Geometroneurodynamics

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Abstract

Hilbert space structure is assumed as a valid geometric description for neurodynamics, i.e., for applying any kind of quantum formalism in brain dynamics. The orientation selectivity of the neurons is used as a justification to construct a type of statistical distance function which is proportional to the usual distance (or angle) between orientations of the neurons. The equivalence between the statistical distance and the Hilbert-space distance is discussed within this framework. It gives rise to the possibility of reanalysing the issue of measurement and information processing in the brain function.

Keywords: Brain Dynamics, Hilbert-space, measurement, statistical distance, orientation selectivity, quantum formalism.

1 Introduction

Recent progress in brain research indicates (Stapp 1993, Penrose and Hemeroff 1996, Tegmark 2000, Pribram, 1991, Hagan et al. 2002) that to apply a wave function formalism used in quantum mechanics, it is necessary to consider Hilbert space structure. For that, one needs to consider first the geometric structure over the cortical surfaces of brain from the anatomical point of view (Roy and Kafatos 2003) and then its relation to Hilbert-space structure. To the best of our knowledge, no systematic attempt has been made so far to construct the geometric structure, starting from the neuronal characteristics over the cortical surface of brain and its connection to Hilbert space.

Amari(2001) studied the geometrical structure of neuromanifold called multilayer perceptrons using information theoretic approach. In this approach, a family of neural networks constitutes of neuromanifold having probability distributions. Using Baysian approach, Amari constructed Reimannian metric tensor and applied this concept to the behavior of learning as well as statistical inferences over this kind of neuromanifold. It is worthmentioning that Pellionisz and Llinás(1982) and Llinás(2000) tried to understand the space-time representation for the internal world. According to their approach, cerebellum acts like a geometric tensor which connects the contravariant vector associated with the motor activities related to brain. A tensor network theory has been proposed by them so as to study the computational details associated to activities to brain. However, in this approach, it is quite difficult to define a smooth metric tensor globally i.e., over all the cortical surfaces of brain due to the existence of nonlinearities in some cortical surfaces.

The aim of this paper is to study the possibility of assigning any geometrical structure over the cortical areas of the brain. To do that, we first approach the problem considering neurodynamics from the physiological point of view and then attempts have been made to construct the corresponding Hilbert structure for the cortical surface of brain. The appropriate wave function for the neurons in the brain can be defined in Hilbert space as a geometric description of different cortical areas of the brain. The main fact lying behind the idea is that cells in different parts of brain, for example, the visual cortex exhibit orientation selectivity(Hubel 1995). The orientiation selectivity of the cells is similar to polarizing filters producing a beam of polarized photons in physics laboratory experiments.

Also, we examine the notion of statistical distance as related to distinguishability of different oriented states of the cells in the cortical areas. It should be emhasized that we are not attempting to explain consciousness problem as such. Rather, we are trying to develop a possible theoretical framework within the quantum view to provide understanding for certain brain processes which do not appear to fit the classical paradigm. In section(2), we examine the orientation selectivity of neurons as well as the relation between statistical distance and Hilbert space. In section(3), the spontaneous activity of neurons is examined. Finally, the possible implications of this work are discussed in section (4).

2   Orientation Selectivity of Neurons and Statistical Distance

There is a large variety as well as a large number of neurons in the brain. Collective effects which can only be accounted for in terms of statistical considerations, are clearly important. Up till now, experimental evidence points to more than 100 different types of neurons in the brain, although the exact number is not yet fully known. It is found that no two neurons are identical, and it becomes very difficult to say whether any particular difference represents more a difference between individuals or a difference between different classes. Neurons are often organized in clusters containing the same type of cell. The brain contains thousands of cluster cell structures which may take the form of irregular clusters or of layered plates. One such example is the cerebral cortex which forms a plate of cells with a thickness of a few millimeters. In the visual cortex itself(Hubel and 1995), certain clear, unambiguous patterns are found in the arrangement of cells with particular responses.

Even though our approach could apply to non-visual neurons, here we limit our study to the neurons in the visual cortex as the visual cortex is smoother preventing non-linear effects. For example, as the measurement electrode is moving at right angles to the surface through the grey matter, cells encountered one after the other have the same orientation as their receptive field axis. It is also found that if the electrode penetrates at an angle, the axis of the receptive field orientation would slowly change as the tip of the electrode is moved through the cortex. From a large series of experiments in cats and monkeys, it was found :

Neurons with similar receptive field axis orientation are located on top of each other in discrete columns, while we have a continuous change of the receptive field axis orientation as we move into adjacent columns.

In the monkey striate cortex, about 70 to 80 percent of cells have the property of orientation specificity. In a cat, all cortical cells seem to be orientation selective, even those with direct genuculate input(Hubel,1995). Hubel and Wiesel found a striking difference among orientation-specific cells, not just in the optimum stimulus orientation or in the position of the receptive field on the retina, but also in the way cells behave. The most useful distinction is between two classes of cells : simple and complex. The two types differ in the complexity of their behavior and one can make the resonable assumption that the cells with the simpler behavior are closer in the circuit to the input of the cortex.

The first oriented cell recorded by Hubel and Wiesel(Hubel,1995) which responded to the edge of the glass slide was a complex cell. The complex cells seem to have larger receptive fields than simple cells, although the size varies. Both type of cells do respond to the orientation specificity. There are certain other cells which respond not only to the orientation and to the direction of movement of the stimulus but also to the particular features such as length, width, angles etc. Hubel and Weisel originally characterized these as hypercomplex cells but it is not clear whether they constitute a separate class or, represent a spectrum of more or less complicated receptive fields.

We now ask how the computational structure or filters can manifest as orientation detectors ? Pribram(1981) discussed the question whether single neurons serve as

feature or channel detectors. In fact, Pribram and his collaborators(1981,1991) made various attempts to classify "cells" in the visual cortex. This proved to be impossible because each cortical cell responded to several features of the input, such as, orientation, velocity, and the spatial and temporal frequency of the drifted gratings. Further, cells and cell groups displayed different conjunctions of selectivities. From these findings and analysis, he concluded that cells are not detectors, that their receptive field properties could be specified but the cells are multidimensional in their characteristics (Pribram 1991). Thus, the pattern generated by an ensemble of neurons is required to encode any specific feature, as found by Vernon Mountcastle's work on the parietal cortex and Georgopoulos data(Pribram 1998) on the motor cortex. So the cells can merely be treated as filters.

Again, when dealing with the problem of perception, Freeman and his collaborators(1991) suggested that perception cannot be understood solely by examining properties of individual neurons i.e., by using the microscopic approach that currently dominates neuroscience research. They claimed that perception depends on the simultaneous, cooperative activity of millions of neurons spread throughout expanses of the cortex. Such global activity can be identified, measured and explained only if one adopts a macroscopic view alongside the microscopic building up view.

2.1   Statistical Distance

To start with, we can define the notion of distance between the "filters" or the orientation selective neurons in the similar manner i.e., to the statistical distance between two quantum preparations, as introduced by Wootters (1981). The statistical distance is most easily understood in terms of photons and polarizing filters : for example, let us consider a beam of photons prepared by a polarizing filter and analyzed by a nicol prism. Let $\theta \in [0, \pi]$ be the angle by which the filter has been rotated around the axis of the beam, starting from a standard position ($\theta = 0$) referring to the filter's preferred axis as being vertical. Each photon, when it encounters the nicol prism, has exactly two options : to pass straight through the prism (with "yes" outcome) or to be deflected in a specific direction characterised by the prism ("no" outcome). If we assume that the orientation of the nicol prism is fixed once and for all in such a way that vertically polarized photons always pass straight through, then, by counting how many photons yield each of the two possible outcomes, an experimenter can learn something about the value of θ via the formula $p = cos^2\theta$, where p is the probability of "yes"(Wootters 1981), as given by quantum theory.

If we follow this analogy in the case of oriented neurons in the brain i.e., as if the filters are oriented in different directions like oriented analyzers, we can proceed to define the statistical distance as shown in the following way:
Suppose the experimenter, in making his determination of the value of θ, has only a limited number of photons to work with, so that precisely n photons actually pass through the filter to be analyzed by the nicol prism. Then, because of the statistical fluctuations associated with a finite sample, the observed frequency of occurence of "yes" is only an approximation to the actual probability of "yes", the typical error

being of the order of $\frac{1}{\sqrt{n}}$. The experimenter's uncertainty in the value of p is

$$\delta p = \left[\frac{p(1-p)}{n}\right]^{1/2} \tag{1}$$

This uncertainty causes the experimenter to be uncertain as to the actual value of θ by an amount

$$\delta\theta = |\frac{dp}{d\theta}|^{-1}\delta p = |\frac{dp}{d\theta}|^{-1}[\frac{p(1-p)}{n}]^{1/2} \tag{2}$$

In this way we can associate with each value of θ, a region of uncertainty be extending from $\theta - \delta\theta$ to $\theta + \delta\theta$. Let us call two neighbouring orientations θ and $\theta\prime$ distinghuishable in n trials if their regions of uncertainty do not overlap, i.e, if

$$|\theta - \theta\prime| \geq \delta\theta + \delta\theta\prime \tag{3}$$

If the uncertainty could be reduced to zero, one would effectively have an infinite number of distinguishable orientations. This is , however, not the case.

We now define the statistical distance $d(\theta_1, \theta_2)$ between any two orientations θ_1 and θ_2 to be

$$d(\theta_1, \theta_2) = lim_{n\to\infty}\frac{1}{\sqrt{n}}[D] \tag{4}$$

where $D =$ maximum number of intermediate orientations each of which is distinghuishable (in n trials) from its neighbours. Here, the statistical distance is obtained by counting the number of distinguishable states. This does not have anything to do, a priori, with the usual notion of distance (or angle) between θ_1 and θ_2 which is, of course, $|\theta_1 - \theta_2|$. However, it can be shown that these two types of distance are the same. From eqns(2)-(4) we can write the statistical distance in terms of $p(\theta)$ as follows:

$$d(\theta_1, \theta_2) = \frac{1}{\sqrt{\pi}}\int_{\theta_1}^{\theta_2}\frac{d\theta}{2\delta\theta} = \int_{\theta_1}^{\theta_2}d\theta\frac{|\frac{dp}{d\theta}|}{2[p(1-p)]^{\frac{1}{2}}} \tag{5}$$

Substituing $p(\theta) = cos^2\theta$ we find

$$d(\theta_1 - \theta_2) = \theta_1 - \theta_2 \tag{6}$$

which is equal to the angle between the two orientations. It should be noted that in order for the proportionality constant between the statistical distance and the actual distance to be unity, one needs to have

$$|\frac{dp}{d\theta}| \propto [p(1-p)]^{1/2} \tag{7}$$

This will not be true if the probability law is different from

$$p(\theta) = cos^2\theta$$

Now, if we demand that the statistical distance be proportional to $|\theta_1 - \theta_2|$, the $cos^2\theta$ dependence of the probability function must necessarily follow and it is possible to define information measure according to above prescription. This can be reduced to

Fisher information measure in the limiting case(Frieden, 1999).

For example, if W be Wootter's information measure and I be the Fisher information measure, then

$$W = cos^{-1}\left[\int d\theta p^{\frac{1}{2}}(\theta + \triangle\theta)\right] \tag{8}$$

So,

$$W^2 \rightarrow \left[\frac{\triangle\theta^2}{4}\right] I$$

Using the identities,

$$\int d\theta \; p'(\theta) = 0 \quad \text{and} \quad \int d\theta \; p^n(\theta) = 0$$

where prime denotes the first and n the nth derivative respectively. It is worthmentioning that Amari(2001) constructed the same distance function in neuromanifold using Fisher information measure but within Baysian framework where the metric concerned is considered to be of Reimannian nature. With these developments regarding the distance measure, the detail study of orientation selectivity of neurons might shed new light on the issue of information measure, suitable for the description of activity of brain and information processing.

2.2   Statistical Distance and Hilbert Space

It can be shown(Wootters, 1981) that the statistical distance, as explained above, between two preparations, is equal to the angle in Hilbert space between the corresponding rays. The main idea is as follows:

Let us imagine the following experimental set up: there are two preparing devices, one of which prepares the system in a specific state, say ψ^1, and the other prepares in ψ^2. Here, the statistical distance between these two states can be thought as the measure of the number of distinguishable preparations between $\psi^1 \& \psi^2$. However, in treating quantum systems, new features should be observed as opposed to, like, just rolling the dice for a classical system. For dice, there is only one possible experiment to perform i.e., rolling the dice, where as for quantum system there are many, i.e., one for each different analyzing device. Furthermore, two preparations may be more easily distinguished with one anlyzing device rather than with another. For example, the vertical and horizontal polarizations of photons can easily be distinguished with an appropriately oriented nicol prism, but can not be distinguished at all with a device whose eigenstates are the right and left handed circular polarizations. Due to this reason, one can speak of the statistical distance between two preparations $\psi^1 \& \psi^2$ to related to a particular measuring device which means the statistical distance is device dependent. The absolute statstical distance between $\psi^1 \& \psi^2$ is then defined as the largest possible such distance i.e. statistical distance between $\psi^1 \& \psi^2$ when the states are anlyzed by the most appropriate or discriminating apparatus.

We can illustrate this point in the following way: Let $\phi^1, \phi^2..... \phi_N$ be the eigenstates of a measuring device A by which $\psi^1 \& \psi^2$ are to be distinguished. It is assumed that these eigenstates are non-degenerate so that there are N-distinct outcomes of each

measurement. The probabilities of various outcomes are $|(\phi^i, \psi^1|^2$ if the apparatus is ψ^1 and $|(\phi^i, \psi^1|^2$, if the apparatus is ψ^2. Then the statistical distance between ψ^i & $\psi^(2)$ with respect to the analyzing device A is

$$d_A(\psi^1, \psi^2) = cos^{-1}\Big[\Sigma_{i=1}|(\phi^i, \psi^1||(\phi^i, \psi^2|\Big] \qquad (9)$$

This quantity attains its maximum value if one of the eigenstates of A (say, ϕ^i). In that case, we get the statistical distance as

$$d(\phi^1, \phi^2) = cos^{-1}\left|(\phi^1, \phi^2\right| \qquad (10)$$

This clearly indicates that the statistical distance between two preparations is equal to the angle in Hilbert space between the corresponding rays. This equivalence between the statistical distance and the Hilbert space distance may appear very surprising at first. But it gives rise to the interesting possibility that statistical fluctuations in the outcome of mesurements might be partly responsible for Hilbert space structure of quantum mechanics. In this way, these statistical fluctuations are as basic as the fact that quantum measurements are probabilistic in their nature.

At this pint, a question may arise for the plausability of this approach for the following reason. It has been found that although representation of orientation of objects in the visual cortex is fairly fine-scaled, visual information regarding the nonstriate visual processing and in superior colliculus is very rough and varies in a non-linear way from that in striate cortex. This has been supported by the recent findings from Van Essen Laboratory who have reconstructed cortical surface of brain from 3-D data. However, for the present case, this kind of nonlinearity is neglected here, as we have adopted statistical consideration which averages out that kind of nonlinearity. And also, we consider the distance between the different clusters of neurons or between the ensemble of neurons only. It is worthmentioning here that the statistical distance has been studied rigourosly also by Schweizer and Sklar(1983)in the context of cluster analysis, from the mathematical point of view.

3   Concept of Measurement and Brain Functions

It is now clear that the statistical distance can be related to the angular distance in Hilbert space by applying the concept of measurement in quantum mechanics. We need to address the issue of measurement in the context of brain function so as to consider Hilbert space structure needed for any kind of quantum formalism. This is intimately related to the information processing in brain function. The information generated by integrated neural processes and its measurement remains one of the central issues of brain dynamics. The measure of information essentially depends on the basis of statistical foundation of information theory. One of the intriguing question arises is how far the statistical aspects of information theory can help one to assign a measure to differentiate the informative character of the neural processes without any reference to an external observer. The issue of external observer is debated in various branches of science and philosophy over the last century, since the birth of quantum

mechanics. In the standard approach, one generally assigns a number to measure the information and probability of states of the system that are distinguishable from the point of view of an external observer. But the brain not only processes the information but also interprets the pattern of activities(Pribram 1991). Therefore, one must avoid the concept of privilaged viewpoint of an external observer to understand the information processing in the neural processes of the brain.

Edelman et al.(2000) discussed this problem in the context of neurobiology and consciousness. The main problem is how to measure the differences within a system , like the brain? He defined a concept of mutual information as considered in Shannon's framework. In this case, first a subensemable is considered and then the mutual information between this subensemable and and the rest of the states is found out. But, choosing the first subensemable, again, remains arbitrary. therefore, it is needed to analyze this situation in a more rigorous way so as to understand the measurement process in brain dynamics.

Wright(1998), too, with the objectives to ascertain the minimal assumptions needed to reproduce the experimental data, proposed a lumped continuum model of the cortex to explain EEG data for synchronic oscillation in the cerebral cortex and object coherence. In his model, synchronicity depends wholly upon relatively long-range excitory connections in a continuum field-dendritic lag summations. Relatively rapid axonal transmission are considered as the essential ingredients while short range excitory/inhibitory interactions appear crucial only to the occurence of oscillation. These characteristics are very similar to that observed in neural network models with interactional couplings observed by Wilson and Bower(1991). The continuum formulation parallels and strengthens the neural network approach by enabling a different insight into the physical nature of synchronicity, since the essential nonlinearity of individual elements is avoided and the stochastic and essentially linear properties of the neuronal mass is retained.

On relatively long-range scales, the $cos^2\theta$ probability law follows from the requirement that the statistical distance in case of neurodynamics is proportional to the usual distance(angle) between orientations of a set of filters or a set of neurons. It may be possible that in case of neurodynamics, the statistical distance equals to the Hilbert space distance. Then we can think of Hilbert space structure over the cortical areas of the brain. Once this kind of Hilbert space structure is envisaged over the cortical areas of the brain, it will be plausible to define quantum processes to be valid underlying the neuronal dynamics.

4   Implications

It is evident from the above analysis that the law of probability related to the statistical distance is similar to the *channel representation* as considered by Granlund(1999).

In the world around us, things generally appear different, whether they irrespective of their exact form depending upon the different measuring parameters with which it is looked through. Still we can recognize most objects at arbitrary positions, orientations, distances, etc. Some aspects of an object are sufficiently familiar in order to

begin the process of recognition even for arbitrary orientations. This may be the reason why one is interested in the simultaneous appearance of similarity and difference in the properties of objects. A representation of similarity requires a metric or characteristic distance measure between items to be defined. Granlund (1999) defined one type of such distance which is related to channel representation. Here, each channel represents a particular property measured at a particular position of the input space. Such a channel can be viewed in the analogy as the output from some band pass filter sensor associated with a specific property of the input. This relates the function of biological neural feature channels.

In biological vision one can think of several examples for the properties like edge, line or orientation detectors. If we consider the channel output as derived from a band pass filter, we can establish a measure of distance or similarity in terms of the properties of the filter. For this channel representation, Granlund (1999) considered the measure for the output of the channel as $cos^2\theta$ where θ denotes the orientation of the filter. Of course he did not derive this analytically. On the other hand our framework is consistent with his view. In conventional linear simple band pass filter, the phase distance between the flanks is a constant times $\pi/2$. Different filters will have different bandwidths, but we can view this as a standard unit of similarity or distance, related to a particular channel. Such a channel filter has the characteristic that it is located in some input space, as well as local in some property space.

The recent debates about the applicability of the wave function formalism in quantum mechanics as well as the relevance of quantum coherence for the information processing in the brain in various regions like visual cortex, auditory regions, olfactory bulbs etc. have raised lots of interest among scientific community. Pribram (1991) developed the first holographic model of the brain. According to Pribram, the brain encodes information on a three dimensional energy field that enfolds time and space, yet allows to recall or reconstruct specific images from the countless millions of neurons, stored in a space slightly smaller than a melon. Together with his collaborators, he studied the implications of Gabor's quanta of information for brain's function and their relation to Shannon's measure on the amount of information contained in the data, obtained in their investigations. Rigorous investigations have been carried out(Jibu et al., 1996) further so as to study how the quantum mechanical processes can operate at the synaptodendritic level.

We have already discussed that Wootter's measure of information (and distance) is related to Fisher information measure which is considered as the mother of all information measure(Frieden, 1999) including that of Shannon's measure. So, their approach regarding information processing in brain should be reanalyzed using the concept of statistical distance function. However, arguments and counter arguments(Tegmark, 2000, Hameroff, 2002) have been raised recently about the applicability of the kind of model which, one way or other, is related to the application of quantum mechanical concepts.It is generally argued that the brain is warm and wet. Recent theoretical and experimental papers(Tegmark, 2000) support the prevailing opinion that the large warm systems will rapidly lose quantum coherence and classical properties will emerge out as a result. This rapid loss of coherence(decoherence problem) would naturally be expected to block any crucial role of quantum theory in explaining the interaction between our conscious experiences and the physical activities of our brain.

However, to mention further, in the quantum theory of mind developed by Stapp(2000a), based on a relativistic version of von Neumann's quantum theory, the efficacy of mental effort is not affected by decoherenece problem. Briefly, Stapp(2000b) relates mind-brain to two separate processes: First, there is the unconscious mechanical brain process governed by the Schrodinger equation which involves processing units that are represented by complex patterns of neural activity (or more generally, of brain activity) and, sub-units within these units that allow "association" i.e., each unit tends to be activated by the activation of several of its subunits. The mechanical brain evolves by the dyanmical interplay of these associative units. Each quasi-classical element of the ensemble that constitutes the brain creates, on the basis of clues, or cues, coming from various sources, a plan for having a possible coherent course of action. Quantum uncertainties entail that a host of different possibilities will emerge. This mechanical phase of the processing already involves some selectivity, because the various input clues contribute either more or less to the emergent brain process according to the degree to which these inputs activate, via associations, the patterns that survive and turn into the plan of action.

The issue of quantum coherence and consciousness has been discussed in detail, also by Hameroff and Penrose(1996) in their model, known as Orchestrated Objective Reduction(OrchOR) model, where the possible scenario for emergence of quantum coherence leading to OOR and conscious events is cellular vision. On the otherhand, Jibu et al.(1994) argued that this kind of processes require quantum coherence in microtubules and ordered water, based on the experimental observations(Albrecht-Buehler, 1992) which states that single cells utilize their cytokeletons in cellular vision-detection, orientation and directional response to beams of red/infrared light supports.

It is clear from the above models of Stapp, Penrose et al. that they used the quantum mechanical wave function to describe both unconscious mechanical brain processes connected with the complex patterns of neuronal activity as well as the conscious activity of the brain. In both the approaches, a Hilbert structure has been assumed in order to produce an evolution equation like Schrodinger equations. We like to emphasize that our approach gives rise to a new possibility to construct the Hilbert space structure over the cortical surface of brain from an anatomical perspective. However, the issues, like concept of measurement, the role of observer and information measures should be thoroughly analyzed in the context of brain before applying any kind of quantum mechanical formalism.

References

1. Amari S.(2001), *IEEE Transaction on Information Theory*,**47**,1701-1711.

2. Albrecht-Buehler G.(1992),*Rudimentary form of Cellular Vision*, Cell. Biology, **89**,8288-92.

3. Donald M.J.(1990),*Quantum Theory and the Brain*, Procd.Roy.Soct. London,**A 427**,43-53.

4. Freeman W J.(1991),*The Physiology of Perception*,Scientific American **264**, 78-85.

5. Grandpierre A.(1995),*Quantum-Vacuum Interaction Dynamics in the Brain* in The Interconnected Universe, by Ervin Laszlo, World Scientific, Appendix I,113-118.

6. Grandpierre A.(1999),*The Nature of Man-Universe Connection*,The Noetic Journal,**2**, 52-66.

7. Granlund Goesta H.(1999),*The Complexity of Vision*,Signal Processing,**74**,101-116..

8. Hagan S, Hameroff S. and Tuszynski J.A.(2002),Phys.Rev.**E**,**65**,061901.

9. Hameroff S.and Penrose R.(1996),*Conscious Events as Orchestrated Space-time Selections*, Journal of Consciousness Studies ,**3**,36-53.

10. Hubel H.David(1995),*Eye, Brain, and Vision*(Scientific American Library, NY).

11. Jibu M. , Hameroff S.,Pribram K H.,Yasue K.(1994),*Quantum Optical Coherence in Cytoskeletol Microtubules : Implication for brain function*,Biosystems **32**,195-209.

12. Jibu M., Pribram K H.,Yasue K.,(1996),*From Conscious Experience to Memory Storage and Retrieval : The Role of Quantum Brain Dynamics and Boson Condensation of Evanescent Photons*,Int.J.Mod.Phys.B,**10**,1735-54.

13. Lindahl B.I.B and Arhem P.(1994),*Mind as a Force Field*,J.Theort.Biology **171**,111-118 and references there in.

14. Llinás R.(2002),*I of the Vortex*, MIT Press.

15. Pellionisz A. and Llinás R.(1982), *Space-time Representation in the Brain Neuroscience*, **7**,2949-2970.

16. Penrose Roger(1994),*Shadows of the Mind* (Oxford Press, Oxford, U.K.).

17. Pribram K., Lassonde M.,Ptito M.(1981),*Classification of Receptive Field Properties in Cat Visual Cortex*, Exp.Brain Res.**43**,119-130.

18. Pribram K.(1991),*Brain and Perception- Holonomy and Structure in Figural Processing* (Lawrence Erlbaum Associates Publishers, NJ).

19. Pribram K.(1998),*The History of Neuroscience in Autobiography*, **2**,335.

20. Schweizer B. and Sklar A.(1983),*Probabilistic Metric Spaces*(North Holland)

21. Roy Sisir(1998),'*Statistical Geometry and Applications to Microphysics and Cosmology*(Kluwer Academic Publishers).

22. Spinelli D N., Pribram K H., Bridgeman B.(1970),*Visual Receptive Field Organization of Single Units in the Visual Cortex of Monkey*,Int.J Neurosci,**1**,67-70.

23. Stapp Henry(1993),*Mind, Matter and Quantum Mechanics*(Springer-Verlag,NY.Inc).

24. Stapp Henry(2000 a),*Decoherence, Quantum Zeno Effect and the Efficacy of Mental Effort*, quant-ph/0003065

25. Stapp Henry (2000 b),*From Quantum Nonlocality to Mind-Brain interaction* , quant-ph/0009062)

26. Van Essen Laboratory : cis.jhu.edu/wu_research/brain.html

27. Von Neumann John(1958),*The Computer and the Brain*(Yale University Press,1979).

28. Wilson M A., and Bower J M.,(1991),*A computer Simulation of Oscillatory Behaviour in Primary Visual Cortex*, Neural Computation **3**,498-509.

29. Wootters W.K.(1981),*Statistical Distance and Hilbert Space*, Phys.Rev D **23**,357-365.

30. Wright J J.,(1998),*Simulation of EEG : Dynamic Changes in Synaptic Efficacy, Cerebral Rythms, and Dissipative and Generative Activity in the Cortex*,(Biological Cybernatics Press).