Projection properties of some orthogonal arrays

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Abstract

In factor screening experiments, one generally starts with a large pool of potentially important factors. However, often only a few of these are really active. Under this assumption of effect sparsity, while choosing a design for factor screening, it is important to consider projections of the design on to smaller subsets of factors and examine whether the projected designs allow estimability of some interactions along with the main effects. While the projectivity properties of symmetric 2-level and a few 3-level fractional factorial designs represented by orthogonal arrays have been studied in the literature, similar studies in respect of asymmetric or, mixed level factorials seems to be lacking. In this paper, we initiate work in this direction by providing designs with good projectivity properties for asymmetric factorials of the type $t \times 2^m$ based on orthogonal arrays. We also note that the results of Cheng (1995) regarding the projectivity of symmetric two-symbol orthogonal arrays do not necessarily extend to arrays with more than two symbols.

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1. Introduction

In the initial stage of experimentation, one generally considers a large number of factors that might be potentially important. Among these, often only a few have large effects or, are active.

Under this assumption of effect sparsity, studying the properties of projections of the design on to small subsets of factors becomes important as, the projected designs might allow the estimability of certain interactions among the projected factors, apart from that of the main effects. According to Box and Tyssedal (1996), a fractional factorial design is said to have projectivity p if in every subset of p factors, a complete factorial, with possibly some repeated runs is produced. Clearly, if a design has projectivity p and the number of active factors is at most p, the projection of the design on to the active factors allows the estimability of all factorial effects involving the active factors. The property of projectivity can be viewed as an extension of the strength of an orthogonal array. An orthogonal array, $\mathrm{OA}(N, n, m_1 \times \cdots \times m_n, g)$ of strength g, $2 \leqslant g < n$ is an $N \times n$ matrix, having $m_i \geqslant 2$ distinct symbols in the ith column, $i = 1, \ldots, n$, such that in any $N \times g$ submatrix, all possible combinations of the symbols occur equally often as a row. When $m_1 = \cdots = m_n = m$, the orthogonal array is called symmetric and is denoted by $\mathrm{OA}(N, n, m, g)$; otherwise, the array is called asymmetric. An $\mathrm{OA}(N, n, m_1 \times \cdots \times m_n, g)$ represents an N-run fractional factorial design for an asymmetric or mixed level $m_1 \times \cdots \times m_n$ experiment, with symbols representing the levels of the factors, columns corresponding to factors and rows representing the runs or, treatment combinations. Similarly, a symmetric orthogonal array $\mathrm{OA}(N, n, m, g)$ represents an N-run fractional factorial design for a symmetric m^n experiment. A fractional factorial plan represented by such an orthogonal array obviously has projectivity g.

In the case of symmetric factorials, an important class of symmetric orthogonal arrays give rise to the so-called regular fractional factorial designs. It is well known that a regular fractional factorial design of resolution R is an orthogonal array of strength $R-1$. Such a regular design has projectivity $R-1$ but cannot have projectivity greater than $R-1$. However, it is possible for a non-regular fractional factorial design represented by an orthogonal array of strength g to have projectivity greater than g. This fact was first observed by Lin and Draper (1992) and Box and Bisgaard (1993), who found that certain Plackett–Burman plans (Plackett and Burman, 1946) for 2-level symmetric factorials have projectivity three, even though it is known that such plans are represented by orthogonal arrays of strength two. Cheng (1995) proved that as long as N is not a multiple of 8, a fractional factorial design represented by an $\mathrm{OA}(N, n, 2, 2)$ has projectivity three. This result of Cheng (1995) extends the patterns observed by Lin and Draper (1992) and Box and Bisgaard (1993) on small Plackett–Burman designs through computer searches. The result of Box and Tyssedal (1996) is a special case of the result of Cheng (1995). Cheng (1995) further proved that as long as N is not a multiple of 16, a fractional factorial design represented by an $\mathrm{OA}(N, n, 2, 3)$ of strength three has projectivity four. However, these results of Cheng do not necessarily extend for (symmetric) orthogonal arrays with more than two symbols, as demonstrated by a counter-example in the next section.

Some non-regular fractions also exhibit a hidden projection property. A fractional factorial plan is said to have hidden projection property of order p if it allows the estimability of the main effects and all or, some two-factor interactions when projected on to any subset of p factors. For example, the 12-run Plackett–Burman plan has projectivity three, but when projected on to any four factors, has the property that all four main effects and two-factor interactions among the four are estimable, when higher-order effects are assumed negligible; see Lin and Draper (1992) and Wang and Wu (1995).

Further theoretical results concerning the projectivity and hidden projection properties of fractional factorial designs based on non-regular orthogonal arrays can be found in Cheng (1998) and Bulutoglu and Cheng (2003). However, a major portion of the available results on projection properties of designs based on orthogonal arrays concern 2-level symmetric designs. Some specific 3-level symmetric designs exhibiting hidden projection properties have been reported by Wang and Wu (1995) and Bulutoglu and Cheng (2003). Projection properties of asymmetric fractional factorial designs do not seem to have received any attention so far.

Asymmetric or, mixed level factorials are inevitable in many experimental situations and thus, it is important to study the projectivity and hidden projection properties of asymmetric fractional factorial designs as well. In this paper, we initiate work in this direction. In Sections 3–5, we provide fractional factorial designs with good projectivity properties for experiments of the type $t \times 2^m$, $t \geqslant 3$. The designs are represented by orthogonal arrays of strength two.

2. A counter-example

As stated in the previous section, Cheng (1995) showed that as long as N is not a multiple of m^{g+1}, an OA(N, n, m, g) with $m = 2$, $n \geqslant g + 2$ and $g = 2, 3$ has projectivity $g + 1$. A natural question then is: does the result hold even if $m > 2$ and/or $g \neq 2, 3$? We show via a counter-example that the result cannot be extended if $m = 3$.

Consider a symmetric orthogonal array, OA(36, 12, 3, 2), displayed in transposed form in Table 1.

Here, $N = 36$, $n = 12$, $m = 3$, $g = 2$, so that the conditions stated above hold. However, the design does not have projectivity three as can be observed by considering, for example, columns 1, 3, 4 of the above orthogonal array. Therefore, it appears that for an arbitrary orthogonal array

Table 1
An OA(36, 12, 3, 2)

Columns	Runs					
1	012012	012120	120012	012120	012201	201012
2	012012	012012	201012	201012	201012	012120
3	012012	120012	012201	120201	012012	120012
4	012012	201201	012120	012012	120120	012012
5	012120	201201	012012	120120	201012	201201
6	012120	201120	201120	201201	201201	120012
7	012120	012012	201201	012201	120120	201201
8	012120	120201	120201	201012	012201	012201
9	012201	120201	120012	012201	201120	120120
10	012201	120012	012120	201120	120201	201120
11	012201	201120	201201	120120	012120	012120
12	012201	012120	120120	120012	120012	120201

OA(N, n, m, g) to have projectivity greater than g, additional conditions on the orthogonal array need to be imposed. The problem of finding such conditions remains open.

A closer look at the above orthogonal array, however, reveals that there is a subset of six columns, forming an orthogonal array OA(36, 6, 3, 2), which has projectivity three. These columns are 1, 2, 3, 5, 6, 8. When projected on to any three columns, there are 20 distinct projected designs. Each of these projected designs consists of a complete 3^3 factorial and a $\frac{1}{3}$ replicate of a 3^3 factorial. However, the $\frac{1}{3}$rd fraction in the projected designs are not the same. In nine designs, the fraction is defined by $(ABC)_0$, in five designs, the fraction is defined by $(ABC)_1$ and in the remaining designs, it is defined by $(ABC)_2$, where A, B, C are the three factors involved in the projected design and for $i = 0, 1, 2$, $(ABC)_i$ means that the defining relation is $x_1 + x_2 + x_3 = i \pmod 3$, with x_1, x_2, x_3, respectively, denoting the levels of A, B, C.

3. Asymmetric designs with projectivity three and four

Non-regular fractional factorial designs based on symmetric orthogonal arrays of strength two can have projectivity greater than two. A similar phenomenon is observed in the case of some asymmetric fractional factorial designs based on orthogonal arrays. We begin with an example of a 12-run fractional factorial design represented by an orthogonal array OA($12, 4, 3 \times 2^3, 2$) of strength two. The design in transposed form is displayed in Table 2.

It can be verified that this design has projectivity three, even though the strength of the orthogonal array is two. Note that in an orthogonal array OA($12, m+1, 3 \times 2^m, 2$), the maximum value of m is four. However, the orthogonal array OA($12, 5, 3 \times 2^4, 2$), reported by Wang and Wu (1992), does not have projectivity three.

The construction method adopted for the 12-run example given above can be generalized. Let H_n be a Hadamard matrix of order n. Recall that a Hadamard matrix H_n is a square matrix of order n with entries ± 1 such that $H_n H_n' = nI_n$, where I_n is an identity matrix of order n and a prime over a matrix denotes its transpose. It is well known that H_n exists for $n = 1, 2$ and a necessary condition for the existence of an H_n, $n > 2$ is that $n \equiv 0 \pmod 4$. A positive integer n is called a Hadamard number if H_n exists. It is easy to see that a Hadamard matrix remains so when any of its rows or columns is multiplied by -1. In view of this, one can always write a Hadamard matrix such that its first column contains only $+1$'s.

Let H_n, $n \geqslant 4$ exist. Write $H_n = [\mathbf{1}_n \vdots B]$, where $\mathbf{1}_n$ is an $n \times 1$ vector of unities and B is an $n \times (n-1)$ matrix of the remaining columns of H_n. It is easy to see that B is an OA($n, n-1, 2, 2$). Let F_1 be a t-level factor and suppose $t = 2m + 1$ is odd, where $m \geqslant 1$ is an integer. Let the levels of

Table 2
An OA($12, 4, 3 \times 2^3, 2$) (transposed)

$$\begin{bmatrix} 0000 & 1111 & 2222 \\ 0011 & 1100 & 0011 \\ 0101 & 1010 & 0101 \\ 0110 & 1001 & 0110 \end{bmatrix}$$

F_1 be coded as $0, 1, \ldots, 2m$. Consider the design

$$d_1 = \begin{bmatrix} \mathbf{0}_n & \vdots & B \\ \mathbf{1}_n & \vdots & B \\ \vdots & & \\ m\mathbf{1}_n & \vdots & B \\ (m+1)\mathbf{1}_n & \vdots & \bar{B} \\ \vdots & & \\ 2m\mathbf{1}_n & \vdots & \bar{B} \end{bmatrix},$$

where $\bar{B}$ is an $n \times (n-1)$ matrix obtained by interchanging the symbols in B and $\mathbf{0}_n$ is an $n \times 1$ null vector. Clearly, this is a fractional factorial design for a $t \times 2^{n-1}$ experiment and the rows of d_1 form an orthogonal array $\mathrm{OA}(m, n, t \times 2^{n-1}, 2)$. To see that d_1 has projectivity three, first observe that since

$$\begin{bmatrix} B \\ \bar{B} \end{bmatrix},$$

obtained by folding over an orthogonal array of strength two, is an orthogonal array, $\mathrm{OA}(2n, n-1, 2, 3)$, of strength three (see e.g., Dey and Mukerjee, 1999, p. 35), the design formed by any three of the last $(n-1)$ columns of d_1 has a complete 2^3 factorial plus some repeated runs. Now, consider the design formed by the t-level column, F_1 and any two of the remaining columns of d_1. Since the last $(n-1)$ columns of d_1 form an orthogonal array of strength two, from the method of construction of d_1, it is clear that the design formed by taking the first column F_1 and any two other columns of d_1 produces a complete $t \times 2^2$ factorial. Thus the claimed projectivity three of d_1 is established.

Next, let $t = 2m$ be even. Then, arguing as earlier, one can see that the design d_2, given below, also has projectivity three. In fact, d_2 is an orthogonal array $\mathrm{OA}(nt, n, t \times 2^{n-1}, 3)$ of strength three.

$$d_2 = \begin{bmatrix} \mathbf{0}_n & \vdots & B \\ \mathbf{1}_n & \vdots & B \\ \vdots & & \\ (m-1)\mathbf{1}_n & \vdots & B \\ m\mathbf{1}_n & \vdots & \bar{B} \\ (m+1)\mathbf{1}_n & \vdots & \bar{B} \\ \vdots & & \\ (2m-1)\mathbf{1}_n & \vdots & \bar{B} \end{bmatrix}.$$

We now show that under certain conditions, each of the designs d_1 and d_2 has projectivity *four*. Cheng (1995) showed that a fractional factorial design represented by an orthogonal array $OA(N, n, 2, 2)$ with $N \neq 0 \pmod 8$, $n \geqslant 4$ has projectivity three, and, an orthogonal array $OA(N, n, 2, 3)$ has projectivity four if $N \neq 0 \pmod{16}$, $n \geqslant 5$.

Since

$$\begin{bmatrix} B \\ \bar{B} \end{bmatrix}$$

is an orthogonal array of strength three, in view of the results of Cheng (1995), it follows that the designs formed by the last $(n-1)$ factors of each of d_1 and d_2 has projectivity four, provided $2n \neq 0 \pmod{16}$, $(n-1) \geqslant 5$, i.e., provided $n \neq 0 \pmod 8$, $n > 5$. Also, if $n \neq 0 \pmod 8$, then the design formed by the first factor at t levels and any three of the remaining factors in either d_1 or d_2 has projectivity four. We have therefore proved the following result.

Theorem 1. *Each of the designs d_1 and d_2 have projectivity three. Furthermore, if $n \neq 0 \pmod 8$, $n > 5$ then each of the designs d_1 and d_2 has projectivity four.*

For instance, starting from a Hadamard matrix of order $n = 12$ and taking $t = 3$, one can obtain a design for a 3×2^{11} factorial in 36 runs that has projectivity four.

4. Designs based on Paley matrices

Paley (1933) constructed a class of Hadamard matrices, called Paley matrices. For completeness, we describe this construction. Let $n \equiv 0 \pmod 4$ be such that $n-1$ is an odd prime power, say q. Let $\rho_0 = 0, \rho_1, \ldots, \rho_{q-1}$ denote the elements of the finite or Galois field, $GF(q)$ of order q. Define a function $\chi : GF(q) \to \{-1, 0, 1\}$ as

$$\chi(x) = \begin{cases} 1 & \text{if } x = y^2 \text{ for some } y \in GF(q), \\ 0 & \text{if } x = 0, \\ -1 & \text{otherwise.} \end{cases}$$

Let $A = (a_{ij})$ be a $q \times q$ matrix, where $a_{ij} = \chi(\rho_i - \rho_j)$ for $i, j = 0, 1, \ldots, q-1$ and

$$P_n = \begin{bmatrix} 1 & -1_q' \\ 1_q & A + I_q \end{bmatrix}.$$

Then, P_n is a Hadamard matrix of order $n = q + 1$ and is known as a Paley matrix of the first kind. By deleting the first column of all ones from P_n, we get an $OA(n, n-1, 2, 2)$, which we call a Paley design and denote it by d_n. Note that for $n = 12$, the Paley design d_{12} is the 12-run Plackett–Burman design.

Bulutoglu and Cheng (2003) recently proved that if d_n is a Paley design of size $n \geqslant 12$, then d_n has

(a) projectivity three and,
(b) hidden projection property of order four, that is, in its projection on to any four factors, the main effects and two-factor interactions are estimable under the assumption of absence of higher-order interactions.

We use a Paley design $d_n, n \geqslant 12$ to get a design for a $t \times 2^{n-1}$ experiment, much the same way as d_1 or d_2 is constructed in the previous section. Let the levels of a t-level factor be coded as $0, 1, \ldots, t-1$. Consider the design

$$d_0 = \begin{bmatrix} \mathbf{0}_n & d_n \\ \mathbf{1}_n & d_n \\ 2\mathbf{1}_n & d_n \\ \vdots & \\ (t-1)\mathbf{1}_n & d_n \end{bmatrix}.$$

Then, we have the following result.

Theorem 2. (i) *The design d_0 has projectivity three and,* (ii) *in its projection on to any four factors, the main effects and two-factor interactions are estimable when all higher-order interactions are assumed to be absent.*

Proof. Since d_n is an orthogonal array of strength two, by the construction of d_0 and result (a) above of Bulutoglu and Cheng (2003) about the projectivity of d_n, part (i) of the result follows. Again, by result (a) of Bulutoglu and Cheng (2003), the design formed by the first factor at t levels and any three of the remaining factors has a $t \times 2^3$ factorial plus some repeated runs. The rest of the result (ii) follows from part (b) of Bulutoglu and Cheng (2003). □

Remark. In each of the above constructions of d_0, d_1 and d_2, suppose t is not a prime and, let $t = t_1 t_2 \ldots t_u$, where, for $i = 1, \ldots, u$, $t_i \geqslant 2$. Then, the t-level factor can be replaced by u factors, having levels $t_1, \ldots, t_u$ to get a design for a $t_1 \times \cdots \times t_u \times 2^{n-1}$ experiment having the same properties as d_i, $i = 0, 1, 2$. For example, let $n = 12$ and $t = 6$ in d_0. Then, following the construction method described above, we get a 72-run design for a 6×2^{11} experiment. Replacing the 6-level factor by two factors at 3 and 2 levels, respectively, one gets a 72-run design for a 3×2^{12} experiment having properties stated in Theorem 2.

The construction procedure described in this section can be adopted to get designs with hidden projection properties of higher order. Suppose A is an orthogonal array, $\mathrm{OA}(N, n, 2, 3)$ of strength three. Then, from Cheng (1995), we know that if $N \neq 0 \pmod{16}$, $n \geqslant 5$, then the fractional factorial design represented by A has projectivity four. Furthermore, Cheng (1998) has shown that when $N \neq 0 \pmod{16}$, the design has hidden projection property of order five, that is, in the projection on to any five factors, all the main effects and two-factor interactions are estimable under the assumption that the higher-order interactions are absent. Utilizing these facts and the construction procedure of d_0 with d_n replaced by A, we get a design, say d_0^*, with hidden projection property of order five for an experiment of the type $t \times 2^m$.

5. Another class of designs with projectivity three

In this section, we give a family of designs for 4×2^m factorials represented by orthogonal arrays of strength two and having projectivity three. Let H_n, $n \geqslant 4$, $n \neq 0 \pmod 8$, be a Hadamard matrix. As before, write $H_n = [\mathbf{1}_n \vdots B]$. Let B be partitioned as $B = [\boldsymbol{c} \vdots C]$, where $\boldsymbol{c}$ is any column of

B and C is an $n \times (n-2)$ matrix of the remaining columns of B. Consider the design

$$d_3 = \begin{bmatrix} \boldsymbol{c} & C & C \\ \boldsymbol{c} & C & -C \\ 3\boldsymbol{c} & C & C \\ 3\boldsymbol{c} & C & -C \end{bmatrix}.$$

It follows from a result of Dey and Ramakrishna (1977) that the rows of d_3 form an orthogonal array OA$(4n, 2n-3, 4 \times 2^{2n-4}, 2)$ of strength two. We then have the following result.

Theorem 3. *The design d_3 has projectivity three.*

Proof. Consider the first half of the 2-level part of d_3, that is, the part given by

$$D = \begin{bmatrix} C & C \\ C & -C \end{bmatrix}.$$

Since both C and $-C$ are orthogonal arrays of strength two and $n \neq 0 \pmod 8$, it follows from Cheng (1995) that the designs consisting of the columns

$$\begin{bmatrix} C \\ C \end{bmatrix}$$

and

$$\begin{bmatrix} C \\ -C \end{bmatrix}$$

have projectivity three separately.

Next, let $\boldsymbol{\alpha}_i, \boldsymbol{\alpha}_j$ be two distinct columns of C. Then, considering these two columns from $[C', C']'$ and one column, say $\boldsymbol{\alpha}_k$, from $[C', -C']'$, we arrive at two cases: (a) $\boldsymbol{\alpha}_i = \boldsymbol{\alpha}_k$ (the case $\boldsymbol{\alpha}_j = \boldsymbol{\alpha}_k$ can be handled similarly). (b) $\boldsymbol{\alpha}_i, \boldsymbol{\alpha}_j \neq \boldsymbol{\alpha}_k$. That the design in case (b) has a complete 2^3 factorial follows from Cheng (1995). Consider case (a) now. Under the columns $\boldsymbol{\alpha}_i, \boldsymbol{\alpha}_j, \boldsymbol{\alpha}_k$ among the first n rows of D given by $[C, C]$, let $f(u, v, w)$ be the frequency of occurrence of the combination (u, v, w) as a row, where $u, v, w = \pm 1$. Then,

$$f(-1,-1,-1) = n/4 = f(-1,1,-1) = f(1,-1,1) = f(1,1,1).$$

Considering now the same columns among the last n rows of D given by $[C, -C]$, we have

$$f(-1,-1,1) = n/4 = f(-1,1,1) = f(1,-1,-1) = f(1,1,-1).$$

This shows that the design formed by taking two columns from $[C', C']'$ and one from $[C', -C']'$ has a complete 2^3 factorial. A similar argument holds when one column is chosen from $[C', C']'$ and two from $[C', -C']'$. Thus, the 2-level part of d_3 has projectivity three.

Let d_{30} denote the design formed by the first $2n$ rows of d_3, i.e.,

$$d_{30} = \begin{bmatrix} \boldsymbol{c} & C & C \\ \boldsymbol{c} & C & -C \end{bmatrix}.$$

Consider the sub-design of d_{30} formed by $[\boldsymbol{c} \;\; C]$, which has projectivity three. It follows then that the design formed by the column $\boldsymbol{c}$ and any two columns from C has projectivity three. Next, consider the sub-design formed by the first column of d_{30}, one column from $[C', C']'$ and one column from $[C', -C']'$. Arguing as in cases (a) and (b) above, it can be seen that this also has projectivity three. Applying the same arguments to the last $2n$ rows of d_3 shows that the sub-design formed by the first column and any two other columns of d_3 has projectivity three. This completes the proof.

The design d_3 is 'better' than the design d_2 (which also in particular yields a design for a 4×2^m experiment) in terms of accommodating more number of factors. On the other hand, under the condition $n \neq 0 \pmod 8$, assumed while constructing d_3, the design d_2 has projectivity four while d_3 has projectivity three only. For example, with $n = 12$, using d_2 one gets a design for a 4×2^{11} experiment in 48 runs with projectivity four, while with d_3, one gets a 48-run design for a 4×2^{20} experiment with projectivity three.

Instead of starting from an arbitrary Hadamard matrix H_n with $n \neq 0 \pmod 8$, one can start with a Paley matrix of order $n \geqslant 12$ to construct the design d_3, that is, in d_3 replace $\boldsymbol{c}$ by any column of the Paley design d_n and C by the remaining $(n-2)$ columns of d_n. Then, from Bulutoglu and Cheng (2003), it follows that d_3 has projectivity three. In using the Paley matrix, one does not need the condition $n \neq 0 \pmod 8$. However, a Paley matrix does not exist for every Hadamard number; for instance there is no Paley matrix for $n = 36$ and yet, H_{36} exists.

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