}$$

where n_i is the number of products produced in the ith region for $i = N, S$. The description of the demand side is similar to that in the GH (1991b) model.

2.2. Production in the North

There are two sectors in the North — a competitive R&D sector and a production sector. In the production sector, n_N firms produce n_N differentiated products; and each of those firms is a monopolist on its own product. Labour is the only input used in both the sectors; and there is perfect intersectoral mobility of labour leading to the same equilibrium wage in all the sectors. In the R&D sector, the blue prints of the new products are developed.

The production function in the R&D sector takes the following form.

$$\dot{n} = \left(\frac{n}{a_N}\right) L_N^R \tag{4}$$

where L_N^R and $\frac{a_N}{n}$ stand for the level of employment and the per unit labour requirement in the R&D sector. Here $a_N > 0$ is a technological parameter. The number of products, n, rises over time if $L_N^R > 0$; and hence the labour productivity, $\frac{n}{a_N}$, rises over time. Northern labour market is competitive; and hence the Northern wage rate, w_N is equal to the value of the marginal productivity of labour in the R&D sector in the North. Value of the Northern firm is normalised to unity.[6] So w_N is proportional to $\frac{n}{a_N}$. Hence

$$\frac{\dot{w}_N}{w_N} = \frac{\dot{n}}{n}. \tag{5}$$

It is assumed that one unit of labour is required to produce one unit of product of any variety produced in the North. Then, using Eq. (4), we can express the labour market clearing equation as

$$L_N = a_N\left(\frac{\dot{n}}{n}\right) + n_N x_N. \tag{6}$$

Here L_N and x_N stand for the level of Northern labour endowment and the level of output of any Northern variety.[7]

[6] GH (1991b) does not make this assumption. We borrow it from Lai (1998). However, major results of the paper are independent of the normalising assumption.

[7] All the commodities in the North are produced in equal quantities because the utility function is symmetric and the technologies are identical.

The monopoly price and the monopoly profit of the Northern firm producing each of the n_N varieties are given by the following.

$$p_N = \frac{w_N}{\alpha}; \tag{7}$$

and

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N. \tag{8}$$

Here the Northern wage rate, w_N, is the marginal cost of production of each of these varieties. It turns out in this model that π_N is constant in the steady state equilibrium. The free-entry condition in the R&D sector in the North is given by

$$\frac{a_N w_N}{n} = \frac{\pi_N}{r+m} \tag{9}$$

where the left hand side of the Eq. (9) is the cost of developing a new variety and its right hand side is the value of the Northern firm defined as the discounted present value of expected stream of its monopoly profits over the infinite time horizon. Here $(r+m)$ is the effective rate of discount; and

$$m = \frac{\dot{n}_S}{n_N}$$

is the rate of imitation in the South. It represents the risk premium to be paid by the Northern firm. Using Eqs. (6), (8) and (9), we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N - a_N\left(\frac{\dot{n}}{n}\right)}{a_N}\right)\left(1+\frac{m}{\dot{n}_S/n_S}\right) = r+m. \tag{10}$$

So far the description of the North is concerned, there is no major difference between the present model and the original GH (1991b) model.

2.3. Production in the South

The South does not innovate but imitates the Northern products. It has a competitive imitative R&D sector and a production sector producing imitated products. The skilled labour whose endowment is assumed to be exogenously given is used in both the sectors and is perfectly mobile. The unskilled labour whose endowment is measured in efficiency unit is used only in the production sector.

$$h = h\left(\frac{w_{SL}}{w_R}\right) \quad \text{with } h'(.)>0, \quad \text{and with } h''(.)>(<)\,0 \text{ for } \left(\frac{w_{SL}}{w_R}\right)<(>)\,\gamma>0.$$

is the wage-efficiency function of the representative unskilled worker. $h\left(\frac{w_{SL}}{w_R}\right)$. L_S is the aggregate endowment of the unskilled labour expressed in efficiency unit and L_S stands for the number of unskilled workers. This wage-efficiency function also satisfies the properties like $h(0)=0$ and $h(1)=1$. Here w_R and w_{SL} are the reference wage and the wage rate of the unskilled worker respectively. We assume the reference wage to be proportional to the skilled worker's wage, w_{SH}. Hence

$$w_R = \phi w_{SH} \quad \text{for } 0<\phi<1.$$

We now try to explain this specification of the wage-efficiency function.[8] The worker works harder as his current wage relative to the reference wage is higher. This reference wage may be either external or internal to the firm.

[8] This is different from the explanations of Mirrlees (1976), Stiglitz (1976), Dasgupta and Ray (1986) etc.

Conventional models like Shapiro and Stiglitz (1984), Salop (1979), Akerlof (1982) etc. interpret the reference wage as external to the firm. It is the average of wages paid in all other firms weighted by probabilities of being employed there plus the unemployment benefit weighted by the probability of remaining unemployed. This is the expected wage of the worker when he is sacked from the present firm. However, some other models like Akerlof and Yellen (1990), Agell and Lundborg (1992, 1995), Danthine and Kurmann (2006) etc. interpret the reference wage as internal to the firm. Price of the complementary factor of production, e.g., wage rate of the co-worker with different skill,[9] rental rate on capital,[10] may be interpreted as the internal reference wage. Danthine and Kurmann (2004) considers the worker's past wage in the same firm as the reference wage. In the present model, the skilled labour is the only complementary factor to the unskilled labour in the production sector of the South. So, following the tradition of the existing literature, the reference wage in the efficiency function of the unskilled worker is assumed to be proportional to the wage-rate of the skilled worker.[11] In GH (1991b) model, $h(.)\equiv 1$. So there is no difference between the endowment of labour and the number of workers.

The production function in the imitative R&D sector is given by

$$\dot{n_S} = \frac{n_S}{a_S} H_R \tag{11}$$

where H_R, $\dot{n_S}$ and (a_S/n_S) stand for the amount of skilled labour used in the imitative R&D sector, the number of new imitated products and the effective labour output coefficient in the imitative R&D sector. Here

$$a_S = a_m + \lambda$$

where a_m is the technology parameter and λ is a policy parameter representing the degree of strengthening the IPR protection in the South. The stronger the IPR protection, the greater is the value of λ and hence the greater is the effective per unit labour requirement[12] in the imitative R&D sector.

All the imitated products in the South are produced under identical technology; and the production function of the representative imitated product is given by

$$x_S = (\delta L_D^{-\rho} + (1-\delta) H_p^{-\rho})^{-\frac{1}{\rho}}.$$

Here $0<\delta<1$ and $\rho>0$ are two technological parameters. Here x_S, L_D, H_p and $\frac{1}{(1+\rho)}$ stand for level of output, level of unskilled labour employment expressed in efficiency unit, amount of skilled labour input and the elasticity of substitution between L_D and H_p. In GH (1991b), $H_p=0$ by assumption. If $\delta=1$, then we can come back to the production function in the GH (1991b) model.

A typical Southern firm, assumed to be a monopolist on its own imitated product, maximises profit given by

$$\pi_S = p_S x_S - \left(w_{SH} H_p + w_{SL} \frac{L_D}{h\left(\frac{w_{SL}}{w_R}\right)} \right)$$

with respect to w_{SL}, H_p and L_D subject to the demand function for x_S given by Eq. (1). Here w_{SL}, w_{SH}, and p_S represent the prices of the unskilled labour, of the skilled labour and of the representative imitated product. From the solution to this optimisation exercise, we obtain following equations.[13]

$$H_p = x_S (\Omega)^{\frac{1}{\rho}}; \tag{12}$$

$$L_D = x_S (\Omega)^{\frac{1}{\rho}} \cdot K^{\frac{-1}{1+\rho}}; \tag{13}$$

$$p_S = \frac{w_{SH}}{(1-\delta)\alpha} (\Omega)^{\frac{1+\rho}{\rho}}; \tag{14}$$

[9] See Akerlof and Yellen (1990).
[10] See Agell and Lundborg (1992, 1995), Danthine and Kurmann (2006) etc.
[11] The results of this model are conditional on this assumption. Additional remarks are made in the conclusion section.
[12] The increase in the labour requirement means the increase in the cost of imitation because skilled labour is the only input. We follow Glass and Saggi (2002) for this kind of definition of IPR protection in the South.
[13] See Appendix B for the detail derivation.

and

$$\frac{h'(.)}{h(.)}\frac{w_{SL}}{w_R} = 1. \tag{15}$$

Here

$$K = \frac{1-\delta}{\delta}\frac{w_{SL}}{h(.)w_R} \quad \text{and} \quad \Omega = \left(\delta K^{\frac{\rho}{1+\rho}} + 1 + \delta\right).$$

Here Eqs. (12) and (13) represent the demand functions for skilled labour and unskilled labour. Eq. (14) shows the equilibrium (monopoly) price for the representative Southern imitated product; and Eq. (15) shows the equilibrium condition of efficiency wage stating that the elasticity of efficiency with respect to relative wage is equal to unity. Note that this equilibrium condition (15) does not exactly correspond to the equilibrium condition of Solow (1979) which is concerned with absolute real wage but not with relative wage. Here Eq. (15) solves for $\left(\frac{w_{SL}}{w_{SH}}\right)$ uniquely because $w_R = \phi w_{SH}$.[14] So w_{SL} and w_{SH} always move proportionately and the equilibrium value of $\left(\frac{w_{SL}}{w_{SH}}\right)$ is independent of any parametric change that does not affect the efficiency function. So the expressions K and Ω, being functions of $\left(\frac{w_{SL}}{w_{SH}}\right)$ only, are also invariant to these parametric changes. Hence Eqs. (12) and (13) show that levels of demand for inputs vary proportionately with the level of output; and Eq. (14) shows that the monopoly price varies proportionately with w_{SH}. Hence we have

$$\frac{\dot{w_{SL}}}{w_{SL}} = \frac{\dot{w_{SH}}}{w_{SH}} = \frac{\dot{p_S}}{p_S}. \tag{16}$$

In the imitative R&D sector, wage rate of skilled labour is equal to its value of marginal productivity given by $v_S.\left(\frac{n_S}{a_S}\right)$ where v_S is the value of a Southern firm. Using Eq. (1) and the North South labour market clearing conditions, it can be shown that $\frac{w_N}{w_{SH}}$ and $\frac{w_N}{w_{SL}}$ are constant in the steady state equilibrium. So using Eq. (5) we have

$$\frac{\dot{w_{SH}}}{w_{SH}} = \frac{\dot{w_{SL}}}{w_{SL}} = \frac{\dot{w_N}}{w_N} = \frac{\dot{n}}{n}. \tag{17}$$

Eq. (17) implies that, the value of the Southern firm (v_S) is time-independent in the steady state equilibrium. The equilibrium condition in the skilled labour market and that in the unskilled labour market is given by

$$H_S = a_S\frac{\dot{n_S}}{n_S} + n_S H_p \tag{18}$$

and

$$L_S = \frac{n_S L_D}{h(.)} + U_S \tag{19}$$

respectively. Here H_S and U_S represent the number of available skilled workers and the number of unemployed unskilled workers respectively. Since the efficiency wage is rigid downwards, there should be a positive level of unemployment of unskilled labour in equilibrium. At a particular point of time, n_S is given. So Eqs. (18) and (19) solve for $\left(\frac{\dot{n_S}}{n_S}\right)$ and U_S because values of (w_{SL}/w_{SH}), L_D and H_p are obtained from Eqs. (12), (13) and (15).

In the GH (1991b) model, $H_S = 0$. So H_p and w_{SH} do not exist. Since $h(.) \equiv 1$, Eq. (15) does not exist too. w_{SL} is flexible and hence $U_S = 0$ in equilibrium. Only one type of labour is employed in production sector and in imitative R&D sector. Eqs. (12) and (18) do not exist. Eqs. (13) and (14) are modified as follows.

$$L_D = x_S;$$

[14] We assume that the solution of $\left(\frac{w_{SL}}{w_{SH}}\right)$ satisfies the condition that $w_{SH} > w_{SL}$. This ensures that the skilled workers do not join the unskilled labour force in equilibrium.

and

$$p_S = \frac{w_{SL}}{\alpha}.$$

Eqs. (11) and (19) are modified as follows.

$$\dot{n_S} = \frac{n_S}{a_S} \cdot L_R;$$

and

$$L_S = n_S L_D + L_R.$$

It can be shown that π_S remains constant in the steady state equilibrium of this model. The free entry condition in the imitative R&D sector in the South is given by

$$\frac{\pi_S}{r} = \frac{a_S w_{SH}}{n_S}. \tag{20}$$

Here the R.H.S. of Eq. (20) represents the cost of imitating a new variety; and the L.H.S. of Eq. (20) represents the value of the Southern firm defined as the discounted present value of its profits from its production over the infinite time horizon when r is the rate of discount. The value of the Southern firm is also time independent in the steady state equilibrium. In GH (1991b), Eq. (20) remains otherwise identical with the only modification that w_{SH} is replaced by w_{SL}. This is so because, in GH (1991b), same labour is used in the production sector as well as in the imitative R&D sector.

Note that the North as well as the South faces the same rate of interest, r, because it is assumed that there exists a competitive world capital market.

2.4. Wide gap vs. narrow gap

We assume that the marginal cost of production of a variety in the South is always lower than that in the North and there is Bertrand price competition between a successful Southern imitator and its Northern counterpart. The Southern firm can charge the monopoly price on its imitated product as given in Eq. (14) if the following condition holds true in equilibrium.

$$p_S = \frac{w_{SH}}{(1-\delta)\alpha}(\Omega)^{\frac{1+\rho}{\rho}} < w_N.$$

This is possible if the North South wage gap is very high. This is the case of wide gap equilibrium. However, when the above inequality is satisfied in the reverse order we have the case of narrow gap equilibrium. Here the equilibrium price charged by a typical Southern firm is given by

$$p_S = w_N < \frac{w_{SH}}{(1-\delta)\alpha}(\Omega)^{\frac{1+\rho}{\rho}}. \tag{21}$$

The possibility of this case arises when the North South wage gap is very narrow. Henceforth, in this paper, North South relative wage will be defined by $\left(\frac{w_N}{w_{SH}}\right)$ which represents the Northern wage rate relative to the wage rate of the Southern skilled labour.[15] The instantaneous profit of the representative Southern firm in the wide gap case is given by

$$\pi_S = \frac{1-\alpha}{\alpha}\frac{w_{SH}}{(1-\delta)}(\Omega)^{\frac{1+\rho}{\rho}} x_S; \tag{22}$$

[15] Ratio of the skilled wage to the unskilled wage in the South is fixed by the efficiency function (see Eq. (15)).

and that in the narrow gap case is given by

$$\pi_S = \left[w_N - \frac{w_{SH}}{(1-\delta)} (\Omega)^{\frac{1+\rho}{\rho}} \right] x_S. \tag{23}$$

From Eq. (1) we have

$$\frac{x_N}{x_S} = \left(\frac{p_N}{p_S} \right)^{-\varepsilon}.$$

This is the relative demand function for the Northern product. In the narrow gap case, using Eqs. (7) and (21), we have,

$$\frac{x_N}{x_S} = \alpha^{\varepsilon}.$$

This implies that the relative demand for Northern product is constant in the narrow gap case. However, it varies with the North South relative wage in the wide-gap case. Using Eqs. (7) and (14) we have

$$\frac{w_N}{w_{SH}} = \frac{\Omega^{\frac{1+\rho}{\rho}}}{1-\delta} \left(\frac{x_N}{x_S} \right)^{\varepsilon}. \tag{24}$$

While analysing the effects of strengthening the IPR protection on the growth rate and on the unemployment level, we shall consider these two cases separately.

2.5. Steady state equilibrium growth

It is assumed that the economy is in the steady state growth equilibrium where n_N and n_S grow at equal rate, g. Hence

$$\frac{\dot{n}}{n} = \frac{\dot{n_N}}{n_N} = \frac{\dot{n_S}}{n_S} = g. \tag{25}$$

Value of g is determined endogenously in this model and so it is an endogenous growth model. We have normalised the value of a Northern firm to unity. This implies that the expenditure in the North would grow at the rate of new product development there.[16] Now using Eqs. (5), (7), (16), (17), and (25) we have

$$\frac{\dot{E_N}}{E_N} = \frac{\dot{E_S}}{E_S} = \frac{\dot{n}}{n} = \frac{\dot{n_S}}{n_S} = \frac{\dot{n_N}}{n_N} = \frac{\dot{w_N}}{w_N} = \frac{\dot{w_{SL}}}{w_{SL}} = \frac{\dot{w_{SH}}}{w_{SH}} = \frac{\dot{p_S}}{p_S} = \frac{\dot{p_N}}{p_N} = g. \tag{26}$$

Eqs. (6) and (25) together imply that $n_N x_N$ is constant in the steady state growth equilibrium. Similarly Eqs. (12), (18) and (25) imply that $n_S x_S$ is also constant in the steady state growth equilibrium. So, in the steady state equilibrium, we have

$$\frac{\dot{x_N}}{x_N} = \frac{\dot{x_S}}{x_S} = -g. \tag{27}$$

In the Appendix A (see Eq. 1E.6), it is shown that

$$U = (n_N x_N^{\alpha} + n_S x_S^{\alpha})^{\frac{1}{\alpha}}.$$

[16] Normalising the value of a Northern firm (which is $\frac{a_N}{m_N}$) to unity, we get $\frac{\dot{w_N}}{w_N} = \frac{\dot{n}}{n}$. We have $E_N = p_N n_N x_N$. Now replacing $n_N x_N$ from Eq. (6) we can easily show that in the steady state $\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} = \frac{\dot{n}}{n} = g$. Also $E_S = p_S n_S x_S$ and from Eq. (18) we get ($n_S x_S$) is constant in the steady state. Now using Eqs. (16) and (17) we get $\frac{\dot{E_S}}{E_S} = \frac{\dot{p_S}}{p_S} = \frac{\dot{n}}{n} = g$.

Then using Eqs. (25) and (27) and also considering the fact that $n_N x_N$ and $n_S x_S$ are constant in the steady state growth equilibrium, we have

$$\frac{\dot{U}}{U} = \left(\frac{1-\alpha}{\alpha}\right) g.$$

Like GH (1991b) model, the above equation implies that the long run rate of product development and the long run growth rate of utility are proportional (see page 1221 in GH, 1991b). Any parametric change should have similar effects on these two.

3. The equilibrium and the comparative statics

We now derive the reduced form equations of the model which can be used to determine the rate of innovation (growth), rate of imitation and the level of unemployment in the steady state equilibrium. Using Eqs. (2) and (26) and $E=(E_N+E_S)$, we have

$$r = \theta + g. \tag{28}$$

Again using Eqs. (12), (13), (18), (19) and (26) we have

$$H_S = a_S g + n_S x_S (\Omega)^{\frac{1}{\rho}}; \tag{29}$$

and

$$L_S - U_S = \frac{n_S x_S (\Omega)^{\frac{1}{\rho}} \cdot K^{\frac{-1}{1+\rho}}}{h(.)}. \tag{30}$$

Also using Eqs. (10) and (26), we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N - a_N g}{a_N}\right)\left(1+\frac{m}{g}\right) = r + m. \tag{31}$$

In the wide gap case, we use Eqs. (20) and (22); and obtain

$$r a_S \frac{\alpha}{1-\alpha} = \frac{n_S x_S}{(1-\delta)} (\Omega)^{\frac{1+\rho}{\rho}}. \tag{32}$$

In the narrow gap case, we use Eqs. (20) and (23) and obtain

$$\left(w_N - \frac{w_{SH}}{(1-\delta)} (\Omega)^{\frac{1+\rho}{\rho}}\right) x_S n_S = a_S w_{SH} r. \tag{33}$$

Again using Eqs. (6) and (29) in the narrow gap case, we have

$$\alpha^e = \frac{x_N}{x_S} = \frac{L_N - a_N g}{H_S - a_S g} \frac{n_S}{n_N} (\Omega)^{\frac{1}{\rho}}. \tag{34}$$

In the wide gap case, we have a set of four Eqs. (29), (30), (31), (32) with four unknowns — $n_S x_S$, g, m and U_S. In the narrow gap case, our Eqs. are (29), (30), (31) and (34). Eq. (33) is used to solve for the North South relative wage, $\left(\frac{w_N}{w_{SH}}\right)$, in the narrow gap case. In the wide gap case, this relative wage is obtained from the equation given by

$$\frac{w_N}{w_{SH}} = \frac{\Omega^{\frac{\alpha+\rho}{\rho}}}{1-\delta}\left(\frac{H_S - a_S g}{L_N - a_N g} \frac{g}{m}\right)^{1-\alpha} \tag{35}$$

which we obtain using Eqs. (6), (24) and (29).

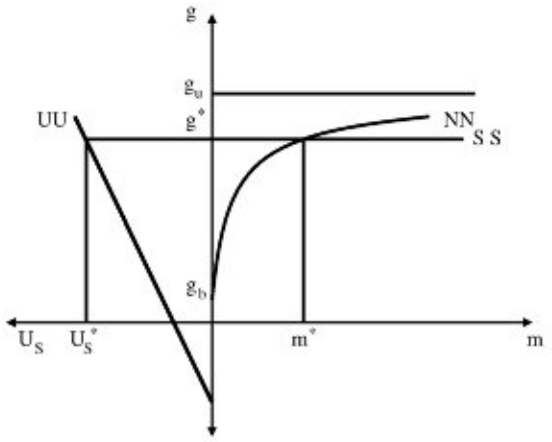

Fig. 1. Wide gap equilibrium.

3.1. The wide gap case

Using Eqs. (28), (29) and (32), we obtain

$$g = \frac{H_S \cdot (\Omega) - \frac{\alpha(1-\delta)}{1-\alpha} a_S \theta}{a_S \cdot \left(\frac{\alpha(1-\delta)}{1-\alpha} + \Omega\right)}. \tag{36}$$

This is the equation of SS curve which represents the relationship between the rate of innovation, g, and the rate of imitation, m, satisfying equilibrium in the South. This curve is drawn horizontal in the Fig. 1. Then, combining Eqs. (28) and (31), we have

$$\frac{1-\alpha}{\alpha}\left(\frac{L_N - a_N g}{a_N}\right)\left(1 + \frac{m}{g}\right) = \theta + g + m \tag{37}$$

which can also be written as

$$m = \frac{\theta + \frac{g}{\alpha} - \frac{1-\alpha}{\alpha}\frac{L_N}{a_N}}{\frac{1-\alpha}{\alpha}\frac{L_N}{a_N g} - \frac{1}{\alpha}}. \tag{38}$$

This is the equation of NN curve which shows the relationship between g and m satisfying equilibrium in the North. Note that $m>0$ for

$$g_b = (1-\alpha)\frac{L_N}{a_n} - \alpha\theta < g < (1-\alpha)\frac{L_N}{a_N} = g_u.$$

NN curve slopes positively in the Fig. 1 starting from the point

$$g_b = (1-\alpha)\frac{L_N}{a_N} - \alpha\theta$$

on the g-axis. This NN curve is identical to that in GH (1991b) because descriptions of the North are same in two models. The equilibrium values of g and m are determined at the point of intersection of NN curve and SS curve.[17]

[17] The existence of this intersection depends on a mild parametric restriction. See Appendix C for the details.

Since $\frac{n_S}{n_N}=\frac{m}{g}$, we now can determine the equilibrium value of $\frac{w_N}{w_{SH}}$ using Eq. (35). Then, using Eqs. (29) and (30), we have

$$\frac{H_S-a_S g}{L_S-U_S}=K^{\frac{1}{1+\rho}}h(.). \tag{39}$$

This is the equation of UU curve which shows the relationship between unemployment level and the growth rate in the South. Here both K and $h(.)$ are functions of the relative wage, $\frac{w_{SL}}{w_{SH}}$, and the equilibrium value of $\frac{w_{SL}}{w_{SH}}$ is uniquely determined by the Eq. (15). So $\frac{w_{SL}}{w_{SH}}$ and hence $h(.)$ and K are independent of the changes in a_S, U_S and g because a_S, U_S and g do not enter the efficiency function. We find a positive relationship between the growth rate and the unemployment level from this Eq. (39). Here $U_S>0$ is guaranteed by the following assumption

$$\left(1+\frac{\Omega}{\frac{\alpha(1-\delta)}{1-\alpha}}\right)\cdot K^{\frac{1}{1+\rho}}\cdot h(.)>\frac{H_S-a_S\theta}{L_S}. \tag{40}$$

This inequality (40) implies that the demand for unskilled labour falls short of this labour endowment when the efficiency wage is binding.

3.1.1. IPR protection

We now turn to analyse the effects of the strengthening of IPR protection on the equilibrium growth rate, g, on the imitation rate, m, and on the level of unemployment, U_S. Here, the strengthening of IPR protection implies an increase[18] in the value of a_S. From Eq. (36), we find that g as well as $a_S g$ varies inversely with a_S From Eq. (38), we find that m and g are positively related. However, a_S does not enter Eq. (38). So an increase in a_S causes the horizontal SS curve to shift downward and does not cause any shift of the positively sloped NN curve. So both g and m take lower values in the new equilibrium shown in the Fig. 1. Note that a_S does enter Eq. (39). So UU curve in the Fig. 1 does shift in this case with changes in the slope as well as in the intercept. From Eq. (36), we find that $a_S g$ varies inversely with a_S. So Eq. (39) shows that U_S falls in the new equilibrium when a_S is increased. In order to find out the effect on the North South relative wage, we can express Eq. (35) as follows.

$$\frac{w_N}{w_{SH}}=\frac{\Omega^{\frac{\alpha+\rho}{\rho}}}{1-\delta}\left(\frac{H_S-a_S g}{(L_N-a_N g)\frac{m}{g}}\right)^{1-\alpha}. \tag{41}$$

Also, using Eq. (37), we have

$$(L_N-a_N g)\frac{m}{g}=\frac{\alpha}{1-\alpha}a_N(\theta+g+m)-(L_N-a_N g). \tag{42}$$

Since an increase in a_S leads to a decrease in both g and m, Eq. (42) shows that $(L_N-a_N g)\frac{m}{g}$ is also reduced in this case. Also Eq. (36) shows that $a_S g$ is reduced due to an increase in a_S. So the North South relative wage is increased in the new steady state equilibrium when a_S is increased.[19] So we can establish the following proposition.

Proposition 1. *The strengthening of IPR protection in the South lowers the rate of growth (innovation) in both the countries in the new steady state equilibrium. It also lowers the rate of imitation and the level of unemployment of the unskilled workers in the South in the new steady state equilibrium. However, it raises the North South relative wage.*

We can provide an intuitive explanation of the above mentioned result. Note that the definition of strengthening IPR protection in this model is similar to that in GH (1991b);[20] and its effects on the rate of innovation and on the rate of imitation are also similar to those in GH (1991b). The profit rate of a typical Southern imitative firm is a negative function of a_S and g and the cost of its capital is a positive function of g.[21] The strengthening of IPR protection in the

[18] Here, $a_S=a_w+\lambda$; and λ takes a higher value when the IPR is stronger.

[19] The wide gap property will not be disturbed in the new steady state equilibrium because increase in a_S raises the North South relative wage.

[20] GH (1991b) defined a tax (subsidy) in the Southern imitative R&D sector as the strengthening (lax) of protection of IPR protection there.

[21] The profit rate is defined as $\frac{\pi_S}{v_S}=\frac{(1-\alpha)\Omega}{(1-\delta)\alpha\, a_S}(H_S-a_S g)$. The cost of capital is $r=\theta+g$.

South raises the cost of imitation there. This lowers the profit rate but leaves the cost of capital unchanged. This results into a decrease in the rate of growth of the imitated products, g.

As the rate of growth falls in the South and as both North and South grow at the same rate in the steady state equilibrium, the profit rate of a typical Northern firm is increased and the cost of its capital is decreased.[22] So the rate of imitation has to fall to equilibrate the profit rate with the cost of capital in the North. This is so because the profit rate of a typical Northern firm is a negative function of g and a positive function of m and the cost of its capital is a positive function of both g and m. Also the effect of a change in m on the profit rate is higher than its effect on the cost of capital. An increase in a_S also causes more firms to stay in the North in the new steady state equilibrium because the ratio $\frac{g}{m}\left(=\frac{n_N}{n_S}\right)$ is increased. This leads to an increase in the demand for labour in the production sector in the North. Hence the relative wage of the North is increased due to stronger protection of IPR in the South.

The size of the imitative R&D sector in the South is also reduced now because the increase in the cost of imitation (per unit skilled labour requirement) in the South causes a substantial reduction in the rate of growth. So more human capital is released to the production sector. The relative skilled unskilled wage in the South is given; and so the demand for skilled labour per unit of unskilled labour is fixed in the production sector of the South. Thus the increase in the skilled labour employment in the production sector is matched by an equal proportionate increase in the unskilled labour employment. So the level of unemployment in the unskilled labour market of the South is reduced. Note that the unemployment problem of skilled labour can never arise in our model because the linear production technology in the imitative R&D sector implies an infinite demand for skilled labour there.

The share of the unskilled workers in the national income of the South is given by

$$\frac{w_{SL}(L_S-U_S)}{p_S n_S x_S}=\frac{w_{SL}}{w_{SH}}\frac{\alpha(1-\delta)}{\Omega h(.)K^{\frac{1}{1+\rho}}}. \tag{43}$$

The right hand side of the above equation is independent of the change in a_S because $\frac{w_{SL}}{w_{SH}}$ is determined by Eq. (15). So the relative income share of the unskilled workers in the South remains unaffected in the wide gap equilibrium when IPR protection is strengthened there.

3.1.2. Factor endowment change

An increase in H_S makes the horizontal SS curve shift upward in the Fig. 1 but leaves NN curve unaffected.[23] Then, in the new steady state equilibrium, both g and m are increased. Also $\frac{m}{g}$ and $(H_S-a_S g)$ are increased in the new steady state equilibrium.[24] Since $(H_S-a_S g)$ is increased, from Eq. (39), we find that (L_S-U_S) also rises proportionately because the R.H.S. of Eq. (39) is independent of H_S. This implies that U_S falls when H_S is increased. From Eq. (41), we have

$$\frac{w_{SH}}{w_N}=\left(\frac{(L_N-a_N g)\frac{m}{g}}{H_S-a_S g}\right)^{1-\alpha}\frac{1-\delta}{\Omega^{1+\frac{\alpha}{\rho}}}. \tag{44}$$

Now, using Eqs. (36) and (37), we have

$$\left(\frac{(L_N-a_N g)\frac{m}{g}}{H_S-a_S g}\right)=\left[\frac{\alpha}{1-\alpha}a_N\left(1+\frac{m}{g+\theta}\right)-\left(\frac{L_N-a_N g}{g+\theta}\right)\right]\frac{(1-\alpha)\Omega}{(1-\delta)\alpha a_S}. \tag{45}$$

Since an increase in H_S raises g, m and $\frac{m}{g}$, the R.H.S. of Eq. (45) is increased in this case. So Eq. (44) implies that $\frac{w_{SH}}{w_N}$ is increased. Since the skilled unskilled relative wage in the South is fixed by the efficiency wage hypothesis, $\frac{w_{SL}}{w_N}$ is also increased in this case.

[22] The profit rate is defined as $\frac{\pi_N}{v_N}=\frac{(1-\alpha)}{\alpha\, a_N}(L_N-a_N g)\left(\frac{m+g}{g}\right)$ where v_N is the value of a typical Northern firm and it is equal to the cost of innovating a new blue print in the North in the equilibrium. The cost of capital is $r+m=\theta+g+m$.

[23] See Eqs. (36) and (37).

[24] From Eq. (36) we obtain $(H_S-a_S g)\Omega=a_S(g+\theta)\frac{\alpha(1-\delta)}{1-\alpha}$. Increase in H_S increases g. So $(H_S-a_S g)$ is increased.

An increase in the unskilled labour endowment in the South, L_S, does not affect g and m because Eqs. (36) and (37) are independent of L_S. Eq. (39) shows that this only raises the level of unemployment of unskilled labour in the South. Since g and m remain unaffected, $\frac{w_S^H}{w_N}$ and $\frac{w_S^L}{w_N}$ also remain unchanged.

Finally, an increase in the Northern labour endowment, L_N, causes NN curve in the Fig. 1 to shift upward and leaves SS curve unaffected. In the new steady state equilibrium, g remains unchanged and m is decreased. U_S also remains unchanged. Eqs. (44) and (45) together show that $\frac{w_S^H}{w_N}$ and $\frac{w_S^L}{w_N}$ are decreased.

We can summarize the results mentioned above in the following proposition.

Proposition 2. *(i) An expansion of the Southern skilled labour endowment raises the rate of innovation, the rate of imitation and the South North relative wage and lowers the level of unemployment in the South in the new steady state equilibrium. (ii) An expansion of the Northern labour endowment lowers the rate of imitation as well as the South North relative wage but keeps the rate of innovation and the level of unemployment unchanged. (iii) An expansion of the unskilled labour endowment in the South raises the unemployment level there but leaves the rate of innovation, the rate of imitation and the South North relative wage unaffected.*

We now explain the intuition behind these results. Given other things unchanged, an increase in the endowment of the skilled labour in the South allocates more labour to the Southern imitative R&D sector; and this raises the rate of imitation in the South. An increase in the rate of imitation raises both the cost of capital and the profit rate of a typical Northern firm. However, its effect on the profit rate dominates its effect on the cost of capital; and so the rate of innovation in the North is increased. An increase in the rate of innovation raises the demand for skilled labour in the Southern manufacturing sector. Given the relative wage rigidity in the South caused by the efficiency wage hypothesis, the demand for unskilled labour is also increased; and so their unemployment level is reduced.

An increase in the Northern labour endowment has no effect on the Southern equilibrium conditions and this raises the profit rate of a typical Northern firm. So the rate of innovation is not changed but the rate of imitation should be reduced to maintain the equality between the profit rate and the cost of capital in the North. Since the rate of innovation remains unchanged, the demand for each type of labour in the Southern manufacturing sector also remains unchanged. So the level of unemployment remains the same.

A change in the unskilled labour endowment in the South cannot alter the allocation of skilled labour between the imitative R&D sector and manufacturing sector due to the rigidity of the efficiency wage above the market clearing level. So the expansion of the unskilled labour endowment does not affect the rate of innovation and the rate of imitation. It only raises the level of unemployment in the South.

Regarding the effects of the changes in the labour endowments (except for the unskilled labour endowment in the South) on the North South relative wage our results are similar to those obtained in GH (1991b). A change in the unskilled labour endowment in the South has no effect on the North South relative wage because it cannot affect the rate of innovation and the rate of imitation and cannot alter the skilled unskilled wage ratio in the South due to rigidity of the efficiency wage there. Our results are different from those in Lai (1995) who, in his full employment model, shows that an increase in the unskilled labour endowment in the South lowers its relative wage.

3.2. The narrow gap case

In this case, our equational structure consists of Eqs. (28), (29), (30), (31), (33) and (34). Since all the equations except Eqs. (33) and (34) are same as those in the wide gap case, the shapes of NN curve and of UU curve remain same as they have in the wide gap case.[25] Since $\frac{n_S}{n_N} = \frac{m}{g}$, using Eq. (34), we obtain the equation corresponding to XX curve as given by

$$\frac{L_N - a_N g}{H_S - a_S g} \cdot (\Omega)^{\frac{1}{\varepsilon}} \cdot \left(\frac{m}{g}\right) = \alpha^{\varepsilon}. \tag{46}$$

We assume that $\frac{L_N}{a_N} < \frac{H_S}{a_S}$. This gives a positive relationship between g and m for $0 < g < \frac{L_N}{a_N}$; and so XX curve in the narrow gap case slopes positively starting from the origin. The slope of XX curve exceeds that of NN curve at any

[25] NN curve is obtained from Eqs. (28) and (31). UU curve is obtained from the Eq. (39) derived from Eqs. (29) and (30).

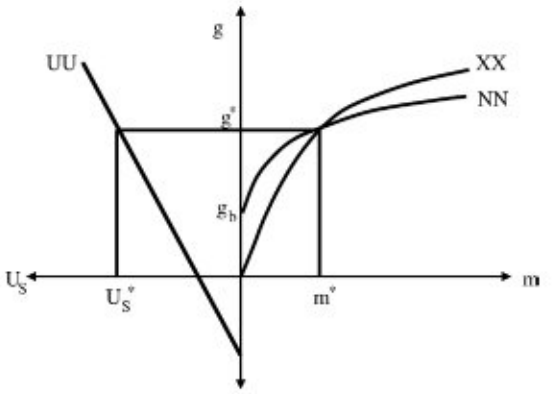

Fig. 2. Narrow gap equilibrium.

common values of g and m.[26] Now Eqs. (38), (39) and (46) solve for the equilibrium values of g, U_S and m simultaneously. Using Eqs. (28), (29) and (33), we have

$$\frac{w_N}{w_{SH}} = \frac{a_S(\theta+g)}{H_S - a_S g}\cdot(\Omega)^{\frac{1}{\rho}} + \frac{1}{1-\delta}(\Omega)^{\frac{1+\rho}{\rho}}. \tag{47}$$

Eq. (47) solves for the equilibrium value of the North South relative wage.[27] A graphical presentation of the steady state equilibrium in the narrow gap case is shown in the Fig. 2. Since NN curve starts from a point on the g-axis and XX curve starts from the origin with a higher positive slope, XX curve must cut NN curve from below at their unique point of intersection.

3.2.1. IPR protection

The strengthening of the IPR protection in the South, i.e., an increase in the value of the parameter, a_S, causes XX curve to shift leftward. However, NN curve does not shift. So, at the new point of intersection, both g and m take lower values. Using Eqs. (37) and (46), we have

$$H_S - a_S g = \alpha^{-\varepsilon}(\Omega)^{\frac{1}{\rho}}\left[\frac{\alpha}{1-\alpha}a_N(\theta+g+m) - (L_N - a_N g)\right]. \tag{48}$$

Eq. (48) shows that $a_S g$ is increased in this case. Eq. (39) shows that U_S is increased when $a_S g$ rises.[28] Eq. (47) shows that $\left(\frac{w_N}{w_{SH}}\right)$ rises in this case.[29] Since $\frac{w_{SL}}{w_R}$ is uniquely determined by Eq. (15) and since w_R is proportional to w_{SH}, then $\frac{w_N}{w_{SH}}$ is also uniquely determined here. Hence $\frac{w_N}{w_{SL}}$ also rises at the same rate in this case. So we have the following proposition.

Proposition 3. *The strengthening of IPR in the South lowers the rate of innovation in the North and the rate of imitation in the South in the new steady state equilibrium. However, it raises the level of unemployment in the South and the North South relative wage.*

Effects of strengthening IPR protection on the growth (innovation) rate and on the imitation rate in this narrow gap case are similar to those in the wide gap case. An increase in a_S lowers the per firm availability of skilled labour for the production sector in the South when g is given. This leads to a decrease in the rate of imitation, given g. This is so because the relative sales volume of a Northern firm to a Southern firm is constant in the narrow gap case. As m is

[26] It is shown in Appendix D.

[27] Determination of $\frac{w_N}{w_{SH}}$ also implies the determination of $\frac{w_N}{w_{SL}}$ because $\frac{w_{SL}}{w_{SH}}$ is uniquely solved from Eq. (15).

[28] A sufficient condition for U_S to be positive is $K^{\frac{1}{1-\alpha}}h(.) > \frac{H_S}{L_S}$.

[29] Note the potential danger of carrying out the comparative static of increasing a_S. Since the North South relative wage increases we may violate the condition for the narrow gap case. We assume that the exogenous changes are sufficiently small so that we don't violate the narrow gap condition.

reduced, this lowers both the profit rate and the cost of capital of a typical Northern firm. The former effect dominates the later; and hence the rate of innovation is decreased. This increase in a_S causes more firms to stay in the North. This leads to an increase in the demand for production labour there; and hence the relative wage of the North is increased. An increase in the skilled labour requirement in the imitative R&D sector in the South lowers the rate product development (growth) there but this negative effect is very weak. So the total skilled labour employment in the imitative R&D sector, by which its size is measured, is increased when the IPR protection is strengthened. So the level of employment of skilled labour in the production sector in the South is reduced. Since the skilled unskilled employment ratio in the production sector in the South is fixed by the rigidity of the efficiency wage, the level of employment of unskilled labour in the production sector also falls at the same proportion. Since no other sector absorbs unskilled workers in the South, their unemployment level is increased.

The share of the unskilled workers in the national income of the South in this narrow gap equilibrium case is given by

$$\frac{w_{SL}(L_S - U_S)}{p_S n_S x_S} = \frac{w_{SL}}{w_N} \frac{\Omega^{\frac{1}{\rho}}}{h(.)K^{\frac{1}{1+\rho}}}. \tag{49}$$

Unlike in the wide gap case, the right hand side of the Eq. (49) is not independent of the change in a_S because $\frac{w_{SL}}{w_N}$ is decreased when a_S is increased. So the relative income share of the unskilled workers in the South is decreased in the narrow gap equilibrium when IPR protection is strengthened there.

3.2.2. Factor endowment change

An increase in H_S causes XX curve in the Fig. 2 to shift rightward and leaves NN curve unaffected. So, in the new steady state equilibrium, g, m and $\frac{m}{g}$ are increased. Eq. (48) shows that $(H_S - a_S g)$ is also increased. Then Eq. (39) shows that U_S is decreased. We express Eq. (37) as

$$\frac{(L_N - a_N g)\frac{m}{g}}{g} = \frac{a_N}{1-\alpha} + \left(\frac{\alpha}{1-\alpha} a_N\right)\frac{m}{g} - \left(\frac{L_N - a_N \frac{\alpha}{1-\alpha}\theta}{g}\right); \tag{50}$$

and Eq. (46) as

$$\frac{\left(\frac{L_N - a_N g^{\frac{\alpha}{g}}}{g}\right)}{\left(\frac{H_S - a_S g}{g}\right)} \cdot (\Omega)^{\frac{1}{\rho}} = \alpha^{\varepsilon}. \tag{51}$$

Since g, m and $\frac{m}{g}$ are increased in this case, the L.H.S. of Eq. (50) is also increased because $L_N > a_N \frac{\alpha}{1-\alpha}\theta$. Hence Eq. (51) shows that $\frac{H_S - a_S g}{g}$ is also increased. Then, from Eq. (47), we find that $\frac{w_N}{w_{SH}}$ and $\frac{w_N}{w_{SL}}$ are decreased in this case.

An increase in L_S does not affect g, m and $\frac{w_{SH}}{w_N}$ because Eqs. (37), (46) and (47) are independent of L_S. So this only raises the level of unemployment of the unskilled labour in the South; and it is shown by Eq. (39).

An increase L_N causes both NN curve and XX curve in Fig. 2 to shift leftward. As in GH (1991b), the extent of the leftward shift of NN curve must be larger at the initial g and that of the XX curve is greater at the initial value of m. In the new steady state growth equilibrium, g is increased and m is decreased. Thus, the long run rate of innovation (imitation) is directly (inversely) related to the size of the labour endowment in the North. Since g is increased, we find that $\frac{w_N}{w_{SH}}, \frac{w_N}{w_{SL}}$ and U_S are also increased.[30] We can summarize the results discussed above in the following proposition.

Proposition 4. *(i) An expansion of the Southern skilled labour endowment raises the rate of innovation, the rate of imitation and the South North relative wage but lowers the level of unemployment in the South in the new steady state equilibrium. (ii) An expansion of the Northern labour endowment raises the rate of innovation, the North South relative wage and the level of unemployment but lowers the rate of imitation. (iii) An expansion of the unskilled labour endowment in the South raises the unemployment level there and leaves the rate of innovation, the rate of imitation and the South North relative wage unaffected.*

[30] See Eqs. (39) and (47).

Effects of changes in the Northern and the Southern skilled labour endowments on the rate of innovation, on the rate of imitation and on the North South relative wage in our model are similar to those obtained from GH (1991b). Effects of these expansions on the unemployment level do not depend on the North South wage gap. In both the wide gap and the narrow gap cases, an increase in the skilled (unskilled) labour endowment in the South lowers (raises) the level of unemployment there.

However, the effect of a change in the Northern labour endowment on the Southern unemployment level crucially depend on the North South wage gap. This raises the level of unemployment in the narrow gap case and keeps it unchanged in the wide gap case. In the narrow gap case, an increase in the Northern labour endowment lowers the level of employment of the Southern skilled labour in the manufacturing sector. Since the skilled unskilled wage ratio is fixed by the rigidity of the efficiency wage, this lowers the demand for unskilled labour in the South which, in turn, raises the level of unemployment.

4. Conclusion

In this paper, we analyse the impact of strengthening IPR protection in the South on the steady state equilibrium rate of growth of the world economy as well as on the rate of imitation and on the level of unemployment in the South. We show that strengthening of IPR in the South lowers both the equilibrium rate of growth and the rate of imitation. However, the level of unemployment of the unskilled labour in the South is increased in the narrow gap case and is decreased in the wide gap case. The level of unemployment in our model is directly related to the size of the imitative R&D sector in the South. It should be noted that, in GH (1991b), the size of the R&D sector in the South is decreased in the wide gap case and is increased in the narrow gap case as the cost of imitation is increased. However, this property is never focused in that paper. We use that unfocused property by relating it to the level of unemployment in the South. It is also shown that the North South relative wage is increased in both the cases.

We also analyse the effects of changes in factor endowments on the steady state equilibrium values of various endogenous variables. Effects of changes in the Northern and the Southern skilled labour endowments on the rate of innovation, on the rate of imitation and on the North South relative wage are similar to those effects obtained in GH (1991b). The level of unemployment of the unskilled labour in the South varies inversely (directly) with the size of its skilled (unskilled) labour endowment and does not depend on the North South wage gap. However, the effect of the change in the Northern labour endowment on the Southern unemployment level depends on the North South wage gap. In the wide gap case, a change in the Northern labour endowment has no effect on the unemployment level. However, its expansion raises the unemployment level in the narrow gap case.

We explain only unemployment of unskilled labour by using the efficiency wage hypothesis. If the efficiency wage is not relevant, then this model is reduced to a full employment model. Many authors have explained unemployment of unskilled labour in less developed countries using the efficiency wage hypothesis. However, they have not considered our formulation in which the reference wage is proportional to some peer group's wage. This particular form applies mostly to those workers who are hard to monitor; and skilled workers are always harder to monitor than unskilled workers. A positive relationship between the worker's efficiency and the wage rate is necessary to explain the existence of an unemployment equilibrium; and this positive relationship exists in any form of the efficiency wage function including the form we consider here. Our results are conditional on the assumption that the reference wage is proportional to the wage of the skilled workers. Due to this assumption, the equilibrium condition of the efficiency wage gives us the unique equilibrium value of skilled unskilled wage ratio which is consistent with the assumption of steady state growth equilibrium where all types of wages are to grow at equal rates. If we drop this assumption we may not be able to prove the existence of a unique time independent skilled unskilled wage ratio. However, the empirical literature on efficiency wage theory has been unable to conclude so far on the most plausible form of efficiency wages and, in particular, on the reference wage level entering the worker's effort decision. In a partial equilibrium shirking model, the outside option is often treated as the reference wage. However, we cannot do this in the present dynamic general equilibrium model. The assumption of perfect mobility of unskilled workers among all Southern firms and the competitive labour market assumption rule out the possibility of an equilibrium with the difference between the worker's actual wage inside the firm and his expected wage in case he is to leave the firm. Danthine and Kurmann (2006) assume firm's productivity as the reference wage. In the present model, the profit maximising average productivity of the unskilled or skilled labour in the production sector is a function of the skilled unskilled wage ratio because the production function of each variety in the South satisfies constant returns to scale. So introducing firm's

productivity as the reference wage, no additional gain is made in this model. However, future research should deal with the robustness of the results to different functional forms of the efficiency function and to different assumptions about the reference wage.

In this model, the consumer does not derive any utility from leisure and so it does not make any optimum labour–leisure allocation. The efficiency of unskilled labour varies only with the skilled unskilled wage ratio and this ratio is uniquely determined by the profit maximising conditions of the firm. Number of workers is exogenous and hence the effective labour supply expressed in efficiency unit also becomes exogenous along the profit maximising path. There is no growth induced income effect on the labour supply decision of the worker. The strengthening of the IPR protection does not cause any shift of the labour supply curve. Modern macroeconomics analyses some growth theoretic issues within a general equilibrium framework with endogenous labour supply where the growth induced income effect shifts the labour supply curve. It is interesting to analyse the issue of IPR protection into a framework with endogenous labour supply; and we plan to do this in our future research.

There are some other important limitations in our model. We ignore the distributional aspect of unemployment here. Our analysis is concentrated only on the steady state equilibrium properties of the model and hence cannot say anything about transitional properties. We do not deal with the unemployment problem in the North and the unemployment problem of the skilled labour in the South. We assume, like many other models in this literature, that the South does not innovate. Nevertheless, the model constructed here sheds new light on the complex interrelationship among innovation, imitation and the unemployment of unskilled labour in the South. No other existing model has analysed this complex inter relationship so far.

Appendix A

The problem of the representative agent (consumer) is to maximise

$$W = \int_t^{\infty} e^{-\theta(\tau - t)} \log[U(\tau)] \mathrm{d}\tau \tag{1E.1}$$

subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r(\tau - t)} E(\tau) \mathrm{d}\tau = \int_t^{\infty} e^{-r(\tau - t)} I(\tau) \mathrm{d}\tau + A(t) \quad \forall t \tag{1E.2}$$

where

$$E = \int_0^n p(z) x(z) \mathrm{d}z \tag{1E.3}$$

and

$$U = \left(\int_0^n x(z)^{\alpha} \mathrm{d}z \right)^{\frac{1}{\alpha}}. \tag{1E.4}$$

The agent can solve this dynamic optimisation problem in two stages. First, it can choose the composition of given level of spending to maximise the instantaneous utility (static part). Then it can optimise W through the time path of spending (dynamic part). In stage 1, the agent's static optimisation exercise is the following.

$$\text{Max} \quad U = \left(\int_0^n x(z)^{\alpha} \mathrm{d}z \right)^{\frac{1}{\alpha}}$$

subject to

$$E = \int_0^n p(z)x(z)\mathrm{d}z.$$

This exercise generates the following demand function for each variety.

$$x(z) = E\frac{p(z)^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}\mathrm{d}u} \quad \forall z \in [0,n]. \tag{1E.5}$$

Using Eqs. (1E.4) and (1E.5), we have the indirect instantaneous utility function given by

$$U = \left(n_N x_N^{\alpha} + n_S x_S^{\alpha}\right)^{\frac{1}{\alpha}} = \frac{E}{P} \tag{1E.6}$$

where

$$P^{(1-\varepsilon)} = \int_0^n p(u)^{1-\varepsilon}\mathrm{d}u = n_N p_N^{1-\varepsilon} + n_S p_S^{1-\varepsilon}.$$

Using Eqs. (1E.1) and (1E.6), we have

$$W = \int_0^{\infty} e^{-\theta(\tau-t)}\log\left[\frac{E(\tau)}{P(\tau)}\right]\mathrm{d}\tau.$$

Differentiating both sides of Eq. (1E.2) with respect to time, τ, we have

$$\dot{A} = I(t) - E(t) + rA(t).$$

Now we express the current value Hamiltonian corresponding to this dynamic optimisation problem as

$$H = \log(U) + j[I(t) - E(t) + rA(t)],$$
$$\text{or,} \quad H = [\log(E) - \log(P)] + j[I(t) - E(t) + rA(t)],$$

where j is the co-state variable. The first order optimality conditions are given by the followings.

$$\frac{\partial H}{\partial E} = \frac{\delta U}{\delta E} - j = 0, \tag{1E.7}$$
$$\text{or,} \quad \frac{1}{E} = j.$$

Differentiating this with respect to time, t, we have

$$\frac{\dot{j}}{j} = -\frac{\dot{E}}{E}. \tag{1E.8}$$

Optimum time path of the co-state variable, j, should satisfy the following equation of motion.

$$\dot{j} = \theta j - \frac{\partial H}{\partial A} = \theta j - rj = (\theta - r)j.$$

The equation mentioned above and Eq. (1E.8) together imply

$$\frac{\dot{E}}{E} = r - \theta.$$

Appendix B

The problem of the representative firm in the South is to maximise

$$\pi_S = p_S x_S - \left(w_{SH} H_p + w_{SL} \frac{L_D}{h\left(\frac{w_{SL}}{w_R}\right)} \right)$$

with respect to H_p, L_D and w_{SL}. The first order optimality conditions are the followings.

$$p_S \frac{\partial x_S}{\partial H_p} = \frac{w_{SH}}{\alpha}; \tag{2E.1}$$

$$p_S \frac{\partial x_S}{\partial L_D} = \frac{w_{SL}}{\alpha h(.)}; \tag{2E.2}$$

and

$$\frac{w_{SL}}{w_{SH}} \frac{h'(.)}{h(.)} = 1. \tag{2E.3}$$

From the production function of x_S, we have

$$\frac{\partial x_S}{\partial H_p} = \frac{x_S}{\delta L_D^{-\rho} + (1-\delta) H_p^{-\rho}} (1-\delta)(H_p)^{-\rho-1}; \tag{2E.4}$$

and

$$\frac{\partial x_S}{\partial L_D} = \frac{x_S}{\delta L_D^{-\rho} + (1-\delta) H_p^{-\rho}} \delta (L_D)^{-\rho-1}. \tag{2E.5}$$

Using these equations, we derive Eqs. (12)–(15) in the text (see page no. 10).

Appendix C. Existence of the wide gap equilibrium

Eq. (36) represents SS curve. Also from NN curve given by Eq. (38), we have

$$\frac{(1-\alpha)L_N}{a_N} > g \geq \frac{(1-\alpha)L_N}{a_N} - \theta\alpha.$$

The inequality mentioned above and Eq. (36) together imply that the existence of the wide gap equilibrium (with both g and m positive) is ensured if

$$\frac{(1-\alpha)L_N}{a_N} > \frac{H_S \cdot (\Omega) - \frac{\alpha(1-\delta)}{1-\alpha} a_S \theta}{a_S\left(\frac{\alpha(1-\delta)}{1-\alpha} + \Omega\right)} > \frac{(1-\alpha)L_N}{a_N} - \theta\alpha. \tag{3E.1}$$

If the inequality mentioned above is satisfied then SS curve and NN curve must have a unique point of intersection in the Fig. 1.

Appendix D. Slope of NN curve and XX curve

Eq. (46) represents XX curve. From this equation, we have

$$\frac{L_N - a_N g}{H_S - a_S g}\left(\frac{m}{g}\right) = \alpha^{\frac{\rho}{}} \cdot (\Omega)^{-\frac{1}{\rho}}.$$

Differentiating its both sides with respect to g we have

$$\frac{-a_N H_S + a_S L_N}{(H_S - A_S g)^2} \cdot \frac{m}{g} + \frac{L_N - a_N g}{H_S - a_S g}\left(\frac{dm}{dg}\frac{1}{g} - \frac{m}{g^2}\right) = 0,$$

or,

$$\left.\frac{dm}{dg}\right]_{XX} = \frac{m}{g} + \frac{\frac{H_S}{a_S} - \frac{L_N}{a_N}}{\frac{H_S}{a_S} - g}\frac{m}{\frac{L_N}{a_N} - g},$$

or,

$$\left.\frac{dm}{dg}\right]_{XX} = \frac{m}{g} + \frac{m}{\frac{L_N}{a_N} - g} - \frac{m}{\frac{H_S}{a_S} - g}. \tag{4E.1}$$

Eq. (38) represents NN curve. From this equation, we have

$$(L_N - a_N g)\left(1 + \frac{m}{g}\right) = (\theta + m + g)a_N \frac{\alpha}{1-\alpha}. \tag{4E.2}$$

Differentiating its both sides with respect to g, we have

$$-a_N\left(1 + \frac{m}{g}\right) + (L_N - a_N g)\left(\frac{\frac{dm}{dg}g - m}{g^2}\right) = \left(1 + \frac{dm}{dg}\right)a_N \frac{\alpha}{1-\alpha},$$

or,

$$-a_N\left(1 + \frac{m}{g}\right) + \frac{L_N - a_N g}{g}\left(\frac{dm}{dg} - \frac{m}{g}\right) = \left(1 + \frac{dm}{dg}\right)a_N \frac{\alpha}{1-\alpha}.$$

From Eq. (4E.2), we have $\frac{L_N - a_N g}{g} = \frac{\theta + g + m}{g + m}\frac{\alpha}{1-\alpha}a_N$. Using this in the equation mentioned above we have

$$\frac{dm}{dg}\left(\frac{\theta + g + m}{g + m} - 1\right) = \frac{g + m}{g\alpha} + \frac{m}{g}\left(\frac{\theta + g + m}{g + m} - 1\right),$$

or,

$$\frac{dm}{dg} = \frac{m}{g} + \frac{g + m}{g\alpha} \cdot \frac{g + m}{\theta}.$$

From Eq. (4E.2), we have $\frac{g + m}{g\alpha} = \frac{\theta + g + m}{(1-\alpha)\left(\frac{L_N}{a_N} - g\right)}$. Using this in the equation mentioned above we have

$$\frac{dm}{dg} = \frac{m}{g} + \frac{1 + \frac{g+m}{\theta}}{(1-\alpha)\left(\frac{L_N}{a_N} - g\right)} \cdot (g + m),$$

or

$$\left.\frac{dm}{dg}\right]_{NN} = \frac{m}{g} + \frac{m}{(1-\alpha)\left(\frac{L_N}{a_N} - g\right)} + \frac{1}{1-\alpha}\frac{g + \frac{(g+m)^2}{\rho}}{\frac{L_N}{a_N} - g}. \tag{4E.3}$$

From Eqs. (4E.1) and (4E.3), it is now clear that

$$\left.\frac{dm}{dg}\right]_{NN} > \left.\frac{dm}{dg}\right]_{XX}$$

for any given common values of m and g. Hence the slope of XX curve drawn in the in the Fig. 2 exceeds that of NN curve. This ensures that their intersection point is unique when it exists.

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