

Technological Asymmetry, Externality, and Merger: The Case of a Three-Firm Industry*

Tarun Kabiraj[†]

Indian Statistical Institute, Calcutta

and

Ching Chyi Lee

The Chinese University of Hong Kong

First Draft November 1999

Revised Draft June 2000

*Correspondence to: Ching Chyi Lee, Department of Decision Sciences and Managerial Economics, The Chinese University of Hong Kong, Shatin, NT, Hong Kong.

E-mail: clee@baf.msmail.cuhk.edu.hk; Fax: (852)2603 6840.

[†]Tarun Kabiraj acknowledges the financial grant received from the Chinese University of Hong Kong.

Abstract

In a set-up of three firms oligopoly with homogeneous goods, we have shown that firms might fail to merge to monopoly even though such a merger maximizes aggregate profits. The role of technological asymmetry of firms and the nature of product market competition are focused in the analysis. In particular, a stable grand coalition can always be formed under price competition, but whether a stable grand merger is possible or not under Cournot competition depends on the extent of asymmetry of the players.

Keywords: Externality, technological asymmetry, bilateral merger, grand coalition, bargaining.

JEL Classification: C71; D43; L13.

1 Introduction

Merger and acquisition are common in business place.¹ If we turn the business pages of the daily newspapers and business magazines, we see that everyday some firms are merging with some other firms, and some firms are negotiating for merger. Still sometimes firms fail to merge. Different characteristics and business strategies may prevent them from merging. Of course, antitrust laws might prohibit horizontal merger. Then if one assumes that antitrust laws are not applicable, at least to some oligopoly industries of our interest, one may then presume that oligopolists, producing homogenous goods, should always merge to avoid competition and prevent dissipation of profits. The nature of product market competition determines the non-cooperative payoffs of the players. Since the monopoly payoff strictly dominates the oligopoly industry payoff, one may tempt to conclude that a grand merger (i.e., a single coalition of all firms) should always take place, because firms can now share a larger profit. This paper examines this hypothesis and proves the invalidity of the statement. Even in a homogenous good industry, firms may not successfully come up with forming a grand coalition. Therefore, the absence of any antitrust rules does not necessarily mean perfect or complete monopolization of the industry. The reason is that firms may fail to agree on a division of pie. In our paper this occurs when firms have asymmetric technologies and merger of a subgroup of firms creates externalities. In such a situation firms might find it even more profitable to stand outside the merger. In particular, externalities under merger occur when the product market is characterized by Cournot type competition. We show that the technological asymmetry and the nature of product market competition may provide a situation where firms cannot agree on a division of profits under grand merger. Hence such a merger will not take place. Since under Bertrand type price competition with homogenous goods, such externalities no longer exist, firms can always sign for a grand merger. The purpose of the paper is to show how the nature of coalition or form of merger and division of pie under possible mergers depend on the asymmetry of technologies and nature of product market competition.

We develop a cooperative model of coalition formation. We assume that firms have complete information about their own and rivals' strategies and payoffs under all possible contingencies. So

¹For instance, see Jacquemin (1990) and Roy (1997).

in the negotiation table players bargain together with perfect communication. When negotiation starts, each one bargains for a larger share, each tries to convince the proposed partners that he can get more by going alone, or he has other better alternatives than to sign the contract on the proposed division. Thus a negotiation quite often takes the form of a sequence of threats and counter threats, or objections and counter objections before they come to settle at a stable outcome (Aumann and Maschler, 1964). If they come to an agreement, it must be essentially a stable outcome in the sense that the constituent members will have no further incentives to deviate unilaterally or as a subgroup. In game theory language, the proposed allocation must be in the ‘core’.² An efficient allocation means that the sum of payoffs under coalition equals the maximum attainable industry payoff. One should also insist on a “fair” division in the sense that equally efficient firms should get equal payoffs, and relatively more efficient or strong firms should get larger payoffs. In the bargaining process, the disagreement payoff of a player is determined by its default payoff that it is expecting to get by going alone. Let us Call this the disagreement payoff or bargaining payoff. Quite naturally, the disagreement payoffs depend on the technological positions of the firms. Hence the technological asymmetry plays a crucial role in our analysis. Then the surplus that comes from coalition, is assumed to be divided by Nash bargaining, that is, the surplus will be divided equally among the players (Nash, 1950). The net payoffs of the players differ to the extent they have different reservation payoffs. Given this rule of division, we study under what situations firms may fail to form a grand merger.

Broadly speaking, our paper is an application of the literature of cooperative game. The main focus of the literature is to study the existence of a feasible set of solutions for different possible coalitions. The value of coalition is given by the characteristic function associated with each possible coalition. The Shapley value provides a rule for division of payoffs (Shapley, 1953), but it is mostly normative in approach. Our first concern is to study how the values of different coalitions are affected by the differences in technologies of the firms, and by the nature of product market competition. We define a rule for “fair division”. Then we examine whether under this rule of division firms agree to a grand coalition. In that sense we have a positivist’s approach to the problem. Because

²The core of a game consists of those utility (payoff) vectors which are feasible for the entire group of players and which cannot be blocked by any coalition.

of the externalities (under Cournot competition) a firm's bargaining payoff becomes larger than its non-cooperative payoff when a sub-group forms a coalition. This gives some firms an extra edge while bargaining. Then there are situations when the group as a whole find it difficult to satisfy the demand of all firms. This gives some other interesting points. Sometimes some firms might have more incentives to be outsiders rather than insiders. As a result there is a possibility that no merger at all will occur. There are also situations when, under non-cooperative competition, some firms cannot operate because of their technological inefficiency, and if these firms would not exist, the other firms could merge to share profits among themselves, but the existence of such inefficient firms might prevent the other firms to merge. In such a situation, the inefficient firms just derive some profits from the grand merger — the efficient firms are to pay to the inefficient firms if to form a grand merger.

It may be mentioned that the result that under some conditions firms will not merge to monopoly, even though such a merger maximizes aggregate profits, is not altogether a new result. This result is referred to in the literature as the hold-up problem. Stigler (1950) was the first to introduce the basic idea of hold-up although he did not formalized it. Kamien and Zang (1990, 1991) and Fridolfsson and Stennek (2000) have formalized the problem. They have non-cooperative models of coalition formation, whereas we have a cooperative approach. All the papers mentioned above assume identical cost function for all firms. We have allowed asymmetry cost although cost functions are assumed linear. Then in our three-firm structure, when all firms are identical, mergers to duopoly are just not profitable. Hence the relevant alternative to monopoly is tripoly. When all firms have asymmetric technologies, there are situations when mergers to duopoly are profitable. In such a case the relevant alternative to monopoly is duopoly. In this case there is hold-up. We focus attention on technology asymmetry and nature of product market competition. In that sense our paper may contribute to our understanding of the hold-up phenomenon, and compared to some earlier work, it may provide a more reasonable theory of the relevant alternative to the grand coalition.

Before we go to the literature of merger let us explain the source of externality.³ We consider linear cost functions with no fixed costs. Then if merger of a subset of firms takes place, it does not

³On bargaining with externalities see Roy Chowdhury (1998), and the references therein.

matter whether firms operate on different plants (but using the same production technology) or on a single plant, we can treat the merged firm as a single competitor. So if there are n firms in the industry and out of them s ($s < n$) firms form a merger, then effectively there are now $(n - s + 1)$ firms in the industry. Therefore, under Cournot competition each of $(n - s)$ outsiders will now derive more than its initial non-cooperative payoff. This has negative effect to the profits of the merged firm. Because of this (negative) externality, a horizontal merger of any subset may not be privately profitable. To the extent merger of asymmetric firms means that some inefficient firms are dropped from operation, one efficient firm's payoff also goes up. If the degree of asymmetry is large enough, merger of a subgroup of firms becomes profitable. In case of price competition under homogenous goods, only the efficient firm survives, and it does not depend on how many inefficient firms are in the industry. Hence under price competition, there is no such externality of merger as in Cournot competition. Let us now briefly look at the theoretical literature on horizontal merger.

Salant, Switzer and Reynolds (1983) provide a model of horizontal merger of any exogenously fixed number of firms when firms compete in quantities. They assume linear demand and identical linear cost functions. Because of the externality, the merged firm needs to have at least 80% market share to be profitable to the constituent members. Hence a bilateral merger in an industry with more than two firms is never profitable. Later, this result has been examined under a more general demand function. It is found that the merged firm requires at least 50% market share for it to be privately profitable (Levin, 1990, Fauli-Oller, 1997). In the works mentioned above there is only concentration effect. Perry and Porter (1985) have shown that if merger is associated with some efficiency gain in the form of cost reduction, then merger of any size can be profitable. Farrel and Shapiro (1990) have extended the model to welfare analysis. The concentration effect reduces output and the efficiency effect increases output. When the efficiency effect dominates the concentration effect, social welfare can be larger.⁴ Kabiraj and Chaudhuri (1999) considers the choice between cross-border mergers and inside-border mergers from the viewpoint of the welfare of the local country. The choice depends on the industry structure, the extent of synergy created through merger and the bargaining power of the local firm under merger with the foreign firm. The Long and Vousden (1995) paper studies

⁴It is possible to have situations when an output-expanding merger reduces welfare, and an output-reducing merger enhances welfare (Cheung, 1992).

the effect of liberalization on the above choice from the firms' perspective. The possibility of merger under price competition with differentiated products is examined in Deneckere and Davidson (1985). Among other papers, Barros (1998) provides a study showing a negative relation between initial market concentration and size of asymmetry of merger participants, and Kabiraj and Mukherjee (1999) examine the relation between cooperation in production and R&D.⁵ In the present paper we discuss the possibility of merger of all firms. We have assumed a three-firm structure. Initially firms are competing either in quantities or in prices. Then across the table all firms together negotiate for a division of profits, if a grand merger is formed. Each firm has its disagreement payoff which it can secure for sure. Then the surplus is divided in a Nash bargaining way. When firms fail to agree, a subset may form a (bilateral) merger, if profitable. The choice of partners is decided optimally. We portray the situations when firms find it difficult to form a grand coalition. The role of technological asymmetry and the nature of product market competition are focused in the analysis..

The organization of the paper is as follows. The second section describes the structure of the model. The third section provides the formation of mergers under all possible assumptions regarding technology asymmetry. The last section is a conclusion.

2 Model

Consider an industry for a homogeneous good with three firms in the industry, 1, 2, and 3. Their production technologies are represented by constant marginal costs of production, c_1 , c_2 and c_3 , respectively. Without loss of generality, assume $0 \leq c_1 \leq c_2 \leq c_3$. Product market is characterised by either Cournot competition or Bertrand competition. Let us first define the model when firms have Cournot conjectures.

Let the non-cooperative payoff under Cournot competition of the i th firm be π_i^N . If firms i and j merge together, the merged firm's payoff is denoted by π_{ij}^0 , and the outsider k 's payoff is π_k^0 . It is always assumed that the merged firm uses the technology which corresponds to the lowest marginal cost available to its constituents.⁶ Then because of the externality under Cournot competition (as

⁵Also see Yi (1995) and Ordover and Willig (1985).

⁶If firm i and firm j merge together, then merged firm's technology (marginal cost of production) is $c_m = \min(c_i, c_j)$.

explained in the introduction), we must have $\pi_k^0 > \pi_k^N \forall k, k = 1, 2, 3$, if i and j have merged.⁷ However, the possibility of a bilateral merger will arise only if such a merger is privately profitable to its members. A bilateral merger between i and j is profitable iff

$$\pi_{ij}^0 > \pi_i^N + \pi_j^N \quad \forall i, j = 1, 2, 3; i \neq j. \quad (1)$$

Note that π_k^0 is defined only when condition (1) holds.

Industry profit is maximized when all firms merge together. We call this a grand merger or grand coalition (a Pareto optimal situation). It will be denoted by G . The industry profit under G is the monopoly profit for c_1 technology. Let $\pi(c_1) = \pi^m$ be the payoff to the grand coalition. Note that π^m is independent of the nature of the product market competition. Our question is: Can we divide π^m among the players in such a way that the allocation is acceptable, and no firm will have any further incentive to leave the coalition? Let v_i be any allocation to the i th player under G . Then for G to be stable, following conditions must hold:

$$(S1) \quad v_1 + v_2 + v_3 = \pi^m$$

$$(S2) \quad v_i + v_j \geq \max[\pi_{ij}^0, \pi_i^N + \pi_j^N]; \quad i, j = 1, 2, 3; i \neq j$$

$$(S3) \quad v_i \geq \max[\pi_i^0, \pi_i^N]; \quad i = 1, 2, 3.$$

The conditions stated above have easy interpretation. The first condition is the division of profits under grand coalition. Any allocation satisfying (S1) is Pareto efficient in the sense that any reallocation implies that at least one firm is worse off. We call this the ‘Pareto optimality’ condition. The left hand side of (S2) is the sum of profits of any two firms, i and j , under G . If (1) is satisfied, by forming a bilateral merger among themselves i and j can together get π_{ij}^0 , and if bilateral merger is not available, their profits will be just the noncooperative profits. Hence the second inequality ensures that, given an allocation in G , any two firms have no incentives to go for a bilateral merger or to compete non-cooperatively. The third condition similarly ensures that individually no firm has any incentive to leave the grand coalition because under grand coalition i gets v_i , whereas if it leaves G and (1) is satisfied for j and k (so that j and k form a bilateral merger), i will get π_i^0 as an outsider,

If all firms merge together, then the merged firm has access to the technology, c_1 .

⁷Under Bertrand competition, as we show later, we shall not get this result.

and if (1) is not satisfied for j and k (so that the market structure is oligopoly of all three firms), i can get just its noncooperative profit, π_i^N . Note that when (1) is satisfied for i and j , we have $\pi_k^0 > \pi_k^N$ under Cournot competition. The second and third conditions may be called respectively ‘group rationality’ and ‘individual rationality’ conditions. Thus the conditions (S1) through (S3) are similar to core, with the exception that we have modified (S2) and (S3) to accommodate externalities, and bilateral merger will occur only when it is profitable.

When the product market competition is characterized by Bertrand competition, we correspondingly denote those variables with ‘tilde’ (‘~’) over them. The condition for a profitable bilateral merger and the conditions for the stable grand merger (i.e., conditions for Pareto optimality, group rationality and individual rationality) are similarly given as above. But there is one important difference. Under Cournot conjectures we have always $\pi_k^0 > \pi_k^N$ when i and j merge. This happens because of the externality we explained in the introduction. But under Bertrand competition there is no such externality, and hence we never get such a result. In fact we have $\tilde{\pi}_k^0 = 0$ always, whereas $\tilde{\pi}_k^N \geq 0$.

Then our problem in the paper is to study whether meaningfully we can allocate pie among the players, given their technologies. We study the effect of the technological asymmetry and the nature of product market competition on this allocation. There are situations where firms cannot agree on a division and hence they fail to form a grand merger.

Regarding the division of payoffs we assume following two rules which appear to us innocuous:

- Identical players will get identical payoffs (symmetry property).
- The surplus payoff, over and above the sum of reservation payoffs, which comes from their cooperation, will be divided equally among the insiders.

The first rule calls for a “fair” division of payoff in the sense that identical firms should get identical payoffs. The second rule specifies that the surplus created due to coalition will be equally divided among the coalition partners. So we are assuming Nash bargaining. Given the technological position, a player’s bargaining power in the process of negotiation is determined by its outside option. Thereafter, firms have equal bargaining power. Let r_i be the disagreement or reservation payoff of the i th player, and S be the surplus under all firms merger. Then following the above rules, the

allocation to the i th player will be

$$v_i = r_i + S/3, \quad i = 1, 2, 3. \quad (2)$$

where

$$S = \pi^m - \sum_i r_i$$

We have to define the reservation payoff or outside option of a player very carefully; not necessarily these are non-cooperative payoffs. When firms have Cournot conjectures, we define r_i as

$$r_i = \max[\pi_i^0, \pi_i^N]. \quad (3)$$

The reason is the following. Consider any allocation under G . Now, if any player, i , wants to leave G , how much it can expect to get depends on the behavior of the other two players. If the other two players, j and k , form a bilateral merger, i gets π_i^0 , and if j and k decide to compete independently, i gets just its non-cooperative payoff, π_i^N . Hence the definition of r_i . When firms have Bertrand conjectures, we must have $r_i = \tilde{\pi}_i^N$, because $\tilde{\pi}_i^0 = 0$. Hence (2) and (3) together imply that (S1) and (S3) are satisfied. For stable grand coalition we have to check (S2) separately. Depending on the technologies of the firms any of the following scenarios is possible: (1) $c_1 = c_2 = c_3$, (2) $c_1 = c_2 < c_3$, (3) $c_1 < c_2 = c_3$, and (4) $c_1 < c_2 < c_3$. In our analysis we assume that the market demand is linear. Let the demand function in inverse form be given by:

$$P = a - \sum_i q_i \quad (4)$$

where P is the product price and q_i is the demand for the i th firm's output.

3 The Structure of Merger

In this section we discuss the possibility of formation of a grand merger and the corresponding allocations for the players under all possible assumptions regarding technology asymmetry, given the allocation rules stated in the previous section. Quite naturally, it depends on the incentives of firms to form subgroup coalitions. As we shall show, the structure of bilateral mergers depends on the technology asymmetry of the players. It is possible to have no bilateral merger, merger between only

the efficient firms, merger between one efficient firm and one inefficient firm, or merger between the inefficient firms. But if the firms compete in prices, then a bilateral merger between the efficient firms is possible, but never with the inefficient firm. The possibility of subgroup mergers creates externalities under quantity competition, and the outsider's bargaining payoff goes up. Then there are situations when the grand coalition cannot meet the demand of all firms, and hence a stable grand coalition may not be formed.

3.1 Assumption 1: $c_1 = c_2 = c_3$

It is a bench mark case and trivial. All the firms are identical. Let $c_i = c \forall i$. Then, given (4), it is easy to get $\pi^m = (a - c)^2/4$, $\pi_i^N = (a - c)^2/16$, $\pi_{ij}^0 = (a - c)^2/9$, $\tilde{\pi}_i^N = 0 = \tilde{\pi}_{ij}^0$.

Lemma 1: *Under assumption 1, bilateral merger is never profitable.*

Proof: Condition (1) is never satisfied both under Cournot and Bertrand conjectures. **QED**

Given the assumption, under Bertrand competition any bilateral merged firm's payoff would be zero because the outsider has the same technology. But under Cournot competition, the outsider gains because of the fall in the number of firms in the industry. Since there is no efficiency gain under merger, the concentration effect is dominated by the negative externality, making the bilateral merger privately unprofitable.⁸ The implication of this result is that a player, by going independently, cannot expect a payoff (r_i) more than its non-cooperative payoff. However, the industry monopoly payoff exceeds the sum of non-cooperative payoffs both under Cournot and Bertrand competition. Since all firms are identical, following our rules of allocation, each firm will get identical payoff under grand coalition. This will be given by $v_i = \pi^m/3 \forall i$. Hence we have the following proposition.

Proposition 1: *When all firms have symmetric technologies, irrespective of the nature of product market competition, the grand coalition can always be formed, with each firm's payoff being one-third of the industry monopoly payoff.*

⁸See Salant, Switzer and Reynolds (1983) for details.

3.2 Assumption 2: $c_1 < c_2 = c_3 \equiv c$

This is the case where we have one efficient firm and two identical inefficient firms. Let P_m be the (unrestricted) monopoly price for c_1 technology. Given the demand function (4), we have $P_m = (a + c_1)/2$. Now given c , if $P_m < c$, i.e., $c_1 < \underline{c} \equiv (2c - a)$, firm 1 is monopoly. There will be no further merger in this case. So assume $c_1 > \underline{c}$. Then under Bertrand competition firm 1 has restricted monopoly, with its payoff, $\tilde{\pi}_1^N = (c - c_1)Q(c)$, where Q is the industry output. All other firms' payoff under price competition is zero. Under Cournot conjectures all firms survive (if $c_1 > \underline{c}$), with $\pi_1^N = (a - 3c_1 + 2c)^2/16$ and $\pi_j^N = (a - 2c + c_1)^2/9$ for $j = 2, 3$.

Lemma 2: *Given assumption 2, (a) if firms have Cournot conjectures, then (i) bilateral merger between the inefficient firms (i.e., between 2 and 3) is never profitable, and (ii) bilateral merger between the efficient firm (firm 1) and one inefficient firm (i.e., either 2 or 3) is profitable iff $c_1 \in (\underline{c}, c^0)$, where $c^0 = (14c - a)/13$ and $\underline{c} < c^0 < c$; (b) but under Bertrand conjectures no bilateral merger is profitable.⁹*

Proof: See Appendix A. **QED**

The results are shown in *Figure 1*. Given Lemma 2, we have a number of subcases depending on the nature of technology asymmetry.

Assumption 2.1: $c_1 \leq \underline{c}$. This is the case of monopoly of the efficient firm, implying that there will be no further merger. Hence $v_1 = \pi^m$ and $v_2 = 0 = v_3$. This result is independent of the nature of product market competition. For all other subcases we first consider quantity competition.

Assumption 2.2: $c^0 \leq c_1 < c$. In this case all firms operate under non-cooperative situation, and there will be no bilateral merger (see Lemma 2). Hence $r_i = \pi_i^N$; $i = 1, 2, 3$. Under G , therefore, $v_i = r_i + S/3$ where $S = \pi^m - \sum_i \pi_i^N$.

Assumption 2.3: $\underline{c} < c_1 < c^0$. In this case all firms operate at positive output levels under non-cooperative situation, and bilateral merger between firm 1 and firm 2 (or 3) is profitable, but not between firm 2 and 3. So if firm 1 leaves G , it cannot get more than its non-cooperative payoff, i.e., $r_1 = \pi_1^N$. But if the j th firm ($j = 2, 3$) leaves G , the remaining two firms will form a bilateral merger, instead of competing independently. Therefore, as an outsider the j th firm gets π_j^0 ; hence,

⁹Part (a) of Lemma 2 is drawn from Kabiraj and Mukherjee (1999).

$$r_j = \pi_j^0; \quad j = 2, 3.$$

Then the question is whether to pay all firms their reservation payoffs at the minimum is feasible at all. It will be feasible if and only if $S = \pi^m - \pi_1^N - 2\pi_j^0 \geq 0$. We may note that S is concave in c_1 , with $S = 0$ at $c_1 = \underline{c}$ and $c_1 = c^{0'} \equiv (82c - 5a)/77 > c^0$. Also $\frac{dS}{dc_1}|_{\underline{c}} > 0$ and $\frac{dS}{dc_1}|_{c^{0'}} < 0$. Hence given assumption 2.3, we have $S > 0$. Now, following the rules of allocation we have, $v_1 = \pi_1^N + S/3$ and $v_j = \pi_j^0 + S/3$; $j = 2, 3$. We can also verify that $v_1 + v_j > \pi_{1j}^0$, given assumption 2.3.¹⁰ Hence, under this assumption, a stable grand merger is possible with the above allocations. Since $\pi_1^N > \pi_j^0$ in this case, the efficient firm is getting the largest payoff under G . In that sense it is a fair division.¹¹

Now consider price competition. Since under assumption 2 there will be no bilateral merger, the non-cooperative payoffs are their reservation payoffs. Under price competition the efficient firm's bargaining payoff is its restricted monopoly payoff and that of other firms is zero. Hence $v_1 = r_1 + S/3$ and $v_2 = v_3 = S/3$, where $r_1 = (c - c_1)Q(c)$ and $S = \pi^m - r_1$.

Proposition 2: *Under assumption 2, grand merger is always formed, and the allocations depend on the technology asymmetry.*

3.3 Assumption 3: $\hat{c} \equiv c_1 = c_2 < c_3$

In this case firm 1 and 2 are equally efficient but firm 3 is inefficient. Let us fix $c_3 = c$, and consider Cournot competition. Then we note that

$$\exists \bar{c} \equiv (3c - a)/2 \mid \hat{c} \leq \bar{c} \Leftrightarrow \pi_3^N = 0;$$

$$\exists \underline{c} \equiv (2c - a) \mid \hat{c} \leq \underline{c} \Leftrightarrow \pi_3^0 = 0.$$

Explanation of the above parameters is quite simple. If firm 1 and 2 each has $mc = \hat{c} < \bar{c}$, the inefficient third firm with $mc = c$ will cease to operate under non-cooperative situation and the market structure will be reduced to duopoly of the efficient firms. However, the inefficient firm will operate at positive output level if it faces a single efficient firm (the merged firm of the two efficient firms), provided that the mc of the efficient firm is not too small (i.e., $\hat{c} > \underline{c}$).

¹⁰Let $Z = v_1 + v_j - \pi_{1j}^0$. Then we can show that Z is strictly concave in c_1 , with $Z = 0$ at $c_1 = \underline{c}$ and at $c_1 = (17a + 38c)/55 > c^0$. Hence $Z > 0$ for $c_1 \in (\underline{c}, c^0)$.

¹¹In fact, $\pi_1^N \leq \pi_j^0$ for $c_1 \geq c^0$, but then there will be no merger.

Lemma 3: *Given assumption 3, (a) under Cournot competition (i) bilateral merger between efficient firms (1 and 2) is profitable iff $\hat{c} < c^*$, and (ii) bilateral merger between one efficient firm i ($i = 1, 2$) and the inefficient firm 3 is profitable iff $\hat{c} \in (\bar{c}, c^{**})$ where $c^* = (c - (\sqrt{2} - 1)a)/(2 - \sqrt{2})$ and $c^{**} = (15c - a)/14$; $\underline{c} < c^* < \bar{c} < c^{**} < c$; (b) under price competition bilateral merger is always profitable, but this is only between the efficient firms.¹²*

Proof: See Appendix B. **QED**

Figure 2 portrays the results of Lemma 3. Again we have many possible subcases. Consider first Cournot competition.

Assumption 3.1: $\hat{c} \leq \underline{c}$. This is a case when firm 3 cannot enter under non-cooperative competition as well as in a situation when firms 1 and 2 merge. So firm 3 has no contribution in merger any way. Hence the optimal merger structure will be the bilateral merger of firms 1 and 2 only. The corresponding payoffs will be: $v_i = \pi_i^N + [\pi^m - 2\pi_i^N]/2$ for $i = 1, 2$, and firm 3 has zero payoff.

Assumption 3.2: $\hat{c} \geq c^{**}$. Since $\hat{c} > \bar{c}$, firm 3 operates at positive output level under Cournot-Nash equilibrium. But no bilateral merger is profitable. Hence non-cooperative payoffs remain to be their reservation payoffs. This gives a payoff to i under G as $v_i = \pi_i^N + [\pi^m - \sum_i \pi_i^N]/3$, $i = 1, 2, 3$.

Assumption 3.3: $c^* \leq \hat{c} \leq \bar{c}$. This case has interesting feature. In this case firm 3 cannot operate under non-cooperative competition (i.e., $\pi_3^N = 0$). So it is a duopoly of firm 1 and 2. However, if firm 1 and 2 would merge, firm 3 could operate profitably. But given \hat{c} in that interval, no bilateral merger is profitable. Hence again non-cooperative payoffs are their reservation payoffs. Then if the grand merger is formed, firm i ($i = 1, 2$) will get, $v_i = \pi_i^N + [\pi^m - 2\pi_i^N]/3$ and $v_3 = [\pi^m - 2\pi_i^N]/3$ where π_i^N is the non-cooperative duopoly payoff of one efficient firm.

Since firm 3's non-cooperative payoff is zero, apparently it seems that firm 3 has no bargaining power, but here the technological asymmetry is such that firm 3 can prevent the efficient firms to merge together. This gives firm 3 some bargaining power and it derives positive payoff under grand merger. Greater is the efficiency of the efficient firms relative to the inefficient firm (i.e., as \hat{c} is closer to c^*), larger the benefits the efficient firm derives.

¹²Part (a) of Lemma 3 is drawn from Kabiraj and Mukherjee (1999).

Assumption 3.4: $\underline{c} < \hat{c} < c^*$. Here firm 3's output is zero under non-cooperative competition, but whenever firm 1 and 2 form a merger, firm 3 operates at positive profit (since $\hat{c} > \underline{c}$). Contrary to the previous case, in this interval merger between efficient firms (only) is profitable. This means, firm 3's reservation payoff goes up to $r_3 = \pi_3^0 > 0$. Since bilateral merger between 1 and 2 is profitable, their reservation payoffs are $r_i = \pi_{12}^0/2$; $i = 1, 2$. Hence under grand coalition each of the efficient firms gets a payoff $v_i = \pi_{12}^0/2 + S/3$ and firm 3 gets $v_3 = \pi_3^0 + S/3$ where $S = [\pi^m - \pi_{12}^0 - \pi_3^0]$.

Note that, compared to the previous case, now the efficient firms have become more efficient, but a part of the profits due to efficiency goes to the inefficient firm. The greater efficiency of the efficient firms pays partially to the inefficient firm.

Assumption 3.5: $\bar{c} < \hat{c} < c^{**}$. This is the case where under non-cooperative competition all firms (including the inefficient firm) make positive profits, but, given the interval, bilateral merger between one efficient firm (i.e., either 1 or 2) and the inefficient firm is profitable, but merger between the efficient firms is not profitable. So if a grand merger is formed, by leaving it firm 3 cannot expect more than its non-cooperative payoff, (i.e., $r_3 = \pi_3^N$). But if the i th firm ($i = 1, 2$) goes out, the j th firm ($j = 1, 2$) and firm 3 can operate as merged firm. Hence by going out, firm i gets $\pi_i^0 > \pi_1^N$, that is, the i th firm's reservation payoff is $r_i = \pi_i^0$, $i = 1, 2$.

Now given the reservation payoffs as stated above, we have to see whether paying each firm at least its reservation payoff is feasible. As before, define $S = \pi^m - 2\pi_i^0 - \pi_3^N$. Then $S = 0$ at $\hat{c} = c^{**'} \equiv (9c - a)/8$ and $\hat{c} = \bar{c} \equiv (9c - 5a)/4$, with $\bar{c}' < \bar{c} < c^{**'} < c^{**}$. Also S is concave, with $\frac{dS}{d\hat{c}}|_{\bar{c}'} > 0$ and $\frac{dS}{d\hat{c}}|_{c^{**'}} < 0$. Therefore, given assumption 3.5 in which case bilateral merger is profitable between one efficient firm and the inefficient firm, the grand coalition (if formed) can pay at least the reservation payoffs to the players iff $\hat{c} \in (\bar{c}, c^{**'}]$. In this interval under grand coalition the optimal allocation will be for $i = 1, 2$, $v_i = \pi_i^0 + S/3$, and for firm 3, $v_3 = \pi_3^0 + S/3$, where S is defined above. But in the other subcase (i.e., $c^{**'} < \hat{c} < c^{**}$), all firms merger under the rules suggested cannot be formed, because with any allocation $\{v_i\}$, $\sum v_i = \pi^m$, at least one firm j ($j = 1, 2$) finds it profitable to go out of the grand coalition and get a larger payoff than what is allocated. In this case the externalities are strong enough that each firm has much to gain from going out of the grand coalition, but the total profits are not large enough to satisfy the demand of each player. Hence grand coalition cannot be formed. One may think of the following allocation. Since firm 3 under this

case has a reservation payoff π_3^N , so suppose $v_3 = \pi_3^N$ and $v_i = [\pi^m - \pi_3^N]/2$ for $i = 1, 2$. But this is not acceptable for two reasons. First, it violates our second rule of allocation, and secondly, even under this allocation, firm i has an incentive to go out of the coalition, because $[\pi^m - \pi_3^N]/2 < \pi_i^0$. While in this case bilateral merger is Pareto superior, but without further assumptions and different rules of the games we cannot determine the structure of the bilateral merger and also the division of payoffs under bilateral merger.

Consider now price competition. Given assumption 3, the inefficient firm cannot prevent the efficient firms to form a bilateral merger. Hence before formation of the grand merger, the efficient firms have more to bargain, although the non-cooperative payoffs of all firms (including efficient and inefficient firms) are zero. Grand merger can, however, be formed with payoff of i ($i = 1, 2$) being $r_i + S/3$ and that of firm 3 being only $S/3$, where $r_i = \bar{\pi}_{12}^0/2$ and $S = [\pi^m - 2r_i]$.

On the basis of the discussion so far we have made we can write the following proposition.

Proposition 3: *Given the two rules of allocation and assumption 3, while under price competition grand merger can always be formed, but under quantity competition grand merger can be formed iff $\hat{c} \notin (c^{**}, c^{**})$.*

3.4 Assumption 4: $c_1 < c_2 < c_3$

We have already defined P_m to be the monopoly price for c_1 technology. Let P_d be the duopoly price when there are two firms with c_1 and c_2 technologies. For the linear demand function, $P_m = (a + c_1)/2$ and $P_d = (a + c_1 + c_2)/3$. Also $P_d < P_m$ when $c_2 < P_m$. When $P_m < c_2$ (i.e., $c_1 > 2c_2 - a$), firm 1 emerges as monopoly. We ignore this case.

Lemma 4: *Given assumption 4, (a) under Cournot competition bilateral merger between any two firms may be profitable depending on the technological asymmetry of the firms, and (b) under price competition bilateral merger only between firms 1 and 2 is profitable.*

Proof of (a): Condition (1) can hold only for one pair or for two pairs or for all pairs depending on the technological asymmetry of the players.¹³

¹³See examples in the following analysis.

Proof of (b): In this case $\tilde{\pi}_1^N = \tilde{\pi}_{13}^0 = (c_2 - c_1)Q(c_2)$, $\tilde{\pi}_2^N = \tilde{\pi}_3^N = 0 = \tilde{\pi}_{23}^0$, but $\tilde{\pi}_{12}^0 = (c_3 - c_1)Q(c_3)$ if $P_m > c_3$ and $\tilde{\pi}_{12}^0 = (P_m - c_1)Q(P_m)$ if $P_m \leq c_3$. But $\tilde{\pi}_{12}^0 > \tilde{\pi}_{13}^0$. Hence condition (1) is satisfied only for bilateral merger between 1 and 2. **QED**

Now, under the assumption of price competition grand merger will always occur, given the technology asymmetry of the firms. We have $r_1 = \tilde{\pi}_1^N + (\tilde{\pi}_{12}^0 - \tilde{\pi}_1^N)/2$, $r_2 = (\tilde{\pi}_{12}^0 - \tilde{\pi}_1^N)/2$, and $r_3 = 0$. Hence $v_i = r_i + S/3$, where $S = [\pi^m - \sum r_i]$.

Consider then Cournot competition. This is the most general case in the sense that all previous results may be possible under assumption 4. We provide the analysis of the remaining section for a general demand function, while giving examples for linear demand (4).¹⁴ It may be recalled that $P_d < P_m$ when $c_2 < P_m$.

Assumption 4.1: $c_1 < P_m \leq c_2 < c_3$. It is monopoly of firm 1. Hence $v_1 = \pi^m$. All other firms are getting zero payoff.

Assumption 4.2: $c_1 < c_2 < P_d < P_m \leq c_3$. It is a duopoly of firms 1 and 2 under non-cooperative competition. While merger between 1 and 2 is possible, but firm 3 can never enter. Therefore, $v_i = \pi_i^N + S/2$, $i = 1, 2$, where π_i^N is the duopoly payoff of firm i and $S = [\pi^m - \sum_i^2 \pi_i^N]$. Firm 3 gets nothing. As for example, given the demand function (4), suppose $a = 10$, $c_1 = 2$, $c_2 = 3$, $c_3 \geq 6$, and our result follows.

Assumption 4.3: $c_1 < c_2 < P_d < c_3 < P_m$. Non-cooperative game is a duopoly of firm 1 and 2. Bilateral merger between 1 and 3 or between 2 and 3 will never occur. But if firm 1 and 2 merge, firm 3 can find entry profitable. Now merger between 1 and 2 is privately profitable iff

$$\pi_{12}^0 > \pi_1^N + \pi_2^N. \quad (5)$$

When (5) does not hold, there will be no bilateral merger, implying that firms have non-cooperative payoffs as their reservation payoffs under grand coalition. Stable grand coalition is always possible under this situation.¹⁵

¹⁴The analysis of the case of price competition in the paper does not at all depend on the linearity of the demand function.

¹⁵To show that (5) may not hold, consider the following example: $a = 10$, $c_1 = 0$, $c_2 = 0.5$, $c_3 = 3.6$. Then, $\pi_{12}^0 = (a + c_3)^2/9 = 184.96/9$ and $\pi_1^N + \pi_2^N = (a + c_2)^2/9 + (a - 2c_2)^2/9 = 191.25/9$.

When (5) holds, we must have $r_i = \pi_i^N + (1/2)[\pi_{12}^0 - \pi_1^N - \pi_2^N]$ for $i = 1, 2$, and $r_3 = \pi_3^0$. Then also note that $S = [\pi_m - (\pi_{12}^0 + \pi_3^0)] > 0$. Hence firms agree on a division of payoffs under grand coalition.

Example: Suppose $a = 10$, $c_1 = 0$, $c_2 = 2$, and $4 < c_3 < 5$.

Then we must get

$$\pi^m = 25, \pi_1^N = 16, \pi_2^N = 4, \pi_3^N = 0, \pi_{12}^0 = \begin{cases} 21.8 & \text{if } c_3 = 4 \\ 25 & \text{if } c_3 = 5 \end{cases} \text{ and } \pi_3^0 = \begin{cases} 4/9 & \text{if } c_3 = 4 \\ 0 & \text{if } c_3 = 5 \end{cases}$$

Hence, $\pi_{12}^0 > \pi_1^N + \pi_2^N$ and $\pi^m > \pi_{12}^0 + \pi_3^0$.

Assumption 4.4: $c_1 < c_2 < c_3 < P_d < P_m$. In this case the non-cooperative profit of each firm is positive. Regarding the structure of bilateral merger we cannot apriori say anything in general. Given the parameters (c_1, c_2, c_3) , let us first consider a case where bilateral merger between any two firms is profitable, i.e.,

$$\pi_{ij}^0 > \pi_i^N + \pi_j^N, \forall i \neq j.$$

Therefore, $r_i = \pi_i^0 > \pi_i^N$. Define $S = [\pi^m - \sum_1^3 \pi_i^0]$. Now if $S \geq 0$, the allocations $v_i = r_i + S/3$ will form a stable grand coalition if and only if (S2) is satisfied at the same time ((S1) and (S2) are necessarily satisfied by construction). The following example describes a scenario where grand merger is formed.

Example: Suppose, $a = 10$, $c_1 = 1$, $c_2 = 2$, and $c_3 = 3$. Then we have, $\pi^m = 20.25$, $\pi_{12}^0 = 13.4$, $\pi_{13}^0 = 11.1$, $\pi_{23}^0 = 5.4$, $\pi_1^N = 9$, $\pi_2^N = 4$, $\pi_3^N = 1.0$, $\pi_1^0 = 11.1$, $\pi_2^0 = 5.4$, $\pi_3^0 = 2.8$, $S = 0.95$. All the relevant conditions of this case are satisfied. Hence $v_1 = 11.42$, $v_2 = 5.72$, $v_3 = 3.11$. Note that the stability conditions are also satisfied.

Assuming that pairwise all bilateral mergers are profitable, the grand coalition cannot, however, be formed if $S \leq 0$, although $\pi^m > \sum_i \pi_i^N$. In the above example if we replace the value of c_3 by $2 + \epsilon$ where ϵ is very small but positive, we shall get $S < 0$. Now, given that the grand coalition is not profitable, our question is: What will be the optimal structure of bilateral merger? It is easy to

see that merger between i and j will be privately optimal if the following condition holds,¹⁶ that is,

$$\pi_{ij}^0 + \pi_k^N > \max[\pi_{ik}^0 + \pi_j^N, \pi_{jk}^0 + \pi_i^N]. \quad (6)$$

Let us now assume that only one bilateral merger, say between i and j , is profitable, and no other bilateral merger is profitable. In this case, a stable grand coalition can be formed because $S = \pi^m - (\pi_{ij}^0 + \pi_k^0) > 0$. In this case, therefore, $r_i = \pi_i^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2$, $r_j = \pi_j^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2$, and $r_k = \pi_k^0$ are the reservation payoffs of firm i , j and k . Under grand merger $v_i = r_i + S/3$ is the payoff for firm i .

Finally, consider that bilateral mergers between i and j and between i and k are profitable, but not between j and k . In this case i cannot get more than its non-cooperative payoff by going out of the grand coalition, but each of j and k can get more than its non-cooperative payoff. So if $\pi^m > \pi_i^N + \pi_j^0 + \pi_k^0$, a stable grand coalition can be formed with an allocation $v_i = r_i + S/3$ iff at the same time following two conditions hold: $v_i + v_j \geq \pi_{ij}^0$ and $v_i + v_k \geq \pi_{ik}^0$, otherwise, there will remain some incentives for a bilateral merger. In our scheme, i has relatively disadvantage to bargain in a sense that its bargaining payoff is stuck up at the non-cooperative level.¹⁷ However, if any of the conditions stated in this case fails to hold, the only stable outcome will be the bilateral merger, and then i has the advantage to choose its partner. i will choose j as its partner if $\pi_{ij}^0 - \pi_j^0 \geq \pi_{ik}^0 - \pi_k^0$; otherwise, partner will be k . In the example below, a stable grand coalition is formed.

Example: Suppose, $a = 10$, $c_1 = 0$, $c_2 = 0.5$, and $c_3 = 3.2$. Then we have, $\pi^m = 25$, $\pi_{12}^0 = 19.36$, $\pi_{13}^0 = 12.25$, $\pi_{23}^0 = 9.0$, $\pi_1^N = 11.73$, $\pi_2^N = 8.56$, $\pi_3^N = 0.05$, $\pi_1^0 = 12.25$, $\pi_2^0 = 9.0$. This means, bilateral mergers between 1 and 3, and between 2 and 3 are profitable, but not between 1 and 2. In this case, $S = \pi^m - \pi_1^0 - \pi_2^0 - \pi_3^N = 3.7$. Then following our rules of allocation, $v_1 = \pi_1^0 + S/3 = 13.48$, $v_2 = \pi_2^0 + S/3 = 10.23$, $v_3 = \pi_3^N + S/3 = 1.29$. Note that the stability conditions are also satisfied, because $v_1 + v_3 > \pi_{13}^0$ and $v_2 + v_3 > \pi_{23}^0$.

From the discussion of this section we can write the following proposition.

¹⁶Player i will prefer player j as its partner iff $\pi_i^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2 > \pi_i^N + [\pi_{ik}^0 - \pi_i^N - \pi_k^N]/2$, and similarly for j , $\pi_j^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2 > \pi_j^N + [\pi_{jk}^0 - \pi_j^N - \pi_k^N]/2$. Condition (6) is derived from these inequalities.

¹⁷Under the scenario described above we cannot rule out the possibility that a relatively efficient firm (say, i) gets under grand merger an allocation which is smaller than that of a relatively inefficient firm (say, j). This occurs if $\pi_i^N < \pi_j^0$.

Proposition 4:

- (a) *A stable grand merger can always be formed if the market is characterized by price competition.*
- (b) *When the market is characterized by Cournot competition, (i) if under non-cooperative situation not all firms survive, a stable grand merger is always possible, but (ii) if under non-cooperative competition all firms survive, whether a stable grand merger will occur or not depends on the extent of asymmetry of the players. In particular, given the technological asymmetry, if a bilateral merger is profitable for only one pair, a stable grand merger can be always formed.*

4 Conclusion

Merger is a business strategy by which firms in an industry consolidate their position to share a larger profit. But the negotiation process for merger is not always smooth enough; it involves lots of threats and counter threats, or objections and counter objections of players. By doing this firms test the bargaining strength of the partners. In this paper we have drawn attention to the role of technology asymmetry and nature of product market competition in evaluating the bargaining position of each firm on the negotiation table. Although each firm knows that there are larger payoffs to share if a grand merger is formed, but often they fail to determine how to divide the payoff in a mutually agreeable way. Since the size of the bigger cake is fixed, the gain of one player necessarily implies the loss of payoff to that extent to the others. No firm wants to give up the gain to others; hence each player tries to prove how important is its contribution to a particular coalition and what it can otherwise gain without being party to the coalition. This tension may lead to disagreement or formation of a coalition of sub-optimal size.

In the process of negotiation, externalities play a very significant role, because firms might gain from the coalition of other firms. In our analysis externalities arise when the product market competition is characterized by Cournot. Hence the nature of product market competition is very important in the process of forming a coalition. As we have seen, under price competition for homogeneous goods a grand coalition is always formed, because price competition induces no externalities. Technological asymmetry is obviously important, because, as we have seen, if all firms are identical, firms can always agree on a grand merger.

It is not just technological asymmetry, but the extent of technology asymmetry is more important. The extent of technology asymmetry determines whether a merger of a sub-group (in our case, a bilateral merger) is profitable. If merger of any sub-group is not feasible, bargaining payoffs are just their non-cooperative payoffs, and through a Nash bargaining firms can divide the total payoff among themselves. The Nash bargaining allocation seems appealing, and hence it is assumed that firms will agree to that division rule. But when sub-group mergers are profitable, some firms' bargaining power goes up because of the externalities. When the effects of externalities are large enough, the grand coalition finds it difficult to meet the demand of all partners, and hence firms fail to agree on a grand merger.

In our paper the extent of technology asymmetry also determines the nature of bilateral merger. There are situations where bilateral merger is possible only between two relatively efficient firms, or between two inefficient firms or between one efficient firm and one inefficient firm. It also determines how many bilateral mergers can be feasible. This gives some other interesting results. There are cases when the inefficient firm cannot just operate because of its inefficient technology, but it can prevent the formation of a (bilateral) merger between other two relatively efficient firms. This gives bargaining power to the inefficient firm and hence it derives positive payoff when a grand merger is formed. The relatively efficient firms are, in a sense, to bribe the inefficient firm if a larger payoff is to be shared under grand merger. Even there might be situations when the efficient firm has the disadvantage to bargain in negotiation, and it comes up with a lower payoff compared to a relatively inefficient firm. Under price competition, however, each firm gets an allocation according to its relative efficiency position. We have considered a three-firm structure, because it is the simplest structure to capture the role of technological asymmetry and externality in the process of negotiation for merger.

Appendix

Appendix A

Proof of Lemma 2(a):

(i) For bilateral merger to be profitable, (1) must hold. Consider the possibility of merger between 2 and 3. Given the demand function by (4), we have, $\pi_{23}^0 = (a - 2c + c_1)^2/9$ and $\pi_2^N + \pi_3^N = (2/16)(a - 2c + c_1)^2$. Hence (1) does not hold.

(ii) Now consider the possibility of merger between firm 1 and firm j , $j = 2, 3$. Let us define

$$S_1(c_1) = \pi_{1j}^0 - [\pi_1^N + \pi_j^N].$$

Then, for $c_1 \leq \underline{c}$, $S_1 = 0$, because $\pi_{1j}^0 = \pi^m$, $\pi_1^N = \pi^m$, and $\pi_j^N = 0$; and $S_1 < 0$ for $c_1 = c$. Also S_1 is continuous and concave for $c_1 > \underline{c}$. Hence

$$\exists c^0 \mid S_1(c_1) > 0 \text{ iff } c_1 \in (\underline{c}, c^0).$$

For the demand function (4), $c^0 = (14c - a)/13$.

Proof of Lemma 2(b):

Under price competition, $\tilde{\pi}_1^N = \tilde{\pi}_{1j}^0$ and $\tilde{\pi}_j^N = 0 = \tilde{\pi}_{23}^0$ for $j = 2, 3$. Hence the result.

Appendix B

Proof of Lemma 3(a):

(i) Bilateral merger between 1 and 2 will be profitable iff

$$S_2(\hat{c}) = \pi_{12}^0 - [\pi_1^N + \pi_2^N] > 0.$$

Now if $\hat{c} > \bar{c}$, firm 3 survives under non-cooperative competition. Given (4), it is easy to show that $S_2 < 0$ for $\hat{c} > \bar{c}$.

When $\hat{c} \leq \underline{c}$, $\pi_3^0 = 0 \Rightarrow \pi_{12}^0 = \pi^m$, i.e., the merged firm becomes monopoly, and $\pi_1^N + \pi_2^N = \pi_1^d + \pi_2^d$ (superscript d stands for duopoly). Then gain from merger becomes

$$S_2(\hat{c}) = \pi^m - [\pi_1^d + \pi_2^d] > 0.$$

Thus if $\hat{c} > \bar{c}$, firms 1 and 2 will never merge, but for $\hat{c} < \underline{c}$, they will always merge. Also for $\underline{c} < \hat{c} < \bar{c}$, S_2 is monotonically decreasing in \hat{c} . So there exists $\hat{c} = c^*$ such that $S_2(\hat{c}) > 0 \Leftrightarrow \hat{c} < c^*$.

(ii) Consider the possibility of merger between firm i ($i = 1, 2$) and firm 3. Define

$$S_3(\hat{c}) = \pi^m - [\pi_i^N + \pi_3^N].$$

Given (4), S_3 has the following properties. S_3 is inverted U-shaped with $S_3(\hat{c}) = 0$ for $\hat{c} \leq \bar{c}$, and $S_3(\hat{c}) < 0$ at $\hat{c} < c$. So there exists $\hat{c} = c^{**}$ at which $S_3(\hat{c}) = 0$ and $S_3 < 0$ for $\hat{c} > c^{**}$. Hence $S_3(\hat{c}) > 0$ for $\bar{c} < \hat{c} < c^{**}$.

Proof of Lemma 3(b):

Consider now price competition. Under non-cooperative situation $\tilde{\pi}_i^N = 0 \forall i$. Also $\tilde{\pi}_{i3}^0 = 0$, but $\tilde{\pi}_{12}^0 = (c - \hat{c})Q(c) > 0$. Hence the result.

References

1. Aumann, R.J. and Maschler, M. (1964) 'The bargaining set for cooperative games', in Dresher, M., Shapley, L.S., and Tucker, A.W. (eds.) *Advances in Game Theory*, Annals of Mathematics Studies, Number 52, Princeton University Press; 443-476.
2. Barros, P.P. (1998), 'Endogenous mergers and size asymmetry of merger participants', *Economics Letters* 60, 113-119.
3. Cheung, F.K. (1992) 'Two remarks on the equilibrium analysis on horizontal merger', *Economics Letters* 40, 119-23.
4. Deneckere, R. and Davidson, C. (1985) 'Incentives to form coalitions with Bertrand competition', *Rand Journal of Economics* 16, 473-86.
5. Farrel, J. and Shapiro, C. (1990) 'Horizontal mergers: an equilibrium analysis', *American Economic Review* 80, 107-26.
6. Fauli-Oller, R. (1997) 'On merger profitability in a Cournot setting', *Economics Letters* 54, 75-79.
7. Fridolfsson, S. and Stennek, J. (2000), 'Holdup of endogenous mergers', mimeo., The Research Institute of Industrial Economics.
8. Jacquemin, A. (1990), 'Horizontal concentration and European merger policy', *European Economic Review* 34, 539-50.
9. Kabiraj, T. and Chaudhuri, M. (1999) 'On the welfare analysis of a cross-border merger', *Journal of International Trade and Economic Development* 8, 195-207.
10. Kabiraj, T. and Mukherjee, A. (1999) 'Cooperation in R&D and production: a three-firm analysis', *Journal of Economics* (forthcoming).
11. Kamien, M.I. and Zang, I. (1990), 'The limits of monopolization through acquisition', *Quarterly Journal of Economics* 105, 465-499.

12. Kamien, M.I. and Zang, I. (1991), 'Competitively cost advantageous mergers and monopolization', *Games and Economic Behavior* 3, 323-338.
13. Levin, D. (1990) 'Horizontal mergers: the 50-percent benchmark', *American Economic Review* 80, 1238-45.
14. Long, N.V. and Vousden, N. (1995) 'The effects of trade liberalization on cost-reducing horizontal mergers', *Review of International Economics* 3, 141-55.
15. Nash, J.F. (Jr) (1950) 'The bargaining problem', *Econometrica* 18, 155-62.
16. Ordover, J.A. and Willig, R.D. (1985) 'Antitrust for high technology industries: Assessing research joint ventures and mergers', *Journal of Law and Economics*, 28, 311-33.
17. Perry, M.K. and Porter, R.H. (1985) 'Oligopoly and the incentive for horizontal merger', *American Economic Review* 75, 219-27.
18. Ray Chowdhury, P. (1998) 'Externalities and bargaining disagreement', *Economics Letters* 61, 61-65.
19. Roy, M. (1997) 'Mergers and takeovers: the Indian scene during the nineties', in A. Bagchi (ed.) *Economy and Organization: Indian Institutions under the Neo-Liberal Regime*; Sage, New Delhi.
20. Salant, S.W., Switzer, S. and Reynolds, R. (1983) 'Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium', *Quarterly Journal of Economics* 48, 185-99.
21. Shapley, L.S. (1953) 'A value for n-person games', in Kuhn, H.W. and Tucker, A.W. (eds.) *Contributions to the Theory of Games II*; Annals of Mathematical Studies, Number 28, Princeton University Press; 307-317.
22. Stigler, G.J. (1950), 'Monopoly and oligopoly by merger', *American Economic Review, Papers and Proceedings* 40, 23-34.

23. Yi, S.S. (1995) 'R&D cooperation, product-market collusion and welfare', *Discussion Paper*, Department of Economics, Dartmouth College, Hanover.