The p_T-Spectra of Some Non-pion Secondaries in High Energy NN to NA/AA Collisions and the Combinational Approach

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Abstract

In continuation of our perusal of the studies on transverse momentum spectra for the main varieties of secondaries from a consistent and comprehensive phenomenological approach, we propose to take up here — after a successful completion of reporting in detail the results (Ref.[4] in the text) on our analyses of the p_T-spectra of pions — the studies specially on production of kaons, protons and antiprotons in several proton-induced and nucleus-involved collisions at high energies. The measured data on inclusive cross sections of kaons, protons and antiprotons from the various published sources have here been assorted first. Next, data on the p_T-spectra of the specific secondaries produced in PP and $P\bar{P}$ reactions have been scanned and analyzed with the help of Hagedorn's model(HM). Thereafter a connector, named here the combinational approach(CA), has been constructed and used to analyze the data on p_T-spectra of some major category of non-pion secondaries produced in nucleus-nucleus(AA/AB) collisions at high energies. And these exercises have, finally, led to the modestly successful interpretations of a wide band of data with the revelation of some insightful physical aspects of high energy interactions. The limitations of the approach have also been precisely pointed out in the end.

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I. INTRODUCTION

The nature of transverse momentum[1, 2] spectra is quite interesting for the following reasons: Firstly, in the form of invariant cross section in terms of p_T, it offers a ready, reliable and very basic observable for both measurements, and for theorization to be attempted. Secondly, the p_T-spectra throw light on the particle production mechanism on the whole; they have also impact and indirect reflections on the predicted features of what is normally projected as the signatures of the conjectured quark-gluon plasma(QGP). Thirdly and finally, the expressions for invariant p_T-spectra lead us to define some other physically significant observables in the domain of particle physics, like the average multiplicity, average transverse momentum etc.

After pions, kaons of all varieties (with $K^{\pm}$ and $K^0/\bar{K}^0$) constitute the second most abundant species among the particle secondaries which are produced in multiparticle processes of hadron-hadron, hadron-nucleus or nucleus-nucleus interactions at high and very high energies. Secondly, kaons are the lightest 'strange' particles for which they have a special status. Besides, kaons are supposed to bear, as told above, some strong reflections on what are viewed as the QGP diagnostics. Lastly, the kaon-pion ratios and their energy-dependences play a very significant role in both the particle physics and non-accelerator physics sectors. So, analyzing the kaon production characteristics, especially the p_T-spectra, in the light of any model assumes a high degree of importance and comes as a challenging task as well. Furthermore, the production of proton-antiprotons, the first family-set of baryonic category, is also important from various physical considerations. The nature of the ratios $\bar{P}/P$ has a strong bearing on astroparticle physics. Besides, the generalized antiparticle to particle ratios are of importance in the study of matter-antimatter symmetry in the universe, since they provide the relative abundance of antiparticle productions[3].

Very recently, spurts of data on the various aspects of particle production from the CERN-SPS and the RHIC-BNL, studying high energy interaction properties in the collisions between $PbPb$ and $AuAu$ respectively, are available. And providing interpretations for both the previous measurements and these oncoming blizzards of new data is really a very significant work.

Our objective here is to try to describe and/or explain the nature of the transverse momentum (p_T)-spectra of kaons, protons and antiprotons in a large variety of proton-induced and nucleus-induced high energy interactions with the help of a combinational approach(CA), called also the grand combination of models (GCM). The present work is prompted by our previous successes in analyzing the p_T-spectra and rapidity-spectra[4, 5] of pions alone in a host of nuclear collisions at high and superhigh energies. In fact, taking up the task of interpreting the p_T-spectra of a set of non-pion secondaries produced in nuclear collisions becomes the logical imperative and the natural follow-up in maintaining a sequence of our studies and with the model of our choice in a self-complete way. In real terms, while proceeding with the present work, we would follow the trajectory of the Ref.[4] here more closely than the other one, as in the former the approach to the studies on p_T-spectra has been rationalized in a better way and much more consistent manner. We have consciously and carefully avoided here to adopt operationally any of the known popular/standard models and have also tried to escape, as far as possible, the buzzwords related to them. And we have singularly been guided by the results of experimental measurements on the nature of p_T-spectra alone in a very dispassionate and objective manner.

This paper is organized here as follows. In the next section (section II) we give the outline of the combinational approach which is to be taken up for this study and the sketch of the physical perspective which prompted us to proceed in the stated direction. In section III we present the essential steps of the methodology of our work, the results of model-based calculations and some brief discussion on the results obtained by this combinational approach. The last section is reserved for summing up the conclusions and pointing out the shortcomings of the approach adopted here.

II. THE BASIC APPROACH AND THE PHYSICS PERSPECTIVE

Following the suggestion of Faessler[6] and the work of Peitzmann[7] and also of Schmidt and Schukraft[8], we propose here a generalized empirical relationship between the inclusive cross-section for production of $K^{\pm}$, P and $\bar{P}$ in nucleon(N)-nucleon(N) collision and that for nucleus(A)-nucleus(A) collision as given below:

$$E\frac{d^3\sigma}{dp^3}\ (AB \to QX)\ \sim\ (A.B)^{\phi(y,\ p_T)}\ E\frac{d^3\sigma}{dp^3}\ (PP \to QX)\ , \qquad (1)$$

where Q stands for $K^{\pm}$, P, $\bar{P}$ and $\phi(y,\ p_T)$ could be expressed in the factorization form, $\phi(y,\ p_T) = f(y)\ g(p_t)$.

While investigating a specific nature of dependence of the two variables(y and p_T), either of them is assumed to remain averaged or with definite values. Speaking in clearer terms, if and when p_t-dependence is studied by experimental group, the rapidity factor is integrated over certain limits and is absorbed in the normalization factor.So, the formula turns into

$$E\frac{d^3\sigma}{dp^3}\ (AB \to QX)\ \sim\ (A.B)^{g(p_T)}\ E\frac{d^3\sigma}{dp^3}\ (PP \to QX)\ , \tag{2}$$

The main bulk of work, thus, converges to the making of an appropriate choice of form for $g(p_T)$. And the necessary choices are to be made on the basis of certain premises and physical considerations which do not violate the canons of high energy particle interactions.

Let us now tentatively propose that the expressions for inclusive cross-section of non-pion secondaries in proton-proton scattering at high energies in eqn.(2) could also be chosen in the same form as that was suggested for pions by Hagedorn[9]:

$$E\frac{d^3\sigma}{dp^3}\ (PP \to QX)\ =\ C_1\ (\ 1\ +\ \frac{p_T}{p_0})^{-n}\ , \tag{3}$$

where C_1 is the normalization constant, and p_o, n are interaction-dependent chosen phenomenological parameters for which the values are to be obtained by the method of fitting. Their $\sqrt{s}$-dependences are here proposed to be given by the following formulations:

$$p_0(\sqrt{s})\ =\ a\ +\ \frac{b}{\sqrt{s}\ \ln(\sqrt{s})} \tag{4}$$

and

$$n(\sqrt{s})\ =\ \acute{a}\ +\ \frac{\acute{b}}{\ln^2(\sqrt{s})} \tag{5}$$

where a, b, $\acute{a}$ and $\acute{b}$ are four constants. The $\sqrt{s}$-dependence of p_0 and n would be shown later diagrammatically in the text and their data-base would also be indicated. The nature and significance of these parameters could be appreciated from the work of Hagedorn[9], and those of Bielich et al[10] and Albrecht et al[11].

The final working formula for the nucleus-nucleus collisions is now being proposed here in the form given below:

$$E\frac{d^3\sigma}{dp^3}\ (AB \to QX)\ \approx\ C_2\ (A.B)^{(\epsilon\ +\ \alpha p_T\ -\ \beta p_T^2)}\ E\frac{d^3\sigma}{dp^3}\ (PP \to QX)\ , \tag{6}$$

with $g(p_T)\ =\ (\epsilon\ +\ \alpha p_T\ -\ \beta p_T^2)$, wherein this quadratic form of parametrization is suggested here tentatively by us for analyzing data and testing its efficacy, if any and is hereafter called De-Bhattacharyya parametrization(DBP). In the above expression C_2 is the normalization term which has a dependence either on the rapidity or on the rapidity-density of the produced secondary; ϵ, α and β are constants for a specific set of projectile and target.

Earlier experimental works[11, 12, 13] showed that $g(p_T)$ is less than unity in the p_T-domain, $p_T < 1.5$ GeV/c. Besides, it was also observed that the parameter ϵ, which gives the value of $g(p_T)$ at $p_T = 0$, is also less than one and this value differs from collision to collision. The other two parameters α and β essentially determine the nature of curvature of $g(p_T)$. However, in the present context, precise determination of ϵ is not possible for the following understated reasons:

(i) To make our point let us recast the expression for (6) in the form given below:

$$E\frac{d^3\sigma}{dp^3}(AB \rightarrow QX) \approx C_2 \ (A.B)^{\epsilon} \ (A.B)^{(\alpha p_T - \beta p_T^2)} \ (\ 1 \ + \frac{p_T}{p_0} \ )^{-n} \tag{7}$$

Quite obviously, we have adopted here the method of fitting. Now, in eqn.(7) one finds that there are two constant terms C_2 and ϵ which are neither the coefficients nor the exponent terms of any function of the variable, p_T. And as ϵ is a constant for a specific collision at a specific energy, the product of the two terms C_2 and $(A.B)^{\epsilon}$ appears as just a new constant. And, it will not be possible to obtain fit-values simultaneously for two constants of the above types through the method of fitting.

(ii) From eqn.(2) the nature of $g(p_T)$ can easily be determined by calculating the ratio of the logarithm of the ratios of nuclear-to-PP collision and the logarithm of the product AB. Thus, one can measure ϵ from the intercept of $g(p_T)$ along y-axis as soon as one gets the values of $E\frac{d^3\sigma}{dp^3}$ for both AB collision and PP collision at the same c.m. energy. In the present study we have tried to consider the various collision systems in as many number as possible. To do so, we have to consider the data on normalized versions of $E\frac{d^3\sigma}{dp^3}$ for many collision systems for which clear $E\frac{d^3\sigma}{dp^3}$-data were not available to us. Furthermore, from these normalized versions we can/could not extract the appropriate values of $E\frac{d^3\sigma}{dp^3}$ as the normalization terms, total inclusive cross-sections(σ_{in}) etc., for these collision systems cannot always be readily obtained. Besides, it will also not be possible to get readily the data on inclusive spectra for PP collisions at all c.m.energies, like e.g., at $\sqrt{s} = 17.8 GeV$(c.m. energy of $Pb + Pb$ collision).

In order to sidetrack these difficulties and also to build-up an escape-route, we have concentrated almost wholly here to the values of α and β for various collision systems; and the effects of C_2 and ϵ have been compressed to a single constant term C_3. Hence, the final expression becomes,

$$E\frac{d^3\sigma}{dp^3}(AB \rightarrow QX) \ \approx \ C_3 \ (A.B)^{(\alpha p_T - \beta p_T^2)} \ (\ 1 \ + \frac{p_T}{p_0} \ )^{-n} \tag{8}$$

with $C_3 = C_2(A.B)^{\epsilon}$.

The exponent term $\alpha p_T - \beta p_T^2$ obviously represents here $[g(p_T) - \epsilon]$ instead of $g(p_T)$ alone. Thus, after obtaining fit-values of α and β, if $[g(p_T) - \epsilon]$ are plotted for various collision systems, all the curves would originate from a single point, i.e. origin; and the systems and processes are then really comparable. In other words, in this convenient way we could study and check the scaling characteristics of $g(p_T)$ with respect to the collision systems.

The expression(8) given above is the physical embodiment of what we have termed to be the grand combination of models(GCM) or the combinational approach(CA) that has been applied here. Firstly, the results of PP scattering are obtained in the above on the basis of an assumption of validity of eqn.(3) for even the non-pion secondaries, though originally the Hagedorn's model(HM) was proposed only for pion secondaries. Secondly, the route for converting the results for the non-pion secondaries produced in NN to NA or AB collisions is built up by the Peitzmann's approach(PA) represented by expression(2). Thirdly, the further input is the De-Bhattacharyya parametrization for the nature of the exponent. Thus, the GCM or the CA provides the combination of HM, PA and the DBP, and thus constitutes a single whole. And this whole is both useful and economical; it is economical in the sense that it can accommodate vast wealth of data with only three arbitrary parameters. Compared to the theoretical apparatus provided by the standard model(SM)[14], this number is quite low.

And the choice of this quadratic form is not altogether a coincidence. In dealing with the EMC effect in the lepton-nucleus collisions, one of the authors here(SB)[15], made use of a polynomial form of A-dependence with the variable x_F(Feynman Scaling variable). This gives us a clue to make a similar choice with both p_T and $y(\eta)$ variable(s) in each case separately. In the recent times, De-Bhattacharyya parametrization is being extensively applied to interpret the measured data on the various aspects[4] of the particle-nucleus and nucleus-nucleus interactions at high energies. In the recent past Hwa et. al.[16] also made use of this sort of relationship in a somewhat different context. The underlying physics implications of this parametrization stem mainly from the expression(6) which could be identified as a clear mechanism for switch-over of the results obtained for nucleon-nucleon(PP) collision to those for nucleus-nucleus interactions at high energies in a direct and straightforward manner. The polynomial exponent of the product term on AB takes care of the totality of the nuclear effects.

The individual model(s), seen in a split manner, is(are) certainly not new; but the combination with the proposed two-factor quadratic p_T-dependence of the exponent (called DBP in the text) for AB or AA interactions at high energies is testably a new proposition offered by us. Besides, the probing of the role of the Hagedorn's model in understanding the production of non-pion secondaries in high energy collisions is another virgin feature in our work. The testing of this mechanism suggested by us is with the views of (i) presenting an integrated approach toward production of various kind of particle-secondaries in high energy collisions and (ii) providing a unified outlook to particle production in all varieties of particle-particle, particle-nucleus or nucleus-nucleus interactions at high energies. Quite understandably, only by intensive analysis of data on several/all sets of collisions somewhat successfully such claims could, at all, be made, and/or later be justified. The trends that emerge from this extensive data-analysis with this grand combination of models(GCM) has been given in the next section.

Obviously,in its approach and method, this work is undoubtedly a continuation of one of our previous works[4]. But, even at the risk of being a bit repetitive, we have to proceed and carry on. Because, without these special efforts at studying on a case-to-case basis for each of the specific variety of the secondaries, we cannot reasonably arrive at any definitive conclusion and also cannot confirm the physical basis, if any, of our work. And the two points made very precisely in the preceding paragraph represent the strong and crucial aspects of our physics motivation behind this proposed formalism. A point is to be made. Our intention here, is surely not just piling up of large bulk of data in a purposeless way; rather we would like to make the fullest possible utilization of them for checking the proposed GCM as extensively as possible with the widest range of available data on diverse set of reactions. Because, such rigorous and intensive checkings only could lead to meaningful and valid conclusions, if any. And, in our opinion, this has helped us to substantiate, to a considerable degree, the functional efficacy and strength of the proposed phenomenological model which is called here the combinational approach. In fact, the revelation of the potency and strength of this phenomenological approach is the other most striking feature which we would surely like to emphasize here.

It is obvious that there are two phenomenological parameters in $g(p_T)$ in expression(8) which need to be physically explained and/or identified. In compliance with this condition we offer the following physical explanations for the occurrence of all these factors. The particle-nucleus or nucleus-nucleus collisions at high energies subsequently gives rise to an expanding blob or fireball with rising temperature. In real and concrete terms this stage indicates the growing participation of the already-expanded nuclear blob. As temperature increases at this stage, the emission of highly energetic secondaries(which are mostly peripheral nucleons or baryons) with increasing transverse momentum is perfectly possible. The coefficient α addresses this particularity of the natural event; and this is manifested in the enhancement of the nuclear contribution with the rise of the transverse momentum. Thereafter, there is a turnabout in the state of reality. After the initial fractions of seconds, the earlier-excited nuclear matter starts to cool down and there is a clear natural contraction at this stage as the system suffers a gradual fall in temperature. Finally, this leads to what one might call 'freeze-out' stage, which results in extensive hadronization, especially in production of hadrons with very low transverse momentum. In other words, the production of large-p_T particles at this stage is lowered to a considerable extent. This fact is represented by the damping or attenuation term for the production of high-p_T particles. The factor β with negative values takes care of this state of physical reality. Thus the function denoted by $g(p_T)$ symbolizes the totality of the features of the expansion-contraction dynamical scenario in the after-collision stage. This interpretation is, at present, only of very suggestive nature. However, let us make some further clarifications.

The physical foundation that has here been attempted to be built up is inspired by thermodynamic pictures, whereas the quantitative calculations are based on a sort of pQCD-motivated power-law formula represented by eqn.(6). This seems to be somewhat paradoxical, because it would be hard to justify the hypothesis of local thermal equilibrium in multihadron systems produced by high energy collisions in terms of successive collision of the QCD-partons(like quarks and gluons) excited or created in the course of the overall process. Except exclusively for central heavy ion collisions, a typical parton can only undergo very few interactions before the final-state hadrons 'freeze out', i.e. escape as free particles or resonances. The fact is the hadronic system, before the freeze-out starts, expands a great deal – both longitudinally and transversally – while these very few interactions take place[17].

But the number of parton interactions is just one of the several other relevant factors for the formation of local equilibrium. Of equal importance is the parton distribution produced early in the collision process. This early distribution is supposed to be a superposition of collective flow and highly randomized internal motions in each space cell which helps the system to achieve a situation close to the equilibrium leading to the appropriate values of collective variables including concerned and/or almost concerned quantities. The parameter α in expression(6) is somehow related to the measure of the ratio of the net binary collision number to the total permissible number among the constituent partons in the pre-freeze out expanding stage identified to be a sort of explosive 'detonation'[17] stage. This is approximated by a superposition of collective flow and thermalized internal motion, which is a function of hadronic temperature manifested in the behaviour of the average transverse momentum. The post freeze-out hadron production scenario is taken care of by the soft interaction which is proportional[13, 18] to the number of participant nucleons, N_{part}, according to almost any variety of wounded nuclear model. The factor β, we conjecture, might have a relationship with the ratio of the actual participating nucleons to the total number of maximum allowable(participating) nucleons. In fact, this sort of physical explanations is reproduced here from some of our previous works[19].

III. PROCEDURAL STEPS, RESULTS AND DISCUSSION

At the very start we study the p_T-spectra for $K^{\pm}$, P and $\bar{P}$ inclusive production in PP collisions at several energies and try to fit the expression in the formula(3) given in the previous section. The graphs are shown in the several diagrams in Fig.1 for kaons, protons and antiprotons. The fits for the average kaon yield and antiproton production in $P\bar{P}$ reactions have also been obtained(Fig.2) on the basis of Hagedorn's model(eqn.(3)). The same could not be done for proton production in $P\bar{P}$ reactions due to unavailability of the measured experimental data in this particular collisions. The obtained fitted-values of the parameters, p_0 and n, for different secondaries produced in $PP/P\bar{P}$ collisions at different energies have been displayed in Table-I - Table-V. The graphs for $\sqrt{s}$-dependences of p_0 and n, as proposed in eqn.(4) and eqn.(5), have been depicted in Fig.3 and the necessary values of the parameters, a, $\acute{a}$, b and $\acute{b}$ have been presented in Table-VI. While plotting graphs we have obviously assumed that at very high energies the proton-proton and proton-antiproton collisions could be treated at par and with the acceptance of equivalence between each other.

Hereafter, in studying the nature of p_T-spectra in all nucleon-nucleus and nucleus-nucleus collisions, the interaction energy in all cases of nucleus-induced reactions is invariably converted first into the c.m. system values, that is expressed in $\sqrt{s_{\rm NN}}$, then the values of p_0 and n are picked up from the graphical plots drawn already and shown by Fig.3. While analyzing nucleus-dependence with expression(8) and trying with the fit parameters C_3, α and β, we have inserted these extracted values of p_0 and n for PP cross section term occurring in the expression(8) given above.

However, the plots presented in Fig.3 deserve special mention from phenomenological points of view. The p_0 and n values for production of the same secondary at various energies ($\sqrt{s}$-values) show very slow fall-off with energy in these graphs. The nearness of values of p_0/n signals a march towards a steady state of hadronization. Once the natures are established by the theoretical procedures adopted for either $PP/P\bar{P}$ reactions at high or very high energies, we would be compelled, for the sake of consistency, to reduce/minimize the degree of arbitrariness in choosing values of p_0 and n for nucleon-nucleus or nucleus-nucleus collisions. As soon as the values of $\sqrt{s_{NN}}$ is given for a specific interaction, the values of p_0 and n are to be obtained from the theoretical plots of them shown in Fig.3(a), Fig.3(b) and Fig.3(c) for different secondaries and/or for different collision(s). The diagrams given in Fig.4 to Fig.8 do actually provide the nature of collected data and also the theoretical descriptions for diverse reactions indicated by the labels in each diagram. The solid curves represent the results arrived at on the basis of the new combination of models(NCM) and the use of De-Bhattacharyya parametrization(DBP). And the totality of this combination of NCM and DBP is called Grand Combination of Models(GCM). And the tables (displayed by Table-VII to Table-IX) provide the necessary parameter values that have entered into each calculation of the theoretical solid curves in all the diagrams of Fig.4-Fig.8 for various reactions. In some cases, due to lack of availability of clear proton data, we have used the net proton($P-\bar{P}$) data with the assumption of the near equivalence and as an approximate measure. The diagrams presented in Fig.9(a) to Fig.9(c) are full of physical import and convey the message of mass number scaling of nuclear reactions.

The deviations, whenever and wherever if any, from this observation could be attributed, in the main, to the following few points: (i) limits of uncertainties in the measurements; (ii) some unavoidable approximations in theoretical calculations; and (iii) availability, in some cases, of only too sparse and rare data on which reliability is certainly questionable.

Obviously, we present here the GCM-based analyses of extensive sets of data on some of the observables measured by the various experimental groups in high energy particle and nuclear physics. The study has, thus, given rise to some crucial observations with revelations of following systematic trends: (a) For the non-pion secondaries produced in nucleus-nucleus collisions at high energies the nature of fits is not as good as in PP reaction, especially at low momentum region, and especially for $E\frac{d^3N}{dp^3}$ vs. p_T plots. The reason(s) for this discrepancy would be discussed later in the last section. (b) The fit parameters p_0 and n for production of secondary kaon(s) and protons are, by ascription, different in the basic PP reactions. And this is consistent with the theoretical expectations. The factor p_0/n has, we recognize, a relationship with the slope parameter of the p_T-spectra. And the slope parameters for the lighter and heavier particles, one knows, are not the same. But, quite interestingly, the nature of $\sqrt{s}$-dependences of p_0 and n values of all three species, kaons, protons and antiprotons, as shown in Fig.3(a)-3(b), are qualitatively quite alike.(c) It is quite evident from the Tables VII, VIII, and IX that the values of α and β are too close. This, in our opinion, reflects the basic fact that the nuclear effects are quite finite and certainly not too pronounced even with the heaviest of the nuclei and the highest available energies.

IV. FINAL COMMENTS AND CONCLUSIONS

In the end let us summarize our main findings from the present study in the following way:

(i) Quite naturally and predictably, the χ^2/ndf values shown in Table-VII-IX present, at times, sharply contrasting nature, because of either very sparse available data and that too for a narrow region of p_T-values, or large ranges of errors and uncertainties in the measurements.

(ii) Still, on the whole, the approach describes modestly well the large bulk of data on kaon-antikaon and proton-antiproton production phenomena in high energy nucleon-nucleus and nucleus-nucleus collisions.

(iii) Related with production of $K^{\pm}$ and $P/\bar{P}$, the c.m. energy-dependences of the parameter values occurring in the basic PP reactions are of the same hyperbolic nature as in the cases of pions. This element of observation on similarity aspect of $K^{\pm}$ and $P/\bar{P}$ with pions is certainly a new finding here made by us, as Hagedorn's early work[9] was made exclusively on pions.

(iv) Besides, the entire energy-dependences, even in purely nuclear collisions, are manifested by only the basic PP interaction; the nuclear geometry exhibits no separate energy behaviour. This is quite significant in the sense that it might be a great hint to and confirmation of the enormous importance of PP interaction even in understanding the nature of heavy ion collisions.

(v) The observation of the close values of α in various sets of nuclear collisions with varying pair of projectile and target and at different energies is another interesting finding. It indicates that enhancement due to nuclear contribution could never assume too large values, even with the heaviest of the nuclei, though the parameters are uniformly non-zero for each and every collision and the values are limited. This does, in effect, imply the very finite degree of the nuclear effects. The constraint would appear more severe and acute, if the diminutive role of the β factor is reckoned with. This might be attributed to the large baryon stopping effect in heavy ion collisions.

Besides, the small positive α-values in any collission represent physically the nuclear enhancement — though certainly not of 'anomalous' nature — called Cronin effect. secondly, the negative β-values might explain the suppressions of productions of hadrons at relatively large values of the transverse momenta.

(vi) Even with some heaviest nuclei, the observations from Fig.9(a) to Fig.9(c) on the validity of a sort of mass-number(A) scaling for production of both strange and non-strange variety of hadrons constitute an interesting and probeworthy point. Physically, this might signal the limit to the finite degree of impact parameter dependence of nuclear collisions. The near-saturation of the parameter value at and after certain magnitude of AB indicates the stringent limit on both the number of binary collisions and the number of participating nucleons in the colliding nuclei.

(vii) The reason for observation of departure of the fits from the data on p_T-spectra produced in nuclear collisions, especially at low p_T region, could be ascribed, in the main, to the following reasons. Firstly, because for the nucleus-involved collisions, the measurements in the experiments on purely soft collisions are very difficult, as hard collision effects are almost unavoidably present there. Secondly, the measurements of the factor $E\frac{d^3N}{dp^3}$ through the array of several detectors have always an intrinsic simulation-based standard-model (mainly the HIJING version) component, for which the obtained data are not the pure products of experimental measurements alone. But the present model can, in no way, accommodate such sorts of superposition effects.

(viii) The property of 'universality' of hadronic, hadronuclear and nuclear reactions at very high energies is also vindicated by the approach adopted here.

The implications of all these are obvious. With the series of such modestly successful ventures, the combinational approach attains a plateau and assumes a potential to claim the status of an alternative approach to understand functionally some aspects of the heavy ion collisions in general, and the nature of p_T/rapidity-spectra in NA or AB collisions in particular.

However, we must not gloss over the gross limitations of the present approach which are as follows: (i) It is a fact that we could, so far, offer no measure to estimate quantitatively the values of α and β. The hints are, uptil now, of only very qualitative nature. (ii) Quite admittedly, the physical explanations outlined here for α and β are also of tentative nature; they have, still, not been identified concretely with any of the known physical observables in the domain of collision dynamics for multiple production of hadrons and of nuclear geometry as well. (iii) The calculations for particle-nucleus or nucleus-nucleus interaction cases are absolutely dependent on the apriori and data-based knowledge of the production characteristics of the same particle species in PP reactions at various energies. Without it, the method is ineffective and that is a major handicap. (iv) The approach fails to respond to the very basic query on actual mechanisms for particle production processes in any collision. (v) And just because of it, the approach is insensitive to the charge-state of the specific secondary produced in any high energy interaction and that is certainly a great difficulty. So, unless these problems could be remedied to a great extent, we accept that we have no reasons for complacency.

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TABLE I: Fit Values of p_0 and n for $PP \rightarrow (K^+ + K^-)/2 + X$ at different energies

$\sqrt{s}_{NN}(GeV)$ [with reference]	Relevant collision-specifics	C_1	p_0(GeV/c)	n	p_0/n(GeV/c)	χ^2/ndf
23[20]		12 ± 3	2.7 ± 0.9	17 ± 4	0.16	1.427
31[20]	$y_{cm} = 0$,	15 ± 2	4 ± 1	22 ± 4	0.18	1.031
45[20]	min. bias	13 ± 3	2.6 ± 0.5	16 ± 2	0.16	1.276
53[20]		15 ± 2	2.4 ± 0.2	15 ± 1	0.16	1.722
63[20]		15 ± 1	1.6 ± 0.2	11 ± 1	0.15	1.831

TABLE II: Fit Values of p_0 and n for $P\bar{P} \rightarrow (K^+ + K^-)/2 + X$ at different energies

$\sqrt{s}_{NN}(GeV)$ [with reference]	Relevant collision-specifics	C_1	p_0(GeV/c)	n	p_0/n(GeV/c)	χ^2/ndf
300[21]		5.7 ± 0.5	1.5 ± 0.3	8.2 ± 0.5	0.18	2.258
540[21]	$\|y_{cm}\| < 2.0$,	4.1 ± 0.1	1.3 ± 0.4	8 ± 2	0.16	2.273
1000[21]	min. bias	0.36 ± 0.02	1.50 ± 0.06	7.0 ± 0.2	0.21	1.971
1800[21]		0.09 ± 0.01	1.55 ± 0.05	8 ± 2	0.19	3.526

TABLE III: Fit Values of p_0 and n for $PP \rightarrow P + X$ at different energies

$\sqrt{s}_{NN}(GeV)$ [with reference]	Relevant collision-specifics	C_1	p_0(GeV/c)	n	p_0/n(GeV/c)	χ^2/ndf
23[20]		11 ± 2	19 ± 6	86 ± 15	0.22	0.984
31[20]		10 ± 2	16 ± 5	70 ± 10	0.23	1.012
45[20]	$y_{cm} = 0$,	9 ± 1	12 ± 3	50 ± 9	0.24	1.359
53[20]	min. bias	10 ± 3	8 ± 1	37 ± 6	0.22	1.022
63[20]		10 ± 1	7 ± 3	34 ± 7	0.21	2.106

TABLE IV: Fit Values of p_0 and n for $PP \rightarrow \bar{P} + X$ at different energies

$\sqrt{s}_{NN}(GeV)$ [with reference]	Relevant collision-specifics	C_1	p_0(GeV/c)	n	p_0/n(GeV/c)	χ^2/ndf
23[20]		$4.2 \pm .4$	15 ± 3	69 ± 9	0.22	1.085
31[20]		$5.2 \pm .5$	13 ± 2	60 ± 11	0.22	2.104
45[20]	$y_{cm} = 0$,	7 ± 0.4	11 ± 4	53 ± 8	0.21	1.734
53[20]	min. bias	6.6 ± 0.7	10 ± 2	45 ± 9	0.22	1.430
63[20]		9.3 ± 0.4	9 ± 3	40 ± 5	0.23	1.226

TABLE V: Fit Values of p_0 and n for $P\bar{P} \rightarrow \bar{P} + X$ at different energies

$\sqrt{s}_{NN}(GeV)$ [with reference]	Relevant collision-specifics	C_1	p_0(GeV/c)	n	p_0/n(GeV/c)	χ^2/ndf
300[21]		2.6 ± 0.2	7.5 ± 0.9	29 ± 3	0.26	1.242
540[21]	$\|y_{cm}\| < 2.0$,	1.4 ± 0.1	7 ± 2	25 ± 6	0.28	1.775
1000[21]	min. bias	0.13 ± 0.03	7.3 ± 0.8	24 ± 3	0.30	1.620
1800[21]		0.025 ± 0.003	7.4 ± 0.9	23 ± 2	0.32	1.483

TABLE VI: Values of $a, b, \acute{a}, \acute{b}$

Secondaries	a	b	$\acute{a}$	$\acute{b}$
$(K^+ + K^-)/2$	1.6	103	3.6	161
P	7	602	5	644
$\bar{P}$	7	478	13	527

TABLE VII: Numerical Values of the parameters: α and β for Kaon production in different high energy collisions

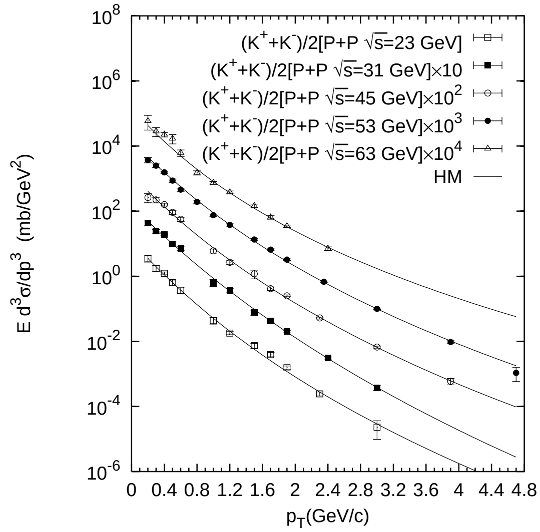
Collision [with reference]	E(GeV)	Relevant collision-specifics	C_3	α (GeV/c)$^{-1}$	β (GeV/c)$^{-2}$	χ^2/ndf
P + D[12]	400	$y = 0$, min. bias	38 ± 6	0.14 ± 0.03	0.04 ± 0.01	2.832
P + Be[24]	450	$2.4 < y < 3.5$, central	1.2 ± 0.4	0.26 ± 0.03	0.04 ± 0.015	0.381
P + S[22]	200	$0.6 < y < 2.4$, central	6.4 ± 0.6	0.23 ± 0.03	0.03 ± 0.01	1.974
P + Au[22]	200	$0.2 < y < 2.0$, central	6.3 ± 0.4	0.21 ± 0.03	0.032 ± 0.005	0.859
P + Pb[24]	450	$2.4 < y < 3.5$, central	1.1 ± 0.1	0.17 ± 0.04	0.04 ± 0.01	0.552
O + Au[22]	$200A$	$0.2 < y < 2.0$, central	63 ± 9	0.24 ± 0.05	0.03 ± 0.01	1.164
S + S[22]	$200A$	$0.6 < y < 2.4$, central	72 ± 10	0.24 ± 0.02	0.03 ± 0.01	2.088
S + Pb[24]	$200A$	$2.3 < y < 2.9$, central	28 ± 3	0.25 ± 0.03	0.027 ± 0.002	2.170
Au + Au[28]	$8450A$	$\|\eta_{cm}\| < 0.35$, min. bias	17 ± 2	0.15 ± 0.03	0.041 ± 0.005	1.005
Pb + Pb[26]	$160A$	$2.4 < y < 3.5$, central	66 ± 2	0.20 ± 0.02	0.038 ± 0.004	2.158

TABLE VIII: Numerical Values of the parameters: α and β for Proton production in different high energy collisions

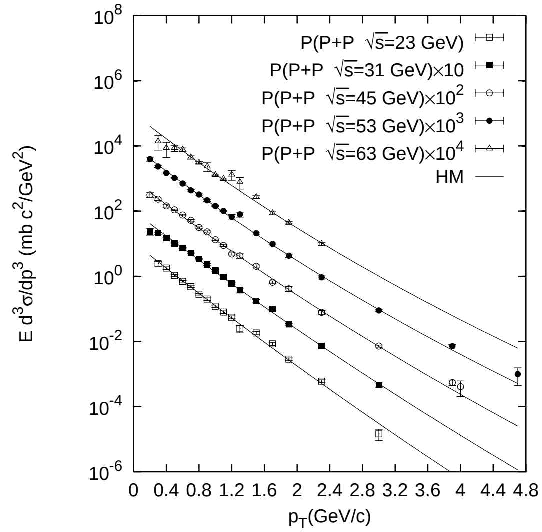
Collision [with reference]	E(GeV)	Relevant collision-specifics	C_3	α (GeV/c)$^{-1}$	β (GeV/c)$^{-2}$	χ^2/ndf
$\mathrm{P+D}$[12]	400	$y=0$, min. bias	28 ± 2	0.18 ± 0.02	0.028 ± 0.002	2.577
$\mathrm{P+Be}$[25]	450	$2.3<y<2.9$, central	0.57 ± 0.02	0.22 ± 0.04	0.04 ± 0.01	2.530
$\mathrm{P+S}$[23]	200	$0.5<y<3.0$, min. bias	16 ± 2	0.15 ± 0.02	0.045 ± 0.008	1.813
$\mathrm{P+Au}$[23]	200	$0.5<y<3.0$, min. bias	47 ± 6	0.14 ± 0.03	0.05 ± 0.02	1.425
$\mathrm{P+Pb}$[25]	450	$2.3<y<2.9$, central	0.8 ± 0.2	0.18 ± 0.02	0.04 ± 0.01	2.763
$\mathrm{D+Au}$[23]	$200A$	$0.5<y<3.0$, min. bias	102 ± 8	0.16 ± 0.02	0.05 ± 0.02	0.535
$\mathrm{O+Au}$[23]	$200A$	$0.5<y<3.0$, min. bias	473 ± 63	0.17 ± 0.01	0.04 ± 0.01	0.866
$\mathrm{S+S}$[23]	$200A$	$0.5<y<3.0$, min. bias	164 ± 30	0.19 ± 0.02	0.03 ± 0.01	1.247
$\mathrm{S+Ag}$[23]	$200A$	$0.5<y<3.0$, min. bias	450 ± 70	0.17 ± 0.01	0.03 ± 0.01	0.992
$\mathrm{S+Au}$[23]	$200A$	$3.0<y<5.0$, min. bias	139 ± 12	0.17 ± 0.03	0.03 ± 0.01	2.063
$\mathrm{Au+Au}$[28]	$8450A$	$\lvert\eta_{cm}\rvert<0.35$, min. bias	7 ± 2	0.17 ± 0.02	0.034 ± 0.001	0.744
$\mathrm{Pb+Pb}$[26]	$160A$	$2.3<y<2.9$, central	33 ± 2	0.19 ± 0.02	0.022 ± 0.002	2.672

TABLE IX: Numerical Values of the parameters: α and β for Antiproton production in different high energy collisions

Collision [with reference]	E(GeV)	Relevant collision-specifics	C_3	α (GeV/c)$^{-1}$	β (GeV/c)$^{-2}$	χ^2/ndf
$\mathrm{P+D}$[12]	400	$y_{cm}=0$, min. bias	7.2 ± 0.6	0.15 ± 0.04	0.035 ± 0.003	1.764
$\mathrm{P+Be}$[25]	450	$2.3<y<2.9$, central	0.16 ± 0.03	0.20 ± 0.03	0.03 ± 0.01	1.362
$\mathrm{P+Pb}$[25]	450	$2.3<y<2.9$, central	$0.18\pm .04$	0.19 ± 0.03	0.032 ± 0.004	1.470
$\mathrm{S+S}$[25]	$200A$	$2.3<y<2.9$, central	1.5 ± 0.2	0.19 ± 0.02	0.03 ± 0.01	2.674
$\mathrm{S+Ag}$[27]	$200A$	$3<y<4$, central	8 ± 1	0.18 ± 0.02	0.03 ± 0.01	1.851
$\mathrm{S+Au}$[27]	$200A$	$3<y<4$, central	5.7 ± 0.6	0.25 ± 0.03	0.024 ± 0.003	1.620
$\mathrm{S+Pb}$[25]	$200A$	$2.3<y<2.9$, central	1.7 ± 0.2	0.23 ± 0.03	0.022 ± 0.003	1.767
$\mathrm{Au+Au}$[28]	$8450A$	$\lvert\eta_{cm}\rvert<0.35$, min. bias	0.42 ± 0.01	0.16 ± 0.03	0.034 ± 0.003	2.081
$\mathrm{Pb+Pb}$[26]	$160A$	$2.3<y<2.9$, central	15 ± 1	0.26 ± 0.02	0.024 ± 0.002	2.838



(a)



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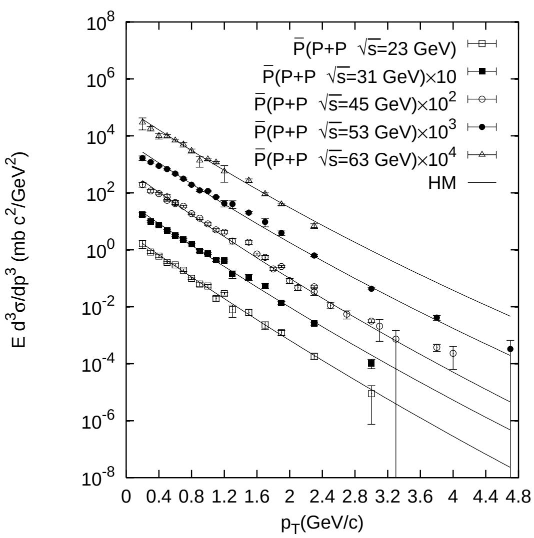
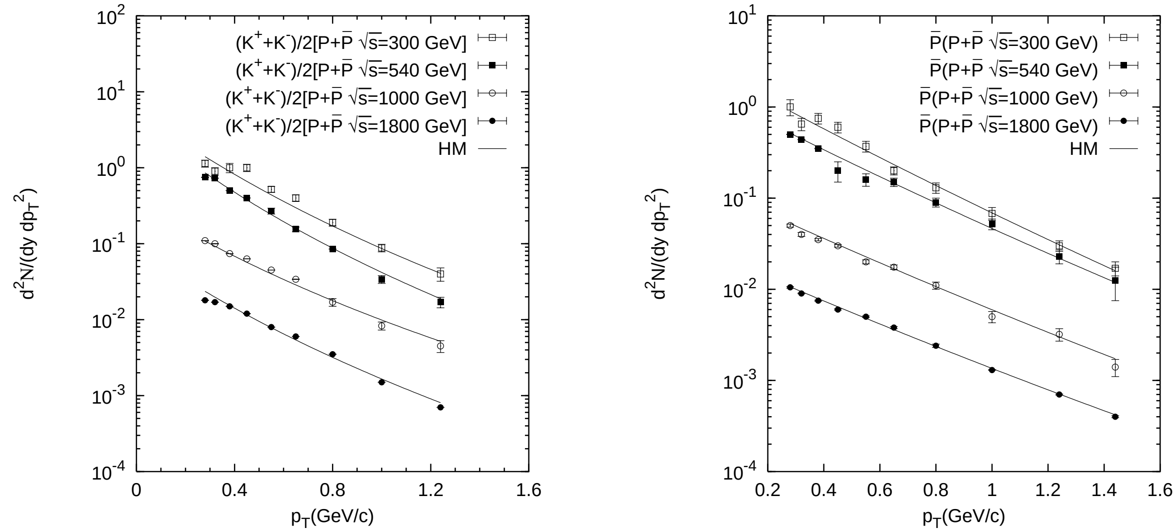


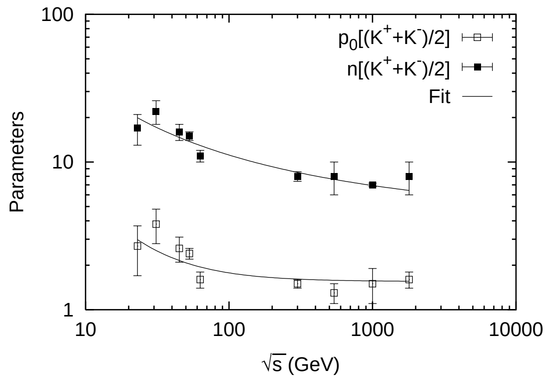
(c)

FIG. 1: Plot of $E\frac{d^3\sigma}{dp^3}$ vs. p_T for various non-pionic secondaries produced in $P+P$ collisions at different c.m. energies. The various experimental points are taken from Ref.[20]. The solid curves give the theoretical fits on the basis of Hagedorn's model(eqn.3).

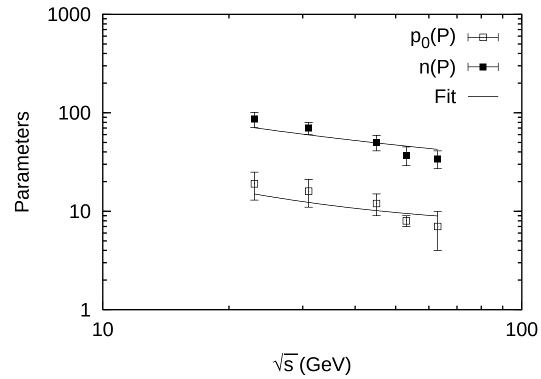


(a) (b)

FIG. 2: The inclusive spectra for secondary $\frac{K^{+}+K^{-}}{2}$ and $\bar{P}$ produced in $P+\bar{P}$ collisions at $\sqrt{s} = 300, 540, 1000$ and 1800 GeV. The various experimental points are from Ref.[21]. The solid curvilinear lines are drawn on the basis of eqn.3.



(a)



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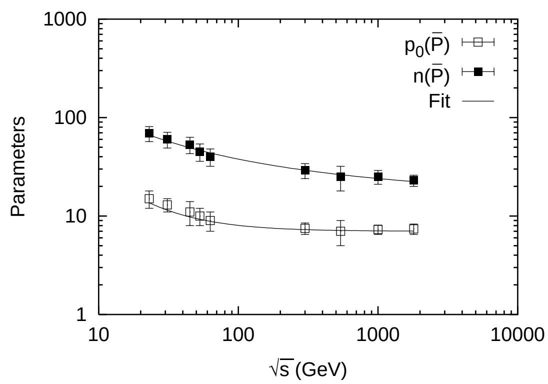


(c)

FIG. 3: Values of p_0 and n as a function of c.m. energy $\sqrt{s}$. Various data points are taken from Table-I to Table-V. The solid curves are drawn on the basis of eqn.4 and eqn.5.

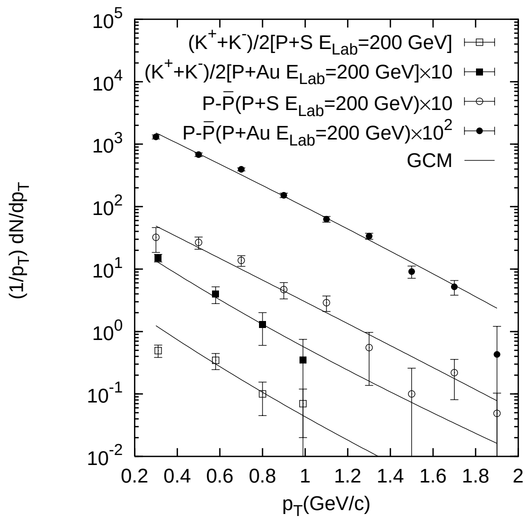


FIG. 4: Plot of $(\frac{1}{p_T})\frac{dN}{dp_T}$ as a function of transverse momentum, p_T for production of kaons and net protons in PS and PAu collisions at $E_{Lab} = 200$ GeV. The various experimental points are taken from Ref.[22, 23]. The solid curves depict the GCM-based results(eqn.8).

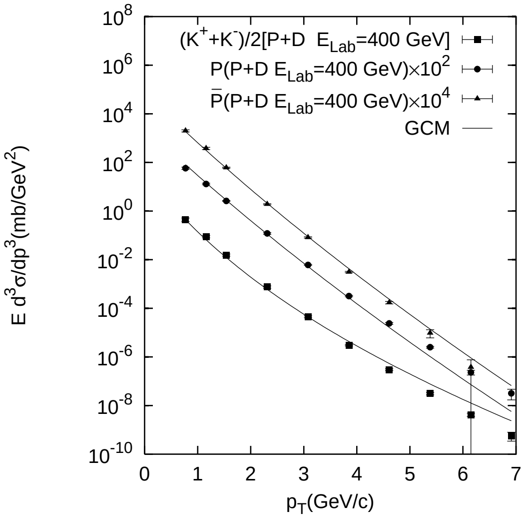


FIG. 5: Plot of $E\frac{d^3\sigma}{dp^3}$ vs. p_T for various secondaries produced in $P + D$ collisions at $E_{Lab} = 400$ GeV. The various experimental points are taken from Ref.[12]. The solid curves provide the fits on the basis of eqn.8.

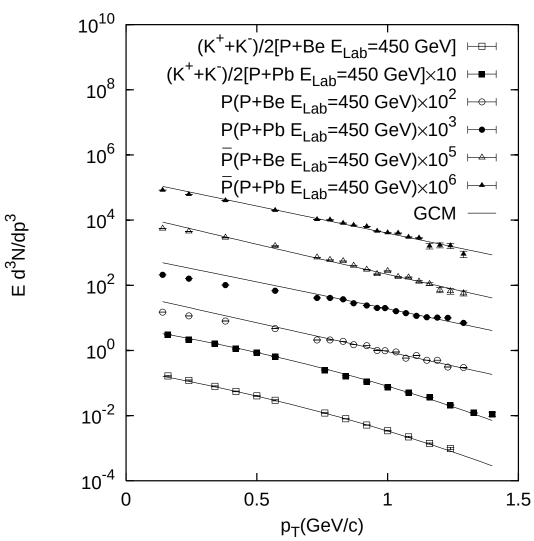
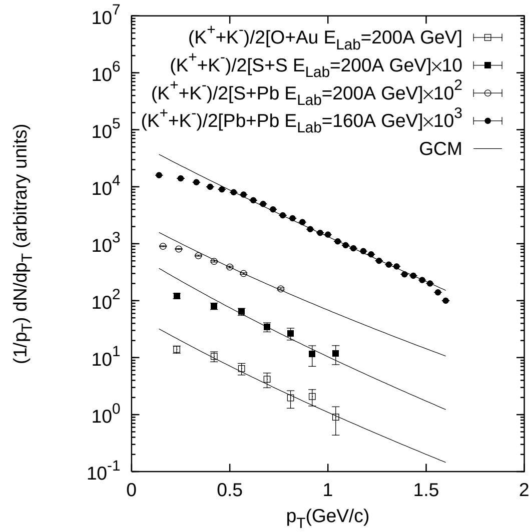
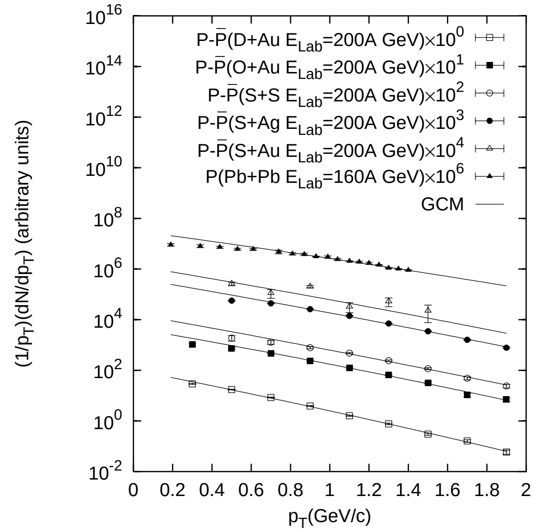


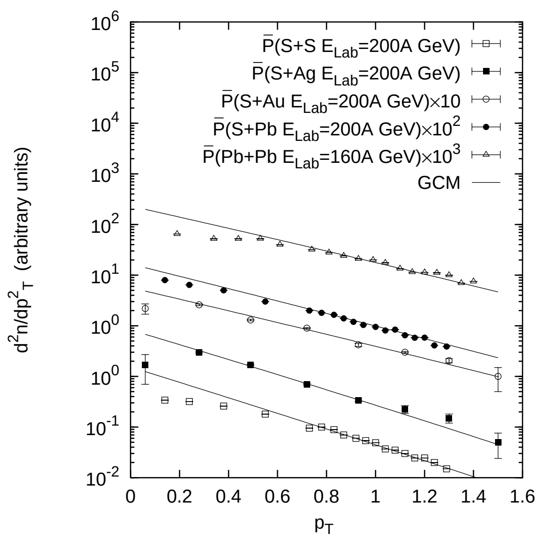
FIG. 6: Nature of Invariant spectra for production of $\frac{K^{+}+K^{-}}{2}$, P and $\bar{P}$ in $P + Be$ and $P + Pb$ collisions at $E_{Lab} = 450$ GeV. Various measured data are obtained from Ref.[24, 25]. The solid curvilinear lines are drawn on the basis of GCM.



(a)



(b)



(c)

FIG. 7: Plots of multiplicity spectra as function of p_T for production of some non-pionic secondaries in different nucleus-nucleus collisions at CERN and SPS energies. Various experimental data are taken from Ref.[22, 23, 24, 25, 26, 27]. The solid curves depict the GCM-based fits(eqn.8).

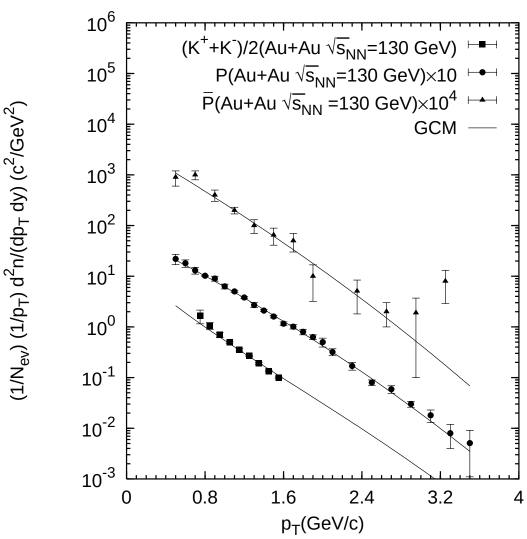
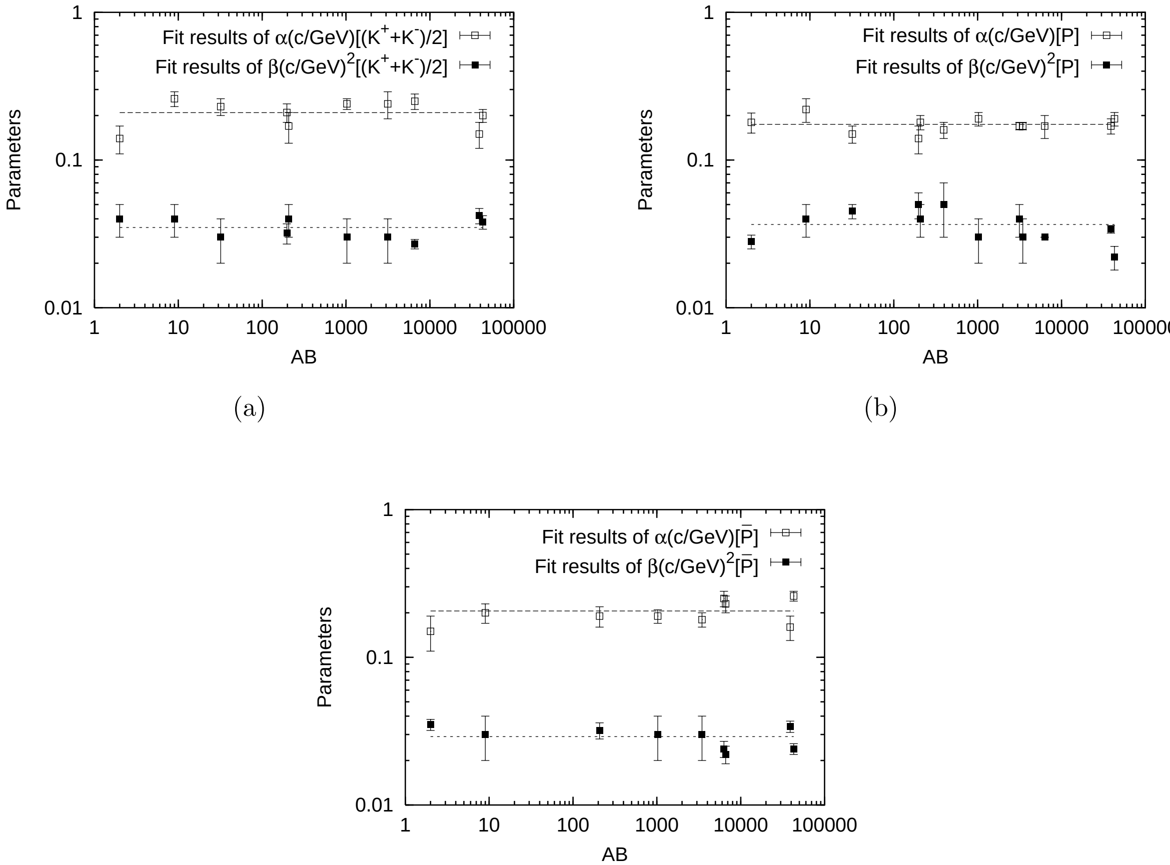


FIG. 8: The nature of invariant spectra for secondary charged kaons, protons and antiprotons produced in $AuAu$ interactions at RHIC energy($\sqrt{s_{NN}} = 130$ GeV). The experimental data are from Ref.[28]. The solid curves provide the present model-based fits.



(a) (b) (c)

FIG. 9: Values of α and β for various non-pionic secondaries produced in different collisions as functions of the product of mass numbers(AB) of the interacting nuclei. The fitted values of α and β, enlisted in Table-VII to Table-IX, are taken as the data points; and are denoted by empty and filled squares respectively. The dashed lines give the average values for $\alpha_{\frac{K^++K^-}{2}} = 0.21 \pm 0.03$, $\alpha_P = 0.17 \pm 0.03$, $\alpha_{\bar{P}} = 0.21 \pm 0.04$, $\beta_{\frac{K^++K^-}{2}} = 0.035 \pm 0.0005$, $\beta_P = 0.037 \pm 0.003$ and $\beta_{\bar{P}} = 0.030 \pm 0.005$.