# Innovation, Imitation and Intellectual Property Rights: A Note on Helpman's Model

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This note analyses the effect of the policy of tightening Intellectual Property Rights (IPR) on the rate of innovation in the North and on the welfare in both North and South in a model which is otherwise identical to Helpman (1993) except in the concept of knowledge capital. We assume that the South based imitated products do not contribute to the knowledge capital in the North. It is shown that the tightening of IPR raises the rate of innovation in the North and may improve the welfare of both North and South. These results are significantly different from those in Helpman (1993).

*Keywords:* innovation, imitation, intellectual property rights, knowledge capital, North–South, economic growth, steady-state growth equilibrium, welfare.

*JEL Classification:* O31, O34, O40.

## 1 Introduction

In a seminal paper, Helpman (1993) analyses the effect of the tightening of "Intellectual Property Rights" (IPR) policies in the South on the growth-rate and welfare in both North and South. He uses a dynamic general equilibrium model of a two country world economy where North innovates and South imitates. Rate of innovation is endogenous[1] while imitation rate is exogenous in this model; and the tightening of the IPR

1 In Sect. 3 of his paper, innovation rate is endogenous.

implies the reduction in the rate of imitation. This tightening of the IPR policy adopted in the South lowers the rate of innovation in the North in the steady-state equilibrium. Also this policy always lowers the welfare of the South; and also lowers the welfare of the North if the rate of imitation is very small.

The basic model of Helpman (1993) has been extended by various authors in various directions[2]. However Helpman (1993) and its various extensions share a common assumption which we want to modify in this present note. In Helpman (1993), the knowledge capital stock in the North is assumed to be proportional to the economy's cumulative research experience measured by the number of product designs already developed. This knowledge capital, treated as the public input into the R&D sector, generates positive externalities and thus lowers the cost of developing new blue prints in the R&D sector. Instead of this so-called Marshall-Arrow-Romer type of knowledge spillover, we consider Jacobs (1969) type of localized knowledge spillover in this note. Now the agglomeration of different production units in one region decreases the cost of doing R&D there. Thus knowledge spillover originates from the presence of producers of different goods in one region rather than the experience of the R&D sector of developing product designs in the past. The researchers might benefit from interactions with the producers of other goods. They observe the production process directly and find it easier to invent new product designs at cheaper cost. These Jacobs (1969) type of externalities[3] in the Northern R&D sector have been considered in the North-South models of Dollar (1986; 1987), Martin and Ottaviano (1999), Baldwin et al. (2001) etc. although they have not analyzed the problem of imitation and IPR protection in the South.

This is the only minor change in assumption we introduce in this present note. However, this gives interesting results. If we introduce this change in an otherwise Helpman (1993) model, we find that the policy of strengthening IPR in the South must raise the rate of innovation in the North in the new steady-state equilibrium. Also, in this case, both North and South may gain in terms of welfare from tightening IPR when the imitation rate is neither very high nor very low. These results are different

2 See, for example, Arnold (2002), Lai (1998), and Grinols and Lin (2006).

3 These types of knowledge spillovers at the level of a city or a region have been documented by Glaeser et al. (1992), Henderson et al. (1995), and also Jacobs (1969).

from those in Helpman (1993); and are interesting in the context of the debate about the enforcement of IPR in less developed countries. While Helpman's (1993) results go against the adoption of such a policy, our results may advocate this. Also it is the extent of the imitation rate which appears to be crucial factor in determining the desired direction of the policy change.

In Sect. 2, we describe the model. In Sect. 3, we analyze the effect of tightening IPR on the steady-state equilibrium rate of growth. In Sect. 4, we analyze the effect of this policy on the welfare of the North and of the South. Concluding remarks are made in Sect. 5.

## 2 The Basic Model

The representative consumer in the North (N) and in the South (S) has the welfare function given by

$$W_i(t) = \int_t^{\infty} e^{-\rho(\tau - t)} log U_i(\tau) d\tau,$$

where $U_i(t)$ is the instantaneous utility function given by

$$U_i(t) = \left( \int_0^{n(t)} x_i(z)^{\alpha} dz \right)^{\frac{1}{\alpha}}; \quad 0 < \alpha < 1$$

for i = N, S. Here $n(\tau)$ stands for the number of varieties available at time $\tau$ and $x_i(z)$ represents the level of consumption of z-th variety by a representative consumer in the i-th region for i = N, S; $\rho$ stand for the rate of time preference and $\alpha$ represents the elasticity of substitution between any two varieties.

A representative Northern consumer maximizes his welfare subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r_N(\tau - t)} E_N(\tau) d\tau = \int_t^{\infty} e^{-r_N(\tau - t)} I_N(\tau) d\tau + A_N(t) \quad \text{for all } t.$$

Here $E_N(\tau)$, $I_N(\tau)$ and $A_N(\tau)$ stand for instantaneous expenditure, instantaneous income and the current value of assets in the North at time $\tau$. $r_N$ stand for the nominal interest rate in the North.

Note that the consumer in the South needs not solve any dynamic optimization problem because South does not have any R&D activity. He maximizes the instantaneous utility function subject to the instantaneous budget constraint which is given by

$$E_S(\tau) = \int_0^{n(t)} p(z)x_S(z)dz.$$

We obtain the following optimality conditions:

$$\frac{\dot{E}_N}{E_N} = r_N - \rho \tag{1}$$

and

$$x_i(z) = E_i(t)\frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon}du} \quad \forall z \in [0, n(t)]. \tag{2}$$

Here Eq. (1) implies the Ramsey rule and Eq. (2) represents the demand function for the z-th variety of a representative consumer in the i-th region for i = N, S. p(z) is the price of the variety z and

$$\varepsilon = \frac{1}{1-\alpha} > 1$$

is the price-elasticity of demand for the z-th variety. Here,

$$n = n_N + n_S$$

and $n_N$ $(n_S)$ is the number of varieties produced in the North (South). North is the innovator country and South is the imitator. Producer of the z-th variety produced in the North is a profit maximising monopolist while the Southern imitators play Bertrand game. One unit of labor can produce one unit of each product[4]. Labor is internationally immobile but is perfectly mobile among all the sectors within a country. So the price of any Northern product is given by

4 This production technology is the same for all Northern and Southern products.

$$p(z) = p_N = \frac{w_N}{\alpha} \tag{3}$$

for all $z \in [0, n_N]$; and the price of an imitated Southern product is given by

$$p(z) = p_S = w_S \tag{4}$$

for all $z \in [0, n_S]$. Here $p_N$ $(p_S)$ and $w_N$ $(w_S)$ represent equilibrium price[5] of any Northern (Southern) variety and equilibrium wage[6] of Northern (Southern) labor, respectively. It is also assumed that

$$w_N > w_S. \tag{A}$$

The labor market equilibrium equation in the North is given by

$$L_N = n_N x_N + L_R, \tag{5}$$

where $L_N$, $n_N x_N$ and $L_R$ stand for the labor endowment, labor employed in production sector[7] and labor employed in the R&D sector. In the South, imitation is costless and hence

$$L_S = n_S x_S \tag{6}$$

is the labor market equilibrium condition there.

R&D sector in the North produces new product designs using labour as the only input; and thus the number of varieties grow over time. This equation of motion is given by

$$\dot{n} = \frac{n_N}{a_N} L_R, \tag{7}$$

where $\frac{a_N}{n_N}$ is the labor requirement to develop a new product-design; and $n_N$ is the knowledge capital. Note that in Helpman (1993), the knowledge capital was assumed to be equal to the total number of blueprints developed by the R&D sector. We follow Dollar (1986; 1987), Martin and

5 Price (quantity) of all the varieties produced in a country take the same equilibrium value because utility function is symmetric and technologies are identical.

6 Wage rate is the marginal cost of production of a variety.

7 This is equal to total production of all the Northern varieties.

Ottaviano (1999), Baldwin et al. (2001), etc. and assume that the knowledge capital is equal to the number of firms currently producing in the North. This is the only change we introduce in an otherwise identical Helpman (1993) model. We consider Jacobs (1969) type of localized knowledge spillovers. Researchers learn by observing the production process directly and interacting with the local producers.

Note that the formulation in Eq. (7) implies that $L_R$ and $\frac{\dot{n}}{n}$ move proportionately in the long-run because the ratio $\frac{n_N}{n}$ is constant in the balanced growth equilibrium. This implication has been criticized by Jones (1995b; 1999) because the observed long-term growth rate has been relatively stable despite upward trends in the number of R&D workers. We do not remove the scale effect from the Helpman (1993) model in the present paper although it is an interesting area of research[8].

Here m stands for the exogenous rate of imitation defined as

$$m = \frac{\dot{n}_S}{n_N} = \hat{m} - \mu, \tag{8}$$

where $\mu$ is a parameter representing the degree of tightening the IPR and $\hat{m}$ is the rate of imitation in the absence of IPR.

Also it can be shown that

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N \tag{9}$$

and in equilibrium

$$v_N = \frac{w_N a_N}{n_N}, \tag{10}$$

where $\pi_N$ and $v_N$ stand for the Northern firm's instantaneous monopoly profit and its discounted present value of profits over the life time, respectively. The standard no-arbitrage condition in the Northern asset market is given by

8 So we can interprete our model and that of Helpman (1993), as one of medium-term growth. For more on non-scale growth models, see Segerstrom (1998), and Arnold (1998). However, we do believe that it would be more interesting (and a much more significant contribution to the literature) to remove the scale effect from the Helpman (1993) model and then to study the effects of strengthening IPR.

$$\frac{\pi_N}{v_N} + \frac{\dot{v_N}}{v_N} = r_N + m. \tag{11}$$

Also we have

$$E_N = p_N n_N x_N, \tag{12}$$

where $E_N$ stands for expenditure of the representative Northern consumer which is equal to the value of total product.

## 3 The Steady-state Growth Equilibrium

Following Helpman (1993), we define

$$\xi = \frac{n_N}{n},$$

$$g = \frac{\dot{n}}{n},$$

and we also define

$$\theta = \frac{g}{\xi},$$

where $\xi$ represents the fraction of goods that have not been imitated so far. So the two equations of motion we can derive[9] are given by the following:

$$\dot{\xi} = \xi\theta - (\xi\theta + m)\xi \tag{13}$$

and

$$\dot{\theta} = \left(\frac{L_N}{a_N} - \theta\right)\left[\rho + \theta - \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - \theta\right)\right]. \tag{14}$$

The explicit solution of these two differential equations are described in Appendix 2. In this section, we analyze the dynamic properties of the model using a phase diagram shown in Fig. 1.

Note that here

9 Derivation is shown in Appendix 1.

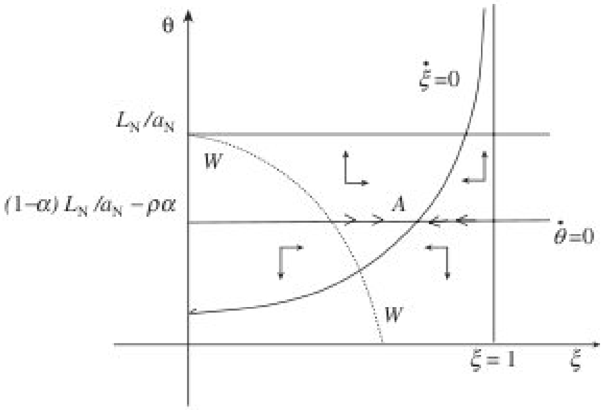


**Fig. 1.** The phase diagram

$$\left(\frac{L_N}{a_N}\right) > \theta \Rightarrow L_N > a_N \frac{\dot{n}}{n_N}$$

and this is always true because, $a_N \frac{\dot{n}}{n_N}$ represents the labor employed in the R&D sector which is always less than the total labor endowment of the North.

So the equation of the $\dot{\theta} = 0$ stationary locus is given by

$$\theta = (1 - \alpha)\left(\frac{L_N}{a_N}\right) - \rho\alpha$$

and so it is a horizontal straight line in the Fig. 1.

The equation of $\dot{\xi} = 0$ locus is given by the following

$$\theta(1 - \xi) = m$$

and this curve slopes positively in Fig. 1 being asymptotic to the $\xi = 1$ vertical straight line and meeting the vertical axis at $\theta = m$.

The point of intersection of these two curves is the steady-state equilibrium point. In Appendix 2, we show that it is a saddle point and the unique saddle path converging to this equilibrium point coincides with the $\dot{\theta} = 0$ locus. This convergence is guaranteed if and only if $\theta(0) = \theta^*$.

The steady-state equilibrium values of $\xi$ and g are given by the following:

$$\xi^* = 1 - \frac{m}{\theta^*} = 1 - \frac{m}{(1-\alpha)(\frac{L_N}{a_N}) - \alpha\rho}$$

and

$$g^* = \theta^* \xi^* = (1-\alpha)\left(\frac{L_N}{a_N}\right) - (\alpha\rho + m).$$

A tightening of IPR means a fall in the effective rate of imitation, m. Hence $g^*$ is increased. In Fig. 1, $\dot{\xi} = 0$ locus shifts downwards and the $\dot{\theta} = 0$ locus remains unchanged. So we have the following proposition:

*Proposition 1:* A policy of tightening IPR in the South raises the steady-state equilibrium rate of innovation in the North.

The result is interesting because this is opposite to what Helpman (1993) obtains[10]. In Helpman (1993), a policy of tightening IPR lowers the rate of growth in the new steady-state equilibrium. We now turn to provide intuitive explanations of why this effect is opposite in nature to the effect in the Helpman (1993) model. A stronger IPR protection leads to a reduction in both the effective cost of capital, $(r_N + m)$, as well as the profit rate, $\frac{\pi_N}{v_N}$, in Helpman (1993) model. Moreover its impact on the effective cost of capital is smaller than the corresponding impact on the profit rate. For this reason the rate of innovation is reduced in his model. However, in the present case, tighter IPR has no effect on the profit rate and only the effective cost of capital is reduced. Thus the positive impact of tightening IPR on the effective cost of capital causes the long run rate of innovation to increase.

We now explain, why, in our model, tightening of IPR has no effect on the profit rate. Let us write the expression of the profit rate, $(\frac{\pi_N}{v_N})$, as

$$\frac{\pi_N}{v_N} = \frac{\frac{1-\alpha}{\alpha} w_N x_N}{\frac{a_N}{n_N} w_N} \Rightarrow \frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha a_N}\left(\frac{n x_N}{\frac{n}{n_N}}\right) \Rightarrow \frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha a_N}\left(\frac{\frac{1}{\xi}(n_N x_N)}{\frac{1}{\xi}}\right).$$

10 Note that Helpman (1993, footnote 19, p. 1261) himself questioned the long run negative relationship between the rate of innovation and the rate of imitation in this product variety framework with a more general functional form of the utility function. However, he was silent about the welfare effect under this more general class of utility function. We offer here a completely different mechanism that leads to a positive relationship between rate of innovation and rate of imitation.

A decrease in m increases $\xi$ in steady state. This decreases per firm profit due to the increased competition among firms in the North on the market share. This also decreases the cost of R&D due to the increased knowledge spillover. However, both the numerator and the denominator move proportionately. Hence the profit rate does not depend on m in the steady-state equilibrium[11].

We obtain the solution of $\theta(t)$ and $\xi(t)$ as

$$\theta(t) = \theta^* \tag{15}$$

and

$$\xi(t) = \xi^* + [\xi(0) - \xi^*]e^{a_{22}t}, \tag{16}$$

where

$$a_{22} = m - \left((1-\alpha)\frac{L_N}{a_N} - \rho\alpha\right) = -g^* = -\theta^*\xi^* < 0.$$

The derivation of these solutions are described in Appendix 2. Since $a_{22} < 0$, $\xi(t) \to \xi^*$, as $t \to \infty$ whatever be the value of $\xi(0)$.

Now $w_N > w_S$ given by the inequality (A) implies that, in the steady-state equilibrium,

$$m < \theta^* \frac{L_S\alpha^{\varepsilon}}{L_S\alpha^{\varepsilon} + L_N - a_N\theta^*}.$$

This condition will always hold true if $L_S$ is large enough relative to $L_N$. Its derivation is given in Appendix 3. Hence $a_{22} < 0$ and $w_N > w_S$ imply that

$$m < \min\left\{(1-\alpha)\frac{L_N}{a_N} - \rho\alpha, \theta^* \frac{L_S\alpha^{\varepsilon}}{L_S\alpha^{\varepsilon} + L_N - a_N\theta^*}\right\}.$$

Since $L_N - a_N\theta^* > 0$, we have $\frac{L_S\alpha^{\varepsilon}}{L_S\alpha^{\varepsilon} + L_N - a_N\theta^*} < 1$; and hence it is clear that the above inequality will be satisfied if

11 In Helpman (1993), the expression of profit rate is $\frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha a_N}\left(\frac{1}{\xi}(n_N x_N)\right)$ and here an increase in $\xi$ (due to a decrease in m) decreases per firm profit only, given the allocation of labor. This results in decreasing the profit rate there.

$$m < \theta^* \frac{L_S \alpha^\varepsilon}{L_S \alpha^\varepsilon + L_N - a_N \theta^*}. \tag{B}$$

This is called the feasibility restriction on the rate of imitation.

Using Eqs. (15) and (16), we obtain:

$$\frac{d\xi(t)}{d\mu}\Big]_{(\theta^*,\xi^*)} = \frac{d\xi^*}{d\mu}(1 - e^{a_{22}t}) > 0 \tag{17}$$

and

$$\frac{d\theta(t)}{d\mu}\Big]_{(\theta^*,\xi^*)} = 0 \tag{18}$$

for all $t > 0$. Since $g(t) = \xi(t)\theta(t)$, we have

$$\frac{dg(t)}{d\mu}\Big]_{(\theta^*,\xi^*)} = (1 - e^{a_{22}t}) > 0 \tag{19}$$

for all $t > 0$. Both the rate of innovation and the fraction of unimitated goods increases at each point of time due to tightening of IPR (except at $t = 0$). Helpman (1993) found that, the rate of innovation is increased in the short run and is decreased in the long run. In our model we do not find any different impact on $g(t)$ in the short run. This is so because, starting from the initial steady-state equilibrium, an increase in $\mu$ does not affect the $\dot{\theta} = 0$ line. Hence the unique equilibrium trajectory which coincides with the $\dot{\theta} = 0$ locus is not changed in our model when the economy attains a new steady-state equilibrium. So $\theta(t)$ remains unchanged and $\xi(t)$ rises for all t until it reaches the new steady-state equilibrium. This ensures that $g(t) = \xi(t)\theta(t)$ will rise for all t till it reaches the new steady-state equilibrium point. In Helpman (1993), there is a new saddle path converging to the new steady-state equilibrium obtained for change in $\mu$; and hence $g(t)$ rises initially to reach the new saddle path. However, $g(t)$ falls in the long-run in his model because the saddle path slopes negatively there.

## 4 Welfare

We now turn to analyze the effects of the policy of tightening IPR in the South on the welfare of a representative worker in the North and in the South. Following Helpman (1993), we define

$$W_N(0) = \int_0^\infty e^{-\rho t} log U_N(t) dt$$

and

$$W_S(0) = \int_0^\infty e^{-\rho t} log U_S(t) dt,$$

where

$$log U_N(t) = \frac{1}{\varepsilon - 1} log(n) + \frac{1}{\varepsilon - 1} log[\xi + (1 - \xi)\left(\frac{p_S}{p_N}\right)^{1-\varepsilon}] + log\left(1 - \frac{a_N \theta}{L_N}\right) \tag{20}$$

and

$$log U_S(t) = \frac{1}{\varepsilon - 1} log(n) + \frac{1}{\varepsilon - 1} log[\xi\left(\frac{p_N}{p_S}\right)^{1-\varepsilon} + (1 - \xi)]. \tag{21}$$

Here $W_i(0)$ is the discounted present value of instantaneous utility flow of the representative worker in the i-th region for i = N, S; and $U_i(t)$ is his instantaneous utility function in the i-th region. Derivation of Eqs. (20) and (21) are described in Appendix 4. In fact, Eqs. (20) and (21) in this note are identical to Eqs. (41) and (16) in Helpman (1993, p. 1265; 1254), respectively. Sign of $(\frac{dW_i(0)}{d\mu})$ represents the nature of the welfare effects in the i-th country due to tightening of IPR. Following Helpman (1993), we obtain

$$\frac{dW_S(0)}{d\mu} = \frac{1}{\varepsilon - 1}(\Delta_N + \Delta_e^S) \tag{22}$$

and

$$\frac{dW_N(0)}{d\mu} = \frac{1}{\varepsilon - 1}(\Delta_N + \Delta_e^N) + \Delta_S^N, \tag{23}$$

where

$$\Delta_N = \int_0^\infty e^{-\rho t} \frac{dlog(n(t))}{d\mu} dt,$$

$$\Delta_e^S = \int_0^\infty e^{-\rho t}\Big[\frac{dlog[\xi(\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]}{d\mu}\Big]dt,$$

$$\Delta_e^N = \int_0^\infty e^{-\rho t}\Big[\frac{dlog[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}]}{d\mu}\Big]dt,$$

$$\Delta_S^N = \int_0^\infty e^{-\rho t}\Big[\frac{dlog(1-\frac{a_N\theta}{L_N})}{d\mu}\Big]dt.$$

The interpretations of $\Delta_N$,$\Delta_e^S$,$\Delta_e^N$ and $\Delta_S^N$ are similar to those given in Helpman (1993, p. 1265). Here the number of products, n, grows over time at the rate g. Hence

$$log(n(t)) = log(n(0)) + \int_0^t g(\tau)d\tau.$$

Now using Eq. (19) and the expression of $\Delta_N$, we have

$$\Delta_N = \frac{-a_{22}}{\rho^2(\rho - a_{22})} > 0 \tag{24}$$

because $a_{22} < 0$. The derivation is shown in Appendix 5. However, $\Delta_N > 0$ implies that the welfare effect via product availability is positive for both countries. This is opposite to what Helpman (1993, p. 1265, proposition 4) obtained because in his model $\Delta_N < 0$. The sign of the welfare effect via product availability is the same as the sign of the growth effect.

We can derive $\Delta_e^S$ as follows while the derivation in detail is given in Appendix 6:

$$\Delta_e^S = \left[\frac{(\frac{p_S}{p_N})^{\varepsilon-1} - 1 - (\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi(\frac{p_S}{p_N})^{\varepsilon-1} + (1-\xi)}\right]\left[\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho - a_{22})}\right], \tag{25}$$

where $\frac{p_S}{p_N}$ and $\xi$ are measured at their steady-state values. Here $\Delta_e^S < 0$ because $a_{22} < 0$; and this implies that the welfare effect of tightening IPR due to changes in both the interregional allocation of production and the terms of trade goes against the South when the economies are initially in the steady state.

Using Eq. (22) and using the expression of $\Delta_N$ and $\Delta_e^S$ given in Eqs. (24) and (25), respectively, we have

$$\frac{dW_S(0)}{d\mu} = \frac{1}{\varepsilon - 1}\frac{-a_{22}}{\rho(\rho - a_{22})}\left[\frac{1}{\rho} + \frac{1}{\theta^*}\frac{\left(\frac{p_S}{p_N}\right)^{\varepsilon-1}\left(1 - \frac{\alpha}{1-\xi^*}\right) - 1}{\xi^*\left(\frac{p_N}{p_S}\right)^{1-\varepsilon} + (1 - \xi^*)}\right]. \qquad (22A)$$

Since $a_{22} < 0$ and $\varepsilon > 1$, the right-hand side of Eq. (22) is positive under the following two sufficient conditions:

(i) $1 - \frac{\alpha}{1-\xi^*} \geq 0$ ; and

(ii) $\theta^*(1 - \xi^*) \geq \rho$.

In the steady-state equilibrium $\theta^*(1 - \xi^*) = m$ and hence conditions (i) and (ii) imply $m \geq Max\{\rho, \alpha\theta^*\}$. Hence using the inequality (B) we have[12]

$$\frac{dW_S(0)}{d\mu} > 0 \ if \ Max\{\rho, \alpha\theta^*\} \leq m < \theta^* \frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}.$$

So we have the following proposition:

*Proposition 2:* If the economies are initially in the steady state, the South gains in welfare due to tightening of IPR if the imitation rate satisfies the following:

$$Max\{\rho, \alpha\theta^*\} \leq m < \theta^* \frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}.$$

Note the contrast between theorem 1 of Helpman (1993, p. 1266) and Proposition 2. In the Helpman (1993) model, the South never gains from tighter IPRs there. However, we prove here that the South gains from tightening of IPR if the imitation rate is neither very low nor very high. This is so because in this case, the positive welfare effect of product availability dominates the negative welfare effect due to change in the interregional allocation of production and the terms of trade. This special

12 See Appendix 7 for details of the derivation.

case does not arise in Helpman (1993) model because both these effects are always negative there.

We now analyze the welfare effect in the North. Note that the North also gains due to greater variety of available products because $\Delta_N > 0$. Also the steady-state equilibrium value of $\theta$ is independent of $\mu$; and hence

$$\Delta_S^N = \int_0^\infty e^{-\rho t}\Big[\frac{dlog(1-\frac{a_N\theta}{L_N})}{d\mu}\Big]dt = 0.$$

The above expression demonstrates that the adjustment of savings rate is not welfare enhancing which contradicts Helpman (1993, eq. (46)). This is because now the size of the R&D sector ($a_N\theta$) is independent of the change in m. Hence there is no intertemporal reallocation of R&D expenditure. The remaining term in Eq. (23) is $\Delta_e^N$ which captures the welfare effect in the North due to change in the terms of trade and due to change in the interregional allocation of production. We can derive $\Delta_e^N$ as follows while the derivation in detail is given in Appendix 8:

$$\Delta_e^N = \Big[\frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}\{1-\frac{\alpha}{\xi}\}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}\Big]\Big[\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho-a_{22})}\Big],$$

where $\frac{p_S}{p_N}$ and $\xi$ take their steady-state values. Since $a_{22} < 0$ we find that $\Delta_e^N > 0$ for $\xi^* \leq \alpha$; and here $\xi^* \leq \alpha$ implies $m \geq (1-\alpha)\theta^*$. Here the welfare effect due to the change in the terms of trade is positive and the welfare effect due to change in the interregional allocation effect is negative. In Appendix 8, we show that the positive terms of trade effect dominates the negative interregional allocation of production effect when $\xi^* \leq \alpha$.

Since $\Delta_S^N = 0$ and $\Delta_N > 0$, we find from Eq. (23) that

$$\frac{dW_N(0)}{d\mu} > 0 \; if \; \xi^* \leq \alpha.$$

So we have the following proposition:

*Proposition 3:* If the economies are initially in the steady state, the North gains due to tightening of IPR if the imitation rate satisfies the following conditions:

$$(1-\alpha)\theta^* \leq m < \theta^* \frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}.$$

Combining Propositions 2 and 3, we find that both North and South gain in welfare due to tightening of IPR if

$$Max\{\rho, \alpha\theta^*, (1-\alpha)\theta^*\} \leq m < \theta^* \frac{L_S\alpha^\varepsilon}{L_S\alpha^\varepsilon + L_N - a_N\theta^*}. \tag{25A}$$

Helpman (1993) does not find any such special case where both North and South may gain. On the otherhand, he finds that both North and South lose in welfare from tightening of IPR policy when the imitation rate is small.

### *4.1 Parameter Simulation*[13]

To have an idea of the interval of the imitation rate provided in the last inequality (25A), we report the results of a simple parameter simulation in Table 1 below. Similar simulation exercises have been done by Helpman (1993, p. 1273), Glass and Saggi (2002), and Lundborg and Segerstrom (2002). We have assumed $L_N = 1$, $L_S = 6$, $\rho = 0.03$, $\alpha = 0.5$, $a_N = 5$, $n(0) = 1$. These values generate the lower limit of m as 0.042 and its upper limit as 0.061 (see Eq. (25A)). To begin with, we take $m = 0.05$ to calculate the steady-state welfare of both the North and the South. The effects of a 0.3 percent reduction in m from 0.05 to 0.047 and from 0.047 to 0.044 on various components of the welfare in steady-state has been shown in the table; and the findings are consistent with our theoretical results.

## 5 Conclusions

This note modifies the Helpman (1993) model assuming that the stock of knowledge capital in the North is measured by the number of firms

13 This is done following the suggestion of one of the referees.

**Table 1.** Steady-state Northern and Southern welfare due to differential IPR protection*

| Welfare due to [15] | | North | | | South | | |
|---|---|---|---|---|---|---|---|
| | | m = 0.05 | m = 0.047 | m = 0.044 | m = 0.05 | m = 0.047 | m = 0.044 |
| 1 | Product availability | 38.89 | 42.22 | 45.56 | 38.89 | 42.22 | 45.56 |
| 2 | Terms of trade and inter-regional allocation of production | 10.05 | 10.41 | 10.72 | −4.35 | −5.02 | −5.75 |
| 3 | Savings rate | −8.01 | −8.01 | −8.01 | – | – | – |
| $W_i(0)$=1+2+3 (i = N, S) | | 40.93 | 44.62 | 48.27 | 34.54 | 37.20 | 39.81 |

* Notes:

(1) $L_N = 1$, $L_S = 6$, $\rho = 0.03$, $\alpha = 0.5$, $a_N = 5$, $n(0) = 1$.

(2) $\frac{1}{m}$ = expected Northern monopoly duration in years.

15 Product availability = $\int_0^\infty e^{-\rho t}[\frac{1}{\varepsilon-1}\log(n(t))]dt$, Terms of trade and interregional allocation of production (in North) = $\int_0^\infty e^{-\rho t}[\frac{1}{\varepsilon-1}\log\{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}\}]dt$, Savings rate = $\int_0^\infty e^{-\rho t}[\log(1-\frac{a_N g}{L_N})]dt$. Analogous expressions can be derived for the South using Eq. (21).

currently producing there. Otherwise the basic Helpman (1993) model with endogenous innovation and without foreign investment remains unchanged in this paper. However this minor modification greatly alters the policy implications.

While the results of Helpman (1993) do not support a policy of tightening IPR in the South, our results may do the opposite. On the basis of the results obtained from this note one may advocate a policy of tightening IPR in the South specially when the rate of imitation is not very small. Both North and South gain in welfare due to tightening of IPR in this case.

So this note reduces the importance of the results of Helpman (1993) who claims that his analysis suggests that the South never benefits from the tightening of IPR and that his result is robust with respect to all the variations that he has examined[14]. This note shows that Helpman's result is not robust at least with respect to the variations in the definition of knowledge capital. This definition of knowledge capital may not be the ideal one. However, this note establishes the importance of further research along those lines with alternative and more meaningful definitions of knowledge capital.

## Appendix 1

*Derivation of Equations (13) and (14)*

Equation (13) can be derived as

$$\frac{\dot{\xi}}{\xi} = \frac{\dot{n_N}}{n_N} - g = \left(\frac{\dot{n}}{n_N} - \frac{\dot{n_S}}{n_N}\right) - \left(\frac{g}{\xi}\right)\xi = (\theta - m) - \theta\xi \Rightarrow \dot{\xi} = \theta\xi - (\theta\xi + m)\xi.$$

To derive Eq. (14) we proceed as follows: We have

$$\frac{\pi_N}{v_N} = \frac{1-\alpha}{\alpha}\left(\frac{L_N}{a_N} - \frac{g}{\xi}\right) \text{ and } \frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - \frac{\dot{n_N}}{n_N} \Longrightarrow \frac{\dot{v_N}}{v_N} = \frac{\dot{p_N}}{p_N} - \left(\frac{\dot{\xi}}{\xi} + g\right);$$

since $n_N = n\xi$.

14 However, see our footnote 10.

Using these two in the no arbitrage condition (11) we solve for $\frac{\dot{p_N}}{p_N}$ as

$$\frac{\dot{p_N}}{p_N} = r_N + m + (\frac{\dot{\xi}}{\xi} + g) - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \frac{g}{\xi}). \quad \text{(A1.1)}$$

From (12), we have $E_N = p_N n_N x_N$. Also from the intertemporal utility maximisation of the representative consumer we get $\frac{\dot{E_N}}{E_N} = r_N - \rho$. Now $E_N = p_N n_N x_N$ imply $\frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} + \frac{(L_N - a_N \dot{\frac{g}{\xi}})}{(L_N - a_N \frac{g}{\xi})}$. Then we have

$$r_N - \rho = \frac{\dot{E_N}}{E_N} = \frac{\dot{p_N}}{p_N} - a_N \frac{(\dot{\frac{g}{\xi}})}{L_N - a_N \frac{g}{\xi}}$$

$$\Longrightarrow \frac{\dot{p_N}}{p_N} = (r_N - \rho) + a_N \frac{\dot{g}\xi - \dot{\xi}g}{(L_N \xi - a_N g)} \frac{1}{\xi}. \quad \text{(A1.2)}$$

Equations (A1.1) and (A1.2) together imply

$$m + \frac{\dot{\xi}}{\xi} + g - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \frac{g}{\xi}) = a_N \frac{\dot{g}\xi - \dot{\xi}g}{(L_N \xi - a_N g)} \frac{1}{\xi} - \rho.$$

Using Eq. (13) and the definition of $\theta$, this last equation implies

$$\dot{\theta} = (\frac{L_N}{a_N} - \theta)[\rho + \theta - \frac{1-\alpha}{\alpha}(\frac{L_N}{a_N} - \theta)].$$

## Appendix 2

*Derivation of the Solution of the Equations of Motions*

Linearizing (13) and (14) around their steady-state values we get:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{\theta}}{\partial \theta}\Big|_{(\theta^*, \xi^*)} & \frac{\partial \dot{\theta}}{\partial \xi}\Big|_{(\theta^*, \xi^*)} \\ \frac{\partial \dot{\xi}}{\partial \theta}\Big|_{(\theta^*, \xi^*)} & \frac{\partial \dot{\xi}}{\partial \xi}\Big|_{(\theta^*, \xi^*)} \end{bmatrix} \cdot \begin{bmatrix} \theta(t) - \theta^* \\ \xi(t) - \xi^* \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} \dot{\theta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \frac{L_N}{a_N} + \rho & 0 \\ \frac{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha - m)m}{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha)^2} & m - ((1-\alpha)\frac{L_N}{a_N} - \rho\alpha) \end{bmatrix} \cdot \begin{bmatrix} \theta(t) - \theta^* \\ \xi(t) - \xi^* \end{bmatrix}.$$

We assume that $m < (1-\alpha)\frac{L_N}{a_N} - \rho\alpha = \theta^*$. Let us denote $a_{11} = \frac{L_N}{a_N} + \rho$, $a_{12} = 0$, $a_{21} = \frac{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha - m)m}{((1-\alpha)\frac{L_N}{a_N} - \rho\alpha)^2}$ and $a_{22} = m - ((1-\alpha)\frac{L_N}{a_N} - \rho\alpha)$.

We have $a_{11} > 0$, $a_{21} > 0$ and $a_{22} < 0$. Since the trace of the matrix on the right-hand side is positive and the determinant is negative, it has one positive root ($a_{11}$) and one negative root ($a_{22}$). This proves that the steady-state equilibrium point is a saddle point. We choose the eigenvector corresponding to $a_{11}$, the positive root, as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and the eigenvector corresponding to $a_{22}$, the negative root, as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. To ensure long run convergence we choose that at time zero $\theta(t)$ takes the value $\theta^*$, i.e., $\theta(0) = \theta^*$. This procedure leads to the solution

$$\begin{aligned} \theta(t) &= \theta^*, \\ \xi(t) &= \xi^* + [\xi(0) - \xi^*]e^{a_{22}t}. \end{aligned} \tag{26}$$

## Appendix 3

*Feasibility Restriction on m*

To find the feasibility restriction on m given in inequality (B) we proceed as follows:

$$\frac{w_N}{w_S} > 1 \Rightarrow \frac{p_N}{p_S} > \frac{1}{\alpha} \Rightarrow (\frac{x_N}{x_S})^{-\frac{1}{\varepsilon}} > \frac{1}{\alpha} \Rightarrow \frac{x_N}{x_S} < \alpha^{\varepsilon} \Rightarrow \frac{L_N - a_N\theta}{L_S}\frac{(1-\xi)}{\xi} < \alpha^{\varepsilon}$$
$$\Rightarrow \frac{1}{\xi} - 1 < \frac{L_S\alpha^{\varepsilon}}{L_N - a_N\theta} \Rightarrow \xi > \frac{L_N - a_N\theta}{L_S\alpha^{\varepsilon} + L_N - a_N\theta}.$$

The values of $(\theta, \xi)$ satisfying the above inequality would ensure $w_N > w_S$. This region in the $(\theta, \xi)$ axis is shown by the area above the WW curve (see Fig. 1). Now in the steady state we have $\xi = \frac{\theta - m}{\theta}$ and then using the above inequality we get:

$$\frac{\theta - m}{\theta} > \frac{L_N - a_N\theta}{L_S\alpha^{\varepsilon} + L_N - a_N\theta} \Rightarrow m < \theta\frac{L_S\alpha^{\varepsilon}}{L_S\alpha^{\varepsilon} + L_N - a_N\theta},$$

where $\theta$ takes its steady-state value.

## Appendix 4

*Derivation of Instantaneous Utility Functions (20) and (21)*

We can write the instantaneous demand function as $x_b(j) = p(j)^{-\varepsilon} \frac{E_b}{P^{1-\varepsilon}}$, where $P = [\int_0^n p(j)^{1-\varepsilon} dj]^{\frac{1}{1-\varepsilon}}$ and b = N, S. Substituting this demand function into the instantaneous utility function we obtain the indirect utility function

$$logU_b = logE_b - logP.$$

Now,

$$P^{1-\varepsilon} = n_N p_N^{1-\varepsilon} + n_S p_S^{1-\varepsilon} \Longrightarrow P = p_S n^{\frac{1}{1-\varepsilon}} [\xi (\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]^{\frac{1}{1-\varepsilon}}$$

or

$$P = p_N n^{\frac{1}{1-\varepsilon}} [\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}.$$

In the South, per capita income is the wage rate $w_S = p_S$. Then $E_S = p_S$ and in North it is $E_N = p_N(1 - \frac{a_N \theta}{L_N})$. Therefore,

$$logU_S = \frac{1}{\varepsilon - 1} log(n) + \frac{1}{\varepsilon - 1} log[\xi (\frac{p_N}{p_S})^{1-\varepsilon} + (1-\xi)]$$

and

$$logU_N = \frac{1}{\varepsilon - 1} log(n) + \frac{1}{\varepsilon - 1} log[\xi + (1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}] + log(1 - \frac{a_N \theta}{L_N}).$$

## Appendix 5

*To Prove $\Delta_N > 0$*

$log\,(n(t)) = log\,(n(0)) + \int_0^t g(\tau) d\tau \Rightarrow \frac{d\,log\,(n(t))}{d\mu} = \int_0^t \frac{dg(\tau)}{d\mu} d\tau = \int_0^t (1 - e^{a_{22}\tau}) d\tau$ *from (19)* $= \int_0^t d\tau - \int_0^t e^{a_{22}\tau} d\tau = t - [\frac{1}{a_{22}} e^{a_{22}\tau}]_0^t = t + \frac{1}{a_{22}} - \frac{1}{a_{22}} e^{a_{22} t}$. Then,

$$\Delta_N = \int_0^\infty e^{-\rho t}\frac{dlog(n(t))}{d\mu}dt = \int_0^\infty e^{-\rho t}\Big(t + \frac{1}{a_{22}} - \frac{1}{a_{22}}e^{a_{22}t}\Big)dt$$
$$+\frac{1}{a_{22}}\int_0^\infty e^{-\rho t}dt - \frac{1}{a_{22}}\int_0^\infty e^{-(\rho-a_{22})t}dt$$
$$= \frac{1}{\rho^2} + \frac{1}{\rho a_{22}} - \frac{1}{(\rho - a_{22})a_{22}} = \frac{-a_{22}}{\rho^2(\rho - a_{22})} > 0.$$

## Appendix 6

*To Prove* $\Delta_e^S < 0$

$$\Delta_e^S = \int_0^\infty e^{-\rho t}\frac{dlog[\xi(\frac{p_S}{p_N})^{\varepsilon-1} + (1-\xi)]}{d\mu}]dt$$
$$= \int_0^\infty e^{-\rho t}\frac{1}{\xi(\frac{p_S}{p_N})^{\varepsilon-1} + (1-\xi)}[(\frac{p_S}{p_N})^{\varepsilon-1}\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu} + \frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{d\mu}\xi].$$

The first two terms in the third bracket of the last expression $[(\frac{p_S}{p_N})^{\varepsilon-1}\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu}]$ captures the effect of interregional allocation of production on welfare (keeping the terms of trade unchanged). Since $\frac{p_S}{p_N} < 1$, $(\varepsilon - 1) > 0$ and $\frac{d\xi(t)}{d\mu} > 0$ the welfare of the South decreases due to interregional allocation of production only. The last term in the third bracket of the last expression $[\frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{d\mu}\xi]$ captures the welfare change due to terms of trade effect only. We have $(\frac{p_S}{p_N})^{\varepsilon-1} = (\frac{L_N - a_N\theta}{L_S}\frac{1-\xi}{\xi})^\alpha$ at the steady state. Then $\frac{d(\frac{p_S}{p_N})^{\varepsilon-1}}{d\mu}\xi = -\frac{d\xi(t)}{d\mu}\frac{\alpha}{1-\xi}(\frac{p_S}{p_N})^{\varepsilon-1}$. This is clearly negative. Hence welfare of the South decreases due to the change in the terms of trade only. The expression for $\Delta_e^S$ is given by

$$\Delta_e^S = \int_0^\infty e^{-\rho t}\frac{1}{\xi(\frac{p_S}{p_N})^{\varepsilon-1} + (1-\xi)}\Big[\Big(\frac{p_S}{p_N}\Big)^{\varepsilon-1} - 1 - \Big(\frac{p_S}{p_N}\Big)^{\varepsilon-1}\frac{\alpha}{1-\xi}\Big]\Big(\frac{d\xi(t)}{d\mu}\Big)dt$$
$$= \frac{(\frac{p_S}{p_N})^{\varepsilon-1} - 1 - (\frac{p_S}{p_N})^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi(\frac{p_S}{p_N})^{\varepsilon-1} + (1-\xi)}[\int_0^\infty e^{-\rho t}\Big(\frac{d\xi(t)}{d\mu}\Big)dt]$$

$$= \frac{\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} - 1 - \left(\frac{p_S}{p_N}\right)^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} + (1-\xi)}\left[\int_0^\infty e^{-\rho t}\frac{1}{\theta^*}(1-e^{a_{22}})dt\right]$$

$$= \frac{\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} - 1 - \left(\frac{p_S}{p_N}\right)^{\varepsilon-1}\frac{\alpha}{1-\xi}}{\xi\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} + (1-\xi)}\left[\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho-a_{22})}\right] < 0,$$

where $\frac{p_S}{p_N}$ and $\xi$ are measured at their steady state.

## Appendix 7

*To prove* $\frac{dW_S(0)}{d\mu} > 0$

$$\frac{dW_S(0)}{d\mu} = \frac{1}{\varepsilon-1}(\Delta_N + \Delta_e^S)$$

$$= \frac{1}{\varepsilon-1}\frac{-a_{22}}{\rho(\rho-a_{22})}\left[\frac{1}{\rho} + \frac{1}{\theta^*}\frac{\left(\frac{p_S}{p_N}\right)^{\varepsilon-1}\left(1-\frac{\alpha}{1-\xi^*}\right)-1}{\xi^*\left(\frac{p_N}{p_S}\right)^{1-\varepsilon} + (1-\xi^*)}\right]$$

$$= \frac{1}{\varepsilon-1}\frac{-a_{22}}{\rho(\rho-a_{22})}\left[\frac{\theta^*\xi^*\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} + \theta^*(1-\xi^*) + \rho\left(1-\frac{\alpha}{1-\xi^*}\right)\left(\frac{p_S}{p_N}\right)^{\varepsilon-1} - \rho}{\rho\theta^*\{\xi^*\left(\frac{p_N}{p_S}\right)^{1-\varepsilon} + (1-\xi^*\}}\right].$$

The numerator in the third bracket of the last expression is positive under the sufficient assumption that $(1-\frac{\alpha}{1-\xi^*}) \geq 0$ and $\theta^*(1-\xi^*) \geq \rho$. Now, $(1-\frac{\alpha}{1-\xi^*}) \geq 0$ if $m \geq \alpha\theta^*$, [note that $1-\xi^* = \frac{m}{\theta^*}$]. Also $\theta^*(1-\xi^*) \geq \rho$ is true if $m \geq \rho$. Therefore, for $m \geq max(\rho, \alpha\theta^*)$ we have $\frac{dW_S(0)}{d\mu} > 0$.

## Appendix 8

*To prove* $\Delta_e^N > 0$

$$\Delta_e^N = \int_0^\infty e^{-\rho t}\frac{dlog[\xi + (1-\xi)\left(\frac{p_S}{p_N}\right)^{1-\varepsilon}]}{d\mu}dt$$

$$= \int_0^\infty e^{-\rho t}\frac{1}{\xi + (1-\xi)\left(\frac{p_S}{p_N}\right)^{1-\varepsilon}}\left[\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu}\left(\frac{p_S}{p_N}\right)^{1-\varepsilon} + (1-\xi)\frac{d\left(\frac{p_S}{p_N}\right)^{1-\varepsilon}}{d\mu}\right]dt.$$

The first two terms in the third bracket of the last expression $[\frac{d\xi}{d\mu} - \frac{d\xi}{d\mu}\left(\frac{p_S}{p_N}\right)^{1-\varepsilon}]$ capture the welfare effect due to changes in the interregional allocation of production only (keeping the terms of trade unchanged). This is clearly negative. The last term $[(1-\xi)\frac{d(\frac{p_S}{p_N})^{1-\varepsilon}}{d\mu}]$ captures the welfare change due to changes in the terms of trade only. We have got $(1-\xi)\frac{d(\frac{p_S}{p_N})^{1-\varepsilon}}{d\mu} = \frac{\alpha}{\xi}(\frac{p_S}{p_N})^{1-\varepsilon}\frac{d\xi}{d\mu}$. This is clearly positive. Then the expression for $\Delta_e^N$ looks like

$$\begin{aligned}\Delta_e^N &= \frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}+\frac{\alpha}{\xi}(\frac{p_S}{p_N})^{1-\varepsilon}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}\int_0^\infty e^{-\rho t}[\frac{d\xi(t)}{d\mu}]dt \\ &= \frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}+\frac{\alpha}{\xi}(\frac{p_S}{p_N})^{1-\varepsilon}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}\int_0^\infty e^{-\rho t}[\frac{1}{\theta^*}(1-e^{a_{22}})]dt \\ &= \frac{1-(\frac{p_S}{p_N})^{1-\varepsilon}\{1-\frac{\alpha}{\xi}\}}{\xi+(1-\xi)(\frac{p_S}{p_N})^{1-\varepsilon}}[\frac{1}{\theta^*}\frac{-a_{22}}{\rho(\rho-a_{22})}],\end{aligned}$$

where $\frac{p_S}{p_N}$ and $\xi$ take their steady-state value.

The numerator of the first term in the last expression is positive if $\xi^* \le \alpha \Rightarrow (1-\frac{m}{\theta^*}) \le \alpha \Rightarrow m \ge (1-\alpha)\theta^*$. The second term in the third bracket of the last expression is always positive. Hence $\Delta_e^N > 0$ under the condition $m \ge (1-\alpha)\theta^*$.

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