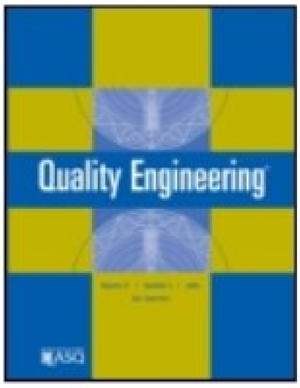

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Evaluation of Nonnormal Process Capability Indices using Generalized Lambda Distribution

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Process capability indices (PCIs) are used to describe a manufacturing process expressing its ability to produce items within the specified limits. These indices are developed under the assumption that the underlying process distribution is normal. In industries, there are many manufacturing processes where process distribution can not be described by a normal distribution. In such cases, those PCIs will give misleading results about the process. The most commonly used approach for analysing a nonnormal process data is to fit a standard nonnormal distribution (e.g., weibull, gamma) or a family of distribution curves (e.g., Pearson, Johnson) to the process data and then to estimate the percentile points from the fitted distribution that can be used to compute generalized PCIs. In this article, we outline the procedure using the generalized lambda distribution (GLD) curve for modeling a set of process data and for estimating percentile points in order to compute generalized PCIs. The four-parameter GLD can assume a wide variety of curve shapes and hence it is very useful for the representation of data when the underlying model is unknown. Compared to the Pearson and Johnson family of distributions, the GLD is computationally simpler and more flexible. The article provides all necessary formulas for fitting a GLD curve, estimating its parameters, performing goodness-of-fit tests, and computing generalized PCIs. An example is used to illustrate the calculations that can be easily performed using spreadsheets.

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Keywords generalized process capability indices, generalized lambda distribution, nonconformance, goodness-of-fit

INTRODUCTION

Process capability indices (PCIs) are used widely in industries for evaluating a manufacturing process whether or not it can produce articles within the specified limits. The most commonly used measures for process capability indices are C_p, C_{pl}, C_{pu}, and C_{pk}. Kane (1986) gave the first comprehensive explanations and interpretations of these indices. Let the process characteristic X is independently and identically distributed as a normal distribution with mean μ and standard deviation σ. Then, these indices are defined by

$$C_p = \frac{U - L}{6\sigma}$$

$$C_{pl} = \frac{\mu - L}{3\sigma}$$

$$C_{pu} = \frac{U - \mu}{3\sigma}$$

$$C_{pk} = \text{Min}\{C_{pl}, C_{pu}\}$$

where L and U are the lower and upper specification limits respectively, of the process and the target T is at the middle of the specification limits. When the process is in statistical control, these indices are estimated by substituting μ and σ with their estimators $\bar{X}$ and s, respectively. For detailed information on process capability indices and recent developments, see Refs. (Johnson and Kotz (1993), Kotz and Lovelace (1998), Kotz and Johnson (2000)).

These capability indices were originally developed under the assumption that the process measurements can be described adequately by a normal distribution. This may not always be an appropriate assumption. In industries, there are many manufacturing processes, where process spread cannot be described by normal distribution curves. Skewed distributions that are bounded on one side are quite common in industries. For example, ovality, concentricity, taper, run-out etc. generally follow a skewed distribution. Munechika (1992) details several examples of machining processes that are inherently nonnormal. For these types of nonnormal processes, one should not use the same formulas for computing process capability indices because those PCIs will give misleading results about the process performance. Gunter (1989) has shown three different distributions with identical values of C_p and C_{pk}, but different proportions of nonconforming parts. Somerville and Montgomery (1997) investigated the effects of nonnormality on the yield of a process that is assumed normal. Kocherlakota et al. (1992) provides additional information on the effects of nonnormality on PCIs.

In literature, there are various methods for analyzing nonnormal data for the computation of process capability indices. The simplest way for dealing with nonnormal data is to transform the data via some mathematical function in such a manner that the transformed data are normal. For example, a skewed distribution may become normal by using a square root transformation. Chou et al. (1998) and Polansky et al. (1998) have proposed using Johnson's system of distribution curves to transform the nonnormal data into normality. The distribution of the transformed data should be tested for normality and if it passes the test, then the transformed data can be used to estimate process capability indices, process nonconformance, etc. Most often, the main difficulty of this approach is to find an appropriate transformation function. Also, some practitioners may feel uncomfortable working with transformed data.

Another approach is to compute the generalized process capability indices that are a simple modification of the normal PCIs. For a nonnormal process, Clements (1989) proposed the use of generalized process capability indices [defined in Eqs. [8]–[11]] that use the values of nonnormal percentile points. Wu (2001) has evaluated the performance of these generalized PCIs for various shapes of nonnormal distribution curves.

These generalized capability indices require estimation of 0.135 and 99.865 percentile points that can be obtained from a distribution curve fitted to the sample data. The easiest way to model a nonnormal process data is to fit a standard probability distribution such as the lognormal or gamma or weibull. Advantages of fitting such a standard nonnormal distribution are parameter estimation for the distribution is fairly straight forward and distribution properties are well documented. Percentile points can easily be obtained for estimating the capability indices. References include Somerville and Montgomery (1997) t, gamma, and lognormal; Mukherjee and Singh (1998) Weibull; Sarkar and Pal (1997) extreme value; and Sundaraiyer (1996) inverse gaussian. This approach of fitting standard nonnormal distribution may not work for all nonnormal processes. In such cases, modeling the process data via a family of distributions may be more appropriate. For example, the Johnson or Pearson family of distributions will be able to fit distributions with a wide variety of shapes. Applications of this kind of method, with various assumed distributional forms, are quite numerous. References include Clements (1989), Rodriguez (1992) Pearson system; Farnum (1997), Polansky et al. (1998), Pyzdek (1995) all Johnson system; Castagliola (1996) Burr distribution; and Kocherlakota et al. (1992) Edgeworth series distributions. There are some potential difficulties associated with these families of distributions Rodriguez (1992). However, this approach of fitting a standard nonnormal distribution or a family of distributions is most commonly used for modeling a nonnormal process data and thereby computing the generalized process capability indices.

In this article, we describe a distribution, namely the generalized lambda distribution (GLD), that can be used for fitting to a set of process data. The generalized lambda distribution can assume a wide variety of curve shapes and hence it is very useful for the representation of data when the underlying model is unknown. The main advantages of fitting GLD to describe a process data are the GLD uses only a single functional form with four parameters and the estimation of these parameters is much simpler. Using the fitted distribution, nonnormal percentile estimates can be obtained for computation of generalized PCIs.

Although the Pearson and Johnson family of distributions cover a large variety of curve shapes, both of these systems incorporate a number of functional forms whereas the GLD uses only one function and is computationally simpler. The Burr distribution also covers a wide range of parameter values, but does not include symmetric distributions. Thus, the GLD has an edge over the other family of distributions while modeling a process data.

The main disadvantages in fitting GLD distribution are the practitioner must have a computer facility and a reference table for estimating the lambda parameters. However, the same disadvantages lie in fitting other family of distributions too.

In the following section, the generalized lambda distribution and some of its properties are described. Then, we describe the method for fitting GLD to process data. The generalized PCIs are defined and their estimation methods using the GLD are discussed. For illustration, an example has been worked out.

THE GENERALIZED LAMBDA DISTRIBUTION

Ramberg and Schmeiser (1974) developed the generalized lambda distribution for the purpose of generating random variables for Monte Carlo simulation studies. The use of GLD as models for data came later. The GLD has been used for a range of different applications, for example, air pollution, Okur; finance, McNichols; climate studies, Abouammoh and Ozturk; and inventory modeling, Nahmris. Excellent literature of the generalized lambda distribution, its applications, and parameter estimation methods appear in Karian and Dudewicz (2000).

A continuous probability distribution is usually defined by its distribution function or by its probability density function. Alternatively, it can be defined by its percentile function that is simply the inverse of the distribution function. The generalized lambda distribution is defined by the percentile function

$$R(p) = \lambda_1 + \frac{[p^{\lambda_3} - (1-p)^{\lambda_4}]}{\lambda_2} \quad (0 \le p \le 1) \qquad [1]$$

where λ_1 is a location parameter, λ_2 is a scale parameter and λ_3 and λ_4 are shape parameters. The distribution function does not exist in "simple closed form".

The density function is given by

$$f(x) = f[R(p)] = \frac{\lambda_2}{[\lambda_3 p^{\lambda_3 - 1} + \lambda_4 (1-p)^{\lambda_4 - 1}]} \quad (0 \le p \le 1) \qquad [2]$$

Ramberg et al. (1979) studied the properties of this distribution in detail. The richness of this four-parameter GLD to fit a wide variety of frequency distributions is also elaborated. Some examples of the density shapes produced by the GLD are shown in the following Figure 1.

Ramberg et al. (1979) also derived the expressions for the mean μ, the variance σ^2, and the third ($\mu_3 = E(X-\mu)^3$) and fourth ($\mu_4 = E(X-\mu)^4$) moments about the mean for this distribution:

$$\mu = \lambda_1 + A/\lambda_2,$$
$$\sigma^2 = (B - A^2)/\lambda_2^2,$$
$$\mu_3 = (C - 3AB + 2A^3)/\lambda_2^3,$$
$$\mu_4 = (D - 4AC + 6A^2B - 3A^4)/\lambda_2^4$$

where

$$A = \frac{1}{1+\lambda_3} - \frac{1}{1+\lambda_4},$$
$$B = \frac{1}{1+2\lambda_3} - 2\beta(1+\lambda_3, 1+\lambda_4) + \frac{1}{1+2\lambda_4},$$
$$C = \frac{1}{1+3\lambda_3} - 3\beta(1+2\lambda_3, 1+\lambda_4) + 3\beta(1+\lambda_3, 1+2\lambda_4) - \frac{1}{1+3\lambda_4},$$
$$D = \frac{1}{1+4\lambda_3} - 4\beta(1+3\lambda_3, 1+\lambda_4) + 6\beta(1+2\lambda_3, 1+2\lambda_4) - 4\beta(1+\lambda_3, 1+3\lambda_4) + \frac{1}{1+4\lambda_4},$$

and $\beta\,(a, b)$ denotes the beta function with parameters a and b. A brief description of the beta function can be obtained from Spiegel (1990).

The skewness and kurtosis, as given by

$$\alpha_3 = \mu_3/\sigma^3, \qquad [3]$$

and

$$\alpha_4 = \mu_4/\sigma^4 \qquad [4]$$

are functions of λ_3 and λ_4, but do not depend upon λ_1 and λ_2.

PARAMETER ESTIMATION AND DISTRIBUTION FITTING

Although there are several methods for estimating the parameters of the GLD in the literature, we will follow the moment-matching method that

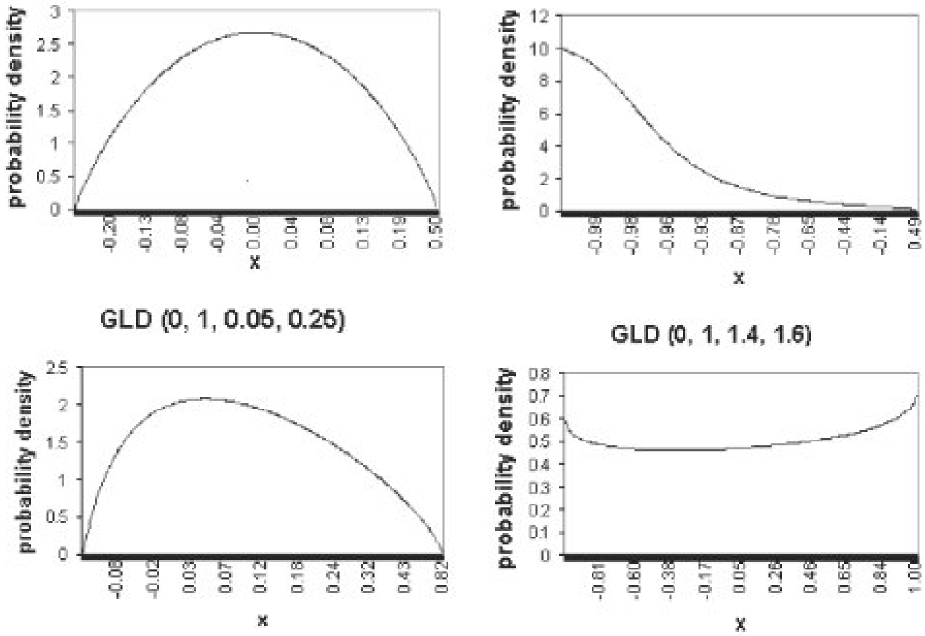

Figure 1. Density curves produced by GLD.

proposed in Ramberg and Schmeiser (1974). This approach for estimating GLD parameters is based on matching the first four moments of the sample data. The method can be described as follows: given the GLD distribution with percentile function $R(p)$, find parameters λ_1, λ_2, λ_3, and λ_4 so that the mean μ, variance σ^2, skewness α_3, and kurtosis α_4 of the GLD match the corresponding mean $\bar{X}$, variance s^2, skewness α_3^*, and kurtosis α_4^* of the sample (i.e., the first four moments of the theoretical GLD match those of the process data). The sample mean, variance, skewness, and kurtosis are computed from the sample data $x_1, x_2, \ldots, x_n$ of size n as

$$\text{Mean } \bar{X} = \sum_{i=1}^{n} \frac{x_i}{n}$$

$$\text{Variance } s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{X})^2$$

$$\text{Skewness } \alpha_3^* = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \bar{X})^3}{s^3}$$

$$\text{Kurtosis } \alpha_4^* = \frac{1}{n} \sum_{i=1}^{n} \frac{(x_i - \bar{X})^4}{s^4}.$$

These are not the maximum likelihood estimators, but correspond to method-of-moments estimators.

Therefore, if $f(x) \equiv f(x;\lambda)$ denotes the probability density function of the random variable X with percentile function $R(p)$, then we estimate the parameters λ such that

$$\begin{aligned} \mu &= \bar{X} \\ \sigma^2 &= s^2 \\ \alpha_3 &= \alpha_3^* \\ \alpha_4 &= \alpha_4^* \end{aligned} \qquad [5]$$

Closed-form solutions of the above equations do not exist and hence the parameters λ_1, λ_2, λ_3, and λ_4 cannot be directly computed. Using numerical methods, approximate solutions can be obtained, but that requires an efficient search algorithm and a good choice of the initial starting point for the search.

Ramberg et al. (1979) provided a table to find the values of λ_1, λ_2, λ_3, and λ_4 for selected values of α_3 and α_4. Karian and Dudewicz (2000) provided a more comprehensive and accurate table that gives lambda parameter values. The lambda values are determined from the table using $|\alpha_3^*|$ and α_4 values as entry points. The lambda values in the table are for a variable with a mean of zero and a variance of one. This means that we use the values $(0, 1, |\alpha_3^*|, \alpha_4^*)$ and obtain a solution $(\lambda_1(0,1), \lambda_2(0,1), \lambda_3, \lambda_4)$ for the Eq. [5]

those equations associated with ($\bar{X}$, s^2, $|\alpha_3^*|$, α_4^*) is obtained by setting

$$\lambda_1 = \lambda_1(0,1)s + \bar{X} \quad [6]$$

$$\lambda_2 = \lambda_2(0,1)/s \quad [7]$$

when $\alpha_3^* < 0$, we interchange the values of λ_3 and λ_4 and change the sign of $\lambda_1(0,1)$. The procedure for fitting a GLD can be summarized as

1. Compute $\bar{X}$, s^2, α_3^*, and α_4^* from sample data.
2. Find the entry point closest to ($|\alpha_3^*|$, α_4^*) in the reference table [see Tables B–I in Appendix B of Karian and Dudewicz (2000)].
3. Using $(|\alpha_3^*|, \alpha_4^*)$, extract $(\lambda_1(0,1),\ \lambda_2(0,1),\ \lambda_3,\ \lambda_4)$ from the table.
4. If $\alpha_3^* < 0$, interchange the values of λ_3 and λ_4 and change the sign of $\lambda_1(0,1)$.
5. Compute λ_1, λ_2 using Eqs. [6] and [7].

More description on this moment-matching method for estimating the lambda parameters of GLD can be obtained from Karian and Dudewicz (2000) and Ramberg et al. (1979).

To verify the goodness-of-fit, one may plot the percentile function and compare it with the sample distribution function being approximated. An approximate χ^2 goodness-of-fit test can also be carried out. The sample data are first arranged in a frequency histogram having k class intervals. Let O_i be the observed frequency and E_i be the expected frequency in the ith class interval. For finding E_i, the p_i values are obtained by solving the percentile function $R(p)$ at each class interval endpoint. Multiplying those p_i-values with the number of observations will give the expected cumulative frequencies from which E_i-values can easily be obtained.

The test statistic

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

has approximately a chi-square distribution with $k-l-l$ degrees of freedom, where l (=4, in this case) represents the number of parameters of the hypothesized GLD distribution. The null hypothesis, which states that the sample is drawn from GLD, is rejected if the calculated value of the test statistic $\chi_0^2 > \chi_{\alpha,k-5}^2$, where α is the significance level. Since the parameters of the model are estimated by the method of moments rather than by the maximum likelihood method, the use of the χ^2 distribution is only approximate.

GENERALIZATION OF CAPABILITY INDICES

We assume that the process is brought under statistical control prior to evaluating its capability. For the nonnormal distributions, the definitions of process capability indices are generalized. The generalized capability indices can always be reduced to, the ''standard'' capability indices when the process data are likely to follow a normal distribution.

The generalized C_p is defined as

$$C_p = \frac{\text{Allowable Spread}}{\text{Process Spread}} = \frac{U-L}{U_p - L_p} \quad [8]$$

where L_p and U_p are respectively the 0.135th and 99.865th percentiles of the distribution used to describe the process. For a normal distribution, $L_p = \mu - 3\sigma$ and $L_p = \mu + 3\sigma$, and so the above equation reduces to the usual definition of C_p.

For the definition of C_{pl} and C_{pu}, the median is used as the measure of the process center. The median is chosen because the median is a better estimator of the population mean than sample average ($\bar{X}$) for nonnormal distributions, especially when the distributions are skewed. The allowable process spreads, and the lower and upper process spreads, are defined with respect to the median. The generalized C_{pl} and C_{pu} are defined as

$$C_{pl} = \frac{\text{Allowable Lower Spread}}{\text{Lower Process Spread}} = \frac{M-L}{M-L_p} \quad [9]$$

$$C_{pu} = \frac{\text{Allowable Upper Spread}}{\text{Upper Process Spread}} = \frac{U-M}{U_p - M} \quad [10]$$

where M is the median of the process distribution. The generalized C_{pk} index is defined as the minimum of the C_{pl} and C_{pu}, that is,

$$C_{pk} = \text{Min}\{\text{C}_{pl}, \text{C}_{pu}\} = \text{Min}\left\{\frac{M-L}{M-L_p}, \frac{U-M}{U_p-M}\right\} \quad [11]$$

For estimating these generalized PCIs, first the percentile points L_p, U_p and M are computed from the fitted GLD which are then used in the above equations. The L_p, U_p and M values are calculated from the percentile function $R(p)$ with p-values 0.00135, 0.99865, and 0.5, respectively.

The estimated process capability indices are compared with a recommended minimum value to decide whether or not the process can be considered as capable. Montgomery (1996) suggested this minimum recommended value as 1.33 for an existing process and 1.50 for a new process where the underlying distribution is normal. We will consider the same values even for a nonnormal process.

The lower and upper proportion nonconformance can be estimated by solving for p-values from the percentile function $R(p)$ with $R(p)$ values equal to L and U, respectively in the following manner.

Lower proportion nonconformance,

$$NC_L = p_1, \text{ where } R(p_1) = L \quad [12]$$

Upper proportion nonconformance,

$$NC_U = 1 - p, \text{ where } R(p_2) = U \quad [13]$$

NUMERICAL EXAMPLE

In this section, we illustrate the method for fitting a GLD curve to sample data. The estimated curve is then used to compute the generalized process capability indices. Let us consider a sample data on the overall length of bolts. Overall length is an important characteristic in the forging process of bolts manufacturing. A sample data of size 200 are collected with an objective of evaluating the performance of the forging process over a short period of time. The randomness of the data are verified to ensure the stability of the process. The sample data are sorted in an ascending order and are given in Appendix 1. The frequency distribution and the summary statistics of the sample data are given in Table 1. The lower and upper specification limits for the overall length of bolt are 6.2 and 7.0, respectively.

Using these α_3^* and α_4^* values, we obtain the lambda values from the reference table provided by Karian and Dudewicz (2000) as $\hat{\lambda}_1(0,1) = -0.753$, $\hat{\lambda}_2(0,1) = 0.187$, $\hat{\lambda}_3 = 0.046$, and $\hat{\lambda}_4 = 0.2281$. The values for $\hat{\lambda}_1(\bar{X}, s)$ and $\hat{\lambda}_2(\bar{X}, s)$ are

$$\hat{\lambda}_1 = 6.507 - 0.753 \times 0.1398 = 6.4021$$

and

$$\hat{\lambda}_2 = 0.187/0.1398 = 1.3396$$

Table 1
Frequency distribution and summary statistics

Class intervals	Frequency	Summary statistics
6.20–6.25	2	Sample size: 200
6.25–6.30	7	
6.30–6.35	13	Average, X: 6.507
6.35–6.40	22	Median: 6.485
6.40–6.45	35	
6.45–6.50	33	Minimum: 6.22
6.50–6.55	22	Maximum: 6.92
6.55–6.60	23	
6.60–6.65	14	Variance, s^2: 0.01945
6.65–6.70	7	Std.Devn., s: 0.1395
6.70–6.75	8	
6.75–6.80	7	Skewness, α_3^*: 0.621
6.80–6.85	4	Kurtosis, α_4^*: 3.103
6.85–6.90	2	
6.90–6.95	1	

Figure 2 shows the relative frequency histogram for the sample data and the fitted probability density curve corresponding to the above lambda values. For verifying the goodness-of-fit of the GLD distribution, χ^2-test is carried out. The expected frequencies are computed and are given in Table 2. The computed value of $\chi^2 = 5.834$ is much less compared to the tabulated value of $\chi^2_{0.05,7} = 14.067$ which indicates that the above GLD curve fits the sample data quite well.

From the fitted GLD curve, we estimate the L_p, M and U_p values by using the p-values as 0.00135, 0.5, and 0.99865, respectively, as

$$L_p = \hat{\lambda}_1 + [(0.00135)^{\hat{\lambda}_3} - (1-0.00135)^{\hat{\lambda}_4}]/\hat{\lambda}_2 = 6.207$$

$$M = \hat{\lambda}_1 + [(0.5)^{\hat{\lambda}_3} - (1-0.5)^{\hat{\lambda}_4}]/\hat{\lambda}_2 = 6.488$$

$$U_p = \hat{\lambda}_1 + [(0.99865)^{\hat{\lambda}_3} - (1-0.99865)^{\hat{\lambda}_4}]/\hat{\lambda}_2 = 6.983$$

Consequently, the generalized capability indices are estimated as

$$\hat{C}_p = \frac{U - L}{U_p - L_p} = 1.031$$

$$\hat{C}_{pl} = \frac{M - L}{M - L_p} = 1.025$$

$$\hat{C}_{pu} = \frac{U - M}{U_p - M} = 1.034$$

$$\hat{C}_{pk} = \text{Min}\{\hat{C}_{pl}, \hat{C}_{pu}\} = 1.025$$

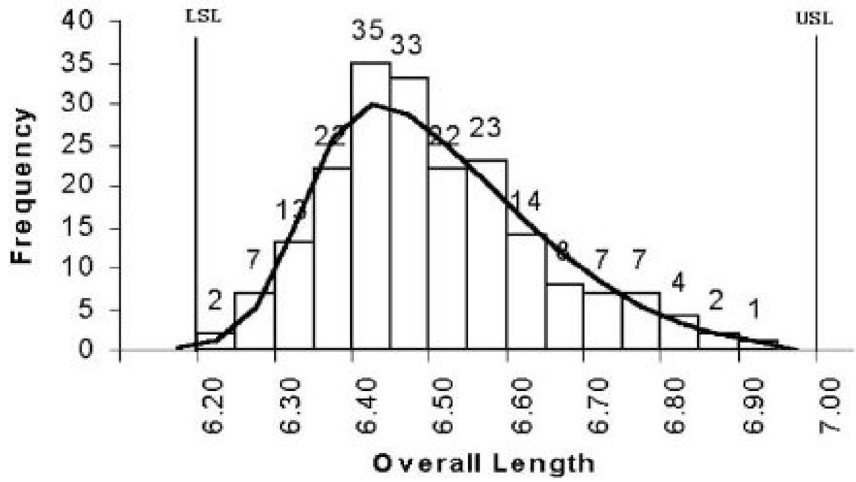

Figure 2. Histogram and fitted GLD density curve.

Table 2
Test for goodness-of-fit

Class interval of life	Observed frequency	Expected frequency	Computed χ^2
Below 6.30	9	6.72	0.774
6.30–6.35	13	15.48	0.397
6.35–6.40	22	25.8	0.56
6.40–6.45	35	30	0.833
6.45–6.50	33	28.7	0.644
6.50–6.55	22	24.96	0.351
6.55–6.60	23	20.34	0.348
6.60–6.65	14	15.7	0.184
6.65–6.70	7	11.62	1.128
6.70–6.75	8	8.22	0.181
6.75–6.80	7	5.46	0.434
Above 6.80	7	7	0
Total	200	200	5.834

At 5% level of significance, tabulated value of $\chi^2_7 = 14.067$.

The estimate of process performance index $\hat{C}_{pk}$ is less than the minimum recommended value of 1.33 and, hence, the process may not be considered as capable.

The estimated proportions of nonconformance are found by solving Eqs. [12] and [13], as

$$NC_k = P(X > k) = 0.104\%$$

$$NC_L = P(X > U) = 0.084\%$$

CONCLUSION

Traditional process capability indices can give misleading indications of a process' ability to meet specification limits when the process measurements cannot be adequately described by a normal distribution. One approach for solving this problem is to model the process data with a standard nonnormal density curve or a family of distribution curves (e.g., Pearson, Johnson) and then to compute generalized process capability indices using the properties of the fitted distribution.

The generalized lambda distribution is very useful for modeling nonnormal process data. This distribution provides a wide variety of curve shapes and uses only one functional form. Compared to the Pearson and Johnson family of distribution curves, this distribution is computationally simpler and more flexible. In this article, we have described the procedure for using this distribution in computing generalized process capability indices for a nonnormal process.

Some practitioners may face a few difficulties in estimating the lambda parameters and in verifying the goodness-of-fit of the lambda distribution with the process data. An Excel or Lotus spreadsheet can be easily designed for this purpose and also for computing the generalized process capability indices. We hope that this problem will be solved with the advancement of statistical process control software programs. We believe that this generalized lambda distribution will be of immense use in the field of nonnormal process capability and process control.

APPENDIX

Data on Overall Length of Bolts.

6.22	6.37	6.425	6.47	6.520	6.58	6.73
6.24	6.37	6.43	6.47	6.520	6.59	6.73
6.26	6.375	6.43	6.47	6.525	6.59	6.74
6.26	6.38	6.43	6.48	6.525	6.60	6.74
6.27	6.38	6.43	6.48	6.53	6.60	6.75
6.27	6.38	6.43	6.48	6.53	6.60	6.75
6.28	6.38	6.43	6.48	6.53	6.6	6.77
6.30	6.38	6.435	6.48	6.53	6.61	6.77
6.30	6.39	6.435	6.48	6.54	6.61	6.77
6.31	6.39	6.44	6.48	6.54	6.61	6.77
6.31	6.39	6.44	6.49	6.54	6.61	6.78
6.32	6.40	6.44	6.49	6.55	6.62	6.79
6.32	6.40	6.445	6.49	6.55	6.62	6.80
6.32	6.40	6.445	6.49	6.55	6.62	6.82
6.33	6.41	6.45	6.49	6.56	6.63	6.83
6.33	6.41	6.45	6.50	6.56	6.63	6.84
6.33	6.41	6.45	6.50	6.56	6.635	6.85
6.33	6.41	6.45	6.50	6.57	6.64	6.86
6.34	6.41	6.45	6.50	6.57	6.64	6.89
6.34	6.41	6.455	6.50	6.57	6.65	6.92
6.35	6.415	6.455	6.50	6.57	6.65	
6.35	6.415	6.46	6.50	6.57	6.66	
6.36	6.42	6.46	6.51	6.57	6.66	
6.36	6.42	6.46	6.51	6.57	6.67	
6.36	6.42	6.46	6.51	6.57	6.67	
6.36	6.42	6.46	6.51	6.57	6.67	
6.36	6.42	6.46	6.51	6.58	6.68	
6.36	6.42	6.46	6.515	6.58	6.68	
6.37	6.425	6.47	6.52	6.58	6.70	
6.37	6.425	6.47	6.52	6.58	6.71	

Specification limits: 6.6 ± 0.40.

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