Electron-acoustic solitary waves and double layers in a relativistic electron-beam plasma system

Biswajit Sahu and Rajkumar Roychoudhury[a)]
Physics and Applied Mathematics Unit, Indian Statistical Institute, Kolkata 700108, India

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The exact Sagdeev's pseudopotential is derived for electron-acoustic waves in a two electron temperature plasma ("cold" and "hot" electrons) in the presence of relativistic electron beam plasma using a vortex-like distribution for trapped electrons. It is found that the relativistic effect restricts the region of existence for solitary waves. It is also seen that different ordering in ϕ, the electric potential, yields different solitary wave solutions. It is also observed that a double layer solution exists even if one keeps terms up to order
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I. INTRODUCTION

Ion-acoustic waves have been studied by many researchers both experimentally and theoretically in nonrelativistic[1–5] and relativistic plasmas.[6–10] However, studies on electron acoustic waves (EAW) are comparatively few in number[11,12] though these waves were discovered experimentally three decades ago.[13–15] Of late there seems to be a great deal of interest in studies on electron acoustic waves.[16–22] Usually one finds EAW[23–25] in a magnetized plasma where the ion temperature is larger than the electron temperature and the waves propagate in a direction perpendicular to the magnetic field. In the case of unmagnetized plasma, EAW have been observed in the laboratory when the plasma consisted of two species of electrons with different temperatures, referred to as "hot" and "cold" electrons. EAW are becoming increasingly important in astrophysical plasmas and the magnetosphere, for example in the interpretation of the electrostatic component of the "broadband electrostatic noise," in the cusp of terrestrial magnetosphere, Earth bow shock, and the heliospheric termination shock.[26–30] It has been argued[31] that when the hot to cold electron temperature ratio is greater than 10, the electron-acoustic mode may be the principal mode of the plasma, while the Langmuir wave is damped. Berthomier *et al.*[32] have shown that where as electron-acoustic solitons exist in a two temperature plasma (with cold and hot electrons), the introduction of an electron beam in such a plasma allows the existence of new electron-acoustic solitons with velocity related to the beam velocity. Nonlinear propagation of electron acoustic waves has been studied by other authors also both in nonrelativistic plasma[18–22] and in relativistic plasma.[16,17] In the latter case, a weak relativistic assumption has been made. In the present paper the propagation of electron acoustic solitary waves (EASW) in a relativistic (unmagnetized) plasma consisting of ions, cold relativistic electrons, hot electrons, and relativistic electron beam is investigated. This study is of importance because of the fact that high energy electrons are found to exist in the solar flare, upstream solar wind, etc.[33,34] In most of the studies referred to earlier (except Refs. 20 and 21) the distribution of hot electrons was taken to be Maxwellian. However, it would be more realistic to take a non-Maxwellian distribution for the hot electrons because of the formation of phase space holes caused by trapped electrons.[35] In this case one has to take a vortex-like distribution. This model was adopted by Mamun *et al.*[20] for obliquely propagating electron acoustic solitary waves. They found that this plasma model considered supports EASW having a positive potential, which corresponds to a hole (hump) in the cold (hot) electron number density. To study the propagation of EASW in relativistic plasma we have used Sagdeev's[36] pseudopotential approach. The motivation for studying nonlinear waves in an electron beam plasma system arises from the fact that introduction of the electron beam may produce the conditions for the existence of double layers. This is because the existence of a beam component permits the existence of a modified Korteweg–de Vries equation with a double layer solution. The organization of the paper is as follows. In Sec. II the basic equations for propagation of electron acoustic waves in an unmagnetized electron beam plasma system are given. In Sec. III the exact pseudopotential for the problem was obtained in terms of ϕ, the electric potential. In Sec. IV we have studied the double layer solutions. Section V is kept for discussion and results, while Sec. VI is kept for conclusion.

II. BASIC EQUATIONS FOR RELATIVISTIC ELECTRON-BEAM PLASMA SYSTEM

We consider a one-dimensional collisionless plasma consisting of ions, cold relativistic electron fluid, hot electrons obeying a trapped distribution, and a relativistic electron beam. Here ions form a motionless neutralizing background and are indicated in the Poisson equation only. For one-dimensional propagation of nonlinear EAW, the dynamics of the relativistic electron fluid can be written as

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_c) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\gamma_c u_c)+u_c\frac{\partial}{\partial x}(\gamma_c u_c)=\alpha\frac{\partial\phi}{\partial x}, \tag{2}$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial}{\partial x}(n_b u_b)=0, \tag{3}$$

$$\frac{\partial}{\partial t}(\gamma_b u_b)+u_b\frac{\partial}{\partial x}(\gamma_b u_b)=\alpha\frac{\partial\phi}{\partial x}-\frac{3\alpha}{\theta}\frac{(1+\alpha+\beta_1)^2}{\beta_1^2}n_b\frac{\partial n_b}{\partial x}, \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2}=\frac{1+\alpha+\beta_1}{\alpha}(n_c+n_h+n_b-1), \tag{5}$$

where

$$\gamma_c=\left(1-\frac{u_c^2}{c^2}\right)^{-1/2},\quad \gamma_b=\left(1-\frac{u_b^2}{c^2}\right)^{-1/2}.$$

In the earlier equations $n_{c,h,b}$ are the densities of the three electron population, $u_{c,b}$ are velocities of the cold and beam electrons, respectively, and ϕ is the electrostatic potential. In Eqs. (1)–(6) these quantities have been normalized to the total unperturbed density $n_0=n_{c0}+n_{h0}+n_{b0}$, the electron acoustic velocity $u_{ea}=(n_{c0}/n_{h0})^{1/2}u_{\mathrm{Th}}$, where $u_{\mathrm{Th}}=(k_BT_h/m_e)^{1/2}$ is the hot electron thermal velocity and to k_BT_h/e, respectively. Time and space variables are normalized, respectively, to the cold electron plasma period $\omega_{\mathrm{pc}}^{-1}=(m_e/4\pi n_{c0}e^2)^{1/2}$ and the hot electron Debye length $\lambda_{\mathrm{Dh}}=(k_BT_h/4\pi n_{h0}e^2)^{1/2}$. We have introduced the following quantities, which will be used in our parametric study:

$$\alpha=\frac{n_{h0}}{n_{c0}},\quad \beta_1=\frac{n_{b0}}{n_{c0}},\quad \theta=\frac{T_h}{T_b}. \tag{6}$$

To model the hot electron distribution in the presence of trapped particles, we employ a vortex-like electron distribution derived by Schamel[35] which solves the electron Vlasov equation. Thus, we have $f_h=f_{\mathrm{hf}}+f_{\mathrm{ht}}$, where

$$f_{\mathrm{hf}}=\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}(v^2-2\phi)\right],|v|>\sqrt{2\phi}, \tag{7}$$

$$f_{\mathrm{ht}}=\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\beta(v^2-2\phi)\right],|v|\leqslant\sqrt{2\phi}. \tag{8}$$

Here v is the hot electron speed normalized to the hot electron thermal speed $v_{\mathrm{Th}}=(k_BT_h/m)^{1/2}$ and $|\beta|=T_h/T_{\mathrm{ht}}$, which is the ratio of the free hot electron temperature T_h to the hot trapped electron temperature T_{ht}, is a parameter determining the number of trapped electrons. We note that $\beta=1(\beta=0)$ represents a Maxwellian (flattopped) distribution, whereas $\beta<0$ represents a vortex-like excavated trapped electron distribution corresponding to an underpopulation of trapped electrons. Integrating the electron distributions over the velocity space, the hot electron number density n_h for $\beta<0$ can be expressed as

$$\frac{1+\alpha+\beta_1}{\alpha}n_h=I(\phi)+\frac{2}{\sqrt{\pi|\beta|}}W_D(\sqrt{-\beta\phi}), \tag{9}$$

where $I(x)=[1-\mathrm{erf}(\sqrt{x})]\exp(x)$ and $W_D(x)=\exp(-x^2)\times\int_0^x\exp(y^2)dy$.

For $\phi\ll 1$ we can express (9) as

$$\frac{1+\alpha+\beta_1}{\alpha}n_h=1+\phi-\frac{4(1-\beta)}{3\sqrt{\pi}}\phi^{3/2}+\frac{1}{2}\phi^2-\frac{8(1-\beta^2)}{15\sqrt{\pi}}\phi^{5/2}+\cdots. \tag{10}$$

III. PSEUDOPOTENTIAL APPROACH

To obtain a solitary wave solution, we make the dependent variables depend on a single independent variable $\xi=x-Vt$, where V is the velocity of solitary wave. Now Eqs. (1)–(4) can be written as

$$-V\frac{dn_c}{d\xi}+\frac{d}{d\xi}(n_cu_c)=0, \tag{11}$$

$$-V\frac{d}{d\xi}(\gamma_cu_c)+u_c\frac{d}{d\xi}(\gamma_cu_c)=\alpha\frac{d\phi}{d\xi}, \tag{12}$$

$$-V\frac{dn_b}{d\xi}+\frac{d}{d\xi}(n_bu_b)=0, \tag{13}$$

$$-V\frac{d}{d\xi}(\gamma_bu_b)+u_b\frac{d}{d\xi}(\gamma_bu_b)=\alpha\frac{d\phi}{d\xi}-\frac{3\alpha}{\theta}\frac{(1+\alpha+\beta_1)^2}{\beta_1^2}n_b\frac{dn_b}{d\xi}. \tag{14}$$

The integration of the earlier equations yield the following:

$$n_c=\frac{1}{1+\alpha+\beta_1}\frac{u_{0c}-V}{u_c-V}, \tag{15}$$

$$\phi=\frac{\gamma_c}{\alpha}(c^2-Vu_c)-\frac{\gamma_{0c}}{\alpha}(c^2-Vu_{0c}), \tag{16}$$

$$n_b=\frac{\beta_1}{1+\alpha+\beta_1}\frac{u_{0b}-V}{u_b-V}, \tag{17}$$

$$\phi=\frac{\gamma_b}{\alpha}(c^2-Vu_b)-\frac{\gamma_{0b}}{\alpha}(c^2-Vu_{0b})+\frac{3}{2\theta}\left[\frac{(u_{0b}-V)^2}{(u_b-V)^2}-1\right], \tag{18}$$

where

$$\gamma_{0c}=\left(1-\frac{u_{0c}^2}{c^2}\right)^{-1/2}\quad\text{and}\quad\gamma_{0b}=\left(1-\frac{u_{0b}^2}{c^2}\right)^{-1/2}.$$

In deriving Eqs. (15)–(18), we have used the following boundary conditions:

$$u_c\to u_{0c},\ n_c\to n_{c0},$$
$$u_b\to u_{0b},\ n_b\to n_{b0},\ \phi\to 0\ \text{as}\ \xi\to\pm\infty.$$

To obtain the pseudopotential $\psi(\phi)$, we notice that Eq. (5) can be expressed as

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2=\frac{1}{\alpha}\int\frac{u_{0c}-V}{u_c-V}d\phi+\frac{1+\alpha+\beta_1}{\alpha}\int n_h d\phi+\frac{\beta_1}{\alpha}\int\frac{u_{0b}-V}{u_b-V}d\phi-\frac{1+\alpha+\beta_1}{\alpha}\phi.$$

The earlier equation can be written as

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+\psi(\phi)=0, \tag{19}$$

where $\psi(\phi)$, the pseudopotential, is given by

$$\psi(\phi)=1+\frac{1+\alpha+\beta_1}{\alpha}\phi+\frac{u_{0c}-V}{\alpha^2}\left(\frac{u_{0c}}{\sqrt{1-\frac{u_{0c}^2}{c^2}}}-\frac{u_c}{\sqrt{1-\frac{u_c^2}{c^2}}}\right)+\frac{\beta_1(u_{0b}-V)}{\alpha^2}\left(\frac{u_{0b}}{\sqrt{1-\frac{u_{0b}^2}{c^2}}}-\frac{u_b}{\sqrt{1-\frac{u_b^2}{c^2}}}\right)+\frac{\beta_1}{\alpha\theta}\left[1-\frac{(u_{0b}-V)^3}{(u_b-V)^3}\right]-I(\phi)-\frac{2}{\sqrt{\pi}}\left(1-\frac{1}{\beta}\right)\sqrt{\phi}-\frac{2}{\beta\sqrt{\pi|\beta|}}W_D(\sqrt{-\beta\phi}). \tag{20}$$

In deriving Eq. (20), the following boundary conditions were used. $n_c\to n_{0c}$, $n_b\to n_{0b}$, $u_c\to u_{0c}$, $u_b\to u_{0b}$, $\phi\to 0$ and $d\phi/d\xi\to 0$ at $\xi\Rightarrow\pm\infty$.

Now from (16) we get

$$u_c=\frac{V-\left(\frac{\phi}{c}+\frac{A_0}{c}\right)\sqrt{A_2+\frac{\alpha^4}{c^4}(2A_0\phi+\phi^2)}}{A_1+\frac{2A_0\alpha^2}{c^4}\phi+\frac{\alpha^2}{c^4}\phi^2}, \tag{21}$$

where A_0, A_1, and A_2 are given by

$$A_0=\frac{\gamma_{0c}}{\alpha}(c^2-Vu_{0c}),$$

$$A_1=\frac{V^2}{c^2}+\frac{\alpha^2A_0^2}{c^4},$$

and

$$A_2=\frac{V^2\alpha^2}{c^2}-\alpha^2+\frac{A_0^2\alpha^4}{c^4}.$$

Now from (18) we get a sixth degree equation in u_b in terms of ϕ, from which u_b can be evaluated.

For solitary wave solutions, the following conditions must be satisfied: $\psi(\phi)=0$, $(\partial\psi/\partial\phi)_{\phi=0}=0$, and $(\partial^2\psi/\partial\phi^2)_{\phi=0}<0$, and $\psi(\phi)<0$ for ϕ lying between 0 and ϕ_m, i.e., either for $0<\phi<\phi_m$ (compressive) or $\phi_m<\phi<0$ (rarefactive).

IV. DOUBLE LAYER

To obtain the double layer solution, we notice that Eq. (5) can be written as

$$\frac{d^2\phi}{d\xi^2}=\frac{1+\alpha+\beta_1}{\alpha}(n_c+n_h+n_b-1). \tag{22}$$

Now Eq. (21) can be written as

$$u_c=u_{0c}-A_3\phi+A_4\phi^2 \tag{23}$$

keeping terms up to $o(\phi^2)$, where A_3 and A_4 are given by

$$A_3=\frac{1}{A_1}\left[\frac{2A_0\alpha^2}{A_1c^4}\left(V-\frac{A_0\sqrt{A_2}}{c}\right)+\sqrt{A_2}\left(\frac{1}{c}+\frac{A_0^2\alpha^4}{A_2c^5}\right)\right]$$

and

$$A_4=\frac{1}{A_1}\left[\left(V-\frac{A_0\sqrt{A_2}}{c}\right)\left(\frac{4A_0^2\alpha^4}{A_1^2c^8}-\frac{\alpha^2}{A_1c^4}\right)+\frac{2A_0\sqrt{A_2}\alpha^2}{A_1c^4}\times\left(\frac{1}{c}+\frac{A_0^2\alpha^4}{A_2c^5}\right)\right]-\frac{\sqrt{A_2}}{A_1}\left(\frac{3A_0\alpha^4}{2A_2c^5}-\frac{A_0^3\alpha^8}{2A_2^2c^9}\right).$$

Now from Eq. (18) we can obtain u_b as

$$u_b=u_{0b}+B_{11}\phi+B_{22}\phi^2 \tag{24}$$

keeping terms up to $o(\phi^2)$, where B_{11} and B_{22} are given by

$$B_{11}=-\frac{-\frac{2A_{01}}{c^2}u_{0b}^6+\frac{8VA_{01}}{c^2}u_{0b}^5+A_{13}u_{0b}^4+A_{15}u_{0b}^3+A_{17}u_{0b}^2+A_{19}u_{0b}+2A_{01}V^4-\frac{3V^2}{\theta}(u_{0b}-V)^2}{-6A_{11}u_{0b}^5+5A_{12}u_{0b}^4+4A_{14}u_{0b}^3+3A_{16}u_{0b}^2+2A_{18}u_{0b}+A_{20}}$$

and

$$B_{22}=-\frac{1}{F_3}(F_4+F_2B_{11}+F_1B_{11}^2),$$

where

$$F_1=-15A_{11}u_{0b}^4+10A_{12}u_{0b}^3+6A_{14}u_{0b}^2+3A_{16}u_{0b}+A_{18},$$

$$F_2=-\frac{12A_{01}u_{0b}^5}{c^2}+\frac{40VA_{01}u_{0b}^4}{c^2}+4A_{13}u_{0b}^3+3A_{15}u_{0b}^2+2A_{17}u_{0b}+A_{19},$$

$$F_3=-6A_{11}u_{0b}^5+5A_{12}u_{0b}^4+4A_{14}u_{0b}^3+3A_{16}u_{0b}^2+2A_{18}u_{0b}+A_{20},$$

$$F_4=-\frac{u_{0b}^6}{c^2}+\frac{4Vu_{0b}^5}{c^2}+u_{0b}^4\left(1-\frac{6V^2}{c^2}\right)+4u_{0b}^3\left(\frac{V^3}{c^2}-V\right)$$
$$+u_{0b}^2\left(6V^2-\frac{V^4}{c^2}\right)-4V^3u_{0b}+V^4.$$

In the earlier expressions we have used the following:

$$A_{01}=\frac{\gamma_{0b}}{\alpha}(c^2-Vu_{0b})+\frac{3}{2\theta},$$

$$A_{11}=\frac{A_{01}^2}{c^2}+\frac{V^2}{\alpha^2},$$

$$A_{12}=\frac{4VA_{01}^2}{c^2}+\frac{1}{\alpha^2}(4V^3+2Vc^2),$$

$$A_{13}=2A_{01}\left(1-\frac{6V^2}{c^2}\right)+\frac{3}{\theta}\frac{(u_{0b}-V)^2}{c^2},$$

$$A_{14}=A_{01}^2\left(1-\frac{6V^2}{c^2}\right)+\frac{3A_{01}}{\theta}\frac{(u_{0b}-V)^2}{c^2}-\frac{1}{\alpha^2}(c^4+6V^4+8V^2c^2),$$

$$A_{15}=8A_{01}\left(\frac{V^3}{c^2}-V\right)-\frac{6V}{\theta}\frac{(u_{0b}-V)^2}{c^2},$$

$$A_{16}=4A_{01}^2\left(\frac{V^3}{c^2}-V\right)-\frac{6VA_{01}}{\theta}\frac{(u_{0b}-V)^2}{c^2}+\frac{1}{\alpha^2}(4Vc^4+4V^5+12V^3c^2),$$

$$A_{17}=2A_{01}\left(6V^2-\frac{V^4}{c^2}\right)+\frac{3}{\theta}(u_{0b}-V)^2\left(\frac{V^2}{c^2}-1\right),$$

$$A_{18}=A_{01}^2\left(6V^2-\frac{V^4}{c^2}\right)+\frac{3A_{01}}{\theta}(u_{0b}-V)^2\left(\frac{V^2}{c^2}-1\right)-\frac{9}{4\theta^2c^2}(u_{0b}-V)^4-\frac{1}{\alpha^2}(6V^2c^4+V^6+8V^4c^2),$$

$$A_{19}=-8A_{01}V^3+\frac{6V}{\theta}(u_{0b}-V)^2,$$

$$A_{20}=-4V^3A_{01}^2+\frac{6VA_{01}}{\theta}(u_{0b}-V)^2+\frac{1}{\alpha^2}(4V^3c^4+2V^5c^2).$$

Now with the help of Eqs. (23) and (24), Eq. (22) can be written as [keeping terms up to $o(\phi^2)$]:

$$\frac{d^2\phi}{d\xi^2}=B_1\phi-B_2\phi^{3/2}+B_3\phi^2=-\frac{d\psi}{d\phi}, \tag{25}$$

where ψ is the Sagdeev potential and B_1, B_2, and B_3 are given by

$$B_1=1-\frac{A_3}{\alpha(V-u_{0c})}+\frac{\beta_1B_{11}}{\alpha(V-u_{0b})},$$

$$B_2=\frac{4(1-\beta)}{3\sqrt{\pi}},$$

$$B_3=\frac{1}{2}+\frac{A_3^2}{\alpha(V-u_{0c})^2}+\frac{A_4}{\alpha(V-u_{0c})}+\frac{\beta_1B_{11}^2}{\alpha(V-u_{0b})^2}+\frac{\beta_1B_{22}}{\alpha(V-u_{0b})}.$$

After integrating Eq. (25), we get

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2+\psi(\phi)=0, \tag{26}$$

where $\psi(\phi)=-(B_1/2)\phi^2+(2B_2/5)\phi^{5/2}-(B_3/3)\phi^3$. Substituting $\phi=\Phi^2$, Eq. (26) transforms as

$$2\left(\frac{d\Phi}{d\xi}\right)^2=\frac{B_1}{2}\Phi^2-\frac{2B_2}{5}\Phi^3+\frac{B_3}{3}\Phi^4=-\psi(\Phi). \tag{27}$$

To get the double layer solution, the modified Sagdeev potential function $\psi(\Phi)$ must satisfy the following boundary conditions: $\psi(\Phi)=0$ at $\Phi=0$ (and at $\Phi=\Phi_m$) and $d\psi/d\Phi=0$ at $\Phi=0$ (and at $\Phi=\Phi_m$) and also the condition $[d^2\psi/d\Phi^2]_{\Phi=\Phi_m}<0$.

Following the earlier boundary conditions, we have $B_1=\frac{4}{5}B_2\Phi_m-\frac{2}{3}B_3\Phi_m^2$ and $B_1=\frac{6}{5}B_2\Phi_m-\frac{4}{3}B_3\Phi_m^2$.

From the earlier two relations, B_1 and B_2 are evaluated as

$$B_2=\tfrac{5}{3}B_3\Phi_m,\quad B_1=\tfrac{2}{3}B_3\Phi_m^2\quad\text{and}\quad 25B_1B_3=6B_2^2. \tag{28}$$

Inserting the values of B_1 and B_2, Eq. (27) could be modified as

$$\frac{d\Phi}{d\xi}=k_1\Phi(\Phi_m-\Phi);\quad\text{with}\quad k_1=\pm\left(\frac{B_3}{6}\right)^{1/2}. \tag{29}$$

Now we use the transformation of "tanh method:"[37,38] $t=\tanh(k\xi)$ and $W(t)=\Phi(\xi)$. Equation (29) then transforms as

$$k(1-t^2)\frac{dW}{dt}-k_1\Phi_mW+k_1W^2=0. \tag{30}$$

It is obvious that Eq. (30) is a Fuchsian-like nonlinear ordinary differential equation. Here we attempt to find a truncated series solution it *viz.* $W(t)=\Sigma_{r=0}^{N}a_rt^r$. The actual number of terms in the series $W(t)$ has to be determined by the leading order of nonlinear terms balancing the order of the differential equation. Now the process evaluates the exact number of N of the series as $N=1$, i.e., the series $W(t)$ have two terms. Thus $W(t)$ is found to be of the form

$$W(t)=a_0+a_1t. \tag{31}$$

Using Eq. (31) we get from (30) the following equations:

$$ka_1-k_1a_0\Phi_m+k_1a_0^2=0,$$

$$-k_1\Phi_ma_1+2a_0a_1k_1=0,$$

$$-ka_1+k_1a_1^2=0.$$

From the earlier equations the unknowns are determined as $a_0=\frac{1}{2}\Phi_m$, $a_1=\pm\frac{1}{2}\Phi_m$, and $k=\pm k_1/2\,\Phi_m$. So the double layer solution is given by

$$\phi(\xi)=\frac{1}{4}\Phi_m^2\left[1\pm\tanh\left(\frac{\xi}{\delta}\right)\right]^2, \tag{32}$$

where $\xi=x-Vt$ and $\delta=(\frac{1}{2}k_1\Phi_m)^{-1}$.

If we consider next higher order in ϕ in Eq. (22) then Eq. (22) can be written as

$$\frac{d^2\phi}{d\xi^2}=B_1\phi-B_2\phi^{3/2}+B_3\phi^2-B_4\phi^{5/2}=-\frac{d\psi}{d\phi}, \tag{33}$$

where B_1, B_2, B_3, are given as before and B_4 is given by

$$B_4=\frac{8(1-\beta^2)}{15\sqrt{\pi}}.$$

Now the integration of Eq. (33) with a transformation $\phi=\Phi^2$, as well as the boundary conditions $d\Phi/d\xi\to 0$, $\Phi\to 0$ at $\xi\to\infty$, leads to

$$2\left(\frac{d\Phi}{d\xi}\right)^2=\frac{B_1}{2}\Phi^2-\frac{2B_2}{5}\Phi^3+\frac{B_3}{3}\Phi^4-\frac{2B_4}{7}\Phi^5=-\psi(\Phi). \tag{34}$$

Differentiating Eq. (34) with respect to ξ we get

$$4\frac{d^2\Phi}{d\xi^2}=B_1\Phi-\frac{6B_2}{5}\Phi^2+\frac{4B_3}{3}\Phi^3-\frac{10B_4}{7}\Phi^4. \tag{35}$$

To use the tanh method, we transform Eq. (35) to a standard form, for which we use the linear transformation $\Phi=\mu F+\nu$ with $\mu=1$ and $\nu=7B_3/30B_4$. Equation (35) then reduces to

$$4\frac{d^2F}{d\xi^2}-PF+\frac{10B_4}{7}F^4=0, \tag{36}$$

where the following relations are used $B_2=\frac{7}{18}B_3^2/B_4$, $B_1=\frac{49}{900}B_3^3/B_4^2$ and $P=B_1-\frac{12}{5}B_2\nu+4B_3\nu^2-\frac{40}{7}B_4\nu^3$.

Now we take the transformation $W(t)=F(\xi)$ with $t=\tanh(k\xi)$. Then Eq. (36) transforms as

$$4k^2(1-t^2)^2\frac{d^2W}{dt^2}-8k^2(1-t^2)\frac{dW}{dt}-PW+\frac{10B_4}{7}W^4=0. \tag{37}$$

Here the Frobenius series solution method requires an infinite series, but whose sum can be calculated and turns out to be

$$W(t)=k_1(1-t^2)^{1/3}. \tag{38}$$

Substituting Eq. (38) into Eq. (37) we get

$$\tfrac{8}{9}k^2(5t^2-3)-P+\tfrac{10}{7}B_4k_1^3(1-t^2)=0. \tag{39}$$

Equation (39) determines the unknowns k and k_1 and finally the solution is given by

$$\phi(\xi)=\left[\frac{7B_3}{10B_4}+\left(\frac{7P}{4B_4}\right)^{1/3}\mathrm{Sech}^{2/3}\left(\frac{\xi}{\delta}\right)\right]^2, \tag{40}$$

where

$$\delta=\left(\frac{9P}{16}\right)^{-1/2}.$$

To obtain double layer solution in this case we proceed as follows. From Eq. (34), we get

$$\left(\frac{d\Phi}{d\xi}\right)^2=-\frac{B_4}{7}\Phi^2\left(\Phi^3-\frac{7B_3}{6B_4}\Phi^2+\frac{7B_2}{5B_4}\Phi-\frac{7B_1}{4B_4}\right). \tag{41}$$

For double layers solution the polynomial on the right-hand side of (41) must have equal roots at a point $\Phi=\Phi_m\neq 0$. The condition for this is

$$\left(\frac{3d_1-d_2^2}{9}\right)^3+\left(\frac{9d_2d_1-27d_0-2d_2^3}{54}\right)^2=0, \tag{42}$$

where d_0, d_1, d_2 are given by $d_0=-7B_1/4B_4$, $d_1=7B_2/5B_4$, $d_2=-7B_3/6B_4$. We can then write Eq. (41) as

$$\left(\frac{d\Phi}{d\xi}\right)^2=-\frac{B_4}{7}\Phi^2(\Phi-\Phi_m)^2(\Phi-\Phi_1), \tag{43}$$

where $\Phi_m=-\frac{1}{3}d_2-(9d_2d_1-27d_0-2d_2^3/54)^{1/3}$, $\Phi_1=-\frac{1}{3}d_2+(9d_2d_1-27d_0-2d_2^3/54)^{1/3}$.

One can solve Eq. (43) in a manner in which ξ is given in terms of ϕ. The expression is given later

$$\xi=\sqrt{\frac{7}{B_4}}\left\{\frac{-2}{\Phi_m\sqrt{\Phi_1}}\tanh^{-1}\left[\frac{\dfrac{\sqrt{\Phi_1-\Phi}-\sqrt{\Phi_1-\Phi_0}}{\sqrt{\Phi_1}}}{1-\dfrac{1}{\Phi_1}\sqrt{(\Phi_1-\Phi)(\Phi_1-\Phi_0)}}\right]\right\}$$

$$+\sqrt{\frac{7}{B_4}}\left\{\frac{2}{\Phi_m\sqrt{\Phi_1-\Phi_m}}\tanh^{-1}\left[\frac{\dfrac{\sqrt{\Phi_1-\Phi}-\sqrt{\Phi_1-\Phi_0}}{\sqrt{\Phi_1-\Phi_m}}}{1-\dfrac{1}{\Phi_1-\Phi_m}\sqrt{(\Phi_1-\Phi)(\Phi_1-\Phi_0)}}\right]\right\}\quad \text{for } \Phi_m<\Phi_1, \tag{44}$$

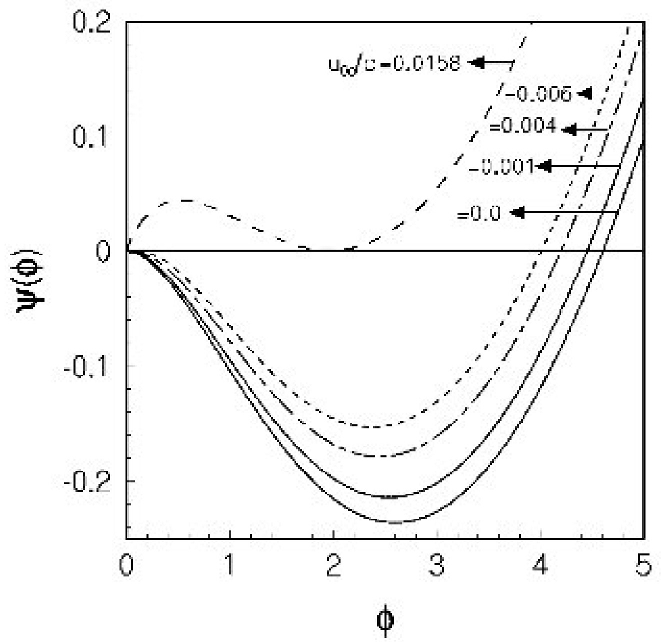

FIG. 1. Sagdeev's potential $\psi(\phi)$ against ϕ is plotted for different values of u_{0c}/c, other parameters are $\alpha=5$, $\beta=-0.5$, $\beta_1=0.01$, $V=1.5$, $u_{0b}/c=0.001$, $\theta=10$.

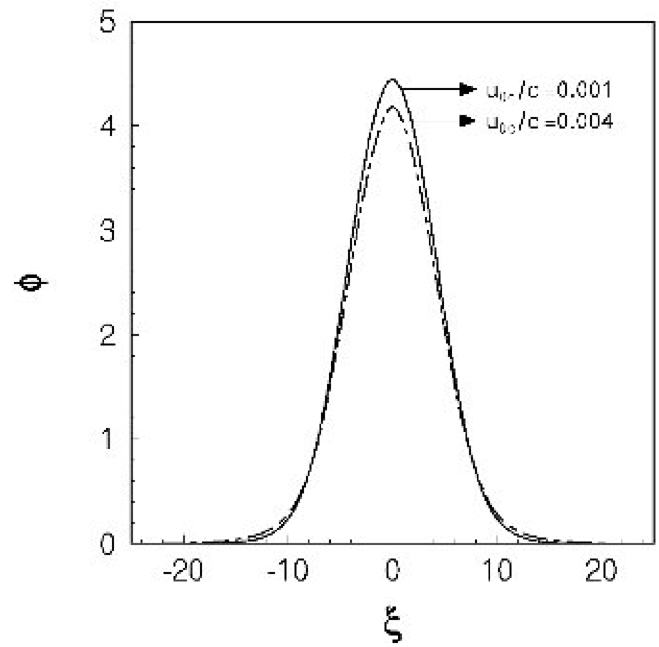

FIG. 2. Plot of ϕ against ξ for different values of u_{0c}/c, other parameters being same as those in Fig. 1.

where we have used the boundary condition: $\xi\to 0$ as $\Phi\to\Phi_0$.

One can proceed taking the nonlinear term to any order in ϕ, and could derive different types of solitary waves. However, it will be increasingly difficult to obtain analytical formula for $\phi(\xi)$ as a function of ξ.

V. RESULTS AND DISCUSSION

Electron-acoustic solitary waves and double layers have been investigated for an unmagnetized relativistic electron-beam plasma system. To facilitate the study, Sagdeev's pseudopotential, $\psi(\phi)$, for this system has been derived analytically. In the absence of an electron beam, our analytical result agrees with that of Mamun it *et al.*[21] provided one neglects the relativistic effect. To make our result physically relevant, numerical studies have been made using plasma parameters close to those values corresponding to the day side auroral zone.[25] It is found that this system supports both solitary waves and double layers provided the relativistic effect is small and other plasma parameters satisfy certain conditions. As a particular case a new double layer solution is obtained. Since one of our motivations was to study the relativistic effect on formation of solitary waves, $\psi(\phi)$ is plotted against ϕ in Fig. 1 for different values of u_{0c}/c including $u_{0c}/c=0$. It is seen that if one takes into account the relativistic effect, the amplitude of the solitary wave becomes reduced. It is also seen that the amplitude of solitary waves gradually decreases with the increase of u_{0c}/c, and when u_{0c}/c crosses a certain limit the soliton ceases to exist. In the present case, with $\beta_1=0.01$ and other plasma parameters being same as those of Mamun it *et al.*,[21] the limiting value of u_{0c}/c is 0.0158. The solitary wave solutions for this case are plotted in Fig. 2. To see the effect of the introduction of the electron beam, $\psi(\phi)$ is plotted against ϕ in Fig. 3 for several values of β_1. When β_1 increases, the solitary waves tend to disappear. The effect of β on the formation of solitary waves can be seen from Fig. 4. Here also, for large negative β, solitary waves disappear. Also, the amplitude of solitary waves decreases with increase of β. In Fig. 5 $\psi(\phi)$ is plotted against ϕ for several values of V, the velocity of solitary waves. Here the amplitude increases with the increase of V.

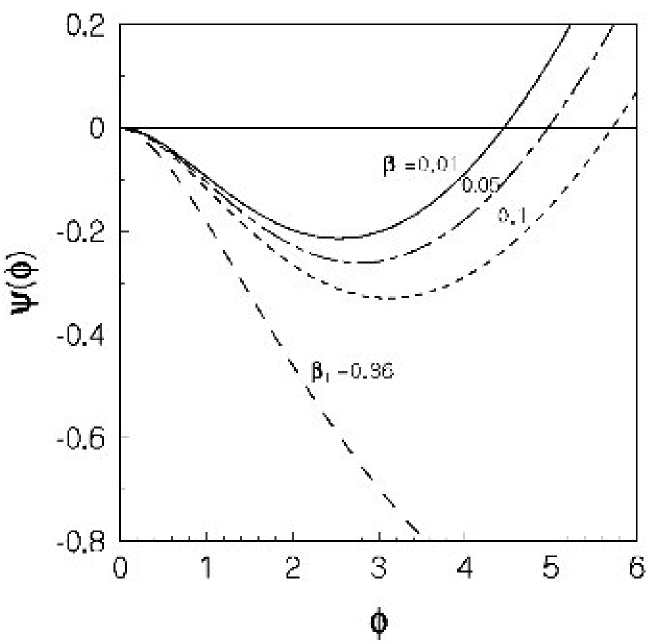

FIG. 3. Plot of $\psi(\phi)$ against ϕ for different values of β_1, where $u_{0c}/c=0.001$ and other parameters being same as those in Fig. 1.

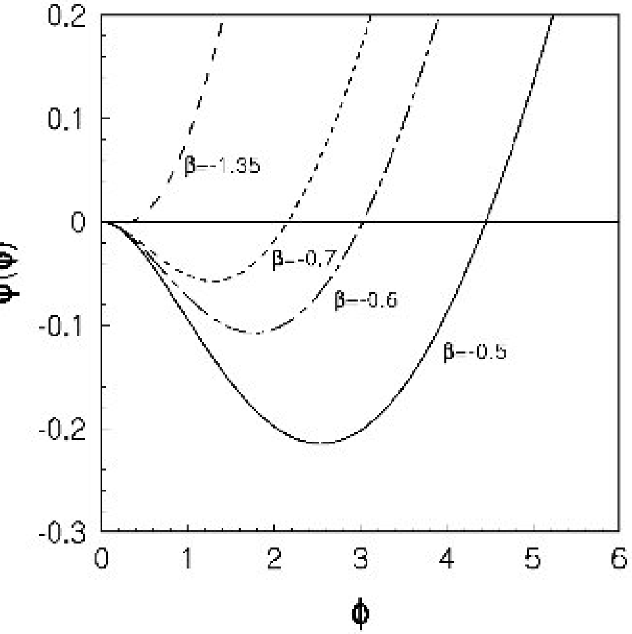

FIG. 4. Plot of $\psi(\phi)$ against ϕ for different values of β, where $u_{0c}/c = 0.001$, other parameters remain same as those in Fig. 1.

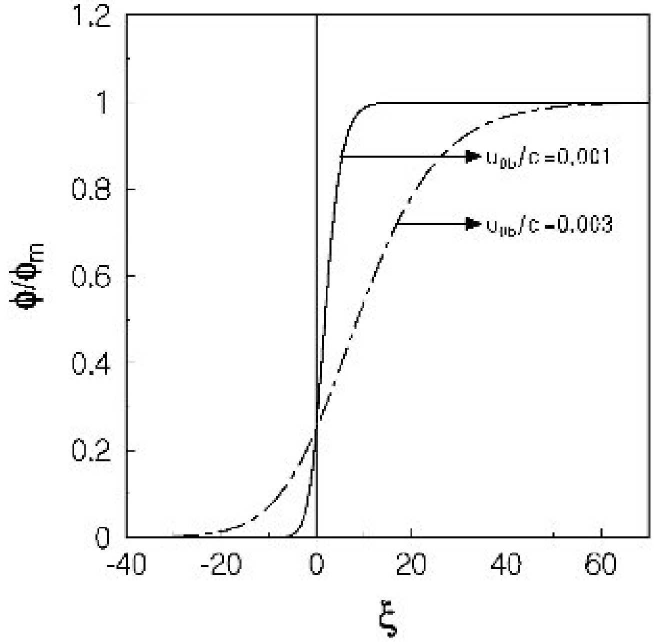

FIG. 6. Plot of ϕ/ϕ_m against ξ [Eq. (32)] for different values of u_{0b}/c, other parameters are $u_{0c}/c=0.001$, $\alpha=5$, $\beta=-0.5$, $\beta_1=0.01$, $\theta=10$, and V is found from condition (28).

However, for $V<V_0$ where V_0 depends on the other plasma parameters ($V_0=0.54$ in the present case) the solitary structure ceases to exist. Double layer solutions are depicted in Figs. 6–8. In Fig. 6 the double layer solution given in Eq. (32) is plotted for several values of u_{0c}/c. A very sharp structure is obtained for $u_{0c}/c=0.001$. Figure 7 depicts the double layer solutions corresponding to the same Eq. (32) for different values of β_1. The sharpest structure is obtained for $\beta_1=0.01$. It is found that double layers persist even when one takes higher order terms, and the solution given in Eq. (44) is plotted in Fig. 8 for $\beta_1=0.01$ and $\beta_1=0.2$, other parameters being same as those of Fig. 7.

VI. CONCLUSION

In this paper we have studied the problem of one-dimensional electron acoustic solitons and double layers in a

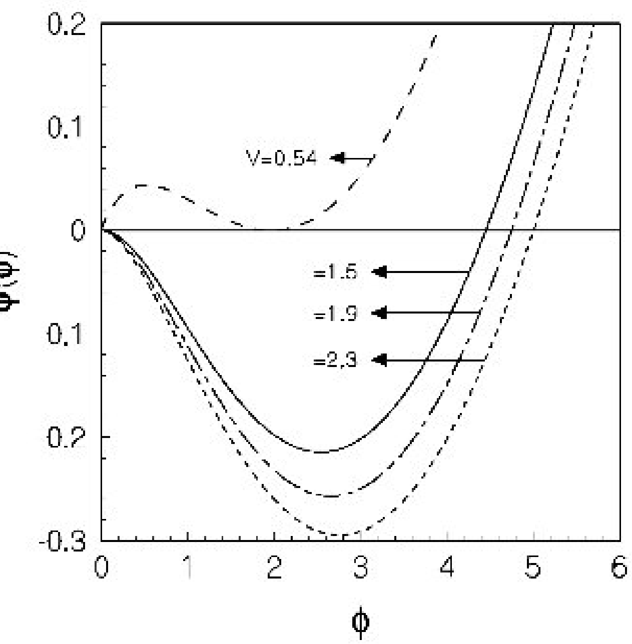

FIG. 5. Plot of $\psi(\phi)$ against ϕ for different values of V, where $u_{0c}/c = 0.001$ and other parameters remain same as those in Fig. 1.

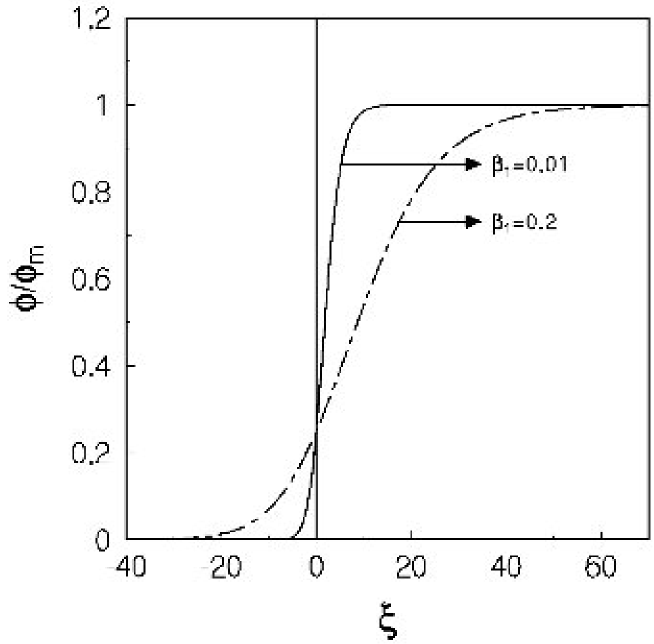

FIG. 7. Plot of ϕ/ϕ_m against ξ [Eq. (32)] for different values of β_1, where $u_{0c}/c=0.001$, other parameters are same as those in Fig. 6.

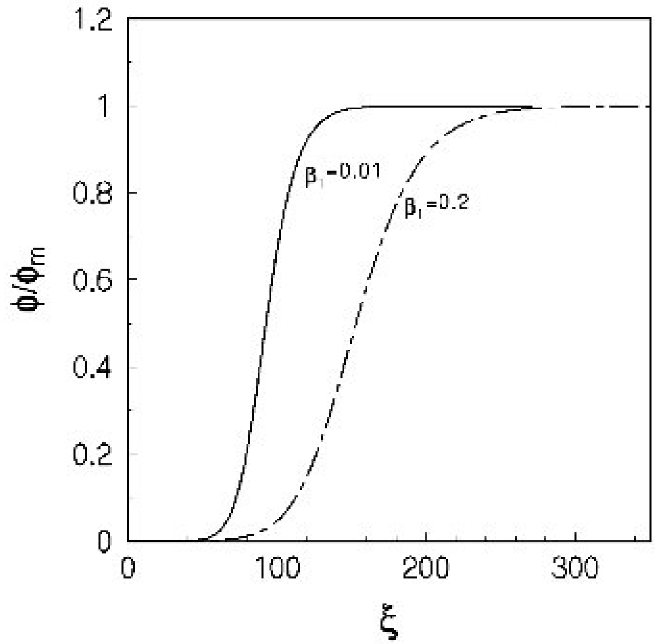

FIG. 8. Plot of ϕ/ϕ_m against ξ for double layer solution [Eq. (44)] for different values of β_1 where $u_{0c}/c=0.001$, $u_{0h}/c=0.001$ $\alpha=5$, $\beta=-0.5$, $\theta=10$. V is found from condition (42).

relativistic electron-beam plasma system. For soliton solutions we have used Sagdeev's pseudopotential approach. We have studied the effect of parameters like u_{0c}, β_1, β, V, on the existence of solitons. It is seen that solitary waves would cease to exist if the relativistic parameter u_{0c} and the beam parameter β_1 exceed certain values, which of course depend on other parameters such as the soliton velocity V, the hot and cold electron density ratio, the hot and cold electron temperature ratio, etc. One interesting fact is that a trapped distribution of hot electrons supports solitary waves with positive electrostatic potential pulses. Using the expansion of Sagdeev's potential, an explicit analytical expression for double layers are obtained. The tanh method transforms the equation of motion obtained for ϕ, the electric potential, to a Fuchsian-like ordinary differential equation, and finally a truncated series solution is employed to find the different solution and double layers. Keeping terms up to order $\phi^{5/2}$ a new analytical double layer solution is obtained which, to the best of our knowledge, has not been reported before.

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