PHYSICAL REVIEW A **69**, 052312 (2004)

Local cloning of Bell states and distillable entanglement

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(Received 15 November 2003; published 13 May 2004)

The necessary and sufficient amount of entanglement required for cloning of orthogonal Bell states by local operation and classical communication is derived, and using this result we provide here some additional examples of reversible as well as irreversible states.

DOI: 10.1103/PhysRevA.69.052312 PACS number(s): 03.67.Hk

I. INTRODUCTION

Nonorthogonal states cannot be cloned exactly [1], whereas orthogonal states can be. But multipartite orthogonal states cannot be cloned by only local operation and classical communication (LOCC) in general without using an entangled ancilla state, as otherwise that would imply creation of entanglement by LOCC. In this scenario, if one allows these far apart parties to share some known entangled states as an extra resource in the form of a blank copy state and machine states (we call them together the *ancilla*), cloning of orthogonal multipartite states (entangled in general) will be possible.

Throughout this paper, we shall consider the four Bell states as

$$|B_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$

$$|B_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$

$$|B_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad (1)$$

$$|B_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle),$$

where one of the particles (i.e., one of the qubits) is held by Alice and the other is held by Bob.

We show in Sec. II that any two Bell states can be cloned by sharing just one ebit of free entanglement in the ancilla state. Cloning of four Bell states is discussed in Sec. III. This has been generalized, in Sec. IV, for $1 \rightarrow N$ copy cloning of Bell states. In Sec. V, we provide some examples of quasipure states [2]. We provide some examples of irreversible states in Sec. VI. We summarize our results in Sec. VII.

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II. 1→2 EXACT CLONING OF TWO BELL STATES

Now consider the simplest case where the state may be any two of the $|B_i\rangle$'s. If they share, as ancilla, two known ebits, they can first locally distinguish the above-mentioned two Bell states, and then they can locally transform the two shared ebit states to two copies of the corresponding Bell states. In fact, one can have the same result using one ebit of the shared ancilla state (although the cloned state will remain unknown). How can we achieve this? Let Alice and Bob share either $|B_1\rangle$ or $|B_3\rangle$ [see Eq. (1)]. If they share (as ancilla) the known state $|B_1\rangle$, then by applying a controlled-NOT (CNOT) operation locally (where the qubit of the unknown Bell state is the source and the qubit of the ancilla is the target, for both Alice and Bob), they will share two copies of the unknown Bell state. The same result can be obtained (modulo some overall phases) for any two Bell states from the set given in Eq. (1), by first locally unitarily transforming (or expressing the local bases in terms of some other local bases) the two Bell states to the states $|B_1\rangle$, $|B_3\rangle$ (expressed in the new basis), then locally cloning these latter two Bell states (using one ebit of entanglement), and finally locally unitarily transforming (or expressing the local bases in terms of some other local bases) $|B_1\rangle^{\otimes 2}$ and $|B_3\rangle^{\otimes 2}$ to the corresponding two copies of the initially given set of two Bell states.

It is to be noted here that one ebit of free (i.e., distillable) entanglement is necessary in the shared ancilla state. If not, let ρ be any shared bipartite ancilla state (where the distillable entanglement of ρ is less than one ebit), for which, under LOCC, any two Bell states $|B_1\rangle$ and $|B_2\rangle$ (say) can be exactly cloned. This immediately shows that the initially shared separable state $\rho_{sep}=\frac{1}{2}(P[|B_1\rangle]+P[|B_2\rangle])$ of Alice and Bob will be transformed (together with the above-mentioned shared ancilla state ρ) as

$$\rho_{sep} \otimes \rho \rightarrow \frac{1}{2}(P[|B_1\rangle \otimes |B_1\rangle] + P[|B_2\rangle \otimes |B_2\rangle]),$$

due to linearity of the $1 \rightarrow 2$ cloning operation [3]. The final state has one ebit of distillable entanglement (across the Alice:Bob cut) [4], while the total initial state has less than one ebit of distillable entanglement—a contradiction. So, whatever shared ancilla state ρ we take (for $1 \rightarrow 2$ cloning of two Bell states), it must have at least one ebit of distillable entanglement.

The above-mentioned argument can be directly generalized to show that $(N-1)$ ebits of free entanglement (i.e., distillable entanglement) in the shared ancilla state are necessary as well as sufficient to have $1\rightarrow N$ exact cloning of any given set of two Bell states [given in Eq. (1)], by LOCC only [5]. This result immediately shows that the distillable entanglement of $\rho_N^{(2)}$ where $\rho_N^{(2)}=\frac{1}{2}(P[|B_1\rangle^{\otimes N}]+P[|B_3\rangle^{\otimes N}])$, in the Alice:Bob cut, is given by $E_D(\rho_N^{(2)})=N-1$.

III. 1→2 EXACT CLONING OF FOUR BELL STATES

Let us now come to the case where Alice and Bob have to clone one (but unknown) of the four Bell states given in Eq. (1). It can be easily shown that if Alice and Bob share three ebits (i.e., three known maximally entangled states of two qubits), they can prepare two copies of that unknown state, locally; because with one shared ebit, Alice can teleport her part (of the unknown Bell state) to Bob, and Bob can distinguish these four Bell states, and hence they can locally transform the remaining two copies of the Bell states to the desired Bell states.

We now show that for cloning any one of the four Bell states given in Eq. (1), locally, two ebits of the shared ancilla state are necessary. Consider the state where two far apart parties Alice and Bob are sharing two copies of one of the four Bell states with equal probabilities. Thus the shared state is

$$\rho_S=\frac{1}{4}\sum_{i=1}^{4}P[|B_i\rangle_{A_1B_1}\otimes|B_i\rangle_{A_2B_2}], \tag{2}$$

where Alice is holding the qubits A_1,A_2, while Bob is holding the qubits B_1,B_2. This state is interestingly separable in the $A_1A_2:B_1B_2$ cut [6]. If cloning of the four Bell states is possible with a shared ancilla state ρ, by using LOCC only, we then have

$$\rho_S\otimes\rho\rightarrow\frac{1}{4}\sum_{i=1}^{4}P[|B_i\rangle\otimes|B_i\rangle\otimes|B_i\rangle].$$

The final state has two ebits of distillable entanglement [7], and hence ρ must have at least two ebits of distillable entanglement.

Now we are going to show that two ebits of distillable entanglement, in the shared ancilla state, are also sufficient for exact cloning of the four Bell states. In order to do this, let us first consider teleportation of two-qubit states (locally) via the state ρ_S, given in Eq. (2). Interestingly, this state can also be written as

$$\rho_S=\frac{1}{4}\sum_{i=1}^{4}P[|B_i\rangle_{A_1A_2}\otimes|B_i\rangle_{B_1B_2}],$$

where the four qubits A_1,A_2,B_1,B_2 might be far apart, where Alice(i) is holding the qubit A_i, and similarly for Bob. Now we consider the teleportation of a two-qubit state shared between Alice(1) and Bob(1). Let Alice(1) and Bob(1) teleport their qubits to Alice(2) and Bob(2) respectively, using the same teleportation protocol, namely the standard teleportation protocol of Bennett *et al.* [8] for exactly teleporting an unknown qubit through the channel state $|B_1\rangle$. This teleportation [i.e., teleportation of any two-qubit state from the Alice(1):Bob(1) cut to the Alice(2):Bob(2) cut, via the shared channel state ρ_S, and using the exact protocol of Bennett *et al.* [8] for the channel state $|B_1\rangle$, for each qubit] can be represented in terms of the following maps (see [9]):

$$\sigma_i^{A_1}\otimes\sigma_j^{B_1}\rightarrow\delta_{ij}\sigma_i^{A_2}\otimes\sigma_j^{B_2}, \tag{3}$$

where $i,j=0,1,2,3$, and where $\sigma_0^{A_1}$ is the identity operator acting on the Hilbert space of the qubit A_1, $\sigma_1^{A_1}$ is the Pauli matrix σ_x acting on the Hilbert space of the qubit A_1, etc. [see Ref. [10] for an explanation of the map (3)]. Using this map, one can easily see that any Bell state [from Eq. (1)] shared between Alice(1) and Bob(1) can be exactly teleported to Alice(2) and Bob(2) [11].

Consider now the following state of six qubits A_i,B_i ($i=1,2,3$):

$$\rho^{(3)}=\frac{1}{4}\sum_{i=1}^{4}P[|B_i\rangle_{A_1B_2}\otimes|B_i\rangle_{A_2B_2}\otimes|B_1\rangle_{A_3B_3}]. \tag{4}$$

This state is a quasipure state [12,13]. It has two ebits of distillable entanglement in the $A_1A_2A_3:B_1B_2B_3$ cut [7], while it can also be created using two ebits of entanglement. Here we are going to provide a simple preparation procedure for this state, starting from two ebits of free entanglement. Let Alice and Bob start with the following shared two-ebit state: $D=P[|B_1\rangle_{A_1B_1}\otimes|B_1\rangle_{A_2B_2}]\otimes\frac{1}{2}(P[|B_1\rangle_{A_3B_3}]+P[|B_2\rangle_{A_3B_3}])$. Now, with equal probability, Bob either applies nothing or he applies the Pauli matrix σ_x on each of his three qubits. Next Alice and Bob both apply locally two consecutive CNOT operations on their respective qubits, taking the third pair of qubits as target (for both of these two CNOT operations), while the first two pairs of qubits act as source. This will directly give rise to the state $\rho^{(3)}$ of Eq. (4).

The two reduced density matrices of $\rho^{(3)}$ corresponding to the subsets $\{A_1,B_1,A_2,B_2\}$ and $\{A_1,B_1,A_3,B_3\}$ are the same Smolin state ρ_S. Thus if Alice(1) and Bob(1) jointly teleport [using the map given in (3)] the unknown Bell state, it will be reproduced both between Alice(2) and Bob(2) as well as between Alice(3) and Bob(3). Thus two ebits of the shared ancilla state [shared between Alice's side (A_1,A_2,A_3) and Bob's side (B_1,B_2,B_3)] are sufficient to have the $1\rightarrow 2$ cloning of one of the four Bell states given in Eq. (1).

IV. 1→*N* EXACT CLONING OF FOUR BELL STATES

Interestingly $1\rightarrow 3$ cloning of an unknown Bell state taken from the set given in Eq. (1) by LOCC also requires two ebits of entanglement. If Alice and Bob share the eight qubit ancilla state $\rho^{(4)}=\frac{1}{4}\Sigma_{i=1}^{4}P[|B_i\rangle_{A_1B_1}\otimes|B_i\rangle_{A_2B_2}\otimes|B_i\rangle_{A_3B_3}\otimes|B_i\rangle_{A_4B_4}]$, using the same procedure as above, they can have the desired $1\rightarrow 3$ cloning of the unknown Bell state. A single copy of this eight-qubit state can also be prepared from two ebits of free entanglement. We now describe a simple process for this state. Alice and Bob first start with the following shared two-ebit state: $D=\frac{1}{2}(P[|B_1\rangle_{A_1B_1}]+P[|B_3\rangle_{A_1B_1}])\otimes P[|B_1\rangle_{A_2B_2}\otimes|B_1\rangle_{A_3B_3}]\otimes\frac{1}{2}(P[|B_1\rangle_{A_4B_4}]+P[|B_2\rangle_{A_4B_4}])$. Alice and Bob both now apply two consecutive CNOT operations, where the first pair of qubits is taken as source, while the second and third pairs of qubits are targets. In the next step, Alice and Bob both apply locally three consecutive CNOT operations on their respective qubits, taking the fourth pair of qubits as target (for both of these two CNOT operations), while the first three pairs of qubits act as source. This will directly give rise to the state $\rho^{(4)}$.

This result can easily be generalized for $1\rightarrow N$ cloning of four Bell states, using the shared ancilla state

$$\rho^{(N+1)}=\frac{1}{4}\sum_{i=1}^{4}P[|B_i\rangle_{A_1B_1}\otimes|B_i\rangle_{A_2B_2}\ldots\otimes|B_i\rangle_{A_{(N+1)}B_{(N+1)}}]. \tag{5}$$

This ancilla state $\rho^{(N+1)}$ has N ebits of distillable entanglement when N is even, and $N-1$ for N odd. But since $\rho^{(N+1)}$ is reversible [12], it can be prepared using N and $N-1$ ebits for even and odd cases, respectively.

V. EXAMPLES OF QUASIPURE STATES

Using the above results of cloning we now provide some examples of quasipure states, each of which is reversible. Consider the following $2N$ party entangled state (where N is odd):

$$\rho(p)=\sum_{i=1}^{4}p_iP[|B_i\rangle^{\otimes N}],$$

where, for all the N copies of the Bell state $|B_i\rangle$, one qubit is with Alice while the other qubit is with Bob, and where $0\leqslant p_i\leqslant\frac{1}{2}$ for all $i=1,2,3,4$. This state can be prepared by the following process. Consider the density matrix $\rho_B=\Sigma_{i=1}^{4}p_iP[|B_i\rangle]$ with the same p_i as above (ρ_B is obviously separable). This state can be prepared locally by Alice and Bob. Alice and Bob also now locally prepare the state $\rho^{(N+1)}$, given as in Eq. (5), using $(N-1)$ ebits of free entanglement [12]. Take ρ_B with $\rho^{(N+1)}$ and apply the previous $1\rightarrow N$ cloning operation, the final state becomes $\rho(p)$. So the entanglement cost [14,15] E_c of the state $\rho(p)$ will be at most $(N-1)$. But now Alice and Bob locally apply $(N-1)$ consecutive CNOT operations on their respective sets of qubits (where the Nth qubit pair is taken as the target and each of the first $(N-1)$ qubit pairs is taken as source), and then they locally distinguish the two sets of Bell states $\{|B_1\rangle,|B_2\rangle\},\{|B_3\rangle,|B_4\rangle\}$ in the target qubit pairs. This will then directly give rise to $(N-1)$ copies of $|B_1\rangle$ or $|B_3\rangle$ (depending upon the measurement outcomes), and hence $(N-1)$ ebits of entanglement can be distilled from $\rho(p)$ (see also [12]). Hence, for this state, the entanglement cost and the distillable entanglement are same [with a value is equal to $(N-1)$], providing an example of reversibility.

VI. EXAMPLES OF IRREVERSIBLE STATES

Now we generalize the example of irreversibility in [16] to the case where N Bell states are involved. It has been shown in [16] that for the state

$$\sigma_1=pP[|B_1\rangle]+(1-p)P[|B_2\rangle],$$

where $0<p<1$ and $p\neq\frac{1}{2}$, we have $E_c(\sigma_1)=H_2[\frac{1}{2}+\sqrt{p(1-p)}]$. But its distillable entanglement is given by $E_D(\sigma_1)=1-H_2(p)$, where, for $0\leqslant x\leqslant 1, H_2(x)=-x\log_2 x-(1-x)\log_2(1-x)$ [14]. Thus we have here

$$E_c(\sigma_1)>E_D(\sigma_1). \tag{6}$$

Next we consider the state

$$\sigma_N=pP[|B_1\rangle^{\otimes N}]+(1-p)P[|B_2\rangle^{\otimes N}],$$

where $0<p<1$ and $p\neq\frac{1}{2}$. We are going to show now that $E_c(\sigma_N)>E_D(\sigma_N)$. First of all, we show that σ_N can be prepared locally from the state σ_1 together with $(N-1)$ ebits of free entanglement $|B_1\rangle^{\otimes(N-1)}$. In this direction, first both Alice and Bob locally apply the Hadamard transformation on the respective qubits of the state σ_1. Thus we will have the following transformation:

$$\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}]$$
$$\rightarrow\{pP[|B_1\rangle]+(1-p)P[|B_3\rangle]\}\otimes P[|B_1\rangle^{\otimes(N-1)}].$$

Now both Alice and Bob use CNOT operations locally and sequentially, where the source qubit belongs to the state σ_1, while the target qubits come from each of the $(N-1)$ Bell states $|B_1\rangle$ (see the case of cloning of the two Bell states $|B_1\rangle$ and $|B_3\rangle$). Thus we have now the following transformation:

$$\{pP[|B_1\rangle]+(1-p)P[|B_3\rangle]\}\otimes P[|B_1\rangle^{\otimes(N-1)}]$$
$$\rightarrow pP[|B_1\rangle^{\otimes N}]+(1-p)P[|B_3\rangle^{\otimes N}]\equiv\sigma_N'\text{ (say)}.$$

Again Alice and Bob locally apply Hadamard transformations on each of the N qubits of their sides, so finally we have the desired state σ_N. As the transformation $\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}]\rightarrow\sigma_N$ has been achieved above by using the local unitary transformation $U_A\otimes V_B$ in the Alice:Bob cut, $E_c(\sigma_N)=E_c(\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}])$ (in the Alice:Bob cut). And, as from Eq. (17) of Ref. [17], it is known that $E_c(\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}])=E_c(\sigma_1)+(N-1)$, then

$$E_c(\sigma_N)=E_c(\sigma_1)+(N-1)=H_2\left(\frac{1}{2}\sqrt{p(1-p)}\right)+(N-1). \tag{7}$$

On the other hand, using the reverse local unitary operation $U_A^{\dagger}\otimes V_B^{\dagger}$, we have the following transformation:

$$\sigma_N\rightarrow\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}]$$

and so $E_D(\sigma_N)=E_D(\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}])$. Now, considering the invariance under local unitary operations and the subadditivity property of the relative entropy of entanglement $E_R(\cdot)$, we have $E_R(\sigma_N)=E_R(\sigma_1\otimes P[|B_1\rangle^{\otimes(N-1)}])\leqslant E_R(\sigma_1)+(N-1)=1-H_2(p)+(N-1)$ [18]. And, as $E_R(\cdot)$ is an upper bound of $E_D(\cdot)$ [19], then, $E_D(\sigma_N) \leq E_R(\sigma_N) \leq E_R(\sigma_1) + (N-1) = E_D(\sigma_1) + (N-1)$ [as, for the state σ_1, we have $E_D(\sigma_1) = 1 - H_2(p)$ [14], and also $E_R(\sigma_1) = 1 - H_2(p)$ [18]]. But as we can distill $E_D(\sigma_1)$ ebits of entanglement from a single copy of σ_1, then from the state $\sigma_1 \otimes P[|B_1\rangle^{\otimes(N-1)}]$ we can distill at least $E_D(\sigma_1) + (N-1)$ ebits of entanglement. So

$$E_D(\sigma_N) = E_D(\sigma_1) + (N-1) = N - H_2(p). \tag{8}$$

Using Eqs. (6), (7), and (8), we have $E_c(\sigma_N) > E_D(\sigma_N)$. Hence the state σ_N is irreversible.

VII. CONCLUSION

In summary, we have shown that the minimum resource (in terms of free entanglement) required to make a $1 \rightarrow N$ copy cloning by LOCC of unknown Bell states, from any known set of two Bell states, is $(N-1)$. On the other hand, it has been shown here that the minimum resource required to make a $1 \rightarrow N$ copy cloning by LOCC of unknown Bell states from a set of four Bell states is different for odd and even cases. In the even case it is just equal to N ebits whereas in the odd case it is $(N-1)$. We have also given here some additional examples of reversible states as well as irreversible states, formed by using multiple copies of Bell states.

Our results show that, in order to have $1 \rightarrow N$ exact cloning of any given set of three Bell states, it is necessary to have at least $(N-1)$ ebits of free entanglement in the shared ancilla state, while N ebits of free entanglement (in the shared ancilla state) is sufficient in this case if N is even and $(1-N)$ ebits of free entanglement (in the shared ancilla state) is sufficient in this case if N is odd. The existence of a tighter bound on the sufficient condition, in this case, is an open question.

ACKNOWLEDGMENT

The authors are thankful to Debasis Sarkar for useful discussions, in particular, about teleportation via the Smolin state.

[1] W. K. Wootters and W. H. Zurek, Nature (London) **229**, 802 (1982); D. Diekes, Phys. Lett. **92A**, 271 (1982); H. P. Yuen, *ibid.* **113A**, 405 (1986).

[2] A bipartite state ρ_{AB}, shared by two far apart parties Alice and Bob, is said to be *quasipure* if it can be reversibly transformed, by LOCC, to the state $\sum_{i=1}^{m} p_i \rho_{A_1B_1}^{(i)} \otimes |\varphi_i\rangle_{A_2B_2}\langle\varphi_i|$, where the $\rho_{A_1B_1}^{(i)}$'s are pairwise orthogonal separable states, which can be exactly distinguished by LOCC, and where A_1, A_2 are subsystems of the system A, while B_1, B_2 are subsystems of the system B.

[3] For any (normalized) pure state $|\psi\rangle$, $P[|\psi\rangle]$ is the projector $|\psi\rangle\langle\psi|$ of $|\psi\rangle$.

[4] S. Ghosh, G. Kar, A. Roy, A. Sen (De), and U. Sen, Phys. Rev. Lett. **87**, 277902 (2001).

[5] This can be achieved in the following way. First, we assume that the two unknown Bell states, which have to be cloned to get N copies, are $|B_1\rangle_{A_1B_1}$ and $|B_3\rangle_{A_1B_1}$. Also let Alice and Bob share the $(N-1)$-ebit ancilla state $|B_1\rangle_{A_2B_2} \otimes |B_1\rangle_{A_3B_3} \otimes \cdots \otimes |B_1\rangle_{A_NB_N}$, where the N qubits $A_1, A_2, \ldots, A_N$ are with Alice and the N qubits $B_1, B_2, \ldots, B_N$ are with Bob. Both Alice and Bob first locally apply a CNOT operation, taking A_1 as source, A_2 as target (for Alice's CNOT), and taking B_1 as source, B_2 as target (for Bob's CNOT). After that, both Alice and Bob locally apply CNOT operations taking A_2 as source, A_3 as target (for Alice's CNOT), and taking B_2 as source, B_3 as target (for Bob's CNOT). Continuing this process, they will finally come up with N copies of the unknown Bell states. For any two Bell states from Eq. (1), in order to have their $1 \rightarrow N$ cloning by LOCC, we shall apply the same procedure (as has been done in the case of $1 \rightarrow 2$ cloning of any two Bell states) of first transforming them locally unitarily to $|B_1\rangle$, $|B_3\rangle$, and then we will have to apply the above-mentioned argument.

[6] J. A. Smolin, Phys. Rev. A **63**, 32306 (2000).

[7] D. Yang and Y.-X. Chen, Phys. Rev. A **69**, 024302 (2004).

[8] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[9] S. Ghosh, G. Kar, A. Roy, D. Sarkar, and U. Sen, Phys. Rev. A **64**, 042114 (2001).

[10] If the standard teleportation protocol of Bennett *et al.* [8] (for exactly teleporting an arbitrary qubit via the channel state $|B_1\rangle$) is used for both the channel states, one shared between A_1, A_2 and another shared between B_1, B_2 [where the total shared state is ρ_S, given by Eq. (2)], any initial two-qubit state $|\psi\rangle$ [shared between Alice(1) and Bob(1)] will suffer the following change:

$$P[|\psi\rangle] \rightarrow \frac{1}{4}\sum_{i=0}^{3} (\sigma_i \otimes \sigma_i) P[|\psi\rangle](\sigma_i \otimes \sigma_i).$$

So the only two-qubit states that remain invariant under this change are nothing but any mixture of the Bell states $|B_1\rangle$, $B_2\rangle$, $|B_3\rangle$, $|B_4\rangle$. The map given in (3) Eq. is nothing but a concise mathematical representation of this fact (of invariance).

[11] One may be surprised that, although there is no entanglement in the [Alice(1), Bob(1)] :Alice(2), Bob(2)] cut of the state ρ_S given in Eq. (2), each of the four Bell states is exactly teleported from the side of Alice(1),Bob(1) to the side of Alice(2),Bob(2). First of all, no entanglement is being created by this teleportation process, which is here a LOCC among the four parties Alice(1), Alice(2), Bob(1), and Bob(2). Second, just four pairwise orthogonal states $|B_1\rangle$, $|B_2\rangle$, $|B_3\rangle$, $|B_4\rangle$ [given in Eq. (1)] from infinitely many states (forming the four-dimensional Hilbert space of two qubits) are being exactly teleported in this way—no other (pure) state can be exactly teleported. We know that this phenomenon holds good even when Alice(1), Bob(1) share a product state with Alice(2), Bob(2).

[12] Y.-X. Chen, J.-S. Jin, and D. Yang, Phys. Rev. A **67**, 014302 (2003).

[13] K. G. H. Vollbrecht, R. F. Werner, and M. M. Wolf, e-print quant-ph/0301072.
[14] C. H. Bennett, D. P. DiVincenzo, J. Smolin. and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[15] P. M. Hayden, M. Horodecki, and B. M. Terhal, J. Phys. A **34**, 6891 (2001).
[16] G. Vidal, W. Dur, and J. I. Cirac, Phys. Rev. Lett. **89**, 027901 (2002).
[17] G. Vidal and J. I. Cirac, Phys. Rev. Lett. **86**, 5803 (2001).
[18] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997); V. Vedral and M. B. Plenio, Phys. Rev. A **57**, 1619 (1998); S. Wu and Y. Zhang, e-print quant-ph/0004018.
[19] E. M. Rains, Phys. Rev. A **60**, 179 (1999).