Electron acoustic solitons in a relativistic plasma with nonthermal electrons

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Electron acoustic solitary waves (EASWs) are studied using Sagdeev's pseudopotential technique for a plasma comprising relativistic ions, cold relativistic electrons, and nonthermal hot electrons. The parametric range considered here is valid for the auroral zone. It is found that the present plasma model supports EASWs having negative potential. It is seen that the relativistic effect significantly restricts the region of existence for solitary waves. The region of existence of solitary waves also depends crucially on α, the parameter that determines the population of the energetic nonthermal electrons. For example, for $\alpha > 0.18$ with the soliton velocity 1.05 and $u_{0c}/c = 0.001$, solitary wave solutions will not exist. We also find that for small values of α, solitary waves would exist for $V < 1$. [DOI: 10.1063/1.2216549]

I. INTRODUCTION

Electron acoustic waves can exist in a two-temperature (cold and hot) electron plasma. They are high-frequency (in comparison with the ion plasma frequency) electrostatic modes[1] in plasmas where a "minority" of inertial cold electrons oscillate against a dominant thermalized background of inertialess hot electrons providing the necessary restoring force.[2] The phase speed of the electron acoustic wave is much larger than the thermal speeds of cold electrons and ions, but is usually smaller than the thermal speed of the hot electron component (in the case of isothermal electrons). The electron acoustic solitary waves (EASWs) can also be generated by electron and laser beams.[3,4] Recently, a great deal of interest has been shown in the studies on the propagation of EASWs, not only because they are excited in space and laboratory plasmas, but also because of their potential importance in interpreting electrostatic component of the broadband electrostatic noise (BEN) observed in the cusp region of the terrestrial magnetosphere,[5,6] in the geomagnetic tail,[7] in the dayside auroral acceleration region,[8,9] etc. The propagation of EASWs in a plasma system has been studied by several investigators in an unmagnetized two-electron plasma,[8,10–12] as well as in magnetized plasma.[13–16] Energetic electron distributions are observed in the different regions of the magnetosphere. Cairns *et al.*[17] used nonthermal distribution of electrons to study the ion-acoustic solitary structures observed by the FREJA satellite. Very recently, Singh and Lakhina[18] studied EASWs with nonthermal distribution of electrons. They showed that inclusion of nonthermal electrons predicts negatively charged potential structures. However, relativistic effects cannot be neglected[19] in the formation of solitary waves when the speed of particles is comparable to that of light. For example, ions with very high speed are frequently observed in the solar atmosphere and interplanetary space. High-energy ion beams also occur in the plasma sheet boundary layers of the Earth's atmosphere and in the Van Allen radiation belts.[20] Another case that may be considered relativistic is the presence of cold electron beams and associated BEN observed upstream of the Earth's bow shock.[19] The beam speeds can be of the order $0.15c$, where c is the speed of light. However, not much theoretical work has been done for nonlinear propagation of EASWs in relativistic plasma. Mace *et al.*[21] and Sah and Goswami[22,23] studied EASWs in weak relativistic plasma. But in their studies the distribution of hot electrons was taken to be Maxwellian. In the present paper, the propagation of EASWs in a relativistic (unmagnetized) plasma, consisting of relativistic ions, cold relativistic electrons, and nonthermal hot electrons is investigated. The parametric range considered here is valid for the auroral zone. This study is of importance because of the fact that high-speed electrons have an influence on the excitation of various kinds of nonthermal waves in interplanetary space and Earth's magnetosphere.[24,25] To study EASWs with nonthermal distribution of electrons in relativistic plasma, Sagdeev's[26] pseudopotential approach has been used. Here we study the relativistic effects on EASWs in an unmagnetized three-component plasma consisting of nonthermal hot electrons, cold relativistic electrons, and relativistic ions. Also the role of α, the nonthermal parameter, on the formation of EASWs has been investigated numerically. The organization of the paper is as follows. In Sec. II the basic equations are given for electron acoustic solitary waves. In Sec. III we derive Sagdeev's pseudopotential. Section IV is kept for discussion and results, while Sec. V is kept for the conclusion.

II. BASIC EQUATIONS

We consider a homogeneous, unmagnetized three-component plasma consisting of nonthermal hot electrons, cold relativistic electrons, and relativistic ions. The basic sys-

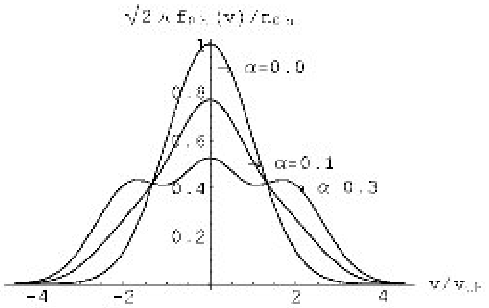

FIG. 1. Plot of $\sqrt{2\pi}f_{oh}(v)/n_{0h}$ against $v/v_{\rm th}$ for different values of α [see Eq. (6)].

tem of equations for one-dimensional propagation of nonlinear EAWs can be written as

$$\frac{\partial n_c}{\partial t}+\frac{\partial}{\partial x}(n_c v_c)=0, \tag{1}$$

$$\frac{\partial}{\partial t}(\gamma_c v_c)+u_c\frac{\partial}{\partial x}(\gamma_c v_c)+3\sigma_c n_c\frac{\partial n_c}{\partial x}=\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0, \tag{3}$$

$$\frac{\partial}{\partial t}(\gamma_i v_i)+v_i\frac{\partial}{\partial x}(\gamma_i v_i)+\frac{3\sigma_i}{m_i}n_i\frac{\partial n_i}{\partial x}=-\frac{1}{m_i}\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_h+n_c-n_i, \tag{5}$$

where

$$\gamma_c=\left(1-\frac{v_c^2}{c^2}\right)^{-1/2},\quad \gamma_i=\left(1-\frac{v_i^2}{c^2}\right)^{-1/2},\quad \sigma_c=\frac{T_c}{T_h},$$

$$\sigma_i=\frac{T_i}{T_h}.$$

In the earlier equations, the suffixes c and i correspond to cold electrons and ions, respectively. The densities are normalized to the total unperturbed density $n_0=n_{0c}+n_{0h}=n_{0i}$, velocities to the thermal velocity of hot electrons, $v_{\rm th}=\sqrt{T_h/m_e}$, and the potential to T_h/e. Time and space variables are normalized, respectively, to the inverse of electron plasma frequency $\omega_{pe}^{-1}=\sqrt{m_e/4\pi n_0 e^2}$ and the hot electron Debye length $\lambda_{Dh}=\sqrt{T_h/4\pi n_0 e^2}$, respectively. As electrons are assumed to be nonthermally distributed, to model the electron distribution with a population of fast particles we choose the distribution function after Cairns *et al.*,[17]

$$f_{0h}(v)=\frac{n_{0h}}{\sqrt{2\pi v_{\rm th}^2}}\frac{\left(1+\frac{\alpha v^4}{v_{th}^4}\right)}{(1+3\alpha)}\exp\left(-\frac{v^2}{2v_{\rm th}^2}\right), \tag{6}$$

where n_{0h} is the hot electron density, $v_{\rm th}$ is the thermal speed of the hot electrons, and α is a parameter that determines the population of energetic nonthermal electrons. α essentially measures the deviation of $f_{0h}(v)$ given in (6) from the Maxwellian case. In Fig. 1, $f_{0h}(v)/n_{0h}$ is plotted to show how the Gaussian form is deformed when $\alpha\neq 0$. The distribution of electrons in the presence of nonzero potential can be found by replacing $v^2/v_{\rm th}^2$ by $v^2/v_{\rm th}^2-2\phi$. Thus integration over the resulting distribution function gives the following expression for the hot electron density:

$$n_h=n_{0h}(1-\beta\phi+\beta\phi^2)\exp(\phi), \tag{7}$$

where $\beta=\frac{4\alpha}{1+3\alpha}$.

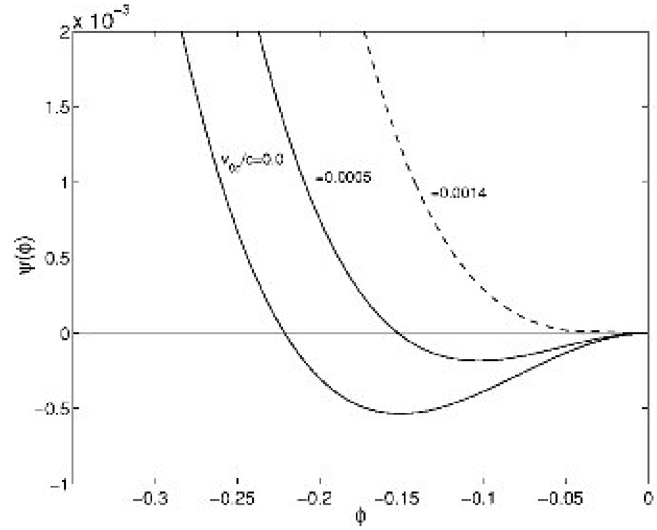

FIG. 2. Plot of $\psi(\phi)$ vs ϕ for different values of v_{0c}/c. The other parameters are $n_{0c}=0.35$, $n_{0h}=0.65$, $V=1.05$, $\sigma_c=\frac{T_c}{T_h}=0.01=\sigma_i=\frac{T_i}{T_h}$, $\alpha=0.1$, $v_{0i}/c=0.001$, $m_i=1836$.

It is clear that (7) expresses the isothermally distributed electrons when $\beta=0$ (i.e., $\alpha=0$).

III. PSEUDOPOTENTIAL APPROACH

To obtain a traveling wave solution, we make all the dependent variables depend on a single independent variable

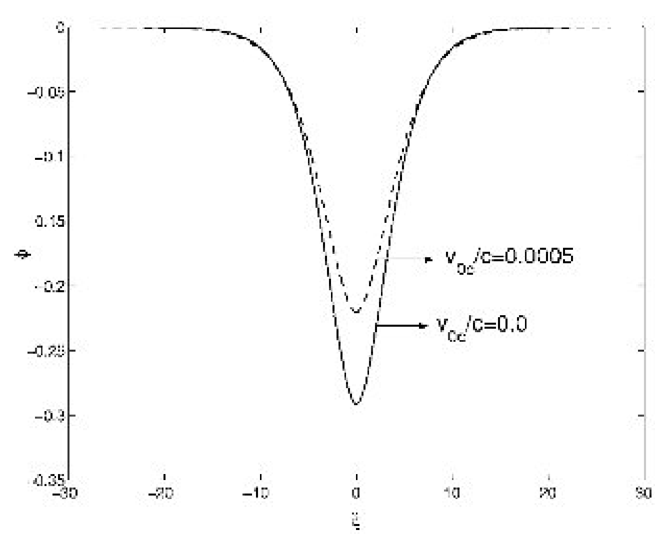

FIG. 3. Plot of ϕ against ξ for different values of v_{0c}/c, where $V=1.1$, other parameters being the same as those in Fig. 2.

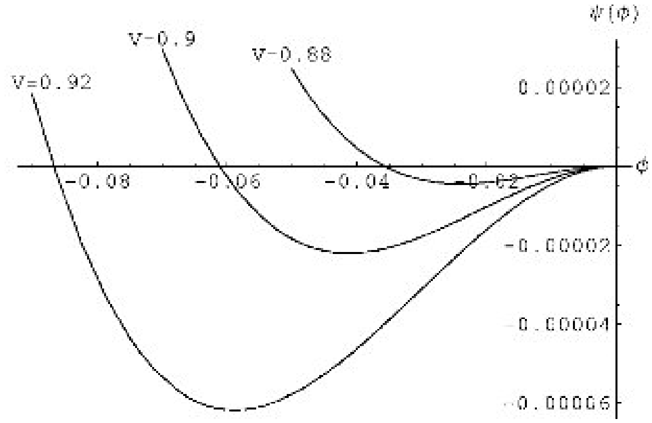

FIG. 4. Plot of $\psi(\phi)$ vs ϕ for different values of V, the soliton velocity, where $v_{0c}/c=0.001$, $\alpha=0.01$, and the other parameters being the same as those in Fig. 2.

$\xi=x-Vt$, where V is the velocity of a solitary wave. Now Eqs. (1)–(4) can be written as

$$-V\frac{dn_c}{d\xi}+\frac{d}{d\xi}(n_c v_c)=0, \tag{8}$$

$$-V\frac{d}{d\xi}(\gamma_c v_c)+v_c\frac{d}{d\xi}(\gamma_c v_c)+3\sigma_c n_c\frac{\partial n_c}{\partial\xi}=\frac{d\phi}{d\xi}, \tag{9}$$

$$-V\frac{dn_i}{d\xi}+\frac{d}{d\xi}(n_i v_i)=0, \tag{10}$$

$$-V\frac{d}{d\xi}(\gamma_i v_i)+v_i\frac{d}{d\xi}(\gamma_i v_i)+\frac{3\sigma_i}{m_i}n_i\frac{\partial n_i}{\partial\xi}=-\frac{1}{m_i}\frac{d\phi}{d\xi}. \tag{11}$$

The integration of the above equations yield the following:

$$n_c=\frac{n_{0c}(v_{0c}-V)}{v_c-V}, \tag{12}$$

$$\phi=\gamma_c(c^2-Vv_c)-\gamma_{0c}(c^2-Vv_{0c})+\frac{3\sigma_c}{2}(n_c^2-n_{0c}^2), \tag{13}$$

$$n_i=\frac{v_{0i}-V}{v_i-V}, \tag{14}$$

$$-\frac{\phi}{m_i}=\gamma_i(c^2-Vv_i)-\gamma_{0i}(c^2-Vv_{0i})+\frac{3\sigma_i}{2m_i}(n_i^2-1), \tag{15}$$

where

$$\gamma_{0c}=\left(1-\frac{v_{0c}^2}{c^2}\right)^{-\frac{1}{2}} \quad \text{and} \quad \gamma_{0i}=\left(1-\frac{v_{0i}^2}{c^2}\right)^{-\frac{1}{2}}.$$

In deriving Eqs. (12)–(15), we have used the following-boundary conditions: $v_c\to v_{0c}$, $n_c\to n_{0c}$, $v_i\to v_{0i}$, $n_i\to 1$, and $\phi\to 0$ as $\xi\to\pm\infty$.

To obtain the pseudopotential $\psi(\phi)$, we notice that Eq. (5) can be expressed as

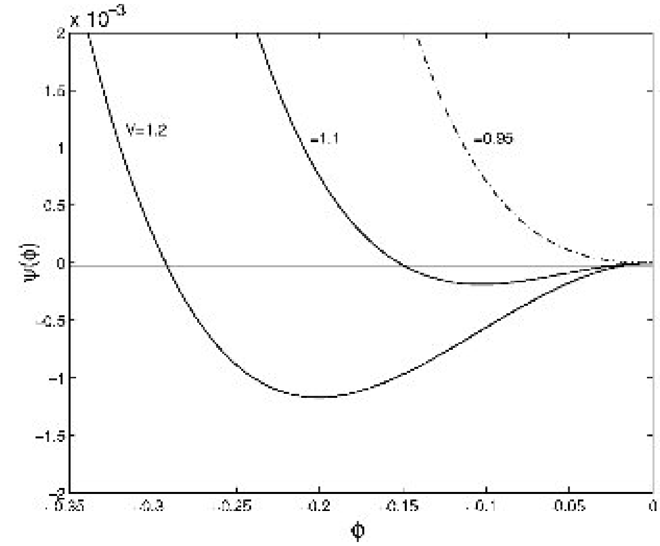

FIG. 5. Plot of $\psi(\phi)$ vs ϕ for different values of V, the soliton velocity, where $v_{0c}/c=0.001$ and the other parameters being the same as those in Fig. 2.

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2=n_{0h}[(1+3\beta)exp(\phi)-3\beta\phi\, exp(\phi)+\beta\phi^2 exp(\phi)]+\int\frac{n_{0c}(v_{0c}-V)}{v_c-V}d\phi-\int\frac{v_{0i}-V}{v_i-V}d\phi. \tag{16}$$

The above equation can be written as

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+\psi(\phi)=0, \tag{17}$$

where $\psi(\phi)$, the pseudopotential, is given by

$$\begin{aligned}\psi(\phi)=&\,n_{0h}[(1+3\beta)-(1+3\beta-3\beta\phi+\beta\phi^2)\exp(\phi)]\\&+n_{0c}(v_{0c}-V)\left\{\frac{v_{0c}}{\sqrt{1-\frac{v_{0c}^2}{c^2}}}-\frac{v_c}{\sqrt{1-\frac{v_c^2}{c^2}}}\right\}\\&+\sigma_c n_{0c}^3\left\{1-\frac{(v_{0c}-V)^3}{(v_c-V)^3}\right\}+m_i(v_{0i}-V)\\&\times\left\{\frac{v_{0i}}{\sqrt{1-\frac{v_{0i}^2}{c^2}}}-\frac{v_i}{\sqrt{1-\frac{v_i^2}{c^2}}}\right\}+\sigma_i\left\{1-\frac{(v_{0i}-V)^3}{(v_i-V)^3}\right\}.\end{aligned} \tag{18}$$

In deriving Eq. (18), the following boundary conditions were used: $n_c\to n_{0c}$, $n_i\to 1$, $v_c\to v_{0c}$, $v_i\to v_{0i}$, $\phi\to 0$, and $\frac{d\phi}{d\xi}\to 0$ at $\xi\to\pm\infty$.

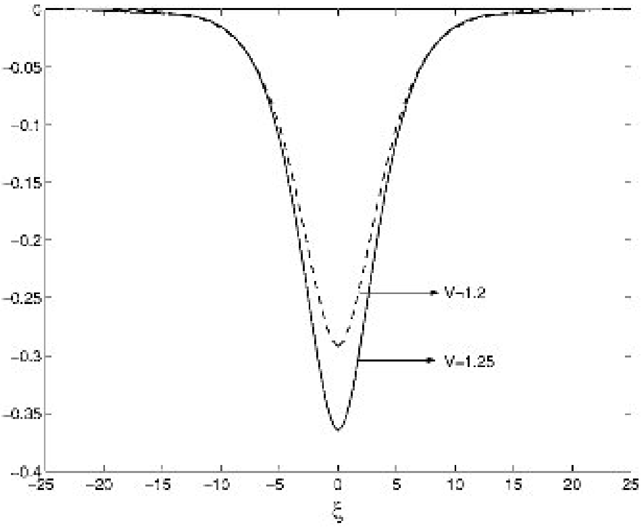

FIG. 6. Plot of ϕ against ξ for different values of V, where $v_{0c}/c=0.001$ and other parameters being the same as in Fig. 2.

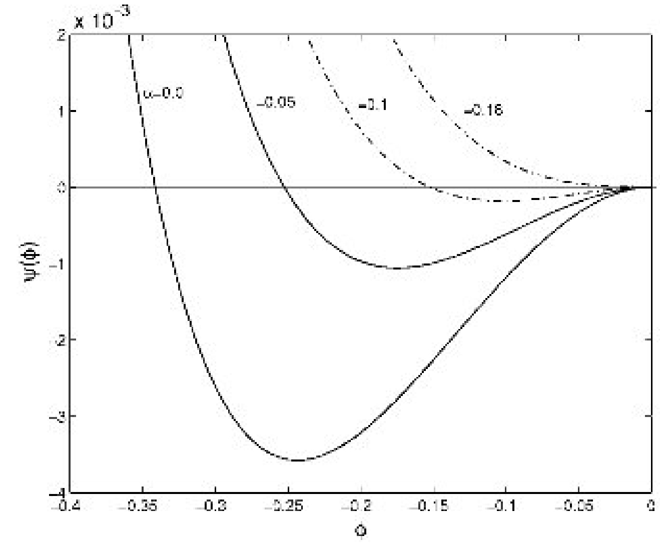

FIG. 7. Plot of $\psi(\phi)$ vs ϕ for different values of α, the nonthermal parameter, where $V=1.1$ and the other parameters being the same as those in Fig. 5.

Now from (13) we get a sixth degree equation in v_c. Exact analytical solution of this equation is not possible. However, one can obtain v_c keeping terms up to $O(1/c^2)$ for the weakly relativistic case,

$$v_c = v_{cn} + \frac{\eta_1}{c^2}, \tag{19}$$

where

$$v_{cn} = V + \left\{ \frac{(V^2+2\phi+2c_1) - \sqrt{(V^2+2\phi+2c_1)^2 - 12\sigma_c n_{0c}(v_{0c}-V)^2}}{2} \right\}^{1/2}$$

and

$$c_1 = \frac{v_{0c}^2}{2} - Vv_{0c} + \frac{3V^2\sigma_c n_{0c}^2}{2(v_{0c}-V)^2}, \quad \eta_1 = \frac{(v_{cn}-V)\left(\frac{3}{4}v_{cn}^4 + v_{cn}^3 + Vv_{0c}^3 - \frac{3}{4}v_{0c}^4\right)}{2(V^2+2\phi+2c_1) - 4(v_{cn}-V)^2}.$$

Similarly, from (15) one gets

$$v_i = v_{in} + \frac{\eta_2}{c^2}, \tag{20}$$

where

$$v_{in} = V + \left\{ \frac{\left(V^2 - \frac{2\phi}{m_i} + 2c_2\right) - \sqrt{\left(V^2 - \frac{2\phi}{m_i} + 2c_2\right)^2 - \frac{12\sigma_i}{m_i}(v_{0i}-V)^2}}{2} \right\}^{1/2}$$

and

$$c_2 = \frac{v_{0i}^2}{2} - Vv_{0i} + \frac{3V^2\sigma_i}{2m_i(v_{0i}-V)^2},$$

$$\eta_2 = \frac{(v_{in}-V)\left(\frac{3}{4}v_{in}^4 + v_{in}^3 + Vv_{0i}^3 - \frac{3}{4}v_{0i}^4\right)}{2\left(V^2 - \frac{2\phi}{m_i} + 2c_2\right) - 4(v_{in}-V)^2}.$$

Equations (18)–(20) are the mai...

For solitary wave solutions, the following conditions must be satisfied:

$$\psi(\phi) = 0, \quad \left(\frac{\partial\psi}{\partial\phi}\right)_{\phi=0} = 0, \quad \text{and} \quad \left(\frac{\partial^2\psi}{\partial\phi^2}\right)_{\phi=0} < 0,$$

and $\psi(\phi) < 0$ for ϕ lying between 0 and ϕ_m, i.e., either for ... $< \phi_m$ (compressive ... $_m < \phi < 0$ ...

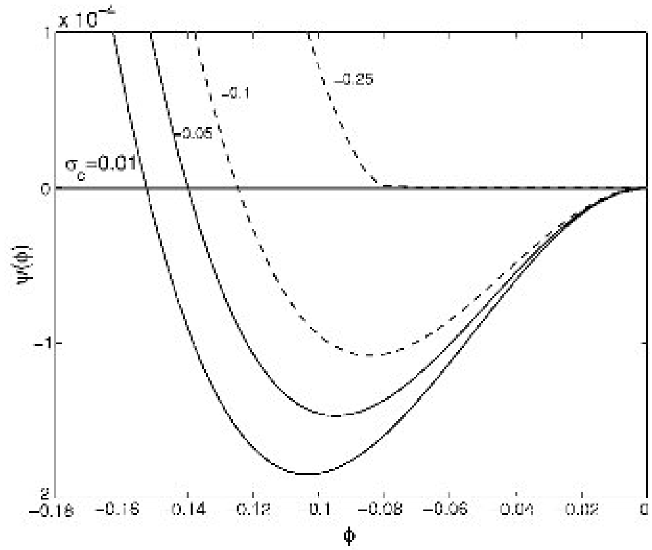

FIG. 8. Plot of $\psi(\phi)$ vs ϕ for different values of σ_c, cold to hot electron temperature ratio, where $V=1.1$ and the other parameters being the same as those in Fig. 5.

IV. RESULTS AND DISCUSSION

Here we have considered the problem of propagation of electron acoustic solitary waves in an unmagnetized collisionless plasma consisting of nonthermal hot electrons, cold relativistic electrons, and relativistic ions. The exact Sagdeev's pseudopotential $\psi(\phi)$ has been derived for this problem taking into account the relativistic effect. We have seen that when drift velocity $v_{0c}\to 0$, $v_{0i}\to 0$, and relativistic parameters $\gamma_c \to 1$ and $\gamma_i \to 1$, our results completely agree with the result of Singh and Lakhina.[18] The typical parameters considered here for numerical evaluation are as follows: cold electron density, $n_{0c}=0.35$, hot electron density, $n_{0h}=0.65$, cold to hot electron temperature ratio, $\sigma_c=\frac{T_c}{T_h}=0.01=\sigma_i=\frac{T_i}{T_h}$ and $\alpha=0.1$, $m_i=1836$. In Fig. 2, $\psi(\phi)$ is plotted against ϕ for different values of v_{0c}/c. It is seen that the amplitude of the solitary waves gradually decreases with the increase of v_{0c}/c. It is also seen that when v_{0c}/c crosses a certain limit, the soliton ceases to exist. This upper limit depends of course on other parameters. However, this limit is always small for the weak relativistic hypothesis to be valid. In Fig. 3, the solitary wave solutions are plotted for this case. For very small values of α, solitary waves exist for $V<1$, which is shown in Fig. 4, and in some cases for $V>1$, depending on the other parameters. However, when α is not too small, solitary waves would exist for $V>1$, which is shown in Fig. 5. This implies that the range of soliton velocity for which solitary wave solutions can exist gets modified when nonthermal electron distribution is considered. This finding is consistent with the one obtained in the case of nonrelativistic plasma. In Fig. 6 we plot the solitary wave solution for different values of V. Figure 7 depicts the plot of $\psi(\phi)$ vs ϕ for different values of α, the parameter for nonthermal electrons. Here it is observed that the amplitude of solitary waves decreases with an increase of α and there exists an upper limit of α, beyond which the soliton solutions are not found in the presence of the relativistic effect. It may be mentioned that inclusion of cold electrons influences the amplitude of solitary waves as observed by Singh and Lakhina.[18] Figure 8 shows the variation of pseudopotential $\psi(\phi)$ with normalized potential ϕ, for several values of σ_c. It is observed that the amplitude of solitary waves decreases with an increase of σ_c, and there is a critical value of σ_c (in the present case it is 0.25) beyond which a solitary structure will not exist.

V. CONCLUSION

In this paper, we have studied the problem of one-dimensional electron acoustic solitons in relativistic plasma composed of relativistic ions, cold relativistic electrons, and nonthermal hot electrons. Sagdeev's pseudopotential approach has been used to find exact large-amplitude solitary wave solutions. An exact pseudopotential has been derived in the presence of the relativistic effect and has been studied numerically for the effect of parameters like v_{0c}, V, α, and σ_c on the existence of solitons. The parametric range taken here is valid for the auroral zone. It is seen that both the relativistic effect and the nonthermality of the electron significantly modifies the region of EASWs. For α, not too small solitary waves can exist for $V>1$. The results obtained here may be useful in understanding the features of EASWs in space and laboratory plasma.

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