

# On solute dispersion in pulsatile flow through a channel with absorbing walls

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Received 10 November 2003; accepted 6 May 2004

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## Abstract

The paper presents the longitudinal dispersion of passive tracer molecules released in an incompressible viscous fluid flowing through a channel with reactive walls under the action of a periodic pressure gradient. A finite-difference implicit scheme is adopted to solve the unsteady advection–diffusion equation based on the Aris–Barton method of moments for all time period. Here it is shown how the spreading of tracers is influenced by the shear flow, lateral diffusion about its mean position due to the action of absorption at both the walls. The analysis has been performed for three different cases: steady, periodic and the combined effect of steady and periodic currents, separately. The results show that for all cases the dispersion coefficient asymptotically reaches a stationary state after a certain critical time and it achieves a stationary state at earlier instant of time, when absorption at the walls increases. The axial distributions of mean concentration are determined from the first four central moments by using Hermite polynomial representation for all three different flow velocities.

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*Keywords:* Longitudinal dispersion; Finite-difference; Channel; Absorption; Concentration distribution

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## 1. Introduction

The basic mechanisms of tracer dispersion in laminar Poiseuille flow were first described by Taylor [1] in his classic paper and subsequently the paper was extended by Aris [2] for Poiseuille flow using the method of moments. They concluded that for sufficiently large time any localized initial configuration of

tracer tends to a Gaussian distribution moving with the mean speed of the flow. Under the pulsating pressure gradient, Aris [3] analyzed the longitudinal dispersion of a solute in an oscillatory flow of a viscous incompressible fluid due to pulsating pressure gradient by using his method of moments. Barton [4] presented an approach for steady flow to resolve the technical difficulties in the Aris method of moments and obtained the solutions of second and third-order moments equations which are valid for all time. Mukherjee and Mazumder [5] extended the Aris–Barton method of moments to study the all time evolution of second

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central moment equation for tracer dispersion in an oscillatory flow through a conduit. The solution was based on the method of separation of variables which depends upon a certain eigenvalue problem with a discrete spectrum of eigenvalues. They confined their analysis only to the case of flow due to a periodic pressure gradient with non-zero mean. Recently, Bandyopadhyay and Mazumder [6] extended the analysis of Mukherjee and Mazumder [5] to study the mean, variance and longitudinal dispersion coefficient of initial tracer material released in an incompressible viscous fluid flowing through a channel. The analysis was performed for three different flow velocities to identify the individual effect of dispersion process due to steady, periodic and periodic with non-zero mean flow. They confined their analysis only upto the second-order moment. Since the pioneering work of Taylor [1], dispersion problems have been extensively studied to cover a great variety of transport problems, but dispersion under the boundary absorption has not received much attention, despite the fact that such problems are of importance in environmental fluid mechanics.

Our aim is to extend the work of Bandyopadhyay and Mazumder [6], and its main objective is to study the effect of boundary absorption or the first-order heterogeneous chemical reaction at the channel walls for all time evolution of first four central moments of tracer dispersion in three different flow velocities. A numerical scheme (Crank–Nicholson implicit type) has been adopted to solve a set of unsteady integral moment equations arising from advection–diffusion equation for all times when the solute is initially uniform over the cross section of the channel. More precisely, results are shown for all times how the spreading of tracers is influenced by the shear flow caused by periodic pressure with non-zero mean, how the spreading of tracers is controlled or protected by reaction parameter, how the center of mass moves and what are the patterns of the mean concentration distributions in three different flow velocities in presence of boundary absorption. The mean concentration distribution is approximated by Edgeworth series expansion using the first four central moments. The recent interest in the analysis of shear dispersion in presence of boundary absorption has been motivated to deposition, transport across a semipermeable membrane, chemical reaction design and depletion of tracers released from chemical plants or industries.

Using Taylor's conceptual model, Gupta and Gupta [7] studied the dispersion of reactive solute in liquid flowing through a channel in presence of first-order heterogeneous reaction for asymptotically large time. Smith [8] used a delay-diffusion equation to study the effect of the boundary reaction on the longitudinal dispersion in shear flows. An attempt was made by Barton [9] to explain the dispersion of reactive solute in pipe Poiseuille flow in presence of boundary reaction for asymptotically large time and presented the results for concentration distribution for weak and vigorous reactions at the boundary. Pumama [10] investigated the case of reaction and retention at the flow boundaries when the tracer is chemically active. Mazumder and Das [11] studied the effect of wall conductance on the axial dispersion in the pulsatile tube flow. Using a numerical scheme they computed the dispersion coefficient for different values of absorption parameter, which is valid for all time.

## 2. Mathematical formulation

Consider an unsteady fully developed viscous, incompressible, two-dimensional laminar flow caused by axial periodic pressure gradient with non-zero mean between two parallel plates separated by a distance  $2L$  apart. We have used a Cartesian coordinate system with  $x^*$ -axis along the flow and  $y^*$ -axis perpendicular to the flow. Plates are situated at  $y^* = \pm L$ . Here, we have considered a unidirectional flow along  $x^*$ -direction and it is a function of  $y^*$  and  $t^*$ .

When the passive solute of constant molecular diffusivity  $D$  is released in time-dependent flow due to periodic pressure gradient in presence of first-order reaction at both the walls of the channel, the concentration  $C(x, y, t)$  of the solute satisfies the non-dimensional convective–diffusion equation of the form

$$\frac{\partial C}{\partial t} + P_e u(y, t) \frac{\partial C}{\partial x} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) C, \quad (1)$$

where the dimensionless quantities are

$$x = \frac{x^*}{L}, \quad y = \frac{y^*}{L}, \quad t = \frac{Dt^*}{L^2}, \quad u = \frac{u^*}{U}, \quad P_e = \frac{UL}{D}.$$

Here the velocity  $u(y, t)$  is the combination of steady velocity  $u_0(y)$  and the periodic velocity

$u_1(y, t)$  due to the imposed perturbed pressure gradient,  $U$  the reference velocity and  $P_c$  the Péclet number, measures the relative characteristic time of the diffusion process ( $L^2/D$ ) to the convection process ( $L/U$ ). In order to formulate Eq. (1), it is assumed that the tracer molecules do not affect the flow of the carrier fluid. This assumption is justified in this case because a few tracer molecules are introduced. But for a true mixture of two or more components, one would have to deal with balance equations for both the constituents which should be coupled. A detailed discussion of the mixture theory can be found in the book [12] as an alternative approach to studying the problem.

The initial and boundary conditions are

$$\left. \begin{aligned} C(x, y, 0) &= \mathcal{C}(x, y), \\ \frac{\partial C}{\partial y} + \beta C &= 0 \quad \text{at } y = 1 \\ &\text{at the upper wall of the channel,} \\ \frac{\partial C}{\partial y} - \beta C &= 0 \quad \text{at } y = -1 \\ &\text{at the lower wall of the channel,} \\ C &\text{ is finite at all points,} \\ \frac{1}{2} \int_{-1}^{+1} \int_{-\infty}^{+\infty} C(0, x, y) dx dy &= 1, \end{aligned} \right\} \quad (2)$$

where  $\beta (= \beta^* L)$  is the first-order reaction rate or absorption parameter corresponding to the catalytic reaction at the wall. The case  $\beta = 0$  is usually referred to as the dispersion of passive contaminant that is neutrally buoyant and chemically inert.

The flow velocity  $u(y, t)$  in (1) is obtained from the solution of approximated Navier–Stokes’s equation subject to the no-slip conditions  $u(\pm 1, t) = 0$  at the walls  $y = \pm 1$  and is given by [6]

$$u(y, t) = u_0(y) + u_1(y, t) \quad (3)$$

with

$$u_0(y) = \frac{1}{2} (1 - y^2), \quad (4)$$

$$\left. \begin{aligned} u_1(y, t) &= -\text{Re} \left[ \frac{i\varepsilon}{\alpha} \left( 1 - \frac{\cosh \sqrt{ixy}}{\cosh \sqrt{ix}} \right) e^{izSt} \right], \\ i &= \sqrt{-1}, \end{aligned} \right\} \quad (5)$$

where  $u = u^*/U$  is the non-dimensional axial velocity ( $U$  being time averaged axial velocity  $P_x^* L^2/4\nu$ ),  $\alpha = \omega L^2/\nu$  is the dimensionless frequency parameter or oscillation Reynolds number,  $S = \nu/D$  is the Schmidt number,  $\varepsilon$  and  $\omega$  are the amplitude and frequency of the pressure pulsation, respectively, and  $\nu$  is the kinematic viscosity of the fluid. Here Eq. (4) represents plane Poiseuille flow and Eq. (5) corresponds to the unsteady part of the flow.

Following the Aris method of moments, the  $p$ th integral moment of the concentration distribution can be defined as

$$C_p(y, t) = \int_{-\infty}^{\infty} x^p C(x, y, t) dx \quad (6)$$

and the concentration distribution of the solute over the cross section of the channel is

$$M_p(t) = \bar{C}_p = \frac{1}{2} \int_{-1}^{+1} C_p(y, t) dy. \quad (7)$$

So, using (6), the diffusion equation (1) with respective initial and boundary conditions becomes

$$\frac{\partial C_p}{\partial t} - \frac{\partial^2 C_p}{\partial y^2} = pu(y, t)P_c C_{p-1} + p(p-1)C_{p-2} \quad (8)$$

with

$$C_p(y, 0) = \mathcal{C}_p(y), \quad (9)$$

where

$$\left. \begin{aligned} \mathcal{C}_p(y) &= 1 \quad \text{for } p = 0, \quad \mathcal{C}_p(y) = 0 \quad \text{for } p > 0, \\ \frac{\partial C_p}{\partial y} + \beta C_p &= 0 \quad \text{at } y = 1, \\ \frac{\partial C_p}{\partial y} - \beta C_p &= 0 \quad \text{at } y = -1, \end{aligned} \right\} \quad (10)$$

and

$$\left. \begin{aligned} \frac{dM_p}{dt} &= pP_c \overline{u(y, t)C_{p-1}} + p(p-1)\overline{C_{p-2}} \\ &\quad - \frac{1}{2} \beta [C_p(-1, t) + C_p(+1, t)] \end{aligned} \right\} \quad (11)$$

with

$$\begin{aligned} M_p(0) &= 1 \quad \text{for } p = 0, \\ M_p(0) &= 0 \quad \text{for } p > 0, \end{aligned} \quad (12)$$

over-bar denotes the cross-sectional mean.

The  $p$ th integral moment of the concentration distribution about the mean can be defined as

$$v_p(t) = \frac{1}{2M_0} \int_{-1}^{+1} \int_{-\infty}^{+\infty} (x - x_g)^p C \, dx \, dy, \quad (13)$$

where

$$x_g = \frac{1}{2M_0} \int_{-1}^{+1} \int_{-\infty}^{+\infty} xC \, dx \, dy = \frac{M_1}{M_0}$$

is the centroid or first moment of the solute which measures the location of the center of gravity of the slug movement with the mean velocity of the fluid, initially located at the source, and  $M_0$  represents the total mass of the reactive solute in the whole volume of the channel. From (13)  $v_2$  represents the variance related to the dispersion of the contaminant about the mean position, the third ( $v_3$ ) and fourth ( $v_4$ ) central moments represent the skewness and kurtosis of the distribution of the solute, respectively. The third moment (skewness) measures the symmetry and the fourth moment (kurtosis) measures the peakedness of the distributions of tracer materials about the mean. The negative value of the skewness indicates an asymmetric profile with a tail to the left of the maximum and the large positive value of kurtosis ( $\beta_3 > 3$ ) indicates a profile with a much narrower peak than the Gaussian distribution.

### 3. Solution procedure

Owing to the complexity of the analytical solution of moment equations ( $p > 1$ ) subject to the initial and boundary conditions for  $\beta \neq 0$ , a finite difference method based on Crank–Nicholson implicit scheme has been adopted to solve the set of integral moment equations. The scheme has been discussed in detail in the work of Mazumder and Das [11]. The derivatives and all other terms have been written at the mesh point  $(i+1, j)$ , where  $i=1$  corresponds to the time  $t=0$  and  $j=1$ , to the lower wall of the channel  $y=-1$ . The mesh point  $(i, j)$  indicates a point where  $t_i = (i-1) \times \Delta t$  and  $y_j = -1 + (j-1) \times \Delta y$ ,  $\Delta t$  and  $\Delta y$  are the increments of  $t$  and  $y$ , respectively. The resulting finite

difference equation becomes a system of linear algebraic equation with a tri-diagonal coefficient matrix,

$$\begin{aligned} P_j C_p(i+1, j+1) + Q_j C_p(i+1, j) \\ + R_j C_p(i+1, j-1) = S_j, \end{aligned} \quad (14)$$

where  $P_j$ ,  $Q_j$ ,  $R_j$  and  $S_j$  are the matrix elements.

The finite difference form of the initial and boundary conditions are

$$C_p(1, j) = \begin{cases} 1 & \text{for } p = 0, \\ 0 & \text{for } p \geq 1, \end{cases} \quad (15)$$

$$C_p(i+1, 0) = C_p(i+1, 2) - 2\beta\Delta y C_p(i+1, 1) \quad (16)$$

at the lower wall of the channel and

$$\begin{aligned} C_p(i+1, M+1) = C_p(i+1, M-1) \\ + 2\beta\Delta y C_p(i+1, M) \end{aligned} \quad (17)$$

at the upper wall of the channel for  $p \geq 0$ ,  $M$  is the value of  $j$  at the upper wall.

This tri-diagonal coefficient matrix has been solved by the method of Thomas algorithm [13] with the help of prescribed initial and boundary conditions. The value of  $M_p$  calculated from (11) by applying Simpson's one-third rule, with the known values of  $u(y, t)$  and  $C_p$  at the corresponding grid points. The values of the variables can be calculated for all time iteratively in the marching direction. Numerical experimentation has been performed for three different velocity profiles to distinguish the individual dispersion processes due to steady, periodic and periodic with non-zero flows. For the steady current, a mesh size ( $\Delta t = 0.0001$ ,  $\Delta y = 0.067$ ) gives satisfactory results. For the oscillatory and periodic with non-zero mean flows, a mesh size ( $\Delta t = 0.00001$ ,  $\Delta y = 0.067$ ) gives satisfactory results for  $\varepsilon = 1.5$  and for the frequencies  $\alpha = 0.5, 1.0$  and  $4.0$ . Actually, the smaller time interval is needed to trap the oscillatory nature in the dispersion process.

### 4. Discussion of results

In order to validate the numerical scheme with the analytical results, a check has been made on the integral moments (mean, variance) of the concentration distribution and longitudinal dispersion coefficient for all three different flow velocities, when the absorption

parameter  $\beta = 0$ . The results are in good agreement with those of Yasuda [14], Mukherjee and Mazumder [5], and Bandyopadhyay and Mazumder [6].

When  $p = 0$ , Eq. (11) becomes

$$M_0(t, \beta) = 1 - \frac{\beta}{2} \int [C_0(-1, t) + C_0(+1, t)] dt, \quad (18)$$

where  $C_0(-1, t)$  and  $C_0(+1, t)$  are found from the solution of Eq. (8). Eq. (18) represents the total mass of the solute, which is a function of  $\beta$  and  $t$ . We observe here that how the total amount of tracer material is depleted over time for a given reaction parameter  $\beta$ . When  $\beta = 0$ ,  $M_0 = 1$ , which represents a constant mass in the whole channel with respect to time. It is seen that the centroid displacement  $x_g$  of the solute mainly depends on the parameters  $\alpha$ ,  $\varepsilon$ ,  $\beta$  and dispersion time  $t$ . When the flow is steady, it is observed that the centroid displacement ( $x_g$ ) increases linearly with time for all  $\beta$  and it advances for a given dispersion time  $t$  and the mean position of the slug moves more rapidly than the average velocity  $\bar{u}$ , when the reaction takes place. For periodic flow, it increases with a wavy nature and for combined flow the centroid of the tracer moves cyclically with the oscillatory nature of the flow and in both the cases amplitude of the oscillation decreases as the boundary reaction  $\beta$  increases. According to Aris, the effective longitudinal dispersion coefficient  $D_a$  may be defined as

$$D_a(\alpha, S, \varepsilon, \beta, t) = \frac{1}{2P_c^2} \frac{dv_2}{dt}. \quad (19)$$

Here  $D_a$  depends on the frequency of oscillation  $\alpha$ , Schmidt number  $S$ , amplitude of the pressure pulsation  $\varepsilon$ , absorption parameter  $\beta$  and the dispersion time  $t$ . The parameter  $\alpha (= \omega L^2/\nu)$  is a measure of ratio of the time ( $L^2/\nu$ ) taken for viscosity to smooth out the transverse variation in vorticity to the period of oscillation ( $1/\omega$ ) or the ratio of the half of channel width  $L$  to the Stokes-layer thickness ( $\sqrt{\nu/\omega}$ ). A small value of  $\alpha$  represents a large viscous layer near the wall compared with a small inviscid core near the center or alternatively, a large oscillation period compared with viscous diffusion time and therefore quasi-steady flow and vice versa for large  $\alpha$ . The Schmidt number  $S (= \nu/D)$  is the ratio of viscous diffusion to the molecular diffusion. When the flow is steady ( $u = u_0(y)$ ),  $D_a$  initially increases and asymptotically reaches a steady

state ( $D_a \sim 0.0023$ ) at dimensionless dispersion time  $t \sim 0.34$ . For a given time  $t$ ,  $D_a$  decreases with increase in absorption parameter  $\beta$ , because the increase in absorption at the boundaries helps to increase the number of moles in the reactive material undergoing chemical reaction and changing the amount of slug material across the channel and hence there is a drop in  $D_a$ . For the oscillatory current ( $u = u_1(y, t)$ ) and when  $P_c = S = 10^3$ , the variations of  $D_a$  with time are depicted for different values of  $\beta$  in Fig. 1(a, b) for  $\alpha = 0.5$ , Fig. 1(c, d) for  $\alpha = 1.0$  and Fig. 1(e, f) for  $\alpha = 4.0$ . It is seen from the figures that for low frequency in periodic flow the dispersion coefficient  $D_a$  changes cyclically with a double-frequency period and it reaches a stationary state after a certain critical time  $t_c$ , which is related to the cross-sectional mixing time ( $L^2/D$ ). When  $\alpha = 0.5, 1.0, 4.0$  and  $\beta = 0$ , the  $D_a$  reaches stationary state at the critical times  $t_c \sim 0.38, 0.43$  and  $0.49$ , respectively. Thus, it is clear from the figures that as  $\alpha$  increases the fluctuation in  $D_a$  increases and reaches a stationary state later than the smaller frequency. But for a given frequency, the critical time  $t_c$  for stationary state achieves in an earlier instant of time as  $\beta$  increases. Furthermore, in the case of low frequency, the amplitudes of oscillation of  $D_a$  during the first and second half of the period of oscillatory flow are approximately equal, but in the case high frequency, period is so short that  $D_a$  initially varies almost single-frequency oscillation as the periodic current and then achieves to a double-frequency period at a large time. However, the situation completely stabilizes after a certain time and the dispersion takes place at a fairly uniform rate with a double-frequency oscillation. The dispersion coefficient  $D_a$  moves cyclically with time even in the stationary state and it is found to be much smaller than  $D_a$  in the steady flow. It is interesting to note that the  $D_a$  decreases with the frequency  $\alpha$ , because the shear effect due to high frequency of the periodic current on  $D_a$  is negligible but is more significant on the steady and quasi-steady current. The fluctuations in the velocity profiles cause the positive and negative dispersion of the solute during the period of oscillation which contracts at each flow reversal during the period of oscillation [15].

The temporal variation of  $D_a$  due to the combined effect of steady and periodic ( $u_0(y) + u_1(y, t)$ ) current for  $\alpha = 0.5, 1.0$  and  $4.0$ ;  $\beta = 0, 1, 3$  and  $10$  with  $P_c = S = 10^3$  have been plotted in Fig. 2(a–f). When

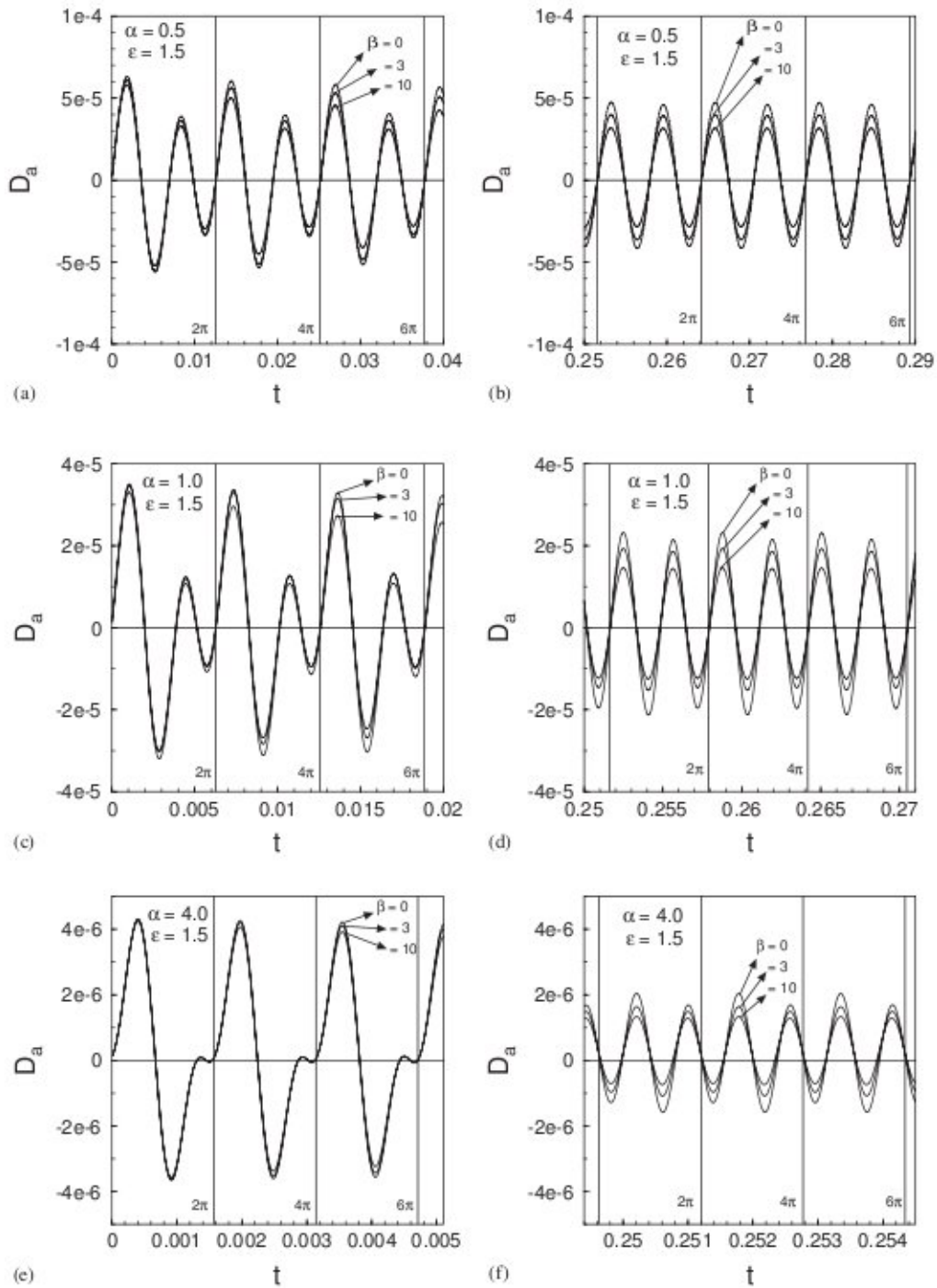


Fig. 1. Dispersion coefficient  $D_a$  due to periodic flow; (a), (c), (e) for small time and (b), (d), (f) for large time for different  $\beta$ , when  $P_e = S = 10^3$ .

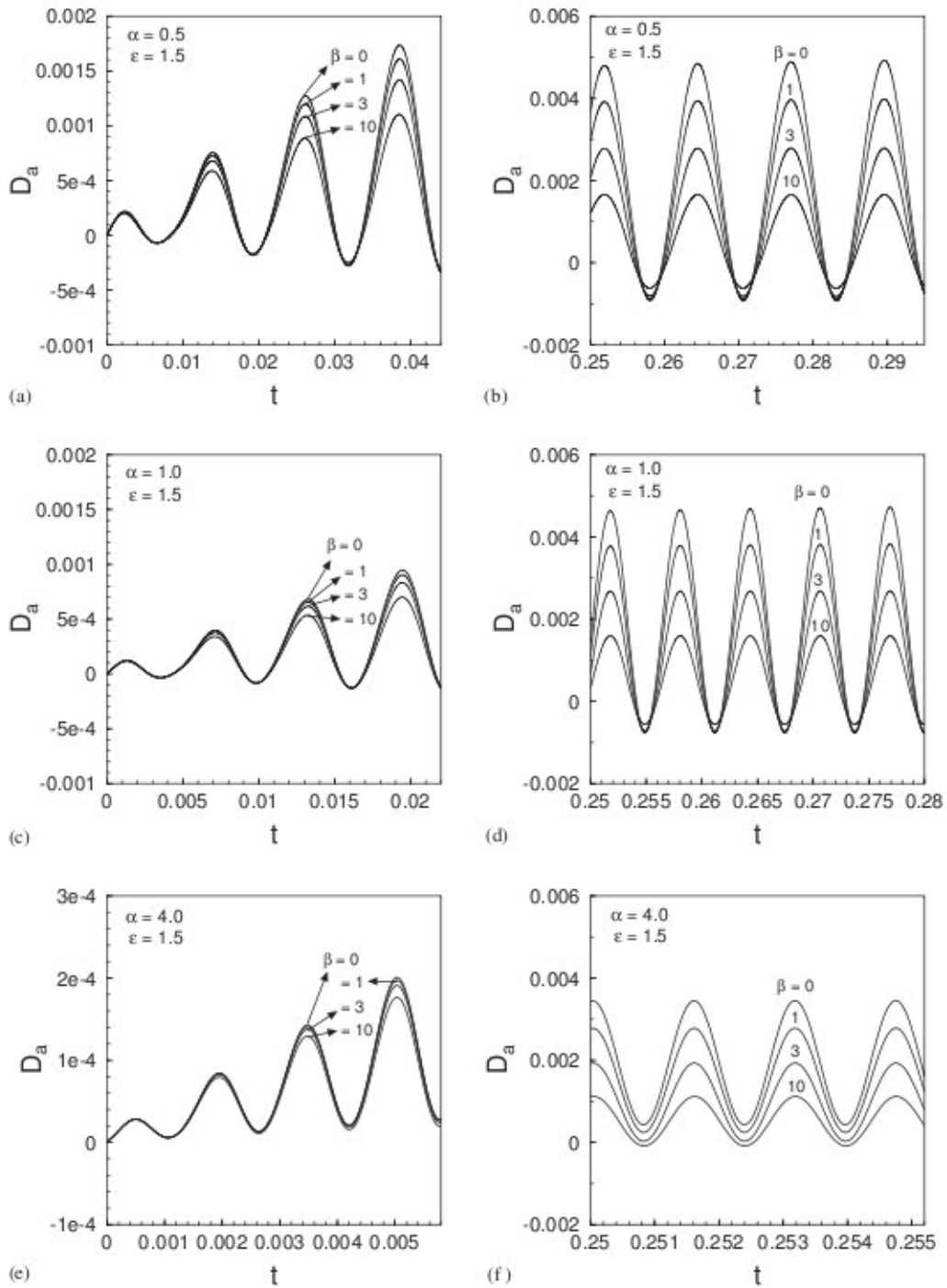


Fig. 2. Dispersion coefficient  $D_a$  due to periodic flow with non-zero mean; (a), (c), (e) for small time and (b), (d), (f) for large time for different  $\beta$ , when  $P_e = S = 10^3$ .

these are compared with  $D_a$  due to periodic current (see Fig. 1),  $D_a$  no longer has the double-frequency period. Moreover, for all cases the amplitude of  $D_a$  increases initially up to a certain extent of time  $t$ , then it becomes stable for a long time. Also, it is observed that for low frequency, the oscillatory nature of  $D_a$  possesses positive and negative dispersion with time, but at high frequency  $D_a$  is always positive for all values of  $\beta$  (Fig. 2(e,f)). As the periodic current moves with the non-zero mean flow in the same direction, the effect of frequency parameter due to periodic pressure gradient becomes less significant. For a given  $\alpha$ ,  $D_a$  decreases with increase in  $\beta$  for all cases. If the longitudinal dispersion  $D_a$  for oscillatory channel flow with non-zero mean is compared with that of pipe flow, it is observed that  $D_a$  is always positive for channel flow for high frequency, whereas for pipe flow there are both positive and negative dispersion [11]. The variation of  $D_a$  has been plotted in Fig. 3(a,b) for periodic and periodic with non-zero mean flows for  $S = 1.0$ ,  $\varepsilon = 1.5$  and  $\alpha = 1.0$  and  $4.0$  with different values of  $\beta$ . It is quite clear that the amplitude and frequency of imposed pressure pulsation exert enormous influence on the tracer dispersion due to oscillatory as well as periodic flow with non-zero mean. It is also noted that for a given  $\alpha$ ,  $D_a$  decreases with increase in  $\beta$  for all cases. It may be mentioned here that there is a remarkable difference in the behavior of dispersion coefficient due to small and large frequency of oscillation. Moreover, it is observed that in channel flow for all three cases: steady, periodic and periodic with non-zero mean flows; the value of the dispersion coefficient  $D_a$  much smaller than that of tube flow.

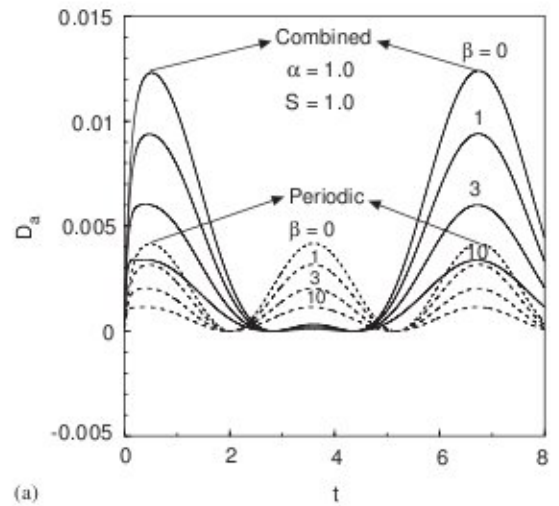
The coefficients of skewness ( $\beta_2$ ) and kurtosis ( $\beta_3$ ) are the important criteria to measure the degree of the symmetry and peakedness of a concentration distribution of a solute, and are defined by

$$\beta_2 = \frac{v_3}{v_2^{3/2}} \quad (20)$$

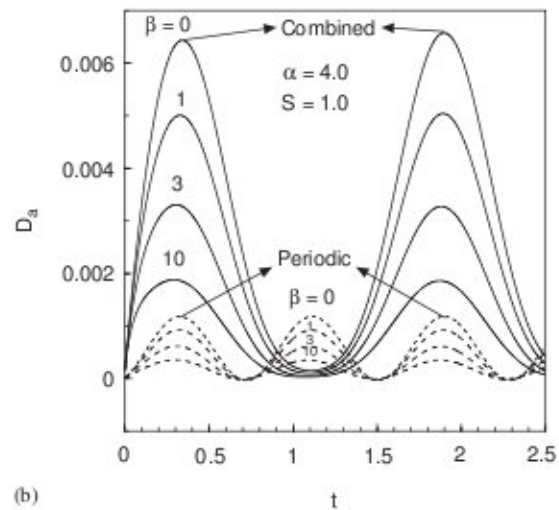
and

$$\beta_3 = \frac{v_4}{v_2^2} - 3. \quad (21)$$

For a Gaussian distribution both  $\beta_2$  and  $\beta_3$  are zero and the non-zero value of skewness and kurtosis will indicate deviations from Gaussian distribution. Fig. 4(a–c) show the temporal variation of skewness for  $\alpha = 0.5$ ,



(a)



(b)

Fig. 3. Dispersion coefficient  $D_a$  against time for  $Pe = 10^3$  and  $S = 1$  for different  $\beta$ ; (a)  $\alpha = 1.0$ , (b)  $\alpha = 4.0$ .

$Pe = S = 10^3$ , and  $\beta = 0, 1, 3$  and  $10$ . It is observed that the skewness in periodic current changes cyclically with a single-frequency period and decreases with dispersion time  $t$ . Also, it is seen that as the boundary reaction  $\beta$  and frequency of the oscillation  $\alpha$  increase, the amplitude of the skewness decreases. In a complete period of oscillatory current, the skewness is approximately zero and therefore the concentration distribution of the solute is essentially symmetrical even at large time. The variation of  $\beta_2$  due to shear effect of



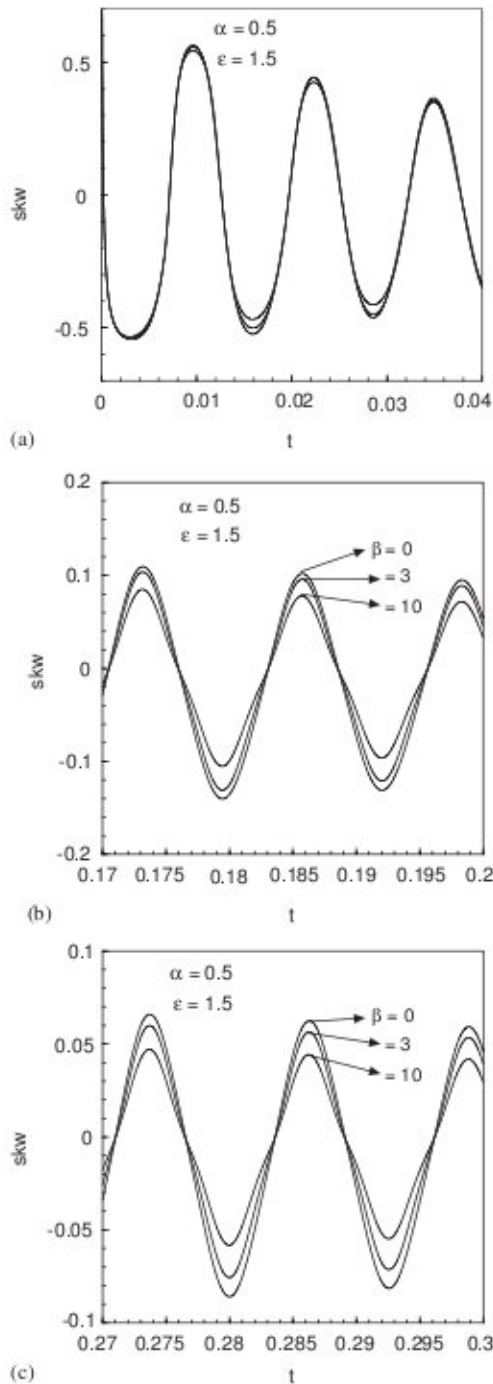


Fig. 4. Skewness ( $\beta_2$ ) of the distribution due to periodic flow for  $P_e = S = 10^3$ ; (a) small time (b) medium time and (c) large time for different  $\beta$ .

periodic current with non-zero mean has been plotted in Fig. 5(a–c) for various values of reaction parameter  $\beta$ . It is seen that the skewness initially decreases with time and moves asymptotically to a constant value after a certain time and have a tendency to reach symmetric distribution at large time. Also, it is seen that for periodic channel flow the skewness of the distribution is initially greater than that of tube flow but after a certain time the skewness values approximately equal in both the conduits. Moreover, it is observed that for periodic with non-zero channel flow the skewness values are negative for all  $\beta$  but in combined tube flow the skewness value becomes negative for large value of  $\beta$ .

The widely used conventional method of moments provides a great deal of information about the concentration distribution across the flow. For any approach to Gaussian distribution, it is worthwhile to use Hermite polynomials with the computed results of higher moments. Once the central moments  $v_2, v_3, v_4$  are known, it is possible to compute the mean concentration distribution  $C_m(x, t)$  with the help of Hermite polynomial representation [16] and is given by

$$C_m(x, t) = M_0(t)e^{-z^2} \sum_{n=0}^{\infty} a_n(t) H_n(x), \quad (22)$$

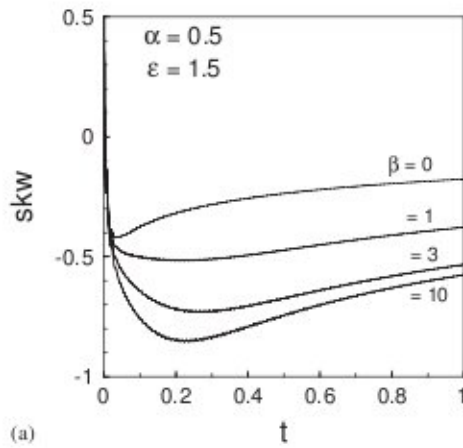
where,  $z = (x - x_g)/(2v_2)^{1/2}$ ,  $x_g = M_1/M_0$  and  $H_i$ , the Hermite polynomials, satisfy the recurrence relation with  $H_0(x) = 1.0$  as

$$H_{i+1}(x) = 2xH_i(x) - 2iH_{i-1}(x), \quad i = 0, 1, 2, \dots \quad (23)$$

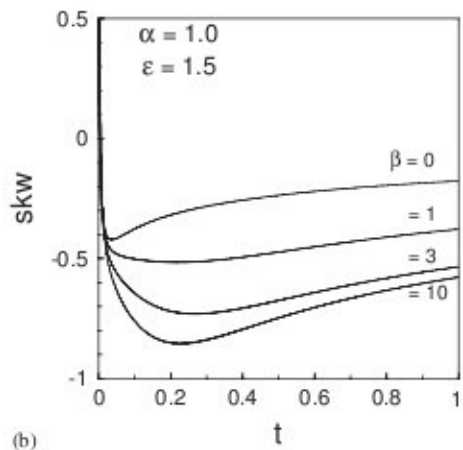
The coefficients  $a_i$  are

$$a_0 = 1/(2\pi v_2)^{1/2}, \quad a_1 = a_2 = 0, \\ a_3 = 2^{1/2} \frac{a_0 \beta_2}{24}, \quad a_4 = \frac{a_0 \beta_3}{96}.$$

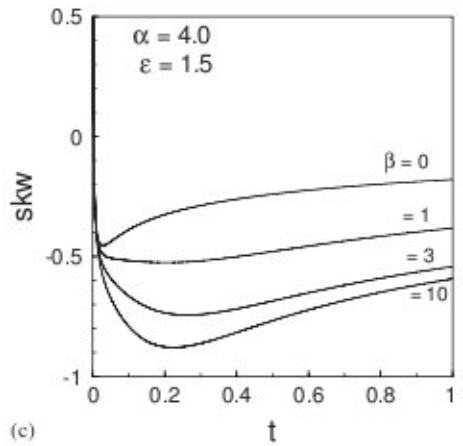
The variation of mean concentration distribution  $C_m(x, t)$  has been presented in Fig. 6(a–c) against the axial distance  $(x - x_g)$  for periodic flow through the channel for various values of  $\beta$  and  $t$ , when  $\alpha = 0.5$  and  $P_e = S = 10^3$ . It is seen from Fig. 6(a, b) that for fixed  $\alpha$ , the tracer material always remains constant for  $\beta = 0$ , whereas for a given reaction parameter  $\beta = 10$  the amount of reactive material is depleted over time; and then it disperses longitudinally with time.



(a)

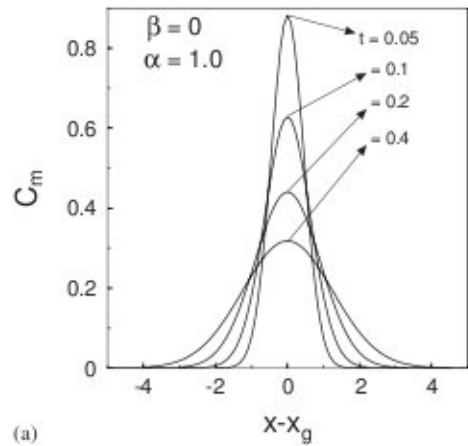


(b)

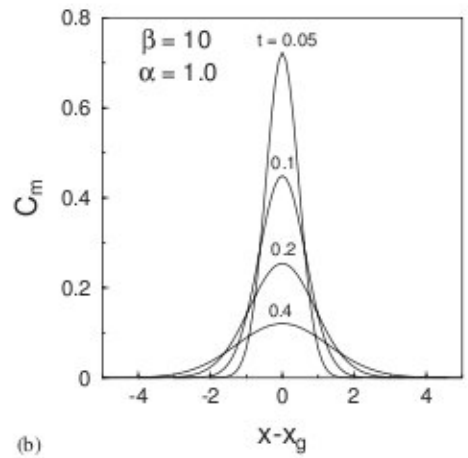


(c)

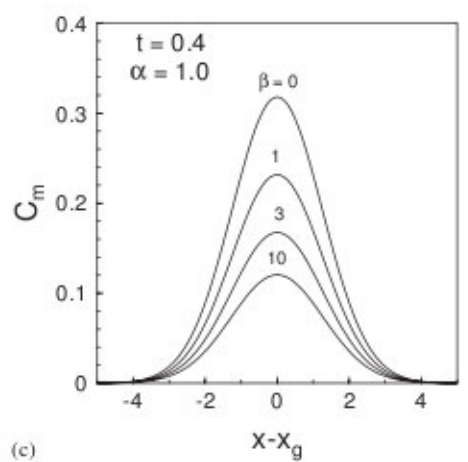
Fig. 5. Skewness ( $\beta_2$ ) of the distribution due to periodic flow with non-zero mean for  $P_e = S = 10^3$  for different  $\beta$ : (a)  $\alpha = 0.5$ , (b)  $\alpha = 1.0$ , (c)  $\alpha = 4.0$ .



(a)



(b)



(c)

Fig. 6. Mean concentration distribution  $C_m(x, t)$  along the periodic channel flow for  $P_e = S = 10^3$ , and  $\alpha = 1.0$  when (a)  $\beta = 0$ , (b)  $\beta = 10$  for different dispersion time  $t$ , and when (c)  $t = 0.4$  with different  $\beta$ .

As the dispersion time  $t$  increases, the peak of the distribution decreases and tends to become flat. Fig. 6(c) shows the depletion of tracer materials over the absorption parameter  $\beta$  for a given dispersion time  $t$ . Fig. 7(a–c) shows the plots of mean concentration distributions  $C_m(x, t)$  in periodic flow for various values of  $\beta$  and  $\alpha$ , when the dispersion time  $t$  is fixed. It is noted from the figures that for a given  $\beta$  the peak of the distribution increases with increase of  $\alpha$ , which implies the reduction of longitudinal dispersion due to high frequency.

Moreover, for comparison, the profiles of  $C_m(x, t)$  along the axial direction are plotted in Fig. 8(a–c) for the periodic flow with non-zero mean, when  $\alpha = 0.5$  and  $P_e = S = 10^3$  for various dispersion time  $t$  and  $\beta$  ( $=0, 1, 3$  and  $10$ ). It is observed that the peak of the distribution gradually decreases with time  $t$  and for small dispersion time  $t$ , the mean concentration distribution  $C_m(x, t)$  remains same even for the boundary absorption parameter  $\beta$ , because the solute has not yet been interacted with the channel boundary. But, it is interesting to note that for the large dispersion time  $t$  and large boundary absorption parameter ( $\beta=10$ ), the uni-modal distribution turns into bi-modality. It is also remarkable that the mean concentration distribution for the case of periodic flow with non-zero mean is not effected by increase of frequency parameter. The above observations show a remarkable similarity between the mean concentration distributions due to boundary absorption in the combined shearing current through the channel and due to periodic flow with non-zero mean through the pipe discussed by Mazumder and Das [11].

## 5. Conclusion

We have presented a numerical solution to study the dispersion process of contaminant molecules on the shear effect due to steady, periodic and combined flow through a channel with conductive walls; and we have compared some specific results due to different shear effects with particular emphasis on the role played by the first-order reaction at the walls. All the investigations have been done for different flow velocities, when the slug is released initially uniform over the cross section of the channel at a large Péclet number. The centroid displacement ( $x_g$ ) of the slug due to the

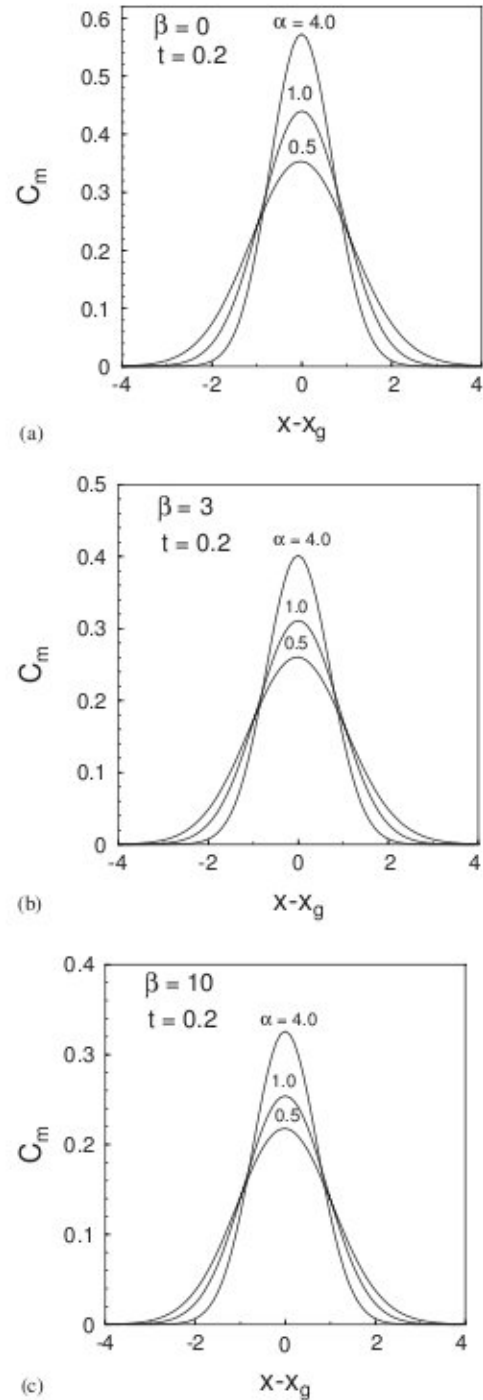


Fig. 7. Mean concentration distribution  $C_m(x, t)$  along the periodic channel flow for  $P_e = S = 10^3$ , and dispersion time  $t = 0.2$  when (a)  $\beta = 0$ , (b)  $\beta = 3$  and (c)  $\beta = 10$  for different  $\alpha$ .

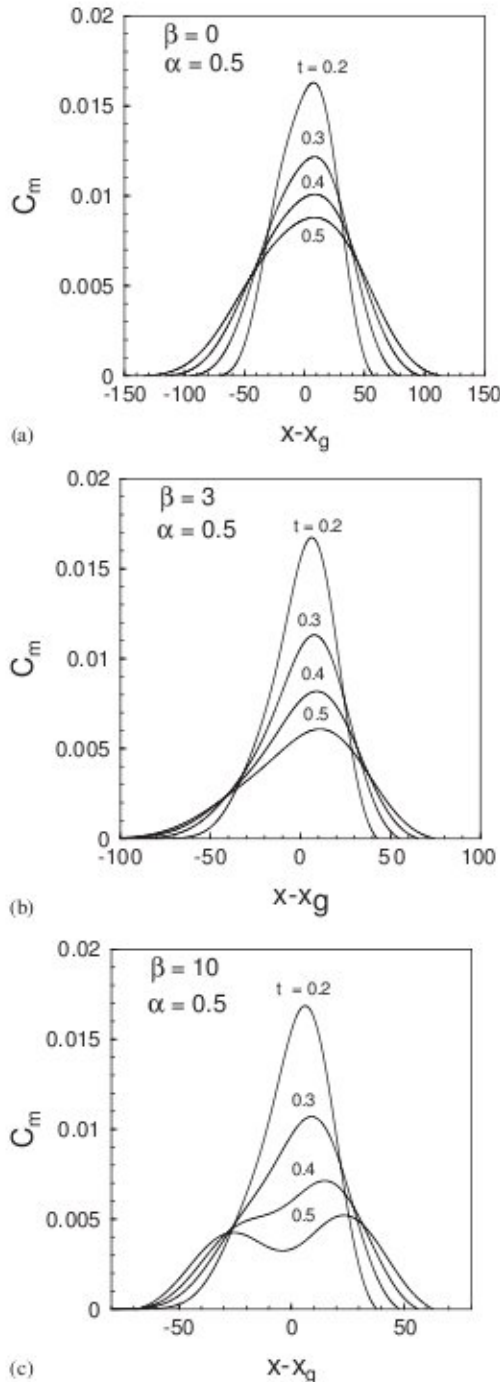


Fig. 8. Mean concentration distribution  $C_m(x, t)$  along the periodic channel flow with non-zero mean for  $P_e = S = 10^3$  and  $\alpha = 0.5$  when (a)  $\beta = 0$ , (b)  $\beta = 3$  and (c)  $\beta = 10$  for different dispersion time  $t$ .

periodic and periodic with non-zero mean flows moves cyclically and after a certain time, it moves asymptotically to a constant value; and the amplitude of the oscillation decreases with increase in absorption parameter  $\beta$ . Also it is seen that the temporal variation of centroid displacement ( $x_g$ ) due to periodic current is much smaller than that of combined flow due to steady and periodic current. The apparent dispersion coefficient  $D_a$  reaches a stationary state after a certain critical time  $t_c$  for all the three different flow velocities: steady, periodic and combined effects due to steady and periodic currents. It is interesting to note that in channel flow for all three cases: steady, periodic and periodic with non-zero mean flows, the value of the dispersion coefficient  $D_a$  is much smaller than that of tube flow. For a fixed  $\alpha$ ,  $D_a$  initially moves with double frequency and ultimately reaches a stationary state and the amplitude of the oscillation decreases with increase in  $\beta$ . It is interesting to note that for high frequency of the periodic current, it appears that  $D_a$  varies cyclically with almost the same frequency as the periodic current during the initial stage; but as time goes on it reaches a stationary state with double-frequency oscillation. In case of combined flow (periodic flow with non-zero mean), the double-frequency oscillation disappears from  $D_a$  for all cases of  $\alpha$  and the amplitude of the oscillation increases till it reaches a stationary state and when  $\beta$  increases, the amplitude of  $D_a$  decreases significantly. The effect of boundary absorption on mean concentration distributions have been studied for oscillatory and combined effect of steady and periodic flow. When the flow is oscillatory, it is seen that for a fixed  $\alpha$ , the peak of the distribution becomes flat with increase in both  $\beta$  and dispersion time  $t$ . Also it is interesting to note that as  $\alpha$  increases, the peak of the distribution gradually increases and the free space expansion gradually decreases. But for periodic flow with non-zero mean, when both the dispersion time  $t$  and the absorption parameter  $\beta$  are large, the uni-modal distribution turns into bi-modality. It is observed that the mean concentration distribution for the case of periodic flow with non-zero mean is not effected by increase of frequency parameter. The above observations show a remarkable similarity between the mean concentration distributions due to boundary absorption in the combined shearing-current through the channel and due to periodic flow with non-zero mean through the pipe.

## Acknowledgements

One of us (Kajal Kumar Mondal) is thankful to the CSIR, India for financial support for pursuing this work.

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