Measuring Ambiguities In Images Using Rough And Fuzzy Set Theory

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Abstract: **Images, in general, are ambiguous in nature. In this paper, we propose the combined use of rough and fuzzy set theory to measure the ambiguities in images. Rough set theory is used to capture the indiscernibility among nearby gray values, whereas fuzzy set theory is used to capture the vagueness in the boundaries of the various regions. A measure called rough-fuzzy entropy of sets is proposed to quantify image ambiguity using which a characteristic measure of an image called the average image ambiguity (AIA) is presented. The rough-fuzzy entropy measure is used to perform various image processing tasks such as object / background separation, multiple region segmentation and edge extraction, and the corresponding performance are compared to those obtained using certain existing fuzzy and rough set theory based image ambiguity measures. Extensive experimental results are given to demonstrate the utility of measuring image ambiguity using the proposed rough-fuzzy entropy measure.**

1. Introduction

In general, ambiguities are inherent in images. The various regions in an image have "fuzzy" boundaries and nearby gray values visually seem to be "roughly" the same. Hence, the study of the ambiguities present in an image is just as interesting as studying the frequency content of an image. Fuzzy set theory [1], where the concept of vague boundaries of subsets in a universe of discourse is considered, has been used in literature to capture the ambiguities in images. Measures of image ambiguity have been presented in terms of index of fuzziness [2], fuzzy entropy [2, 3] and fuzzy correlation [4], and then applied to certain image processing tasks.

In the year 1982, Zdzislaw Pawlak came up with the rough set theory [5], which focuses on ambiguity as the limited discernibility of subsets in the domain of discourse. The rough set theory like the fuzzy set theory can be used to measure the ambiguities in images. Recently, such an application has been demonstrated in [6].

Fuzzy set theory can only be used to capture the ambiguity in an image due to the vague boundaries between various regions. However ambiguity is also present in an image due to the limited discernibility between nearby gray values. From the underlying concept of rough set theory, we find that such an ambiguity can be precisely captured using rough sets. With this rationale, we propose the use of rough set theory along with fuzzy set theory for various image processing tasks.

Rough set theory deals with the approximation of a certain crisp set due to the limited discernibility among various subsets in the universe of discourse [5]. We may generalize the rough set theory by considering the approximation of a fuzzy set and not that of a crisp set, and in this process arrive at the concept of rough-fuzzy sets [7-9]. Rough-fuzzy sets can be used for image processing tasks in such a way that we get the advantages of both the fuzzy and rough set theory in capturing different types of ambiguities the image.

In this paper, we propose an entropy measure of a set of elements called the rough-fuzzy entropy. We use this proposed entropy measure to quantify the image ambiguity and then perform various tasks such object / background separation, multiple region segmentation and edge extraction. A characteristic measure of an image called the average image ambiguity (AIA) is also presented using the proposed rough-fuzzy entropy measure. Experimental results obtained using the rough-fuzzy entropy and comparisons with those obtained using the existing measures of fuzziness [2, 4] and rough entropy [6], are given to demonstrate the effectiveness and utility of the proposed rough-fuzzy entropy measure.

The organization of the paper is as follows. Rough and rough-fuzzy sets of gray values are defined and explained in Section II. Section III contains the definition of the rough-fuzzy entropy measure and demonstration of the use of rough and rough-fuzzy entropy measure in measuring image ambiguity. Experimental results are given in Section IV and the paper concludes with Section V outlining the contributions made.

2. Rough and Rough-Fuzzy Sets of Gray Values

Consider an L-level (gray level) image I of size $S1 \times S2$. Let Λ be the universe of discourse of the L gray values with the elements represented by l_i, $i = 1, 2, \cdots, L$.

A. *Rough Set of Gray Values [6]*

Let us first define two mutually exclusive sets A_b and B_b respectively representing the 'bright' and 'dark' gray values in Λ as

$$A_b = \{l_i \mid l_i \in \Lambda : l_i > b\} \quad (1)$$

$$B_b = \{l_i \mid l_i \in \Lambda : l_i \leq b\} \quad (2)$$

It can be easily seen that A_b and B_b are defined such that $A_b \cup B_b = \Lambda$. The gray value b is called the cross-over point.

Next, we define granules [5] or induced equivalence classes (of equal size) by partitioning the set Λ based on the indiscernibility created by the concept of perceptual similarity between nearby gray values (see Fig. 1). Let ω denote the size of the granules. As mentioned in [6], the granules are

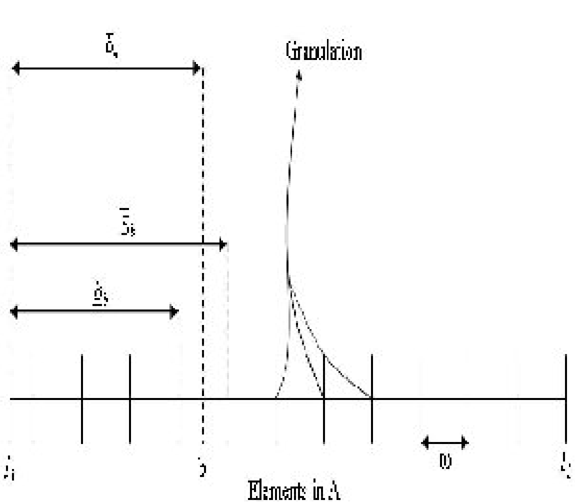


Fig. 1. Rough set of gray values.

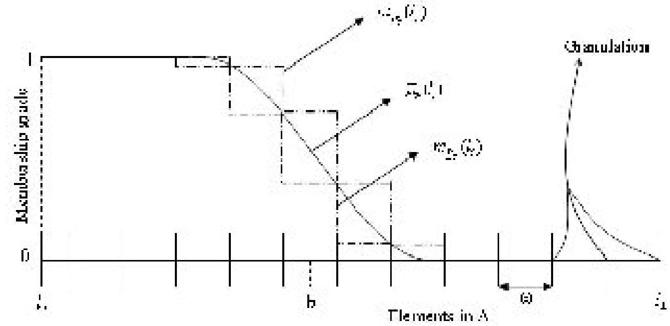


Fig. 2. Rough-fuzzy set of gray values

obtained such that the gray value b is never at the boundary of a granule. We now consider the ambiguity in Λ due to the limited discernibility in its subsets created by the concept of perceptual similarity in gray values and then use the rough set theory [5] to obtain the lower approximations of A_b and B_b, which are given by

$$\underline{A_b} = \{l_i \mid l_i \in \Lambda : [l_i]_\omega \subseteq A_b\} \quad (3)$$

$$\underline{B_b} = \{l_i \mid l_i \in \Lambda : [l_i]_\omega \subseteq B_b\} \quad (4)$$

and the upper approximations of A_b and B_b, which are expressed as

$$\overline{A_b} = \{l_i \mid l_i \in \Lambda : [l_i]_\omega \cap A_b \neq \emptyset\} \quad (5)$$

$$\overline{B_b} = \{l_i \mid l_i \in \Lambda : [l_i]_\omega \cap B_b \neq \emptyset\} \quad (6)$$

where $[l_i]_\omega$ stands for the granule of size ω containing the element l_i.

We now have two rough sets of gray values. The rough set $[\underline{A_b}, \overline{A_b}]$ represents the 'bright' gray values in Λ, whereas, the rough set $[\underline{B_b}, \overline{B_b}]$ represents the 'dark' gray values in Λ. The rough set $[\underline{B_b}, \overline{B_b}]$ is shown in Fig. 1.

B. Rough-Fuzzy Set of Gray Values

Note that the subsets A_b and B_b are considered as crisp sets in (1) and (2). However, as explained in Section I, various regions in an image have "fuzzy" boundaries and hence it is more suitable to consider A_b and B_b as fuzzy sets rather than crisp sets.

Therefore, let us now define two fuzzy sets A_b and B_b respectively representing the 'bright' and 'dark' gray values in Λ as

$$A_b = \{(l_i, \mu_b(l_i)) \mid l_i \in \Lambda\} \quad (7)$$

$$B_b = \{(l_i, \bar{\mu}_b(l_i)) \mid l_i \in \Lambda\} \quad (8)$$

where $\bar{\mu}_b(l_i) = 1 - \mu_b(l_i)$. Again, it can be easily seen that A_b and B_b are defined such that $A_b \cup_F B_b = \Lambda$, where $\cup_F$ stands for a fuzzy union such that the membership of every element in Λ is unity. We calculate the $\mu_b(l_i)$ values using Zadeh's S-function [1] as given below

$$\mu_b(l_i) = \begin{cases} 0 & l_i \le a \\ 2\left[\dfrac{(l_i - a)}{(c - a)}\right]^2 & a \le l_i \le b \\ 1 - 2\left[\dfrac{(l_i - c)}{(c - a)}\right]^2 & b \le l_i \le c \\ 1 & l_i \ge c \end{cases} \quad (9)$$

where the gray value b is called the cross-over point and a measure $\Delta b = b - a = c - b$ is called the bandwidth of the underlying fuzzy set.

Again we define granules in the set Λ as explained earlier (see Fig.2) and consider the ambiguity in Λ due to the limited discernibility in its subsets created by the concept of perceptual similarity in gray values and then use the rough set theory to obtain the lower approximations of the fuzzy sets A_b and B_b given in (7) and (8) [8] as

$$\underline{A_b} = \{(l_i, m_{\underline{A_b}}(l_i)) \mid l_i \in \Lambda\} \quad (10)$$

$$\underline{B_b} = \{(l_i, m_{\underline{B_b}}(l_i)) \mid l_i \in \Lambda\} \quad (11)$$

where

$$m_{\underline{A_b}}(l_i) = \inf_{y \in [l_i]_\omega} \mu_b(y) \quad (12)$$

$$m_{\underline{B_b}}(l_i) = \inf_{y \in [l_i]_\omega} \bar{\mu}_b(y) \quad (13)$$

and the corresponding upper approximations [8] as

$$\overline{A_b} = \{(l_i, m_{\overline{A_b}}(l_i)) \mid l_i \in \Lambda\} \quad (14)$$

$$\overline{B_b} = \{(l_i, m_{\overline{B_b}}(l_i)) \mid l_i \in \Lambda\} \quad (15)$$

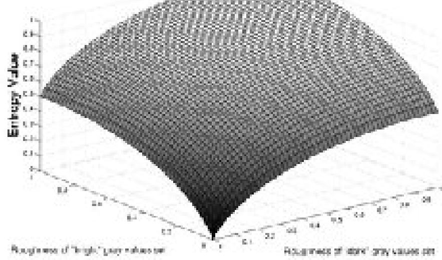
where

$$m_{\overline{A_b}}(l_i) = \sup_{y \in [l_i]_\omega} \mu_b(y) \quad (16)$$

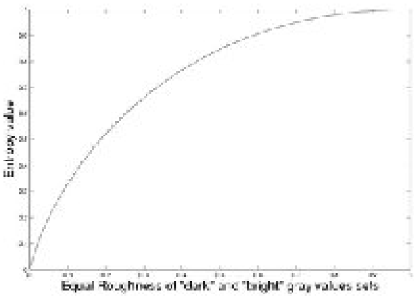
$$m_{\overline{B_b}}(l_i) = \sup_{y \in [l_i]_\omega} \bar{\mu}_b(y) \quad (17)$$

Again $[l_i]_\omega$ stands for the granule of size ω containing the element l_i. Note that the upper and lower approximations are fuzzy sets like the sets A_b and B_b which are being approximated.

We now have two rough-fuzzy sets of gray values. The rough-fuzzy set $[\underline{A_b}, \overline{A_b}]$ represents the 'bright' gray values



(a) Plot of rough entropy for various roughness values (R_{A_b} and R_{B_b})



(b) Plot of rough entropy for roughness values ($R_{A_b} = R_{B_b}$)

Fig. 3. Plots of the proposed entropy measure of set using rough and fuzzy set theory

in Λ, whereas the rough-fuzzy set $[\underline{B_b}, \overline{B_b}]$ represents the 'dark' gray values in Λ. The rough-fuzzy set $[\underline{B_b}, \overline{B_b}]$ is shown in Fig. 2

Note that, in this paper, we have defined the rough and rough-fuzzy sets of gray values in Λ considering two regions, namely the "bright" and "dark" regions, in an image. It is obvious that different regions instead of the "bright" and "dark" can also be used. In such a case the sets A_b and B_b should be defined accordingly.

3. Measuring Image Ambiguity

In this section, we shall define the rough-fuzzy entropy of a set, use the proposed rough-fuzzy entropy to measure image ambiguity and present a characteristic measure of an image called the average image ambiguity.

A. Rough and Rough-Fuzzy Entropy

In the previous section, we have obtained a pair of rough sets considering crisp regions in an image and a pair of rough-fuzzy sets considering fuzzy regions in an image. In both the cases we have denoted the sets as $[\underline{A_b}, \overline{A_b}]$ and $[\underline{B_b}, \overline{B_b}]$.

We now present an entropy measure of the set Λ as given below

$$RE_b(\Lambda) = -\frac{1}{2}[R_{A_b}\log_e(\frac{R_{A_b}}{e}) + R_{B_b}\log_e(\frac{R_{B_b}}{e})] \quad (18)$$

where

$$R_{A_b} = 1 - \frac{|\underline{A_b}|}{|\overline{A_b}|}, R_{B_b} = 1 - \frac{|\underline{B_b}|}{|\overline{B_b}|} \quad (19)$$

In the above, $|\underline{A_b}|$, $|\overline{A_b}|$, $|\underline{B_b}|$ and $|\overline{B_b}|$ are the cardinalities of the sets $\underline{A_b}$, $\overline{A_b}$, $\underline{B_b}$ and $\overline{B_b}$, respectively.

Now, when $[\underline{A_b}, \overline{A_b}]$ and $[\underline{B_b}, \overline{B_b}]$ represent a pair of rough sets we shall call the entropy measure as rough entropy [] and when $[\underline{A_b}, \overline{A_b}]$ and $[\underline{B_b}, \overline{B_b}]$ represent a pair of rough-fuzzy sets we shall call the entropy measure as rough-fuzzy entropy. Note that in the expression of rough-fuzzy entropy, the cardinalities in (19) are calculated using any method [1] to calculate the cardinality of a fuzzy set.

The value of RE_b lies between 0 and 1. Note that RE_b the minimum value of zero only when $R_{A_b} = R_{B_b} = 0$ and the maximum value of unity only when $R_{A_b} = R_{B_b} = 1$. This concept is consistent with the fact that maximum information (entropy) is available when the uncertainty is maximum, which is the case when the roughness values given in (19) are unity. The plot of RE_b for various values of R_{A_b} and R_{B_b} is given in Fig.3.

B. Image Ambiguity Measure

Let us now consider a portion X of an image I having N elements denoted by x_n, $n = 1, 2, \cdots, N$. Let $H_X(l_i)$ represent the number of elements in the array X having the gray value l_i (that is, $x_n = l_i$). In order to find the rough-fuzzy entropy measure of ambiguity in X, we define the following define the following measures

$$R_{A_b}^X = 1 - \frac{\sum_{l_i \in \Lambda} m_{\underline{A_b}}(l_i) \cdot H_X(l_i)}{\sum_{l_i \in \Lambda} m_{\overline{A_b}}(l_i) \cdot H_X(l_i)}, R_{B_b}^X = 1 - \frac{\sum_{l_i \in \Lambda} m_{\underline{B_b}}(l_i) \cdot H_X(l_i)}{\sum_{l_i \in \Lambda} m_{\overline{B_b}}(l_i) \cdot H_X(l_i)}$$

(20)

We then obtain the expression for the rough entropy measure of ambiguity in X as

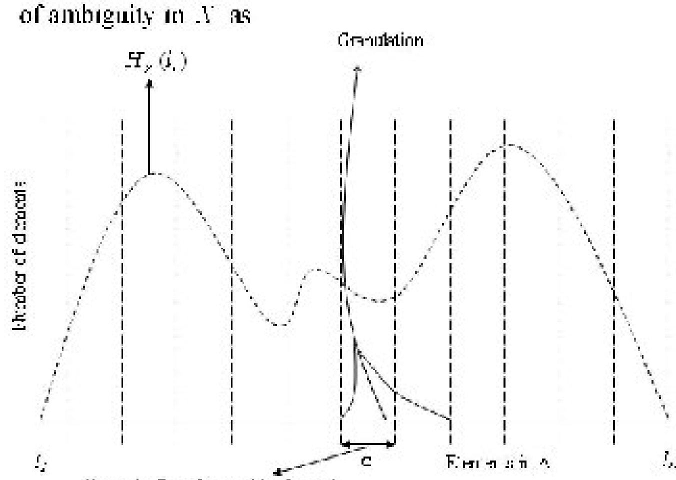


Fig. 4. Graylevel granulation in an image histogram

$$RE_b^X(A) = \frac{1}{2}[R_{A_b}^X \log_e(\frac{R_{A_b}^X}{e}) \quad R_{B_b}^X \log_e(\frac{R_{B_b}^X}{e})] \qquad (21)$$

The graylevel granulation in the histogram representing the number of elements at different gray values in X is shown in Fig.4.

Note that when we find the rough entropy measure of ambiguity in X, the expressions in (20) reduces to

$$R_{A_b}^X = 1 - \frac{\sum_{l_i \in \underline{A}_b} H_X(l_i)}{\sum_{l_i \in \overline{A}_b} H_X(l_i)}, R_{B_b}^X = 1 - \frac{\sum_{l_i \in \underline{B}_b} H_X(l_i)}{\sum_{l_i \in \overline{B}_b} H_X(l_i)} \qquad (22)$$

C. Average Image Ambiguity (AIA):

We shall now present a characteristic measure of an image I using the rough-fuzzy entropy proposed earlier in this section. Let us consider that we have obtained the value of $RE_b^I(A)$ in (21) for all possible values of the cross-over point b and by considering the rough-fuzzy sets $[\underline{A}_b, \overline{A}_b]$ and $[\underline{B}_b, \overline{B}_b]$. The average image ambiguity measure (AIA) of an image is then defined as

$$AIA = \frac{1}{L}\sum_{b=l_1}^{l_L} RE_b^I(A) \qquad (23)$$

That is, AIA is obtained by calculating the average of $RE_b^I(A)$ over all values of b. Note that rough-fuzzy sets are preferred over rough or fuzzy sets in defining the AIA measure as rough-fuzzy sets capture the ambiguities in images in a better manner, as explained earlier. It is evident from (23) that the AIA measure lies in the range [0, 1]. When the measure is zero for an image it implies that the image is unambiguous, whereas when the measure is unity it indicates that the image is maximally ambiguous.

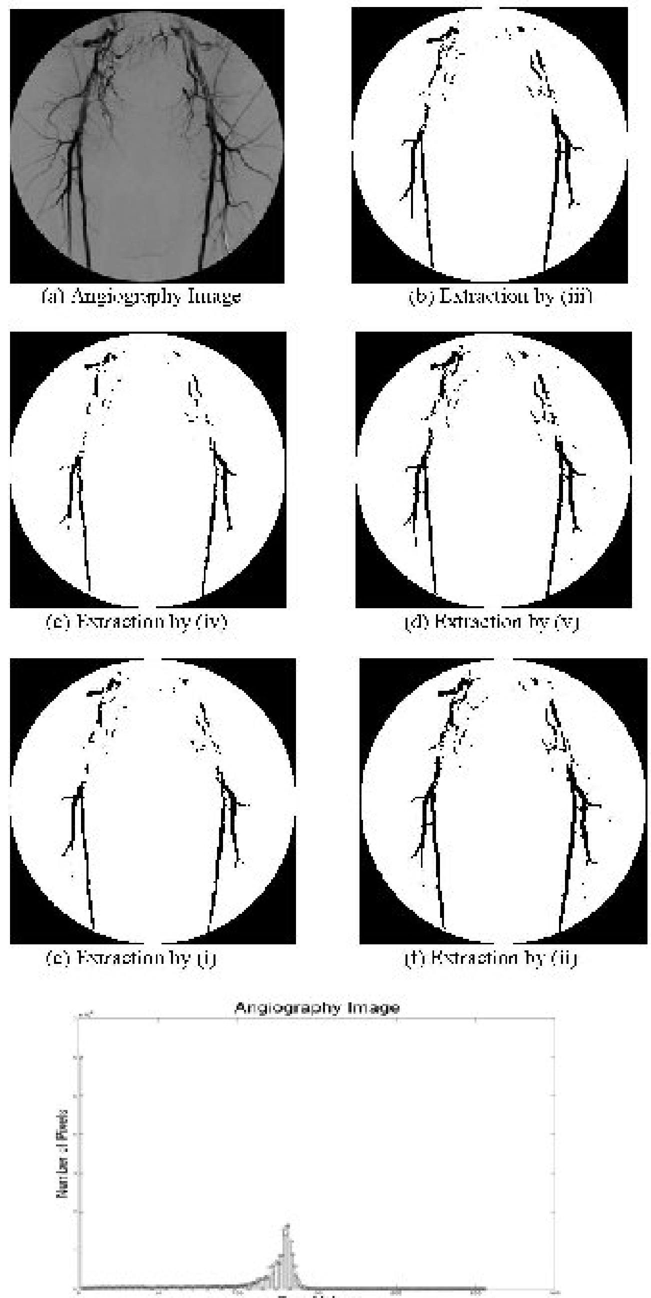
4. Applications and Experimental Results

In this section, we provide experimental results using a few images in order to demonstrate the efficacy of the proposed rough-fuzzy entropy in image processing applications and its superiority over the use of certain existing fuzzy set theory based measures. Qualitative evaluation of performance is considered in this paper. Quantitative evaluation is avoided as there is no globally accepted objective measure present for evaluating a thresholding process.

Thresholding operations in order to perform object / background separation, edge extraction and multiple region segmentation are considered as the image processing tasks to be carried out using the proposed and other measures of image ambiguity.

A. Threshold Determination using Ambiguity Measures

We shall now explain in brief the process of threshold selection in a histogram using ambiguity measures. Let us, for example, consider the proposed ambiguity measure $RE_b^I(A)$ of an image I. Like all other existing fuzzy and rough set theory based ambiguity measures [2, 4, 6], the proposed measure depends on the cross-over point b. In order to determine a threshold from a histogram, the value of b is varied over all possible element values, that is, l_i, $i = 1, 2, \cdots, L$ [2, 4, 6]. The value of l_i at which $RE_b^I(A)$ (the ambiguity measure) attains a global optimum is considered as the threshold. In order to select more than one threshold, one should search for the required number of element values where $RE_b^I(A)$ attains a local optimum. Note that in our case.



(a) Angiography Image (b) Extraction by (iii)

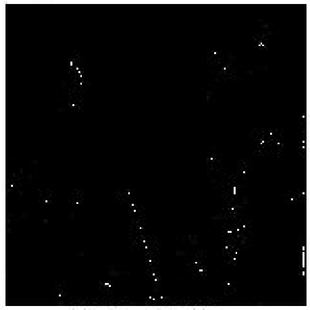
(c) Extraction by (iv) (d) Extraction by (v)

(e) Extraction by (i) (f) Extraction by (ii)

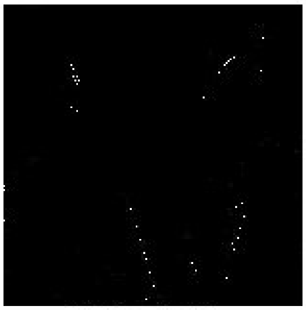
(g) Graylevel histogram

Fig. 5. Qualitative results obtained using the various techniques to perform blood vessel extraction in an angiography image.

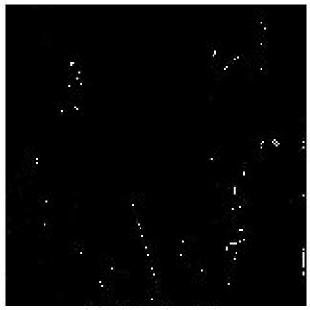
(a) "Montreal" Image



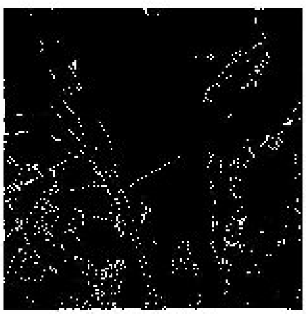
(b) Edges by (iii)



(c) Edges by (iv)

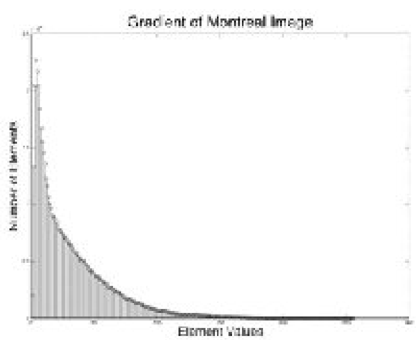


(d) Edges by (v)



(e) Edges by (i)

(f) Edges by (ii)



(g) Gradient magnitude histogram of the "Montreal" image

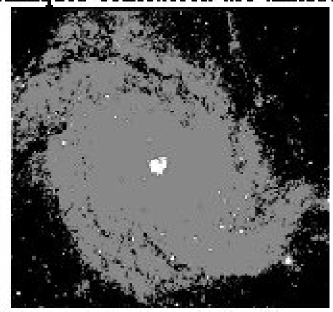
Fig. 6. Qualitative results obtained using the various techniques to perform edge extraction on the gradient image of the "Montreal" image

the term optimum refers minimum.

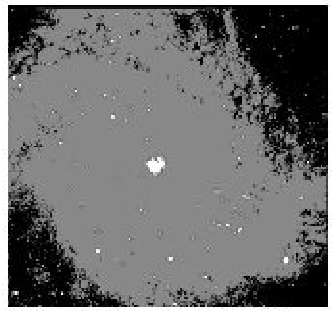
The techniques considered for comparison are (i) thresholding using the rough entropy measure [6], (ii) thresholding using the proposed rough-fuzzy entropy measure, (iii) thresholding using linear index of fuzziness [2], (iv) thresholding using fuzzy entropy measure [2], and (v) thresholding using fuzzy correlation measure [4]. These methods will henceforth be referred using their corresponding numbers in the paper.

The application of the mentioned techniques to blood vessel extraction in an angiography image is shown in Fig. 5. The performance of all the techniques considered are almost

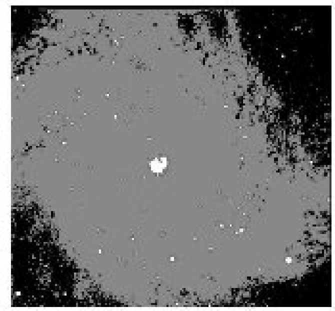
(a) Galaxy Image



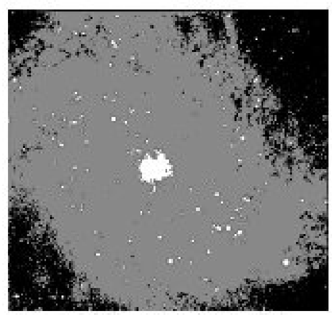
(b) Segmentation by (iii)



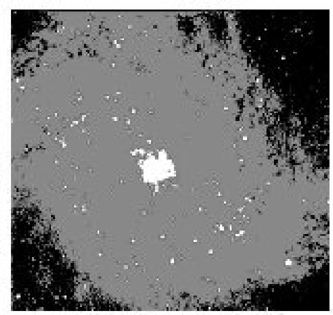
(a) Segmentation by (iv)



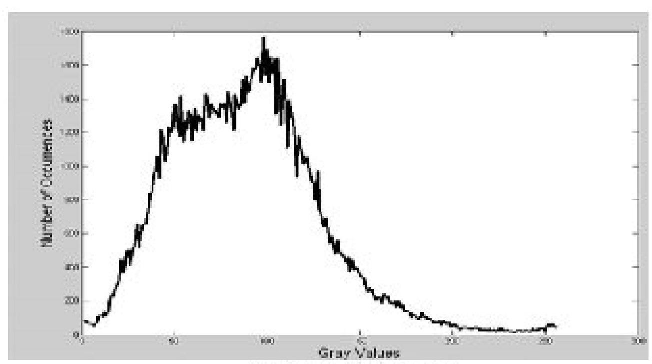
(b) Segmentation by (v)



(a) Segmentation by (iv)



(b) Segmentation by (v)



(g) Graylevel histogram

Fig. 7. Qualitative results obtained using the various techniques to perform multiple region segmentation on a galaxy image

the same. However, we notice that the proposed rough-fuzzy set based technique does slightly better than the techniques using only rough or fuzzy set theory based measures.

Next, we consider an edge extraction problem in an image of the city of Montreal in Canada, which is shown in Fig. 6. Fig. 6(g) gives the histogram of gradient magnitudes of the image. The gradient values are obtained using the Canny's gradient operator [10]. Note that as a gradient histogram is now under consideration for thresholding, the element values l_i will now be gradient magnitude values and not gray values. It is evident from the figure that rough set and rough-fuzzy set based thresholding operation outperforms the other in extracting the edges. We also notice that the proposed rough-fuzzy set based edge extraction gives clearly the best performance.

An image of a galaxy is considered in Fig. 7. The graylevel histogram of this image shown in Fig. 7(g), is almost unimodal in nature and hence extracting multiple regions from it is a non-trivial task. We consider the proposed ambiguity measure based multiple threshold determination technique and the others to find out the total extent and the core region of the galaxy. It is evident from the figure that although the rough set theory based segmentation does quite well, the proposed rough-fuzzy set theory based segmentation outperforms all the others considered in achieving the objective.

In Section III, we have put forth a characteristic measure of an image, called the average image ambiguity (AIA). We now put down the AIA measure of the three images considered in this section. The AIA measures are

Angiography image : 0.39501
"Montreal" image : 0.49631
Galaxy image : 0.51039

A casual look on the images in Figs. 5, 6 and 7 will explain the values obtained above. In the angiography image, we are able to distinguish between the blood vessels and the background visually without much difficulty and hence we have a low AIA measure. However distinguishing different regions in the "Montreal" image and the galaxy image are not that easy and hence we have a higher AIA measure.

Thus we demonstrate the usefulness of the proposed rough-fuzzy entropy measure and conclude that it successfully captures the ambiguous nature of an image.

5. Conclusion

The use of rough and fuzzy set theory together to measure the ambiguities in images has been proposed in this paper. Rough set theory has been used to capture the whereas fuzzy set theory is used to capture the vagueness in the "fuzzy" boundaries of the various regions. A measure called rough-fuzzy entropy of sets has been proposed to quantify image ambiguity. The rough-fuzzy entropy measure has been used to perform various image processing tasks such as object / background separation, multiple region segmentation and edge extraction, and define a characteristic measure of an image. The efficacy of the proposed rough-fuzzy entropy measure in capturing image ambiguity has been demonstrated with the help of extensive experimental results and comparisons.

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