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Optimal Inventory Policy for Welding Electrode

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Abstract

This successful case study shows how OR tools can be incorporated in traditional shops where they were not used up to now. It deals with finding the optimal inventory policy for welding electrode in a large ferro-alloys manufacturing unit. This inventory problem was modeled using a probabilistic reorder point lot size system with constant lead time. It was decided to take the reviewing period to be one day. The optimal inventory policy is determined. The plant accepted the results and implemented the same. Their misplaced apprehension that shortages would arise was found to be incorrect.

Keywords: Reviewing Period, Probability distribution function, Probabilistic Models.

1. Introduction

Proper inventory planning is of key importance for the success of almost every industry in general and of engineering industry in particular. Large inventory essentially means blocking of precious (and perhaps, scarce) capital. Whitin [1] reported that inventories are referred to as the "graveyard" of American business, as surplus stocks have been a principal cause of business failure. In contrast, the Japanese industries have reduced inventories as much as possible to increase productivity by the Just-intime Management (JITM) control system (see, for example, Hall [2] and O'Connor [3]). On the other hand, inadequate inventory can lead to shortages. These shortages can be very critical leading to huge tangible losses (e.g. monetary losses) and intangible losses (e.g. loss of goodwill). Between these two extremes, there are situations where inventory can be maintained at a balanced level. It is the aim of the science of inventory theory to manage the inventory position at the "optimal" level. The simplest model is the deterministic model where the nature of demand is known with certainty. However, in real life situations, such a phenomenon is rarely met with in practice. Demand is usually probabilistic in nature.

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Many models have been proposed for such a situation, see for example Sharma [4], Axsäter [5] and Silver et al. [6]. These models get more involved when lead-time is present, which may not be constant. Magson [7] describes such a situation. The models get even more complicated when reviewing period is allowed to be another parameter. Donaldson [8] discusses such a situation. Cheng and Sethi [9] consider a periodic review inventory model where demand is influenced by promotion decision.

This case study is based on a probabilistic reorder point lot size system with constant lead-time. It describes the application of scientific inventory management at the Stores Department of an engineering unit, which had hitherto ***not*** used any OR principles. This unit manufactures ferro-alloys used in the production of alloy steel. It had a large number of items in the stores. Careful scrutiny of records showed that there were items that had been lying unused even for very long periods of time. In fact some 5% of the items had not been used even for the past seven years.

No shortage of any technically important item had ever been reported. Rather, there were problems of storage and warehousing. Table I gives the proportion of unused items and the corresponding time for which they had been lying unused. It clearly implies that a large volume of capital is locked up by inventory. Since no scientific inventory management procedure was in place, it was felt that inventory management based on scientific principles need to be introduced systematically, *but slowly.* It is with this background that this study was taken up. It was decided to develop a scientific inventory policy for one high value critical item as a confidence-building exercise. Initially, an important item, *welding electrode* $-$ $4mm$, was chosen for the study. This welding electrode is a critical item used for welding purposes in the blast furnace. If any problem in the blast furnace is reported, it must be attended to immediately, and hence no shortages can be permitted for this item. During the eight month data collection period it was observed that the lead time for this item is only 8 days; however, a high inventory of 248 packets was maintained even though the maximum daily demand (during this data collection period of eight months) was only 13 packets, and more importantly in about 20% of the time there was no demand. Hence it is worthwhile to model for this inventory problem and implement the same at the shop floor.

The rest of the paper is organized as follows. Section 2 introduces the problem, describes the available data and presents elementary preliminary analysis. Section 3 describes the model used. Section 4 gives the results. Section 5 concludes the paper with a discussion.

Table I. Proportion of Items Lying Unused over Years.

Proportion of Items Lying Unused	Time (yrs)
5%	7
8%	5
10%	2

2. The Problem – Description And Formulation

The data were collected from the Stores Department. The company maintains a record of the number of welding electrodes consumed on any particular date (i.e. instantaneous demand) and the stock position (after satisfying the demand). The instantaneous demand may be zero. Data were collected for the period January 1, 2002 to August 31, 2002. Summarizing these data (so as to highlight the number of welding electrodes bought during the period under review), we get Table II. From Table II it is evident that a high level of inventory is carried for this item. This assertion can be upheld by analysing the demand figures.

Table II. Welding Electrode – Stock Profile.

Date	Instantaneous Demand	Quantity Bought	Stock Level after meeting Instantaneous Demand
	On Given Date		
01.1.2002	0	0	157
07.2.2002	7	100	114
27.2.2002	0	200	248
22.5.2002	2	240	244
02.6.2002	4	32	235
14.8.2002	6	100	107

For any scientific study on inventory management, other relevant information is necessary. Discussion with the stores and purchase personnel revealed the following: –

(a) This is a critical item and shortages are *not* permitted.
(b) The item is purchased locally from the state capital.
(c) Lead time as revealed from the records may be assumed to be a constant – 8 days.
(d) The cost of each packet is Rs 281/-.

(e) There are no discounts for bulk orders.

(f) The replenishment (ordering) cost (c_3) is Rs 250/- per order.

(g) Inventory carrying cost (c_1) is 16% per annum of the cost price.

On the basis of the figures given, the carrying cost, c_1 is calculated to be Re 0.12317 per packet per day. A preliminary survey of the daily demand data showed that the minimum demand is 0, while the maximum demand is 13. Table III shows the frequency distribution of the observed demand and the empirical demand distribution. It is clear from Table III that demand is probabilistic in nature.

Thus what is required is a model, which can incorporate the probabilistic nature of demand with constant lead time, and yet be simple enough for implementation at the shop floor. Inventory management with probabilistic demands and deterministic lead times has been wellresearched (see, for example, Sharma [4], Axsäter [5], Silver et al. [6] Hadley and Whitin [10], Naddor [11], and Taha [12]). However, it was felt that a *Probabilistic Reorder Point Lot Size System with Constant Lead Time* might be the most appropriate model. Since the plant is primitive in the usage of scientific inventory management principles, it was decided to review the stock daily. Thus the reviewing period (w_p) of one day was chosen.

Table III. Frequency Distribution of Observed Demand.

Daily Demand (x)	Frequency (f)	Probability
0	48	0.198
1	9	0.037
2	54	0.222
3	28	0.115
4	43	0.177
5	15	0.062
6	19	0.078
7	14	0.058
8	8	0.033
9	2	0.008
10	2	0.008
13	1	0.004
TOTAL	**243**	**1.000**

3. The Model Used

A brief discussion of the *Probabilistic Reorder Point Lot Size System with Constant Lead Time* is in order. Borrowing the notation of Naddor [11], this is a type $(1,3)$ system (i.e. carrying costs are balanced against replenishing costs) in which demand is probabilistic and the optimal lot size (q_0) has to be determined. In other words this is a (z,q) system, where z is the reorder point and q is the order quantity (to be determined).

Let Q designate the amount in inventory at the beginning of each reviewing period. (The quantity Q refers to the amount in inventory after a replenishment, if any). We shall show that the probability distribution of Q is the uniform distribution. Thus Q is equally likely to be any amount between $z+u$ and $z+q$. Note that, whenever the amount on hand is equal to or below the reorder point z, a lot of size q is scheduled for a replenishment. Since the electrode packets are discrete in nature, we consider the basic unit to be u. (In our case $u=1$).

Let $P(x)$ be the probability distribution of demand during the reviewing period, where $x = 0, u, 2u, \ldots, x_{\max}$. Let $H(Q)$ be the probability distribution of the amount in inventory at the beginning of each reviewing period. We, therefore, want to show that

$$H(Q) = \frac{1}{q/u} = \frac{u}{q} \qquad Q = z+u, z+2u, \ldots, z+q. \tag{1}$$

The amount Q may be regarded as a state in Markov chain. The transition probabilities of going from one inventory state Q to another are given in Table IV.

The transition matrix represented by the Table IV is a doubly stochastic matrix. However, we know that the steady state probabilities of a doubly stochastic matrix are equal (Feller [13]), p 358). This proves the result in (1) above.

Let x be the demand during the reviewing period w_p. Let $\overline{x}$ be the average demand and let $x_{\max}$ be the maximum demand. Let w be an instantaneous demand equivalent to the uniform demand x during the reviewing period. Let the demand during the lead time be v and let its probabilistic distribution be $F(v)$. Let y be a demand equal to the sum of the demand of w and v, i.e. $y = w + v$.

Since no shortages are allowed, the reorder point (z_p) is essentially prescribed, namely

$$z_p = x_{\max} + \max\{v_{cum}\} - u, \tag{2}$$

(since for larger values of z unnecessary inventories will be carried), where $\max\{v_{cum}\}$ is the maximum of the cumulative demands during the lead times. Let $P_e(y)$ be the probability distribution of the equivalent demand $y = w + v$. Then the expected average amount in inventory is given by

$$\begin{aligned} I_1 &= \sum_{Q=z_p+u}^{z_p+q} \sum_{y=0}^{y_{\max}} (Q-y)P_e(y)H(Q) \\ &= z_p + \frac{u+q}{2} - \overline{y} \\ &= x_{\max} + \max\{v_{cum}\} - u + \frac{u+q}{2} - \left(\frac{\overline{x}}{2} + \overline{v}\right) \\ &= x_{\max} + \max\{v_{cum}\} + \frac{q}{2} - \frac{u+\overline{x}}{2} - \overline{v} \qquad (3) \end{aligned}$$

where v is the average demand during the lead time.

Table IV. The Transition Probabilities of Going from One State of Inventory, at the Beginning of a Reviewing Period, to Another State, at the Beginning of the Next Period.

To → From ↓	$z+q$	$z+q-u$	$\cdots$	$z+2u$	$z+u$
$z+q$	$P(0)$ $+P(q)$ $+\cdots$	$P(u)$ $+P(q+u)$ $+\cdots$	$\cdots$	$P(q-2u)$ $+P(2q-2u)$ $+\cdots$	$P(q-u)$ $+P(2q-u)$ $+\cdots$
$z+q-u$	$P(q-u)$ $+\cdots$	$P(0)+P(q)$ $+\cdots$	$\cdots$	$P(q-3u)$ $+P(2q-3u)$ $+\cdots$	$P(q-2u)$ $+P(2q-2u)$ $+\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$z+2u$	$P(2u)$ $+\cdots$	$P(3u)$ $+\cdots$	$\cdots$	$P(0)+P(q)$ $+\cdots$	$P(u)$ $+P(q+u)$ $+\cdots$
$z+u$	$P(u)$ $+\cdots$	$P(2u)$ $+\cdots$	$\cdots$	$P(q-u)$ $+\cdots$	$P(0)$ $+P(q)$ $+\cdots$

We need to determine the expected number of replenishments per unit of time I_3. Let h denote the probability of the occurrence of a replenishment at the beginning of a reviewing period. Then $I_3 = \frac{h}{w_p}$.

Consider any state $Q = z_p + r$. A replenishment will occur whenever the demand is equal to or larger than r. Hence, by equation (1), we have,

$$\begin{aligned} h &= \sum_{Q=z_p+u}^{z_p+q} \sum_{x=Q-z_p}^{x_{\max}} P_e(x)H(Q) \\ &= \sum_{Q=z_p+u}^{z_p+q} \Big[1 - F(Q-z_p-u)\Big]\frac{u}{q} \\ &= 1 - \frac{u}{q}\sum_{x=0}^{q-u} F(x) \\ &= 1 - \frac{u}{q}B(q-u) \end{aligned} \tag{4}$$

where

$$B(x) = \sum_{t=0}^{x} F(t) = \sum_{t=0}^{x}\sum_{k=0}^{t} P_e(k). \tag{5}$$

Then the expected total cost of the Probabilistic Reorder Point Lot Size System with Constant Lead Time is given by

$$\begin{aligned} C(q) &= c_1 I_1 + c_3 I_3 \\ &= c_1\Big(x_{\max} + \max\{v_{cum}\} + \frac{q}{2} - \frac{u+\overline{x}}{2} - \overline{v}\Big) + c_3\frac{h}{w_p} \\ &= \frac{c_1 q}{2} - \frac{c_3 u B(q-u)}{q w_p} + c_1\Big(x_{\max} + \max\{v_{cum}\} - \frac{u+\overline{x}}{2} - \overline{v}\Big) + \frac{c_3}{w_p} \end{aligned} \tag{6}$$

where the function $B(\cdot)$ is as in equation (5) above.

For the optimal lot size (q_0), we must have

$$C(q_0) \le C(q_0+u), \qquad C(q_0) \le C(q_0-u).$$

After simplification we obtain

$$R(q_0-u) \le \frac{2c_3}{c_1 w_p} \le R(q_0) \tag{7}$$

where

$$R(q) = \frac{q(q+u)}{\sum_{x=0}^{q} x P_e(x)} \qquad q = u, 2u, \ldots \tag{8}$$

We have thus proved the following theorem:-

Theorem. *For the Probabilistic Reorder Point Lot Size System with Constant Lead Time, the optimal lot size q_0 is obtained as the solution to the following inequation:-*

$$R(q_0-u) \le \frac{2c_3}{c_1 w_p} \le R(q_0)$$

where $R(q)$ *is defined by* (8) *above.*

4. Results Obtained

In the present case the value of the decision criterion is given by

$$2c_3/(c_1 w_p) = 2(2 \times 250)/(0.12317 \times 1) = 4059$$

Using (7) and (8), the optimal lot size q_0 is found to be 114.
The reorder point is given by,

$$\begin{aligned} \text{Reorder point } z_p &= x_{\max} + \max\{v_{cum}\} - u \\ &= 13 + 32 - 1 = 44 \end{aligned}$$

Thus the recommended inventory policy is the following:-
Review the stock position daily; if the amount in inventory at any point becomes 44 *packets or less, order* 114 *packets.*

It should be noted that it is not difficult to review the stock position on a daily basis as their materials handling system is computerised. Each transaction triggers an immediate updating of the status. Thus transaction reporting ensures that the stock status is always known. The expected reduction in costs in using the recommended scientific inventory policy is over 23%. (When there is no planned inventory management, the associated inventory carrying cost (c_1) incurred during the data collection period is Rs. 3587 and the ordering cost is (c_3) is Rs. 1250. If scientific inventory management principles were applied during this period then, the inventory carrying cost (c_1) incurred would have been Rs. 2420 while the ordering cost (c_3) would have been Rs. 1500. Thus the use of scientific inventory management principles would have resulted in a saving of Rs. 917, which represents more than 23%). When these results were presented to the management, they were impressed and decided to accept this scientific policy on an experimental basis.

5. Discussion

The company has adopted this policy of daily review, and placing an order only when dictated by the inventory policy. It has been in operation for more than three years and no stock out has occurred. Encouraged by these findings, the company decided to implement scientific inventory policies for other high value items.

It may be mentioned in passing that as the company is very primitive in knowledge of scientific principles of inventory management, daily review was chosen. This gives the consoling satisfaction that there is a chance for ordering a fresh consignment daily and no stockout will be possible. However, perhaps one can obtain better results by varying the reviewing period, and choosing that period for which the expected total cost is minimum; and is at the same time operationally simple at the shop floor.

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