SATYA R. CHAKRAVARTY – PIETRO MULIERE

Welfare indicators: a review and new perspectives. 2. Measurement of poverty

Summary - The purpose of this paper is to present significant results on welfare theoretic approaches to the measurement of poverty. Alternative forms of indices are analyzed. The problem of ranking income distributions in terms of welfare, graphical techniques, different forms of equalizing transfers, stochastic dominance have been studied extensively.

Key Words - Poverty; Indices; Welfare; Orderings.

1. Introduction

Promotion of higher equality is an important policy issue in many countries. Similarly, in many societies poverty reduction is an important goal of public policy. The technical literature on evaluation and measurement of economic inequality and related issues has grown remarkably over the last thirty years or so. However, there does not exist unambiguous agreement about how to measure concepts like inequality and poverty in an accurate way.

Chakravarty and Muliere (2003), in this journal, surveyed the literature on the normative approach to the measurement of income inequality, which includes derivation of inequality indices that are based on reasonable and ethically attractive social value judgements. An analysis of dominance reasoning as well as other normative priorities has also been presented. The present paper, which can be regarded as a sequel to this survey, discusses some of the remaining important topics along this line. More precisely, it is a survey of the literature on ethical indicators of poverty. In order to make the social judgement concerning the indices explicit, each index is assumed to correspond to a social welfare function in a particular way. The problem of ranking alternative income distributions using different types of dominance conditions is also investigated. We start the presentation by assuming that welfare depends absolutely on income.

Received February 2004.

But income often as the sole indicator of well-being is inappropriate and should therefore be supplemented by other attributes of well-being, e.g., literacy, public goods, etc. Therefore, we also present a review of suggested multidimensional indicators in this context. As noted, a second attractive feature of the survey is its coverage of a wide range of topics. From these perspectives this survey is quite exhaustive. None of the recent contributions along this line appears to be informative to such a large extent ([1]).

The next section of the paper presents the background material. Section 3 sets out the different postulates by which indices of poverty can be selected, discusses suggested indicators of poverty and makes a rigorous discussion on poverty and welfare dominance. Finally, Section 4 concludes.

2. The background

Let (Ω, A, P) be a probability space, and X be a non-negative random variable defined on it that has finite positive expectation $\mu(X)(\mu)$. Ω may be seen as a population of individuals or households and $X(\omega)$ as the income of the individual (household) $\omega \in \Omega$. To facilitate presentation we speak of individuals and their incomes, but do not rule out other possibilities of interpretation. We may regard the income distribution as a non-negative random variable X with distribution function $F(x) = P(X \leq x)$ and finite positive expectation μ. For an n-person economy, that is, if $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ and P gives equal mass $\frac{1}{n}$ to each ω_i, we write $x_i = X(\omega_i)$ and $X = (x_1, x_2, \ldots, x_n)$, for short. The vector X is an element of D^n, the nonnegative orthant of the n-dimensional Euclidean space R^n with the origin deleted. Deletion of origin from the domain ensures that there is at least one person with positive income. The set of all income distributions is $D = \bigcup_{n \in N} D^n$, where N is the set of natural numbers. For any function $\tau : D \rightarrow R^1$, the restriction of τ on D^n will be denoted by τ^n. For all $n \in N$, $X \in D^n$, we will write $\hat{X} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ for the illfare ranked permutation of X, that is, $\hat{x}_1 \leq \hat{x}_2 \leq \ldots, \hat{x}_n$. For all $n \in N$, 1^n will stand for the n-coordinated vector of ones. Sometimes we will use D^n_+, the strictly positive part of D^n or R^n_+, nonnegative part of R^n, as the set of income distributions in an n-person population. The corresponding sets of all income distributions will be denoted by D and R_+ respectively.

Let F be any income distribution function on $[0, \infty)$. The left continuous version of the inverse of F is defined by

$$H(y) = F^{-1}(y) = \inf\{x : F(x) \geq y\}) \tag{1}$$

([1]) Some of these issues have been discussed in several recent books and surveys. See, for instance, Kakwani (1980a, 1999), Seidl (1988), Chakravarty (1990), Johnson (1996), Foster and Sen (1997), Zheng (1997, 1999, 2000a), Silber (1999), Lambert (2001) and Dutta (2002).

where $0 \leq y \leq 1$. While $F(x)$ represents the cumulative proportion of persons with income less than or equal to x, $H(y)$ gives the income of the $100y$-percent poorest individuals in the distribution X and is referred to as the quantile function. For any income distribution function F, we also define

$$F^{r+1}(t) = \int_0^t F^r(u)du \tag{2}$$

for all $t \in [0, \infty)$, where $r \geq 1$ is any positive integer. Obviously $F^1 = F$.

As we will see, this expression makes clear the connection between stochastic dominance and poverty ranking for a given poverty line t.

A fundamental order or dominance principle in this context is the Lorenz order, which relies on the Lorenz function $L_X : [0, 1] \to [0, 1]$, where

$$L_X(t) = \frac{1}{\mu} \int_0^t H(y)dy \tag{3}$$

with $0 \leq t \leq 1$. The graph of the Lorenz function is the Lorenz curve which indicates the cumulative proportion of income enjoyed by the bottom $t(0 \leq t \leq 1)$ proportion of the population. By scaling up the Lorenz curve of a distribution by its mean income, we get the generalized Lorenz curve of the distribution. Formally, the generalized Lorenz curve of X is defined as

$$GL_X(t) = \mu L_X(t)$$

for all $0 \leq t \leq 1$.

Definition 2.1. Given two income distributions X and Y, with distribution functions F_X and F_Y respectively, we say that:

(a) X Lorenz dominates Y, which we write $X \geq_L Y$, if

$$L_X(p) \geq L_Y(p)$$

for all $p \in [0, 1]$, with $>$ for some p, that is, the Lorenz curve of X is nowhere below that of Y and strictly above at some places (at least).

(b) X generalized Lorenz dominates Y, $X \geq_{GL} Y$, for short, if

$$GL_X(p) \geq GL_Y(p)$$

for all $p \in [0, 1]$ with $>$ for some p.

If the means of the distributions are the same, the Lorenz and the generalized Lorenz dominations coincide.

The following definition will be necessary for formulating a central property of various indices.

Definition 2.2. For all $n \in N$ we say that $Y \in D^n$ is obtained from $X \in D^n$ by a regressive transfer if there exists two persons i and j such that $x_l = y_l$ for all $l \neq i, j$; $y_i - x_i = x_j - y_j > 0$; $x_j \leq x_i$.

That is, X and Y are identical except for a positive transfer of income from person j to person i who has a higher income than j. We can equivalently say that X has been obtained from Y by a progressive transfer.

The next definition will be helpful in stating some postulates and results.

Definition 2.3. For all $n \in N$, we say that $X \in D^n$ is obtained from $Y \in D^n$ by an increment if for some i, $x_i = y_i + c$ and $x_j = y_j$ for all $j \neq i$, where $c > 0$. Equivalently, we say that Y is obtained from X by a decrement.

The dominance rule stated below will also be useful for analyzing our results.

Definition 2.4. Given any two income distributions X and Y with distribution functions F_X and F_Y, we say that X rth order stochastic dominates Y, which we denote by $X \geq_r Y$, if $F_X^r(t) \leq F_Y^r(t)$ for all $t \in [0, 1]$, with $<$ for at least one t, where r can be equal to any finite positive integer. For weak rth stochastic dominance we need only the weak inequality above.

Thus, for first order stochastic dominance between X and Y we need inequality between the corresponding distribution functions. Similarly, X second order stochastic dominates Y if we have $F_X^2(t) \leq F_Y^2(t)$ for all $t \in [0, 1]$ with $<$ for some t. The condition $X \geq_r Y$ is equivalent to the requirement that the expected utility underF_X is greater than that under F_Y, where all odd order derivatives of the utility function U through r are positive and all even order derivatives are negative, that is

$$\int_0^\infty U(t)dF_X(t) > \int_0^\infty U(t)dF_Y(t) \tag{4}$$

where $(-1)^{j+1}U^j > 0$, U^j, being the jth order derivative of U, $j = 1, 2, \ldots, r$ (see Fishburn (1980) and Fishburn and Willig (1984)). Thus, efficiency preference or preference for higher incomes, ceteris paribus, is the main distinguishing characteristic for first order dominance. On the other hand, $X \geq_r Y$ holds for $r = 2$, that is, X second order stochastic dominates Y if and only if X is preferred to Y by all utilitarians who approve of both efficiency and equity. Third order stochastic dominance is characterized by efficiency, equity and transfer sensitivity which demands that the transfers that occur lower down in the distribution should have greater impact. Examples of utility functions identified in (4) are $U(t) = t^c$, $0 < c < 1$ and $U(t) = 1 - e^{-t}$. (See Muliere and

Scarsini (1989), Mosler and Muliere (1998), Moyes (1999) and Chakravarty and Muliere (2003) for further discussion.)

Since many of the indices we discuss are based on the Atkinson (1970) - Kolm (1969) - Sen (1973) equally distributed equivalent (EDE) income or its related indices, we make a brief discussion on them also. We assume that moral judgements on alternative distributions of income are summarized by the social welfare function $W : D \to R^1$, where W is ordinally significant. It is further assumed that for all $n \in N$, W^n is regular, that is, W^n continuous, increasing and strictly S-concave. Continuity ensures that minor observational errors on incomes does not give rise to abrupt jump in the value of the social welfare function. Increasingness means that if we increase any income, keeping the remaining fixed, social welfare increases. Increasingness is analogous to the strong Pareto preference condition. Strict S-concavity demands that a rank preserving transfer of income from a person to anybody who has a lower income increases social welfare(2).

Assuming that W^n is regular, given any $X \in D^n$, the Atkinson-Kolm-Sen (AKS) EDE income x_e is defined as that level of income which if given to everybody will make the existing distribution X ethically indifferent (indifferent as measured by W^n). Thus, x_e is implicitly defined by

$$W^n(x_e 1^n) = W^n(X)\,. \tag{5}$$

Given regularity conditions on W^n, we can solve (5) uniquely for x_e

$$x_e = E^n(X)\,. \tag{6}$$

By continuity of W^n, E^n is a continuous function. Furthermore, E^n is a specific numerical representation of W^n, that is,

$$W^n(X) \geq W^n(Y) \Leftrightarrow E^n(X) \geq E^n(Y) \Leftrightarrow x_e \geq y_e\,. \tag{7}$$

Thus, one income distribution is socially better than another if and only if its EDE income is higher.

To define the AKS inequality index, we assume that W^n is regular and homothetic. Homotheticity of W^n demands that it can be expressed as an ordinal transform of a linear homogeneous function. Formally, W^n is called homothetic if it can be written as

$$\phi(\tilde{W}^n) \tag{8}$$

where ϕ is increasing in its argument and $\tilde{W}^n$ is linear homogenous. This means that the entire set of social indifference curves can be generated by

(2)All strictly S-concave functions are symmetric, that is, welfare remains unchanged under any permutation of incomes. A function is called strictly S-convex if its negative is strictly S-concave.

radial expansion or contraction by means of rays emanating from the origin of a single curve (see Chakravarty (1990)).

The AKS index of inequality is defined by $I_{\text{AKS}} : D \to R^1$, where for all $n \in N,\ X \in D^n$,

$$I^n_{\text{AKS}}(X) = 1 - \frac{E^n(X)}{\mu} \tag{9}$$

or

$$I^n_{\text{AKS}}(X) = 1 - \frac{\tilde{W}^n(X)}{\mu \tilde{W}^n(1^n)} \tag{10}$$

I_{AKS} is continuous, symmetric and decreasing under a rank preserving progressive transfer (due to strict S-concavity of W), and bounded between zero and one, where the lower bound is achieved whenever incomes are equally distributed. It gives the fraction of aggregate income that could be saved without any welfare loss if society distributed incomes equally. I_{AKS} can also be interpreted as the proportional welfare loss that arises due to existence of inequality. Homotheticity of W^n ensures that I_{AKS} is a relative index-it remains invariant under equal proportionate changes in all incomes. Note that given a functional form for I_{AKS}, we can recover W^n using (10), (6) and (5). Thus, the AKS relative inequality index is normatively significant or exact in the sense that it implies and is implied by a social welfare function.

Clearly, to each homothetic social welfare function there corresponds a relative inequality index and they differ depending on the form of the welfare function. We may explain this by two examples. For this, first suppose that the welfare evaluation is done with respect to the Gini social welfare function $E_G : D \to R^1$, where for all $n \in N,\ X \in D^n$

$$E^n_G(X) = \frac{1}{n^2}\sum_{i=1}^{n}(2(n-i)+1)\hat{x}_i\,, \tag{11}$$

which is a rank order weighted average of individual incomes, where the weight attached to the ith ranked income is independent of the income distribution. The resulting AKS index of inequality is the well-known Gini index:

$$I^n_G(X) = 1 - \frac{1}{n^2\mu}\sum_{i=1}^{n}(2(n-i)+1)\hat{x}_i\,. \tag{12}$$

As a second example we consider the Bonferroni social welfare function $E_B : D \to R^1$, where for all $n \in N,\ X \in D^n$,

$$E^n_B(X) = \frac{1}{n}\sum_{i=1}^{n}\mu_i = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{i}\frac{\hat{x}_j}{i} \tag{13}$$

where μ_i is the ith partial mean, that is,

$$\mu_i = \sum_{j=1}^{i} \frac{\hat{x}_j}{i}.$$

The resulting AKS index becomes the Bonferroni index:

$$I_B^n(X) = 1 - \frac{1}{n\mu} \sum_{i=1}^{n} \mu_i. \tag{14}$$

Strictly speaking, Bonferroni (1930) suggested the use of the transformed index

$$\frac{(n-1)I_B^n}{n}$$

as an index of inequality. Nygard and Sandstrom (1981) referred to I_B^n as the Bonferroni index. For a fixed number of persons $(j-i-1)$ between the donor j and the recipient i, a progressive transfer is valued more by them if the transfer occurs at lower income levels. However, given the difference $(j-i)$, the Gini index is equally sensitive to transfers at all income positions ([3]).

An alternative to the invariance condition satisfied by I_{AKS} is translation invariance, which requires inequality to remain unchanged under equal absolute changes in all incomes. Such indices are called absolute indices. To present the most commonly used ethical absolute inequality index we assume that for all $n \in N$, $W^n : R_+^n \to R^1$, is regular and translatable. Translatability of W^n means that it can be written as $\phi(\widehat{W^n}(x))$, where ϕ is increasing in its argument and is $\widehat{W^n}$ unit translatable, that is,

$$\widehat{W^n}(X + c1^n) = \widehat{W^n}(X) + c \tag{15}$$

where c is a scalar such that $X + c1^n \in R_+^n$.

Assuming that W^n is regular and translatable, the Blackorby - Donaldson (1980a) - Kolm (1976a, 1976b) (BDK) absolute index of inequality is defined by $A_{\text{BDK}} : R_+ \to R^1$, where for all $n \in N$, $X \in R_+^n$,

$$A_{\text{BDK}}^n(X) = \mu - E^n(X) = \mu - \hat{W}^n(X) + \hat{W}^n(01^n) \tag{16}$$

A_{BDK} is continuous, strictly S-convex and bounded from below by zero, where this bound is achieved whenever incomes are equal. It gives the per capita income that could be saved if society distributed incomes equally without any

([3]) For further discussion on different properties of the Bonferroni index, see Giorgi (1984, 1998), Tarsitano (1990), Giorgi and Mondani (1994, 1995) and Giorgi and Crescenzi (2001a, 2001b, 2001c).

welfare loss. It also determines the size of absolute welfare loss associated with the existence of inequality. From policy point of view, the absolute index A^n_{BDK} determines the total cost of per capita inequality in the sense that it tells us how much must be added in absolute terms to the income of every member in an n-person society to reach the same level of social welfare that would be achieved if everybody enjoyed the mean income of the current distribution. Given a functional form for A^n_{BDK}, we can recover W^n using (16), (6) and (5). In order to illustrate A_{BDK}, we may derive the absolute indices corresponding to the Gini and Bonferroni welfare functions since they are both homothetic and translatable. Evidently, we can also generate examples of A_{BDK} where the welfare function satisfies translatability but not homotheticity. One such welfare function is the Kolm (1976a) - Pollak (1971) function ([4]).

3. Measurement of poverty

3.1. *Poverty axioms: definitions and discussion*

Poverty has been in existence for many years and continues to exist in a large number of countries. Therefore, targeted poverty alleviation remains an important policy issue in many countries. In order to understand the threat that the problem of poverty poses, it is necessary to know its dimension and the process through which it seems to be aggravated. A natural question here is how to quantify the extent of poverty. Sen (1976) noted that evaluation of poverty requires solution to two distinct problems:

(a) the problem of identification, that is, identifying the set of poor persons, and
(b) the problem of aggregation-aggregating the characteristics of the poor into an indicator of poverty.

The identification problem involves the specification of a poverty line representing the level of income necessary to maintain a subsistence standard of living and a person with income not exceeding the subsistence income level is called poor. (See Ravallion (1994), on issues related to determination of poverty line.) We assume that the poverty line $z > 0$ is given exogenously and takes values in Z, a subset of the real line. For any $n \in N$, $X \in R^n_+$, let $Q(X) = \{i | x_i \leq z\}$ be the set of poor persons. This is the strong definition of the set of poor. The weak version will require replacement of the inequality $\leq$ in $Q(X)$ by $<$. In the literature the former definition is more common. Person i is called non-poor or rich if he is not poor. For any $n \in N$, $X \in R^n_+$, let

([4]) Similarly, an example of a homothetic social welfare function which is not translatable is the symmetric mean of order $k(< 1)$ (see Chakravarty and Muliere (2003)).

X^p be the income distribution of the poor and hence its illfare ordering will be denoted by $\hat{X}^p$.

A poverty index P is a real valued function defined on $R_+ \times Z$. Thus, given any income distribution $X \in R_+^n$ and a poverty line $z \in Z$, $P^{n,1}(X, z)$, will indicate the level of poverty associated with X. A poverty index is called a relative index or an absolute index according as it satisfies a scale invariance condition or an absolute invariance condition. Formally,

Definition 3.1. We say that $P : R_+ \to R^1$ is a relative poverty index if for all $n \in N$, $X \in R_+^n$, $z \in Z$, $P^{n,1}(X, z) = P^{n,1}(cX, cz)$, where $c > 0$ is any scalar.

Definition 3.2. We say that $P : R_+ \to R^1$ is an absolute poverty index if for all $n \in N$, $X \in R_+^n$, $z \in Z$, $P^{n,1}(X, z) = P^{n,1}(X + c1^n, z + c)$, where c is a scalar such that $(X + c1^n) \in R_+^n$ and $(z + c) \in Z$.

That is, a relative index remains unchanged under equal proportionate changes in all incomes and the poverty line, whereas an absolute index does not alter under equal absolute addition to all incomes and the poverty line. Thus, while in the former case a poor person views his deprivation in terms of relative shortfall of his income from the poverty line z, in the latter case deprivation can be taken as the absolute gap between z and the income.

The following postulates have been suggested for an arbitrary poverty index P, whether relative or absolute. Unless specified, we assume that z is arbitrarily given.

Focus Axiom (FOC). For all $n \in N$, $X, Y \in R_+^n$, if $Q(X) = Q(Y)$ and $x_i = y_i$ for all $i \in Q(X)$, then $P^{n,1}(X, z) = P^{n,1}(Y, z)$.

Weak Monotonicity Axiom (WMN). For all $n \in N$, $X, Y \in R_+^n$, if Y is obtained from X by a decrement in a poor person's income, then $P^{n,1}(X, z) < P^{n,1}(Y, z)$.

Strong Monotonicity Axiom (SMN). For all $n \in N$, $X, Y \in R_+^n$, if X is obtained from Y by an increment in a poor person's income, then $P^{n,1}(X, z) < P^{n,1}(Y, z)$.

Minimal Transfer Axiom (MTR). For all $n \in N$, $X, Y \in R_+^n$, if Y is obtained from X by a regressive transfer between two poor persons with no one becoming rich as a result of the transfer, then $P^{n,1}(X, z) < P^{n,1}(Y, z)$.

Weak Transfer Axiom (WTR). For all $n \in N$, $X, Y \in R_+^n$, if Y is obtained from X by a regressive transfer from a poor person with no one becoming rich as a result of the transfer, then $P^{n,1}(X, z) < P^{n,1}(Y, z)$.

Strong Transfer Axiom (STR). For all $n \in N$, $X, Y \in R_+^n$, if Y is obtained from X by a regressive transfer from a poor person to someone who is richer, then $P^{n,1}(X, z) < P^{n,1}(Y, z)$.

Symmetry Axiom (SYM). For all $n \in N,\ X, Y \in R^n_+$, if Y is obtained from X by a permutation of incomes, then $P^{n,1}(X, z) = P^{n,1}(Y, z)$.

Increasing Poverty Line Axiom (IPL). For all $n \in N,\ X \in R^n_+,\ P^{n,1}(X, z)$, is increasing in z.

Population Principle Axiom (POP). For all $n \in N,\ X \in R^n_+,\ P^{n,1}(X, z) = P^{mn,1}(Y, z)$, where Y is the m-fold replication of X, that is, $Y = (X^{(1)}, X^{(2)}, \dots, \dots, X^{(m)})$ with each $X^{(i)}$ being X.

Continuity Axiom (CON). For all $n \in N,\ X, Y \in R^n_+,\ P^{n,1}(X, z)$, is a continuous function of X.

The axioms **FOC**, **WMN**, **WTR** and **STR** were suggested by Sen (1976, 1979, 1981).

The axiom **FOC** demands that poverty index should be independent of the incomes of non-poor. However, it does not assume that the poverty index cannot depend on the number of non-poor persons. A poverty index satisfying **FOC** will be called focused.

The axiom **WMN** says that a reduction in a poor person's income, holding other incomes constant, must increase poverty. A stronger version of **WMN** is **SMN**, which was suggested by Donaldson and Weymark (1986).

According to **SMN**, poverty should decrease under an increment in the income of a poor. Thus, it includes the possibility that the beneficiary of the income increase may become rich. Therefore, for either definition of the poor, **SMN** implies **WMN**. The axiom **MTR** requires poverty to increase if there is a regressive transfer between two poor, the set of poor persons remaining the same. The axiom **WTR** has the same spirit as **MTR**, but it allows the possibility that the recipient of the transfer may be a rich person.

In **STR** it is possible that the transfer recipient crosses the poverty line as a result of the transfer. Sen earlier suggested **STR**, but in later works opted for **WTR**. Clearly, for either definition of the poor, **STR** implies **WTR**, which in turn implies **MTR**.

Symmetry means that for a given poverty line, poverty remains unchanged under any reordering of incomes. Thus, any characteristic other than income, e.g., the names of the individuals, is irrelevant to the measurement of poverty. One implication of symmetry is that we can define an index of poverty directly on ordered distributions. It may be noted that under **SYM**, we allow only rank preserving transfers of income between individuals. The axiom **IPL**, which was introduced by Clark, Hemming and Ulph (1981) and Chakravarty (1983a), is reasonable because between two identical societies the one with higher poverty line should have higher poverty.

The axiom **POP** was considered in the context of poverty measurement by Chakravarty (1983a) and Thon (1983). It demands that if a population is replicated several times, then for a given poverty line, the poverty levels of the original and the replicated populations are the same. In other words, **POP** views poverty as an average concept. Clearly, using this principle, we can make inter-population as well as inter-temporal comparisons of poverty. This is because using replications we can convert two income distributions with different population sizes into distributions with the same population size and **POP** keeps poverty unchanged under replications. It may be noted that **POP** is a property of all poverty indices that are defined on the continuum. Continuity ensures that the index will not be over sensitive to income measurement errors.

Kakwani (1980b) argued that a poverty index should be more sensitive to what happens among the bottom poor. He therefore suggested three axioms, one on income reduction and two on income transfer.

Monotonicity Sensitivity Axiom (MNS). For all $n \in N$, $X \in R^n_+$, $P^{n,1}(Y^1, z) - P^{n,1}(X, z) > P^{n,1}(Y^2, z) - P^{n,1}(X, z)$, whenever $Y^1, Y^2 \in R^n_+$ are obtained from X by the same amount of decrement to poor incomes x_i and x_j, where $x_i < x_j$.

According to this axiom a poverty index should increase by a higher amount due to a reduction in a poor person's income the poorer the poor is. It may be noted that this axiom is identical to **MTR** (Kakwani (1980b) and Zheng (1997)). Therefore, minimal transfer can be justified by monotonicity sensitivity as well. Kakwani's next axiom argues that greater weight should be attached to transfers lower down the income scale. More precisely, a poverty index should assign more weight to a regressive transfer between individuals with a given income difference if the incomes are lower than when they are higher.

Diminishing Transfer Sensitivity Axiom (DTR). For all $n \in N$, $X \in R^n_+$, if Y is obtained from X by a regressive transfer of income from the person with income x_i to the person with income $x_i + h$, then for a given $h > 0$, the magnitude of increase in poverty $P^{n,1}(Y, z) - P^{n,1}(X, z)$ is higher the lower is x_i, with no one becoming rich as a result of the transfers ([5]).

Earlier, Kolm (1976a, 1976b) suggested this axiom for inequality indices. A generalization of this postulate was suggested by Shorrocks and Foster (1987) which requires inequality to decrease under a favorable composite transfer, composed of a progressive transfer and a regressive transfer, the former taking place at lower incomes than the latter such that the variance of the distribution does not change. Kakwani's third sensitivity axiom, the principle of positional transfer sensitivity, is a positional version of the diminishing transfers principle, which requires that a transfer from any person to someone who has a higher income, given that there is a fixed proportion of population between

([5]) Clearly, the notion of transfer considered here corresponds to the one described in **WTR**. We can have formulations similar to **DTR** for the types of transfer considered in **MTR** and **STR**.

them, should attach more weight at the lower end of the distribution (see also Mehran (1976) and Zoli (1999)).

Given $\hat{X} \in R^n_+$, let $\Delta P^n_{i+t,i}(\hat{X}, z)$ be the increase in poverty due to a (rank preserving) regressive transfer of δ units of income from the poor person with rank i to the person with rank $(i + t)$, where $t > 0$ is an integer.

Positional Transfer Sensitivity Axiom (PTS). For all $n \in N$, $\hat{X} \in R^n_+$ and for any pair of individuals i and j, $\Delta P^n_{i+t,i}(\hat{X}(\delta), z) > \Delta P^n_{j+t,j}(\hat{X}(\delta), z)$, where $j > i$, with nobody crossing the poverty line as a result of the transfers.

Note that for convenience **PTS** has been defined on ordered distributions.

The next two axioms shows how for any partitioning of the population into subgroups, overall poverty is related to subgroup poverty levels.

Subgroup Consistency Axiom (SUC). For all $m, n \in N$, $X^1, X^2 \in R^m_+$; $Y^1, Y^2 \in R^n_+$, if $P^{m,1}(X^1, z) = P^{m,1}(X^2, z)$ and $P^{n,1}(Y^1, z) < P^{n,1}(Y^2, z)$, then

$$P^{m+n,1}(X^1, Y^1, z) < P^{m+n,1}(X^2, Y^2, z)\,.$$

Subgroup Decomposability Axiom (SUD). For $X^i \in R^n_+$, $i = 1, 2, \ldots, m$, we have

$$P^{n,1}(X, z) = \sum_{i=1}^{m} \frac{n_i}{n} P^{n_i,1}(X^i, z) \tag{17}$$

where $X^i \in R^{n_i}_+$, $i = 1, 2 \ldots, m$, $X = (X^1, X^2, \ldots, X^m)$ and $\sum_{i=1}^{m} n_i = n$.

The axiom **SUC**, which was suggested by Foster and Shorrocks (1991), is analogous to **MON**. While the latter is concerned with change in poverty due to a change in an individual's income, **SUC** is about the impact of a change in a subgroup's poverty, where the subgroups are formed by partitioning the population with respect to some homogeneous characteristic, e.g. age, sex, race, religion, region etc. It shows that subgroup poverty and national poverty should move along the same direction. The axiom **SUD**, which was considered by Hamada and Takayama (1977), Anand (1977), Kakwani (1980a), Chakravarty (1983c), Foster, Greer and Thorbecke (1984) and Foster and Shorrocks (1991) in different senses, is stronger than **SUC**. It says that for any partitioning of the population into subgroups, overall poverty is the population share weighted average of subgroup poverty levels. The amount $(\frac{n_i}{n} P(X^i, z))$ is the contribution of subgroup i to total poverty, the amount by which national poverty will decrease if poverty in the subgroup is eliminated.

$(\frac{100 n_i}{(nP(X,Z))} P(X^i, z))$ is the percentage contribution of subgroup i to total poverty. **SUD** thus allows us to identify the subgroups that are more afflicted by poverty and hence to formulate anti-poverty policy. Note that repeated application of **SUD** enables us to write the poverty index as

$$P^{n,1}(X, z) = \frac{1}{n} \sum_{i \in Q(X)} p(x_i, z) \tag{18}$$

where $p(x_i, z) = P^{1,1}(x_i, z)$ is the individual poverty function.

Kundu and Smith (1983) suggested two population monotonicity axioms, one for poverty growth and one for non-poverty growth.

Poverty Growth Axiom (PGR). For all $n \in N$, $X \in R^n_+$, if $Y \in R^{n+1}_+$ is obtained from X by adding a poor person to the population, then

$$P^{n,1}(X, z) < P^{n+1,1}(Y, z)\,.$$

Non-Poverty Growth Axiom (NPG). For all $n \in N$, $Y \in R^n_+$, if $X \in R^{n+1}_+$ is obtained from Y by adding a rich person to the population, then

$$P^{n+1,1}(X, z) < P^{n,1}(Y, z)\,.$$

The axiom **PGR (NPG)** says that when a poor (non-poor) person is added to the population, poverty should increase (decrease). Note that **NPG** requires a poverty index to be a decreasing function of the non-poor population size. Thus, a focused poverty index satisfying **NPG** is independent of non-poor incomes but dependent on their population size ([6]).

We now look at certain implications of the postulates stated above. First, following Donaldson and Weymark (1986), Chakravarty (1990) and Zheng (1997) we summarize some interesting observations about the behavior of P in the following theorem:

Theorem 3.1.

(a) *For the weak definition of the poor*, **WMN** *and* **CON** *imply* **SMN**.

(b) *Given the set* $Q(X)$, *under strong definition of poor*, **SMN** *implies that* P *achieves its lower bound if all the poor persons are at the poverty line.*

(c) *For strong definition of the poor, a focused poverty index which is continuous at the poverty line cannot satisfy* **SMN**.

([6]) For further discussions on different axioms, see Chakravarty (1983a, 1990), Foster (1984), Donaldson and Weymark (1986), Cowell (1988), Seidl (1988), Foster and Shorrocks (1991), Zheng (1994, 1997, 2000a, 2000b) and Bourguignon and Fields (1997).

When the weak definition of poor is adopted, **FOC**, **SYM**, **WMN** and **WTR** are equivalent to requiring that for a fixed number of poor, the poverty index is a decreasing, strictly S-convex function of the incomes of the poor. That is, the poverty index corresponds to an increasing, strictly S-concave welfare function in a negative monotone way, which in turn means that it agrees with the generalized Lorenz domination. Following Hardy, Littlewood and Polya (1934), Kolm (1969), Marshall and Olkin (1979), Shorrocks (1983), Foster (1984) and Chakravarty (1990), in the following theorem we state several seemingly unrelated equivalent conditions for poverty ranking.

Theorem 3.2. *For all $n \in N$, let $X, Y \in R^n_+$ be arbitrary, where $[Q(X) = Q(Y)]$. Then under the weak definition of the poor, the following statements are equivalent:*

(a) *$\hat{Y}^p$ can be obtained from $\hat{X}^p$ by a sequence of rank preserving decrements and regressive transfers.*
(b) *$P^{n,1}(X, z) < P^{n,1}(Y, z)$ for all focused poverty indices $P^{n,1}$ that satisfy* **SYM**, **WMN** *and* **WTR**.
(c) *$P^{n,1}$ is decreasing and strictly S-convex in the incomes of the poor, where $P^{n,1}$ is any arbitrary poverty index.*
(d) *$W^q(X^p) > W^q(Y^p)$, where W^q is any increasing, strictly S-concave social welfare function defined on the set of income distributions of the poor.*
(e) *$\sum_{i \in Q(X)} U(x_i) > \sum_{i \in Q(Y)} U(y_i)$ for any increasing, strictly concave individual income utility function U of the poor.*
(f) *$X^p \geq_{GL} Y^p$, that is, X^p generalized Lorenz dominates Y^p.*
(g) *$X^p \geq_2 Y^p$, that is, X^p second order stochastic dominates Y^p.*

The next two theorems, which were demonstrated by Donaldson and Weymark (1986), show implications of **WTR** and **STR** respectively.

Theorem 3.3. *For either definition of the poor, if a focused poverty index satisfies* **CON** *and* **WTR**, *then it satisfies* **STR**.

Theorem 3.4. *For weak definition of the poor, if a focused poverty index fulfils* **CON** *and* **STR**, *then it reduces under a progressive transfer to a poor even if the transfer recipient crosses the poverty line as a consequence of the transfer, assuming that the donor does not become poorer than the recipient after the transfer.*

Since **CON** and **WTR** have been justified to be quite reasonable, Theorem 3.3 shows that **STR** can also be justified. Therefore, we can use **STR** as a basic property for poverty indices. Theorem 3.4 shows that for weak definition of the poor, **STR** along with **CON** implies a stronger condition, which Donaldson and Weymark (1986) called **Strong Downward Transfer Axiom (SDT)**. But if we use the strong definition, no focused poverty index can meet **SDT**.

Kundu and Smith(1980) showed that even if the weak definition of the poor is adopted, there is no poverty index that fulfils **STR**,**NPG** and **PGR**.

The intuition behind the conflict between **NPG**, **PGR** and **STR** is that they involve poverty changes in opposite directions as a result of line crossing. For weak definition of the poor, **NPG** drops out as an implication of **POP**, **SMN**(or **WMN** and **CON**) and **FOC**, while **PGR** will hold if the entrant has income not more than that of the poorest poor (Zheng (1997)). Hence it is not necessary to consider **NPG** as an axiom separately. It is difficult to justify why addition of a poor person with income sufficiently close to the poverty line to a society, where all poor persons have almost zero income will increase poverty. Therefore, there is no strong reason to maintain **PGR** as a poverty axiom.

In view of the above discussion we can say that the main axioms for a poverty index are: **FOC**, **WMN**, **STR**, **SYM**, **IPL**, **POP**, **CON**, **DTR** (or its positional version **PTS**) and **SUC**.

Choice of **DTR** or **PTS** will depend on how we view deprivation. While in **DTR**, deprivation depends on the income difference between the recipient and the donor, in **PTS** it is viewed in terms of proportion of persons in between them. These axioms are core axioms in that they are independent and they imply several others.

3.2. Poverty indices

Perhaps the most widely used index of poverty is the head-count ratio, the proportion of poor persons in the population. Let q be the number of poor in $X \in R_+^n$, that is, the cardinality of the set $Q(X)$. Then the head - count ratio is defined as

$$P_1^{n,1}(X, z) = \frac{q}{n} \tag{19}$$

P_1 satisfies a joint invariance property, it is an absolute index as well as a relative index. In fact, Foster and Shorrocks (1991) showed that the only subgroup consistent population replication invariant poverty indices that are both relative and absolute and that satisfy continuity in individual incomes (restricted continuity) are continuous, increasing transformations of P_1. Zheng (1994) strengthened this result by showing that the only focused, restricted continuous poverty indices that are both relative and absolute are related to P_1. It may be noted that P_1 is a violator of all monotonicity and transfer axioms. Therefore, there is no distribution sensitive poverty index that can be both relative and absolute. Zheng (1997) further demonstrated that a linear transformation of P_1 is the only subgroup decomposable index that satisfies **PGR** and **NPG**. Because of insensitivity of P_1 to changes and redistribution of income, this makes the population monotonicity axioms further unattractive.

It has also been common to measure poverty using the poverty gap ratio, the average of the relative income shortfall of the poor from the poverty line.

This index is defined as

$$P_2^{n,1}(X,z) = \frac{\sum_{i \in Q(X)} (z - x_i)}{qz}. \tag{20}$$

This is a summary measure of depth of poverty $(z - x_i)$ of different individuals in the society. From policy point of view qzP_2 gives us the total amount of money required to put all the poor persons at the poverty line. Like P_1, P_2 is also a violator of transfer axioms, although it meets **WMN**.

Sen (1976) suggested a more sophisticated index of poverty. He began by assuming that the poverty index P_3 is the weighted average of income shortfalls of the poor:

$$P_3^{n,1}(X,z) = a(X,z) \sum_{i \in Q(X)} (z - x_i)\nu_i(X,z) \tag{21}$$

where $\nu_i(X,z) > 0$ is the weight attached to the income gap $(z - x_i)$ of poor person i and $a(X,z)$ is a normalization coefficient. Assuming that z is a reference point for the poor, $(z - x_i)$ can be regarded as the extent of deprivation felt by poor person i. In order to attach higher weight to higher deprivation, Sen assumed that the weight on poverty gap $(z - x_i)$ of person i is equal to his rank in the income distribution of the poor.

Independently P_1 and P_2 are subject to many shortcomings. Sen argued that in the special case when all the poor persons have the same income, the two might give an adequate picture of poverty. In this case he sets

$$P_3^{n,1}(X,z) = P_1^{n,1}(X,z)P_2^{n,1}(X,z),$$

the income gap ratio. These specifications yield the Sen index:

$$P_3^{n,1}(X,z) = \frac{\sum_{i=1}^{q} (z - \hat{x}_i)(q + 1 - i)}{(q+1)nz}. \tag{22}$$

Assuming that $\mu(X^p) > 0$, for a large q, P_3 can be written as

$$P_3^{n,1}(X,z) = P_1^{n,1}[P_2^{n,1} + (1 - P_2^{n,1})G_p^q] \tag{23}$$

where G_p^q is the Gini index of the income distribution of the poor (7). Equation (23) explicitly shows that by including G_p^q as a component, Sen's poverty index incorporates distributional sensitivity.

(7) Pattanaik and Sengupta (1995) characterized P_3 using a continuity condition along with variants of Sen's normalization and ordinal weight axioms.

Blackorby and Donaldson (1980b) noted that we can rewrite P_3 as

$$P_3^{n,1}(X,z) = P_1^{n,1}\left[1 - \frac{E_G^q(X^p)}{z}\right] \tag{24}$$

where $E_G^q(X^p)$ is the EDE income of the poor evaluated according to the Gini welfare function.

This motivated them to suggest the following generalization of P_3:

$$P_4^{n,1}(X,z) = P_1^{n,1}\left[1 - \frac{E^q(X^p))}{z}\right] \tag{25}$$

where $E^q(X^p)$ is the EDE income of the poor evaluated according to a regular, homothetic social welfare function.

Thus, P_4 is the product of the population in poverty P_1 and the relative gap between the poverty line z and the EDE income of the poor $E^q(X^p)$. This general index (hence the Sen index) possesses the following nice properties:

(i) it is sensitive to the head-count ratio,
(ii) it is sensitive to how poor the poor are (because of its dependence on the relative gap) and
(iii) it is sensitive to the amount of inequality among the poor (since under the assumption that $\mu(X^p) > 0$, we can rewrite (25) in terms of the AKS inequality index of the poor using their EDE income).

The Sen-Blackorby-Donaldson indices satisfy **FOC**, **WMN**, **SYM**, **IPL** and **WTR**. However, they violate **CON**, **STR**, **POP** and **SUC** (Chakravarty (1983a, 1990, 1997), Foster and Shorrocks (1991) and Zheng (1997)).

Given a welfare function of the poor, we have a corresponding poverty index of the type P_4. For instance, if we assume that the EDE income is of the form $\frac{\sum_{i=1}^{q} \hat{x}_i (q+1-i)^r}{i^r}$, where $r > 0$, the resulting index becomes the one suggested by Kakwani (1980b),which is given by

$$P_K^{n,1}(X,z) = \frac{q}{nz\sum_{i=1}^{q} i^r}\sum_{i=1}^{q}(z-\hat{x}_i)(q+1-i)^r\,. \tag{26}$$

The Sen index corresponds to the case $r = 1$. When $r = 0$, the index is simply $P_1 P_2$. The Kakwani index was introduced with the objective that it meets **DTR**. For a given distribution a positive r exists for which this objective is fulfilled. However, it meets **PTS** for all $r > 1$. But for any given r there is a population size for which P_K does not satisfy **DTR**. Other major limitations of P_K are its violation of **CON**, **STR**, **POP** and **SUC** (Chakravarty (1990) and Foster and Shorrocks (1991)).

Giorgi and Crescenzi (2001c) replaced the Gini index in (23) by the Bonferroni inequality index and derived a variant of P_3. We can rewrite it in terms of the EDE income using the Bonferroni welfare function of the poor as

$$P_{GC}^{n,1}(X,z) = P_1^{n,1}\left(1 - \frac{1}{qz}\sum_{i=1}^{q}\sum_{j=1}^{i}\hat{x}_j\right). \tag{27}$$

This index retains all the three nice properties of P_4 (hence P_3). It has the additional advantage of satisfying **PTS**. However, the Sen index is a violator of this property because it involves the Gini index as the indicator of inequality among the poor. Since the Bonferroni index is not additively decomposable, P_{GC} does not fulfil **SUC**. Since P_{GC} is a particular case of P_4, it will behave in the same way as P_4 with respect to **STR**, **CON** and **POP**. Given that the use of the Bonferroni index in measuring poverty index is rather new, a worthwhile exercise will be to characterize P_{GC} using **SYM**, **WMN**, **WTR** and restricted continuity since it satisfies them.

Blackorby and Donaldson (1980b) also suggested an absolute poverty index, which is defined by

$$P_5^{n,1}(X,z) = q(z - E^q(X^p)) \tag{28}$$

where E^q is evaluated according to a regular, translatable social welfare function of the poor. If each poor in the society were given $(z - E^q(X^p))$ amount of money then P_5 will be zero (since E^q is unit translatable) at an aggregate cost of $q(z - E^q(X^p))$. Therefore, this index determines the monetary cost poverty. However, like its relative counterpart the index does not fulfil **CON**, **SUT**, **SUC** and obviously **POP** (see also Bossert (1990)).

An alternative way of obtaining poverty indices from inequality indices was proposed by Hamada and Takayama (1977) and Takayama (1979) using censored income distributions. In a censored income distribution each non-poor income is replaced by the poverty line z. Formally, Takayama (1979) defined the censored income corresponding to the income level x_i by

$$x_i^* = \min\{x_i, z\}. \tag{29}$$

The censored income distribution associated with X is denoted by X^*. Takayama then defined the Gini index of X^* as a poverty index. More precisely, the Takayama index is given by

$$P_6^{n,1}(X,z) = 1 - \frac{1}{n^2\mu(X^*)}\sum_{i=1}^{n}(2(n-i)+1)\hat{x}_i^*, \tag{30}$$

where $\mu(X^*) > 0$. Hamada and Takayama (1977) also suggested various censored income distribution based inequality indices as poverty indices. But all such indices violate **WMN** (Chakravarty (1983a, 1990)).

Chakravarty (1983a) made an application of the censored income distribution. He suggested the use of the proportionate gap between the poverty line z and the EDE income $E(X^*)$ based on the censored income distribution X^* as an index of poverty, where E is calculated using a regular, homothetic social welfare function. Formally, this index is given by

$$P_7^{n,1}(X,z) = 1 - \frac{E^n(X^*)}{z} \tag{31}$$

P_7 meets **FOC**, **WMN**, **STR**, **SYM**, **IPL**, **CON**. Depending on the form of the welfare function, it may or may not verify **POP**, **SUC** and **DTR/PTS**. It is bounded between zero and one, where the lower bound is achieved when there is no poor person in the society. On the other hand, the index attains its upper bound in the extreme case when everybody possesses zero income.

Assuming that $\mu(X^*) > 0$, we can rewrite P_7 as

$$P_7^{n,1}(X,z) = 1 - \frac{\mu(X^*)(1 - I^n_{\mathrm{AKS}}(X^*))}{z}. \tag{32}$$

Thus, P_7 is a fairly natural translation of the AKS relative inequality index of a censored income distribution into a poverty index. Given a poverty line, for two censored income distributions X^* and Y^* with the same mean, we have

$$I^n_{\mathrm{AKS}}(X^*) \geq I^n_{\mathrm{AKS}}(Y^*) \Leftrightarrow P_7^{n,1}(X,z) \geq P_7^{n,1}(Y,z). \tag{33}$$

That is, for a given poverty line and under equality of means of censored income distributions, ranking of the distributions by the AKS inequality index is same as that generated by P_7. Pyatt (1987) investigated the class P_7 using affluence and basic income, and examined the implications when the society equivalent income is the sum of equivalent basic income and equivalent income of affluence. It is clear that to every homothetic social welfare function we have a corresponding poverty index in (31). In order to illustrate this, suppose that social evaluation is done with respect to the Gini welfare function. Then P_7 becomes

$$P_S^{n,1}(X,z) = \frac{1}{n^2 z} \sum_{i=1}^{q} (z - \hat{x}_i)(2n + 1 - 2i) \tag{34}$$

the continuous extension of the Sen index suggested by Shorrocks (1995). In addition to **POP**, P_S fulfils all the postulates that are fulfilled by P_7 in its general form. But it does not meet **SUC** and **DTR/PTS**. Earlier, Thon (1979) suggested an index which has similar properties as P_S. In fact, the failure of P_3 to satisfy **SUT** motivated Thon to propose his index. The Thon index, denoted by P_T, is obtained by using the EDE income

$$\sum_{i=1}^{n} x_i^* \frac{n+1-i}{n(n+1)}$$

in (31). The resulting formula becomes

$$P_T^{n,1}(X,z) = \frac{2}{(n+1)nz} \sum_{i=1}^{q} (z - \hat{x}_i)(n+1-i). \tag{35}$$

The difference between P_S and P_T is the weighting function on the income gap of a poor. While Thon used the rank of the poor person in the total population, Shorrocks used Gini type weight. Note that the simple change in weighting function in P_3 makes P_S and P_T satisfy several axioms which P_3 violates.

If we employ the symmetric mean of order $k(< 1)$ welfare function in (31), we get the second Clark, Hemming and Ulph (1981) index, which is defined by:

for $k < 1$, $k \neq 0$

$$P_k^{n,1}(X,z) = 1 - \frac{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i^*)^k\right]^{\frac{1}{k}}}{z} \tag{36}$$

and for $k = 0$

$$P_k^{n,1}(X,z) = 1 - \frac{\prod_{i=1}^{n}(x_i^*)^{\frac{1}{n}}}{z} \tag{37}$$

where $X \in D_+^n$ and $n \in N$ are arbitrary. This is one of the most satisfactory indices of poverty. It satisfies **DTR** for all $k < 1$ and also all the remaining core axioms. However, it is not subgroup decomposable. The parameter k determines the curvature of the social indifference surfaces. For any finite value of $k < 1$, the welfare contour becomes strictly convex to the origin and the degree of convexity increases as k decreases. For a given X, $P_k^{n,1}$ is decreasing in k. As the value of k decreases greater weight is attached to transfers at the lower end of the profile. As $k \to -\infty$, the EDE income approaches $\min_i\{x_i^*\}$, the Rawlsian maximin social welfare function (Rawls (1971)) and the corresponding poverty index becomes,

$$1 - \min_i \left\{\frac{x_i^*}{z}\right\}$$

the relative maximin index. On the other hand, as $k \to 1$, P_k becomes simply $P_1 P_2$, which ignores distributional consideration.

Chakravarty (1983a) also suggested an absolute poverty index, which is defined by

$$P_8^{n,1}(X, z) = (z - E^n(X^*)) \tag{38}$$

where E^n is evaluated according to a regular, translatable social welfare function. If in the censored income distribution X^* each person were given $(z - E^n(X^*))$ amount of money, then P_8 will be zero (since E^n is unit translatable) at an aggregate cost of $n(z - E^n(X^*))$. Hence nP_8 determines the total monetary cost of poverty. Since except the invariance property the behavior of P_8 is similar to that of P_7, we are not going for further discussion on P_8.

Some authors advocated the use of deprivations $(z - x_i)$ of the poor directly in constructing the poverty index. Chakravarty (1983b) considered the illfare function of the poor F^q using individual deprivations as the arguments. F^q is assumed to be decreasing, strictly S-convex and homothetic in the income gaps $(z - x_i)$ of the poor. The representative deprivation g_e is defined as that level of deprivation which if suffered by each poor will make the existing deprivation distribution socially indifferent. As a general relative index of poverty he suggested the use

$$P_9^{n,1}(X, z) = P_1^{n,1} \frac{g_e}{z}. \tag{39}$$

This focused index satisfies **WMN**, **WTR** and **SYM**, but not **STR**, **SUN** and **CON**.

For $F = \sum_{\delta \in Q(X)} g_i^{\delta}$, where $\delta \geq 1$, P_9 becomes the first Clark-Hemming-Ulph index defined by

$$P_{\delta}^{n,1}(X, z) = \frac{q}{nz} \left[\frac{1}{q} \sum_{i=1}^{q} (z - \hat{x}_i)^{\delta} \right]^{\frac{1}{\delta}}. \tag{40}$$

If $\delta > 1$, then the index P_{δ} exhibits transfer sensitivity. As $\delta \to \infty$, P_{δ} approaches the product of the head-count ratio and the relative maximin index. As we will see an attractive feature of indices of the type P_9 is that they can be used for poverty ordering of distributions via the absolute rotated Lorenz curve ([8]).

Of particular interest are the subgroup consistent and subgroup decomposable indices. Foster and Shorrocks (1991) showed that a relative poverty index satisfying **FOC** ,**WMN**, **STR**, **SYM**, **POP**, **CON**, **SUC** and a normalization

([8]) Chakravarty (1983b) also suggested an absolute index defined by $\frac{q g_e}{n}$. Note that since the illfare function F is defined directly on income deprivations, for this to be an absolute index we do not need translatability of F.

which demands that the level of poverty is zero if there is no poor person in the society must be of the form:

$$P_{10}^{n,1}(X, z) = \phi \left[\frac{1}{n} \sum_{i \in Q(X)} f\left(\frac{x_i}{z}\right) \right] \tag{41}$$

where ϕ is continuous and increasing, $f : R^1_+ \to R^1$ is continuous, decreasing, strictly convex and $f(t) = 0$ for all $t \geq 1$. If ϕ is the identity mapping, then P_{10} becomes subgroup decomposable. More precisely, the entire family of subgroup decomposable relative poverty indices is given by

$$P_{11}^{n,1}(X, z) = \frac{1}{n} \sum_{i \in Q(X)} f\left(\frac{x_i}{z}\right). \tag{42}$$

If we choose the functional form $f(t) = 1 - t^e$, $0 < e < 1$ then P_{11} coincides with the additively decomposable index characterized by Chakravarty (1983c):

$$P_e^{n,1}(X, z) = \frac{1}{n} \sum_{i \in Q(X)} \left[1 - \left(\frac{x_i}{z}\right)^e \right] \tag{43}$$

P_e satisfies **DTR** and all higher order sensitivity axioms. It has a clear link with the normalized Theil (1967) entropy index when one replaces z by the mean income, normalizes the index and sums over uncensored income distributions (Chakravarty (1990) and Zheng (1997)). It may be noted that for $0 < k < 1$, P_k can be expressed as an increasing transformation of P_e. Thus, they give rise to the same poverty ranking of income distributions.

Next, for the specification $f(t) = -\log t$, $t > 0$, P_{11} becomes the Watts (1968) index:

$$P_W^{n,1}(X, z) = \frac{1}{n} \sum_{i \in Q(X)} \log\left(\frac{z}{x_i}\right) = P_1^{n,1}[I_T^q(X^p) - \log(1 - P_2^{n,1})], \tag{44}$$

where

$$I_T^q(X^p) = \frac{1}{q} \sum_{i \in Q(X)} \log\left(\frac{\mu(X^p)}{x_i}\right) \tag{45}$$

is the second Theil inequality index of the income distribution of the poor. Thus, for a given P_1 and P_2, an increase in the inequality index in (45) is equivalent to an increase in the Watts index and vice versa. Zheng (1993) interpreted this index as the size of absolute welfare loss due to poverty and characterized it as the only such index under a set of axioms. Tsui (1996) noted that the change in this index can be neatly decomposed into growth and redistributive components. Another interesting observation about this index is that for a given poverty line it is ordinally equivalent to P_k when $k = 0$. Therefore, the two indices generate the same poverty ranking of income distributions.

Finally, if we assume that $f(t) = (1-t)^{\alpha}$, where $\alpha > 1$, then P_{11} becomes the Foster-Greer-Thorbecke (1984) index:

$$P_{\alpha}^{n,1}(X,z) = \frac{1}{n}\sum_{i\in Q(X)}\left(\frac{(z-x_i)}{z}\right)^{\alpha}. \tag{46}$$

With the exception of P_e all poverty indices proposed after Sen (1976) and prior to P_{α} are not subgroup decomposable. The difference between P_3 and P_{α} is that the latter uses the $(\alpha - 1)$th power of the income gap ratio $\frac{(z-x_i)}{z}$ of a poor as the weight on this ratio itself instead of his relative rank, as used in the former. For $\alpha > 2$, P_{α} satisfies all the axioms satisfied by P_e. (See Ebert and Moyes (2002), for a characterization of P_{α} using deprivations of the poor.) As $\alpha \to 0, 1$; P_{α} tends respectively to P_1 and P_1P_2. For $\alpha = 2$, P_{α} can be written as

$$P_{\alpha}^{n,1}(X,z) = P_1^{n,1}[(P_2^{n,1})^2 + (1-P_2^{n,1})^2(C_p^q)^2], \tag{47}$$

where C_p^q is the coefficient of variation of poor incomes. This explicitly shows that for $\alpha = 2$, P_{α} fails to exhibit transfer sensitivity. As $\alpha \to \infty$, P_{α} tends to $\frac{q_0}{n}$ with q_0 being the number of persons with zero income, while the transformed index $(P_{\alpha})^{\frac{1}{\alpha}}$ approaches the relative maximin index of poverty.

Vaughan (1987) derived poverty indices which can be viewed as the loss of welfare that results from the existence of poverty. He incorporated social welfare function directly into the construction of poverty indices and suggested the following relative and absolute welfare poverty indices:

$$P_{VA}^{n,1}(X,z) = 1 - \frac{W^n(X)}{W^n(\tilde{X})}, \tag{48}$$

and

$$P_{VA}^{n,1}(X,z) = W^n(X) - W^n(\tilde{X}), \tag{49}$$

where $\tilde{X}$ is derived from X by setting all poor incomes at the poverty line. These two indices are quite general and, as Vaughan pointed out, many indices may be embedded into them. But fulfillment of many axioms will require additional restrictions on W.

Hagenaars (1987) proposed a Dalton - type poverty index using poverty threshold concept. When in (48) we replace X by X^* and assume that the social welfare function is the sum of identical individual utility functions, the

Vaughan relative index becomes the Hagenaars index. More precisely, the Hagenaars index is defined by

$$P_H^{n,1}(X,z) = 1 - \frac{1}{n} \sum_{i \in Q(X)} \frac{U(x_i)}{U(z)} \tag{50}$$

where U is the identical individual utility function. Since U is cardinal, P_H should remain invariant under affine transformations of U. Clearly, P_H is a violator of this property. If U is increasing, strictly concave and has a strictly convex marginal, then P_H fulfils **IPL**, **WMN**, **STR** and **WDT**. If we impose scale invariance condition in (50), P_H coincides with P_e. The specific poverty index Hagenaars gave is obtained by setting $U(x_i) = \log x_i$ in (50), which is neither relative nor absolute.

The recent emphasis on basic needs and human development has put into focus the inadequacy of income as the sole attribute of well-being and argued that other attributes of welfare such as health, housing, environment, public goods, and literacy should be considered along with income. Composite indices of well-being have been developed for the purpose of interpersonal and international comparisons ([9]). Therefore, some authors argued about the inappropriateness of using only income for measuring poverty and insisted on looking at the problem from a multidimensional perspective. In this approach a threshold is specified for each quality of life attribute and shortfalls of different attributes from corresponding thresholds for different individuals are aggregated to arrive at a multidimensional poverty index (see Chakravarty, Mukherjee and Ranade (1998), Bourguignon and Chakravarty (1999, 2003), Tsui (2002) and Bibi (2003)).

Suppose that the well-being of a person depends on m attributes. Let x_{ij} be the quantity of attribute j possessed by person i and $\underline{z} \in \underline{Z}$ be a vector of thresholds or minimally acceptable levels for different attributes, where $\underline{Z}$ is a subset of R_+^n (see Sen (1992)). Let B denote the $n \times m$ matrix of different attribute quantities. Person i is said to be poor with respect to attribute j if $x_{ij} \leq z_j$, where z_j is the threshold level of attribute j. In such a case we also say that attribute j is meagre for person i. Otherwise, the attribute is said to be non-meagre for him. In order to identify the poor persons in such a framework, following Bourguignon and Chakravarty (2003), we define a poverty indicator variable as follows:

$$\rho(x_{i.}, \underline{z}) = 1 \tag{51}$$

([9]) See, for example, Kolm (1977), Atkinson and Bourguignon (1982), Sen (1987), UNDP (1990-2003), Slottje (1991), Mosler (1994), Dardanoni (1995), Koshevoy (1995, 1998), Tsui (1995), Koshevoy and Mosler (1996, 1997), Maasoumi (1999), Atkinson (2003), Chakravarty (2003) and Weymark (2003)).

if $\exists j \in \{1, 2, \ldots, m\}$: $x_{ij} \leq z_j$ and

$$\rho(x_{i.}, \underline{z}) = 0$$

otherwise, where $x_{i.} = (x_{i1}, x_{i2} \ldots x_{im})$. Then the number of poor is simply given by

$$G^{n\times m,m}(B, \underline{z}) = \sum_i \rho(x_{i.}, \underline{z}) . \tag{52}$$

Postulates for a multidimensional poverty index are straight generalizations of the corresponding postulates for a single dimensional index. It may be noted that the **Focus Axiom**, which requires the poverty index to be independent of non-meagre attribute quantities, rules out any trade off between meagre and non-meagre attribute quantities of a person. This of course does not exclude the possibility of a trade off when both attributes are meagre. We now consider an issue which is of very much practical importance and which takes care of essence of multidimensional measurement. It is that of correlation between attributes and the way it affects poverty. By taking into account the association between attributes, as captured by the degree of correlation between them, this property also underlies the difference between single and multidimensional poverty measurement. Redistributing the two attributes so as to keep the marginal distributions constant and increase the correlation between them should increase or decrease poverty according as the attributes are substitutes or complements. In order to understand this, let us consider the two-attribute, two-person case, where $x_{ij} \leq z_j$ for $i, j = 1, 2$. Suppose that initially $x_{11} > x_{12}$ but $x_{21} < x_{22}$. Now, consider a switch of attribute 1 between the two persons. Thus, person 2 who had more of attribute 2 has more of attribute 1 also after the switch and as a result correlation between the two attributes has gone up. Evidently, poverty should increase or decrease under a correlation increasing switch according as the two attributes display the similar or different aspects of poverty, that is, whether they are substitutes or complements (Atkinson and Bourguignon (1982) and Bourguignon and Chakravarty (2003)).

As an illustrative example, Bourguignon and Chakravarty (1999) proposed the following extension of the P_α index which, in addition to respecting all postulates of a multidimensional poverty index, allows for substitutability and complementarity among attributes:

$$P^{n\times 2,2}_{\alpha,\beta}(B, \underline{z}) = \frac{1}{n} \sum_i \left[\left(1 - \frac{x_{i1}}{z_1}\right)^\beta + b^{\frac{\beta}{\alpha}} \left(1 - \frac{x_{12}}{z_2}\right)^\beta \right]^{\frac{\alpha}{\beta}} \tag{53}$$

where it is assumed that the number of attributes $m = 2$, and $\alpha \geq 1$, $\beta \geq 1$, $b > 0$ are parameters. The condition $b > 0$ shows the importance attached to poverty associated with attribute 2 relative to that attached to attribute 1. The restrictions $\alpha \geq 1$, $\beta \geq 1$ ensure that the index obeys multidimensional transfers principle, that is, poverty decreases under an equalizing operation among meagre attributes. For $1 \leq \beta \leq \alpha (\beta > \alpha)$ the two attributes are substitutes (complements) and the index increases (decreases) under a correlation increasing switch. An increase in the value of β makes the individual poverty function more convex to the origin. The elasticity of substitution between two relative shortfalls $\frac{(1-x_{i1})}{z_1}$ and $\frac{(1-x_{i2})}{z_2}$ is given by $\frac{1}{(\beta-1)}$. Thus, β represents substitutability between the two attributes. For $\beta = 1$, there is perfectly elastic trade off between the attributes and as $\beta \to \infty$, the resulting index becomes

$$P_{\alpha,\infty}^{n\times 2,2}(B, \underset{\approx}{z}) = \frac{1}{n}\sum_i \left(1 - \min\left(1, \frac{x_{i1}}{z_1}, \frac{x_{i2}}{z_2}\right)\right)^{\alpha}. \tag{54}$$

In this case the isopoverty contours are of rectangular shape-the two attributes are perfect complements.

3.3. Poverty orderings

The preceding discussion shows that there is a large number of poverty indices satisfying different axioms. Evaluations of two distributions can certainly be conflicting by different indices. A major source of disagreement in evaluations is variation in poverty line (since the choice of the poverty line is subjective). The determination of an appropriate poverty line has been an issue of debate for a long time. Quite often significant degree of arbitrariness is involved in the construction of the poverty line. A poverty index may rank two income distributions differently for two distinct poverty lines. Therefore, it becomes useful to see if two income distributions can be ranked unanimously by a given poverty index by all poverty lines in some reasonable interval. This notion of ordering of distributions by a given index for a range of poverty lines is called poverty- line ordering (Zheng 1999)).

Foster and Shorrocks (1988a, 1988b) derived conditions under which unambiguous poverty comparisons can be made by members of P_α when poverty lines vary. Formally,

Theorem 3.5. *For two income distributions X and Y with distribution functions F_X and F_Y respectively and a non-negative integer α, the following statements are equivalent*:

(a) $P_\alpha(X.z) \leq P_\alpha(Y, z)$ *for all* $z \in (0, \infty)$ *with* $<$ *for some* z.
(b) $X \geq_{\alpha+1} Y$.

That is, for any α, ordering of two distributions by P_α for all poverty lines is same as $(\alpha + 1)$th degree stochastic dominance. Thus, the ordering condition for the head-count ratio is the first order stochastic dominance, which is equivalent to dominance by all symmetric utilitarian social welfare functions, where the identical individual utility function is increasing. This anonymous Pareto dominance is also same as rank dominance (Saposnik (1981, 1983)). Likewise, second order stochastic dominance is necessary and sufficient for dominance with respect to the income-gap ratio. Each of these statements also corresponds to the generalized Lorenz domination, which is equivalent to symmetric utilitarian welfare ordering that shows unambiguous preference for efficiency and equity. Next, note that when $\alpha = 2$, the induced ordering is identical to third order stochastic dominance, which implies and is implied by a symmetric utilitarian welfare dominance that exhibit preference for efficiency, equality and also greater emphasis for transfers at lower incomes. An implication of the P_α ordering is that if two distributions can be ranked by P_α with $\alpha = \alpha_1$, then they can also be ranked by P_α for $\alpha = \alpha_2$ in the same direction if $\alpha_1 \leq \alpha_2$. Foster and Shorrocks (1988b) also derived analogous result when we set an upper bound on the poverty line.

In order to present the next result, we consider the following definition, which is taken from Chakravarty (1983c) and Hagenaars (1986,1987).

Definition 3.3. For any income distribution $X \in D^n$ and poverty line z, the utility gap poverty index is defined as

$$P^{n,1}(X, z, U) = \frac{a(z)}{n} \sum_{i=1}^{n} (U(z) - U(x_i^*)) \tag{55}$$

where U is the individual utility function and $a(z) > 0$ is a normalization factor.

$P(X, z, U)$ contains the Watts index P_W $(U(x) = \log x, a(z) = 1)$, the Hagenaars index P_H $(U(x) = \log x, a(z) = \frac{1}{\log z})$, the Chakravarty index $(U(x) = x^e, a(z) = z^e)$ (hence monotone transformation of the second Clark-Hemming-Ulph index), as special cases. For any $X \in D^n$, let $U^X = (U(x_1), U(x_2), \ldots$ $\ldots, U(x_n))$ be the utility distribution of X.

The following theorem of Foster and Jin (1998) characterizes the poverty line ordering based on the utility gap indices.

Theorem 3.6. *Let U be continuous and increasing, and $X, Y \in D^n$ be arbitrary. Then the following conditions are equivalent*:

(a) $P^{n,1}(X, z, U) \leq P^{n,1}(Y, z, U)$ *for all* $z \in (0, \infty)$ *with* $<$ *for some* z.
(b) $U^X \geq_{GL} U^Y$, *that is,* U^X *generalized Lorenz dominates* U^Y.

Since the utility gap index and the generalized Lorenz curves are population replication invariant, we can extend Theorem 3.6 to the variable population case.

The second goal of research on poverty ordering, which is known as poverty-measure ordering, refers to the ordering for a class of poverty indices with a fixed poverty line (Zheng (2000a)). As we have seen, the axiomatic framework of poverty-index construction that Sen (1976) pioneered and adopted by the following researchers does not yield a unique poverty index. For a set of axioms there exists several satisfactory poverty indices. Since the choice of a particular index is made arbitrarily, the conclusions based on it will be arbitrary as well. We can reduce the degree of arbitrariness by choosing all poverty indices that fulfil a set of reasonable axioms. That is, instead of choosing individual poverty indices we are choosing a set of criteria which in turn determines a class of indices. One can then check whether it is possible to rank two distributions unambiguously by members of this class for a given poverty line. In a sense this kind of research has grown out of presence of too many poverty indices.

Atkinson (1987) derived conditions on *poverty-measure ordering* for subgroup decomposable poverty indices with a common poverty line. He specified a range $[z_{\min}, z_{\max}]$ with $Z_{\min} > 0$ and $Z_{\max} < \infty$ being the minimum and maximum poverty lines and let the poverty line arbitrarily vary within this range. Instead of focussing on a single poverty index, he considered a given class of poverty indices. In the next two theorems we summarize the poverty ordering conditions developed by Atkinson (1987). The presentation is based on Zheng (1999).

Theorem 3.7. *Suppose that the poverty index P satisfies* **SUD**. *Let X and Y be two income distributions with distribution functions F_X and F_Y respectively. Then the following conditions are equivalent*:

(a) $P(X, z) < P(Y, z)$ *for all P that satisfy* **WMN** *and* **CON**, *and for all poverty lines* $z \in [z_{\min}, z_{\max}]$.
(b) *X weakly first order stochastic dominates Y over* $[z_{\min}, z_{\max}]$ *and* $X \geq_1 Y$ *over* $[0, z_{\min}]$.

Theorem 3.8. *Suppose that the poverty index P satisfies* **SUD**. *Let X and Y be two income distributions with distribution functions F_X and F_Y respectively. Then the following conditions are equivalent*:

(a) $P(X, z) < P(Y, z)$ *for all P that satisfy* **WMN**, **WTR** *and* **CON**, *and for all poverty lines* $z \in [z_{\min}, z_{\max}]$.
(b) *X weakly second order stochastic dominates Y over* $[z_{\min}, z_{\max}]$ *and* $X \geq_2 Y$ *over* $[0, z_{\min}]$.

Zheng (1999) showed that if in part (a) of Theorem 3.8 we also include the axiom **DTR**, then the equivalent condition in (b) will be weak third dominance over $[z_{\min}, z_{\max}]$ and third order dominance over $[0, z_{\min}]$. The most significant implication of these theorems is that if a dominance relation holds, then no individual poverty index with appropriate properties needs to be consulted in ranking distributions. The dominance conditions are easy to implement statistically and they have nice welfare interpretations.

Spencer and Fisher (1992) considered the criterion for ranking one distribution as having more hardship than another and established a dominance condition involving the absolute rotated Lorenz curve. Hardship of a person with income x is measured by an increasing, convex transformation of $(z-x^*)$, where x^* is the censored income corresponding to x. The aggregate hardship of income distribution function F is defined as

$$AH(F, z) = \int_0^z h(z - x^*)dF(x)\,, \tag{56}$$

where h is increasing and convex. The absolute rotated Lorenz curve corresponding to F is a plot of the normalized sum of the largest $100t$ per cent of the poverty gaps,

$$\int_0^t (z - H(t))dt$$

against the cumulative population proportion t, where $0 \leq t \leq 1$. The absolute rotated Lorenz curve is concave up to a certain point and then becomes a flat line. The following theorem of Spencer and Fisher (1992) can now be stated.

Theorem 3.9. *Let X and Y be two income distributions with distribution functions F_X and F_Y respectively. Then the following conditions are equivalent*:

(a) $AH(F_X, z) \leq AH(F_Y, z)$ *for each z.*
(b) *The absolute rotated Lorenz curve of X lies nowhere above that of Y.*

Jenkins and Lambert (1993, 1998a, 1998b) and Shorrocks (1995, 1998) analyzed this dominance relation in greater detail and provided some new insights. Jenkins and Lambert showed that many poverty indices can be expressed as hardship functions. They also pointed out that the absolute rotated Lorenz curve depicts three aspects of poverty: the incidence of poverty (the population proportion from which the curve becomes flat is the head-count ratio), the intensity of poverty (the maximum height of the curve represents the gap between the poverty line and the mean of the censored distribution) and inequality among the poor (the curvature of the curve between the origin and the head-count ratio is an indicator of inequality). That is why they renamed the curve as the TIP (three I's of poverty) curve. They also showed that TIP curve dominance is equivalent to censored generalized Lorenz dominance and hence can be used to check poverty ordering for a large class of poverty indices. Atkinson (1992) and Jenkins and Lambert (1993) considered poverty-measure orderings when the poverty line is adjusted for differences in family composition (see also Chambaz and Maurin (1998) and Zoli and Lambert (2003)).

Shorrocks (1998) viewed the absolute rotated Lorenz curve as an application of a more general deprivation ordering and called the corresponding curve 'deprivation profile'. He used it for ranking 'bads' such as wage discrimination and unemployment. Shorrocks (1995) showed that P_S can be interpreted as the area under the deprivation profile.

Recently several authors attempted to derive conditions for poverty ordering in multidimensional context. Bourguignon and Chakravarty (2002) restricted attention to a two dimensional framework and argued how utility should change under a correlation increasing switch of attributes between two individuals. Using this as a postulate and applying the Atkinson-Bourguignon (1982) dominance results to the comparison of two dimensional head-count ratio they showed that if the attributes are substitutes, then the comparison should be made only in the region in which the individuals lack both the attributes, that is, in the intersection of the sets in which quantities of the two attributes remain below the corresponding thresholds. On the other hand, if the attributes are complements the comparison should be in the union of these sets. They also developed a framework for ranking bivariate distributions via poverty gap in a restricted set up. Duclos et al. (2002) derived bivariate poverty orderings under a different set of assumptions. They regarded the attributes only as substitutes and allowed dependence of poverty line of one attribute on that of the other and vice-versa.

4. Concluding remarks

This essay is a survey of the literature on the design of ethical indices for measuring poverty. Ethical indices, which are based on notions of social welfare making the underlying concepts of value judgements quite explicit, contrast with descriptive indices that are derived without any concept of welfare. Needless to say, ethical indices are not meant to supplant descriptive indices, rather they are constructed with different aims. For the sake of completeness, the survey also presents a discussion on the descriptive indices suggested in the relevant areas. A discussion on dominance results which say how one distribution can be preferred to another on welfare ground has been made as well.

Acknowledgments

The research of Muliere was partially supported by CNR. This paper was started when Chakravarty was a Visiting Professor at the Bocconi University. He expresses his sincere gratitude to the Bocconi University for support and hospitality.

REFERENCES

ANAND, S. (1977) *Inequality and Poverty in Malaysia*, Oxford University Press, Oxford.

ATKINSON, A.B. (1970) On the measurement of inequality, *Journal of Economic Theory*, 2, 244-263.

ATKINSON, A.B. (1987) On the measurement of poverty, *Econometrica*, 55, 749-764.

ATKINSON, A.B. (1992) Measuring poverty and differences in family composition, *Economica*, 59, 1-16.

ATKINSON, A.B. (2003) Multidimensional deprivation: contrasting social welfare and counting approaches, *Journal of Economic Inequality*, 1, 51-65.

ATKINSON, A.B. and BOURGUIGNON, F. (1982) The comparison of multi-dimensioned distributions of economic status, *Review of Economic Studies*, 49, 183-201.

BIBI, S. (2003) *Measuring poverty in a multidimensional perspective, a review of literature*, University of Laval.

BLACKORBY, C. and DONALDSON, D. (1980a) A theoretical treatment of indices of absolute inequality, *International Economic Review*, 21, 107-136.

BLACKORBY, C. and DONALDSON, D. (1980b) Ethical indices for the measurement of poverty, *Econometrica*, 48, 1053-1060.

BONFERRONI, C. (1930) *Elementi di statistica generale*, Libreria Seber, Firenze.

BOSSERT, W. (1990) Population replications and ethical poverty measurement, *Mathematical Social Sciences*, 20, 227-238.

BOURGUIGNON, F. and CHAKRAVARTY, S.R. (1999) *A family of multidimensional poverty measures*, In: *Advances in Econometrics, Income Distribution and Scientific Methodology, Essays in Honor C.Dagum*, D. J. Slottje (eds.), Physica -Verlag, Heidelberg.

BOURGUIGNON, F. and CHAKRAVARTY, S.R. (2002) *Multidimensional poverty orderings*, DELTA, Paris.

BOURGUIGNON, F. and CHAKRAVARTY, S.R. (2003) Measurement of multidimensional poverty, *Journal of Economic Inequality*, 1, 25-49.

BOURGUIGNON, F. and FIELDS, G.S. (1997) Discontinuous losses from poverty, generalized measures, and optimal transfers to the poor, *Journal of Public Economics*, 63, 155-175.

CHAKRAVARTY, S.R. (1983a) Ethically flexible measures of poverty, *Canadian Journal of Economics*, 16, 74-85.

CHAKRAVARTY, S.R. (1983b) Measures of poverty based on the representative income gap, *Sankhya B*, 69-74.

CHAKRAVARTY, S.R. (1983c) A new index of poverty, *Mathematical Social Sciences*, 6, 307-313.

CHAKRAVARTY, S.R. (1990) *Ethical Social Index Numbers*, Springer Verlag, New York.

CHAKRAVARTY, S.R. (1997) On Shorrocks' reinvestigation of the Sen poverty index, *Econometrica*, 65, 1241-1242.

CHAKRAVARTY, S.R. (2003) A generalized human development index, *Review of Development Economics*, 7, 99-114.

CHAKRAVARTY, S.R., MUKHERJEE, D., and RANADE, R. (1998) On the family of subgroup and factor decomposable measures of multidimensional poverty, *Research on Economic Inequality*, 8, 175-194.

CHAKRAVARTY, S.R. and MULIERE, P. (2003) Welfare indicators: a review and new perspectives .I. Measurement of inequality, *Metron*, 61, 3, 457-497.

CHAMBAZ, C. and MAURIN, E. (1998) Atkinson and Bourguignon's dominance criteria: extended and applied to measurement of poverty in France, *Review of Income and Wealth*, 44, 497-513.

CLARK, S., HEMMING, R., and ULPH, D. (1981) On indices for the measurement of poverty, *Economic Journal*, 91, 515-526.

COWELL, F.A. (1988) *Poverty measures, inequality and decomposability*, In: *Welfare and Efficiency in Public Economics*, D. Boss, M. Rose, C. Seidl (eds.), Springer Verlag, Heidelberg.

DARDANONI, V. (1995) On multidimensional inequality measurement, *Research on Economic Inequality*, 6, 201-207.

DAVIDSON, R. and DUCLOS, R. (2000) Statistical inference for stochastic dominance and for the measurement of poverty and inequality, *Econometrica*, 68, 1435-1465.

DONALDSON, D. and WEYMARK, J. A. (1986) Properties of fixed population poverty indices, *International Economic Review*, 7, 667-688.

DUCLOS, J.-Y., SAHN, D., and YOUNGER, S. D. (2002) *Robust multidimensional poverty comparisons*, University of Laval.

DUTTA, B. (2002) *Inequality, poverty and welfare*, In: *Handbook of Social Choice*, K. J. Arrow, A. K. Sen and K. Suzumura (eds.), Vol. 1, North-Holland, Amsterdam.

EBERT, U. and MOYES, P. (2002) An axiomatic characterization of the Foster, Greer and Thorbecke poverty index, *Journal of Public Economic Theory*, 4, 455-473.

FISHBURN, P.C. (1980) Continua of stochastic dominance relations for unbounded probability distributions, *Journal of Mathematical Economics*, 7, 271-285.

FISHBURN, P.C. and WILLIG, R. D. (1984) Transfer principles in income distribution, *Journal of Public Economics*, 25, 323-328.

FOSTER, J.E. (1984) *On economic poverty: a survey of aggregate measures*, In: *Advances in Econometrics*, R. Basmann and G. Rhodes (eds.), Vol. 3, JAI Press, Connecticut.

FOSTER, J.E., GREER, J., and THORBECKE, E. (1984) A class of decomposable poverty measures, *Econometrica*, 52, 761-766.

FOSTER, J.E. and JIN, Y. (1998) *Poverty orderings for the Dalton utility-gap measures*, In: *The Distribution of Household Welfare and Household Production: International Perspectives*, S. P. Jenkins, A. Kapteyn and B.M.S. van Praag (eds.), Cambridge University Press, Cambridge.

FOSTER, J.E. and SEN, A.K. (1997) *On Economic Inequality, with a substantial annexe after a quarter century by J.E. Foster and A. K. Sen*, Calrendon Press, Oxford.

FOSTER, J.E. and SHORROCKS, A.F. (1988a) Poverty orderings, *Econometrica*, 56, 173-177.

FOSTER, J.E. and SHORROCKS, A.F. (1988b) Poverty orderings and welfare dominance, *Social Choice and Welfare*, 5, 179-198.

FOSTER, J.E. and SHORROCKS, A.F. (1991) Subgroup consistent poverty indices, *Econometrica*, 59, 687-709.

GIORGI, G.M. (1984) A methodological survey of recent studies for the measurement of inequality of economic welfare carried out by some Italian statisticians, *Economic Notes*, 13, 145-157.

GIORGI, G.M. (1998) Concentration index, Bonferroni, *Encyclopedia of Statistical Sciences, Wiley, New York*, 141-146.

GIORGI, G.M. and CRESCENZI, M. (2001a) A look at the Bonferroni inequality measure in a reliability framework, *Statistica*, 41, 571-583.

GIORGI, G.M. and CRESCENZI, M. (2001b) Bayesian estimation of the Bonferroni index from a Pareto- type I population, *Statistical Methods and Applications*, 10, 41-48.

GIORGI, G.M. and CRESCENZI, M. (2001c) A proposal of poverty measures based on the Bonferroni inequality index, *Metron*, 59, 3-15.

GIORGI, G.M. and MONDANI, R. (1994) The exact sampling distribution of the Bonferroni concentration index, *Metron*, 52, 5-41.

Giorgi, G.M. and Mondani, R. (1995) Sampling distribution of the Bonferroni inequality index from exponential population, *Sankhya B*, 57, 10-18.

Haagenars, A.J.M. (1986) *The Perception of Poverty*, North-Holland, Amsterdam.

Haagenars, A.J.M. (1987) A class of poverty measures, *International Economic Review*, 28, 583-607.

Hamada, K. and Takayama, N. (1977) Censored income distribution and measurement of poverty, *Bulletin of the International Statistical Institute*, 47, 617-632.

Hardy, G.H., Littlewood, J.E., and Polya, G. (1934) *Inequalities*, Cambridge University Press, Cambridge.

Jenkins, S.P. and Lambert, P.J. (1993) Three 'I's of poverty curves with an analysis of UK poverty trends, *Oxford Economic Papers*, 49, 317-327.

Jenkins, S.P. and Lambert, P.J. (1998a) Three 'I's of poverty curves and poverty dominance: TIPS for poverty analysis, *Research on Economic Inequality*, 8, 39-56.

Jenkins, S.P. and Lambert, P.J. (1998b) Ranking poverty gap distributions: further TIPs for poverty analysis, *Research on Economic Inequality*, 8, 31-38.

Johnson, D.T. (1996) *Poverty, Inequality and Social Welfare in Australia*, Physica-Verlag, Heidelberg,

Kakwani, N.C. (1980a) *Income Inequality and Poverty: Methods of Estimation and Policy Applications*, Oxford University Press, London.

Kakwani, N.C. (1980b) On a class of poverty measures, *Econometrica*, 48, 437-446.

Kakwani, N.C. (1999) Inequality, welfare and poverty: three interrelated phenomena, In: *Silber* (1999), 599-628.

Kolm, S.C. (1969) *The optimal production of social justice*, In: *Public Economics*, J. Margolis and H. Guitton (eds.), Macmillan, London.

Kolm, S.C. (1976a) Unequal inequalities I, *Journal of Economic Theory*, 12, 416-442.

Kolm, S.C. (1976b) Unequal inequalities II, *Journal of Economic Theory*, 13, 82-111.

Kolm, S.C. (1977) Multidimensional egalitarianism, *Quarterly Journal of Economics*, 91, 1-13.

Koshevoy, G.A. (1995) Multivariate Lorenz majorization, *Social Choice and Welfare*, 12, 93-102.

Koshevoy, G.A. (1998) The Lorenz zonotope and multivariate majorizations, *Social Choice and Welfare*, 15, 1-14.

Koshevoy, G.A. and Mosler, K. (1996) The Lorenz zonoid of a multivariate distribution, *Journal of the American Statistical Association*, 91, 883-892.

Koshevoy, G.A. and Mosler, K. (1997) Multivariate Gini indices, *Journal of Multivariate Analysis*, 60, 252-276.

Kundu, A. and Smith, T.E. (1983) An impossibility theorem on poverty indices, *International Economic Review*, 24, 423-434.

Lambert, P.J. (2001) *The Distribution and Redistribution of Income*, Third edition, Manchester University Press, Manchester.

Maasoumi, E. (1999) *Multidimensional approaches to welfare analysis*, In: *Silber* (1999).

Marshall, A.W. and Olkin, I. (1979) *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York.

Mehran, F. (1976) Linear Measures of income inequality, *Econometrica*, 44, 805-809.

Mosler, K. (1994) *Multidimensional welfarisms*, In: *Models and Measurement of Welfare and Inequality*, W. Eichhorn (eds.), Springer-Verlag, New York.

Mosler, K. and Muliere, P. (1998) Welfare means and equalizing transfers, *Metron*, 11-52.

Moyes, P. (1999) Stochastic dominance and Lorenz curve, In: *Silber* (1999), 199-219.

MULIERE, P. and SCARSINI, M. (1989) A note on stochastic dominance and inequality measures, *Journal of Economic Theory*, 49, 314-323.

NYGARD, F. and SANDSTROM, A. (1981) *Measuring Income Inequality*, Almqvist and Wicksell International, Stockholm.

PATTANAIK, P.K. and SENGUPTA, M. (1995) An alternative axiomatization of Sen's poverty measure, *Review of Income and Wealth*, 41, 73-80.

POLLAK, R. (1971) Additive utility functions and linear Engel curves, *Review of Economic Studies*, 38, 401-414.

PYATT, G. (1976) The interpretation and disaggregation of the Gini coefficients, *Economic Journal*, 86, 243-255.

PYATT, G. (1987) Measuring welfare, poverty and inequality, *Economic Journal*, 97, 459-467.

RAVALLION, M. (1994) *Poverty Comparisons*, Harwood, Chur.

RAWLS, J. (1971) *A Theory of Justice*, Harvard University Press, Cambridge.

SAPOSNIK, R. (1981) Rank dominance in income distribution, *Public Choice*, 36, 147-151.

SAPOSNIK, R. (1983) On evaluating income distributions: rank dominance, the Suppes-Sen grading principle of justice and Pareto optimality, *Public Choice*, 40, 329-336.

SEDIL, C. (1988) *Poverty measurement: a survey*, In: *Welfare and Efficiency in Public Economics*, D. Boss, M. Rose, C . Seidl (eds.), Springer Verlag, Heidelberg.

SEN, A.K. (1973) *On Economic Inequality*, Clarendon Press, Oxford.

SEN, A.K. (1976) Poverty: an ordinal approach to measurement, *Econometrica*, 44, 219-231.

SEN, A.K. (1979) Issues in the measurement of poverty, *Scandinavian Journal of Economics*, 81.

SEN, A.K. (1981) *Poverty and Famines: An Essay on Entitlement and Deprivation*, Oxford University Press, Oxford.

SEN, A.K. (1987) *Standard of Living*, Cambridge University Press, Cambridge.

SEN, A.K. (1992) *Inequality Reexamined*, Harvard University Press, Cambridge, MA.

SHORROCKS, A.F. (1983) Ranking income distributions, *Economica*, 50, 3-17.

SHORROCKS, A.F. (1995) Revisiting the Sen poverty index, *Econometrica*, 63, 1225-1230.

SHORROCKS, A.F. (1998) *Deprivation profiles and deprivation indices*, In: *The Distribution of Household Welfare and Household Production: International Perspectives*, S.P. Jenkins, A. Kapteyn and B.M.S. van Praag (eds.), Cambridge University Press, Cambridge.

SHORROCKS, A.F. and FOSTER. J. E. (1987) Transfer sensitive inequality measures, *Review of Economic Studies*, 54, 485-497.

SILBER, J. (1999) *Handbook of Income inequality Measurement*, Kluwer Academic, Boston.

SLOTTJE, D.J. (1991) Measuring quality of life across countries, *Review of Economics and Statistics*, 73, 684-893.

SPENCER, B.D. and FISHER, S. (1992) On comparing distributions of poverty gap, *Sankhya B*, 54, 114-126.

TAKAYAMA, N. (1979) Poverty ,income inequality and their measures: Professor's Sen's axiomatic approach reconsidered, *Econometrica*, 47, 747-759.

TARSITANO, A. (1990) *The Bonferroni index of income inequality*, In: *Income and Wealth Distribution, Inequality and Poverty*, C. Dagum and M. Zenga (eds.), Springer-Verlag, Heidelberg.

THEIL, H. (1967) *Economics and Information Theory*, North-Holland, Amsterdam.

THON, D. (1979) On measuring poverty, *Review of Income and Wealth*, 25, 429-440.

THON, D. (1983) A poverty measure, *Indian Economic Journal*, 30, 55-70.

Tsui, K.Y. (1995) Multidimensional generalizations of the relative and absolute inequality indices: the Atkinson-Kolm- Sen approach, *Journal of Economic Theory*, 67, 251-265.

Tsui, K.Y. (1996) Growth-equity decomposition of a change in poverty: an axiomatic approach, *Economics Letters*, 50, 417-423.

Tsui, K.Y. (2002) Multidimensional poverty indices, *Social Choice and Welfare*, 19, 69-93.

Undp (1990-2003) *Human Development Report*, Oxford University Press, Oxford.

Vaughan, R. (1987) Welfare approaches to the measurement of poverty, *Economic Journal*, 98, 160-170.

Watts, H.W. (1968) *An economic definition of poverty*, In: *On Understanding Poverty* D.P. Moynihan (eds.), Basic Books, New York.

Weymark, J. A. (2003) *The normative approach to the measurement of multidimensional inequality*, Vanderbilt University.

Zheng, B. (1993) An axiomatic characterization of the Watts poverty measure, *Economics Letters*, 42, 81-86.

Zheng, B. (1994) Can a poverty index be both relative and absolute?, *Econometrica*, 62, 1453-1458.

Zheng, B. (1997) Aggregate poverty measures: a survey, *Journal of Economic Surveys*, 11, 123-162.

Zheng, B. (1999) On the power of poverty orderings, *Social Choice and Welfare*, 16, 349-371.

Zheng, B. (2000a) Poverty orderings: a review, *Journal of Economic Surveys*, 14, 427-466.

Zheng, B. (2000b) Minimum distribution-sensitivity, poverty aversion and poverty orderings, *Journal of Economic Theory*, 95, 116-137.

Zoli, C. (1999) Intersecting generalized Lorenz curves and the Gini index, *Social Choice and Welfare*, 6, 183-196.

Zoli, C. and Lambert, P.J. (2003) *Sequential procedures for poverty gap dominance*, University of Nottingham.

SATYA R. CHAKRAVARTY
Economic Research Unit
Indian Statistical Institute
203 B.T. Road
Kolkata 700108 (India)
satya@isical.ac.in

PIETRO MULIERE
Istituto Metodi Quantitativi
Università L. Bocconi
viale Isonzo, 25
20135 Milano (Italy)
pietro.muliere@uni-bocconi.it