# Product Development, Imitation and Economic Growth: A Note

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ABSTRACT *A dynamic North–South general equilibrium model of international product cycle is presented in this paper. The qualitative effects of strengthening intellectual property rights (IPR) on the balanced growth rate of the world economy is studied in two alternative cases: (i) imitation is direct from North to South; (ii) multinationalization is the channel of product transfer.*

KEY WORDS: Product variety, imitation, intellectual property rights, steady-state growth, multinational, knowledge capital, weights

## Introduction

Technological change plays the most important role in determining the rate of economic growth of a country. Strengthening the Intellectual Property Rights (IPR) is an important factor that motivates technological change. This issue has received much attention in recent times. The agreement on the Trade Related Intellectual Property issues (TRIPs) under the GATT-WTO of 1994 requires that the developing countries should strengthen their intellectual property rights (IPR) protection. Formal scientific studies on the effects of strengthening IPR on the rate technological progress/economic growth are also available in the theoretical as well as empirical literature of economics; and there has been an ongoing debate on this issue.

Existing theoretical literature is based on either one of the two alternative frameworks provided by Grossman & Helpman (1991a, 1991b) – product variety framework and Quality ladder framework.[1] Models developed by Grossman & Helpman (1991b), Helpman (1993), Lai (1998) etc. are based on product variety approach according to which technological change is viewed as product development. In these models, R&D sector develops new product designs (technology) using labour as input and thus the number of products

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(varieties) grow over time. In addition, all these models assume a world consisting of an innovative North and an imitating South and consider a steady-state growth equilibrium of the entire world economy. In Grossman & Helpman (1991b), the imitation rate in South is endogenously determined. However, this rate of imitation is exogenous in Helpman (1993) and in Lai (1998). Grossman & Helpman (1991b), and Helpman (1993) did not consider the multinationalization of Northern firms and in their model imitation is assumed to be direct. In Lai (1998), the Southern firms can imitate only after multinationalization of the Northern firms.

The nature of the effect on the rate of innovation due to strengthening IPR also varies from model to model. In Grossman & Helpman (1991b) and in Helpman (1993), strengthening IPR in the South leads to a fall in the rate of innovation in the North. However, Lai (1998) has shown that the rate of imitation will fall due to stronger IPR protection if multinationalization is the channel of production transfer.

In all these mentioned papers the production function in the Northern R&D sector has a very simple structure. It has two important characteristics. One, there is scale effect and that is questioned by Jones (1995). Second, all the past innovations receive equal weight in the knowledge spillover specification although the different innovations have taken place at different dates. The innovations of the recent past are always more important in the knowledge spillover than the innovations that took place long ago. So we question this second characteristic in this paper and modify the concept of knowledge capital such that South-based products have relatively lower weights than North-based products. This makes sense if the products imitated and produced in the South are older than those being produced in the North. It should be noted that Dollar (1986, 1987) in his North–South model of product cycle assumes that the Southern products do not receive any weight in the knowledge spillover specification. However, Dollar (1986, 1987) does not analyse the effect of strengthening IPR on the endogenous growth rate. According to Vernon (1966) a Northern innovating firm will think for a transfer of its production to lower wage compared with a Southern country once its products have been standardized in the home country. This takes some time.

This is the only minor change in assumption we introduce in this present note. However, this gives interesting results. If we introduce this change in an otherwise Helpman (1993) model, we find that the policy of strengthening IPR will raise the rate of innovation in the North if the weight attached to the South-based products is very small compared with that attached to the North-based products. This result is completely opposite to that found in Helpman (1993). However, we get the Helpman (1993) result when the weight attached to the South-based products is close enough to that attached to the North-based products. When multinationalization is the channel of production transfer then the rate of innovation in North will fall due to stronger protection in the South if South-based products have very low weights. This result is opposite to that obtained in Lai (1998). However the result of Lai (1998) is valid if the weights given to the South-based products and North-based products are close to each other.

We define the knowledge capital in the following section. In the section after, we consider the benchmark model which is otherwise identical to Helpman (1993). In the fourth section, we introduce multinationalization as the channel of production transfer in the basic model; and so the model presented in this section is otherwise similar to that in Lai (1998). Concluding remarks are given in the fifth section.



## Knowledge Capital

In the existing literature, knowledge capital to which labour productivity in the R&D sector in the North is proportional, is defined as the sum of all products produced in North and South. So $(n_N + n_S)$ is the knowledge capital where $n_N$ and $n_S$ are the numbers of products produced in North and imitated and produced in South, respectively. This means that innovation of all the old products contributes to human capital development at an equal rate irrespective of the date of innovation. This is a restrictive assumption when the blueprints of different products develop at different points of time. Any innovation of the recent past should contribute to the knowledge capital development at a higher rate than that of an the innovation that occurred long ago. Ideally $\int_0^t \dot{n}(\tau)\omega(\tau)d\tau$ should be defined as knowledge capital where $\dot{n}(\tau)$ is the number of blueprints developed at point $\tau$, and $\omega(\tau)$ is the weight given to them. Here $\omega'(\tau) < 0$ implies that the older innovations receive lower weights. If $\omega'(\tau) = 0$, then $\int_0^t \dot{n}(\tau)\omega(\tau)d\tau$ is proportional to $n(t) = n_N(t) + n_S(t)$; and we find this in the existing literature.

We modify the knowledge capital as $(n_N + \lambda n_S)$ where $0 \leq \lambda \leq 1$. Here, all the non-imitated products and all the imitated products receive equal treatment within their own group. However, $\lambda < 1$ implies that the representative non-imitated product contributes to the knowledge development at a higher rate than the representative imitated product. This definition is not ideal but less crude than the existing one because generally the older products are imitated. If $\lambda = 0$, then our definition of knowledge capital is identical to that in Dollar (1986, 1987).

## The Basic Model

This model is identical to that in Helpman (1993) with the only change being in the definition of knowledge capital. The representative consumer maximizes welfare given by

$$W = \int_t^{\infty} e^{-\rho(\tau - t)} log U(\tau) d\tau$$

subject to the intertemporal budget constraint given by

$$\int_t^{\infty} e^{-r(\tau-t)}E(\tau)d\tau = \int_t^{\infty} e^{-r(\tau-t)}I(\tau)d\tau + A(t) \quad \text{for all } t$$

Here $U(t)$ is the instantaneous utility function given by

$$U(t) = \left(\int_0^{n(t)} x(z)^{\alpha}dz\right)^{\frac{1}{\alpha}}; \quad 0 < \alpha < 1$$

Here, $E(\tau)$, $I(\tau)$, $n(\tau)$ and $A(\tau)$ stand for instantaneous expenditure, instantaneous income, number of existing variety and the current value of assets respectively at time $\tau$. $x(z)$ represents the level of consumption (production) of the $z$th variety; $\rho$ and $r$ stand for the rate of time preference and the nominal interest rate respectively. $\alpha$ represents the elasticity of substitution between any two varieties.

Solving this problem of a price-taker consumer we obtain the following optimality conditions:

$$\frac{\dot{E}}{E} = r - \rho \tag{1}$$

and

$$x(z) = E(t)\frac{p(z)^{-\varepsilon}}{\int_0^{n(t)} p(u)^{1-\varepsilon}du} \quad \forall z \in (0, n(t)) \tag{2}$$

Here, equation (1) implies the Ramsey rule and equation (2) represents the demand function for the $z$th variety. $p(z)$ is the price of the variety $z$ and

$$\varepsilon = \frac{1}{1-\alpha} > 1$$

is the price-elasticity of demand for the $z$th variety. Here,

$$n = n_N + n_S$$

and $n_N$ ($n_S$) is the number of varieties produced in the North (South). North is the innovator country and South is the imitator. The producer of the $z$th variety produced in the North is a profit maximizing monopolist while the Southern imitators play the Bertrand game. One unit of labour can produce one unit of each product.[2] Labour is internationally immobile but is perfectly mobile among all the sectors within a country. So the price of any Northern product is given by

$$p(z) = p_N = \frac{w_N}{\alpha} \tag{3}$$

for all $z \in [0, n_N]$; and the price of an imitated Southern product is given by

$$p(z) = p_S = w_S \tag{4}$$

for all $z \in [0,\ n_S]$. Here $p_N$ ($p_S$) and $w_N$ ($w_S$) represent equilibrium price[3] of any Northern (Southern) variety and equilibrium wage[4] of Northern (Southern) labour respectively.

In the North, labour is employed in the production of Northern varieties and in the R&D sector. So the labour market equilibrium in the North is given by

$$L_N = n_N x_N + L_r \tag{5}$$

where $L_N$, $n_N x_N$ and $L_r$ stand for the labour endowment, labour employed in production sector[5] and labour employed in the R&D sector. In the South, imitation is costless and the entire Southern labour, $L_S$, is employed in producing the imitated varieties. Hence

$$L_S = n_S x_S \tag{6}$$

is the labour market equilibrium condition in the South.

R&D sector in the North produces new product designs using labour as the only input; and thus the number of varieties grow over time. This equation of motion is given by

$$\dot{n} = \frac{n_N + \lambda n_S}{a_N} L_r \tag{7}$$

where $\frac{a_N}{n_N + \lambda n_S}$ is the labour requirement to develop a new product-design; and $(n_N + \lambda n_S)$ is the knowledge capital. Here $0 \leq \lambda \leq 1$; and if $\lambda = 1$, then we return to Helpman (1993) where this equation of motion is

$$\dot{n} = \frac{n}{a_N} L_r$$

Using equation (7) we have

$$\frac{\partial \dot{n}}{\partial L_r} = \frac{n - (1 - \lambda) n_S}{a_N}$$

This means that the marginal productivity of labour in the R&D sector varies directly with $\lambda$. The marginal product of labour is maximum for $\lambda = 1$ and minimum for $\lambda = 0$ given $n_N$ and $n_S$.

The free-entry condition in the R&D sector in the North is given by

$$\frac{a_N}{n_N + \lambda n_S} w_N = \frac{\pi_N}{r + m} \tag{8}$$

where the left-hand side of equation (8) is the cost of developing a new variety and the right-hand side is the present value of the expected stream of profits of a Northern monopolist producing the representative variety. Here $\pi_N$ is the maximum profit of the Northern monopolist producing any variety; and using equation (3) we have

$$\pi_N = \frac{1 - \alpha}{\alpha} w_N x_N \tag{9}$$

Here $m$ stands for the exogenous rate of imitation defined as

$$m = \frac{\dot{n}_S}{n_N}$$

The world is in a steady-state growth equilibrium and hence

$$\frac{\dot{n}}{n} = \frac{\dot{n}_N}{n_N} = \frac{\dot{n}_S}{n_S} = g$$

where $g$ is the balanced growth-rate. The market value of each firm is normalized to unity and hence

$$\frac{\dot{E}}{E} = \frac{\dot{n}}{n}$$

The Walras law is to be satisfied and labour-endowments $L_N$ and $L_S$ are exogenously given. So, in the steady-state equilibrium, we have

$$g = \frac{\dot{n}}{n} = \frac{\dot{n}_N}{n_N} = \frac{\dot{n}_S}{n_S} = \frac{\dot{w}_S}{w_S} = \frac{\dot{w}_N}{w_N} = \frac{\dot{E}}{E} \tag{10}$$

This completes the equational structure of the model. Now we solve for the long run rate of innovation, $g$.

Using equations (5), (7) and (10), we have

$$n_N x_N = L_N - a_N g \left( \frac{g + m}{g + \lambda m} \right) \tag{11}$$

which shows the steady-state equilibrium supply (availability) of labour to the Northern production sector and it varies inversely with $g$ and positively with $\lambda$.

Next, using equations (1), (8), (9) and (10), we have

$$n_N x_N = \frac{a_N g(\rho + g + m)}{g + \lambda m} \frac{\alpha}{1 - \alpha}. \tag{12}$$

This is the steady-state equilibrium demand function for labour in the Northern production sector, which is obtained from the free-entry condition in the R&D sector, which equates the profit rate of the firm to the effective cost of capital. This labour demand varies positively with $g$ and $m$ and inversely with $\lambda$.

Then equating the right-hand side of the two equations derived above we obtain the following equation

$$g^2\left(\frac{a_N}{1-\alpha}\right) - g\left[L_N - \left(\frac{\rho\alpha a_N}{1-\alpha}\right) - \left(\frac{m a_N}{1-\alpha}\right)\right] - L_N \lambda m = 0 \tag{13}$$

This is a quadratic equation having two roots. In Appendix A, we show that one root is negative and the other is positive. The negative root implies negative growth-rate in the steady-state equilibrium, which does not make sense because employment in the R&D sector cannot take a negative value. So we consider the positive root given by

$$g = \frac{x + \sqrt{x^2 + 4\frac{a_N L_N}{1-\alpha}\lambda m}}{2\frac{a_N}{1-\alpha}} \tag{14}$$

where $x = (L_N - \frac{a_N \rho\alpha}{1-\alpha} - \frac{m a_N}{1-\alpha})$.

Using equation (14) it can be shown that

$$\frac{\partial g}{\partial m} = -\frac{1}{2} + \frac{\lambda L_N - \frac{x}{2}}{\sqrt{x^2 + 4(\frac{a_N}{1-\alpha})L_N \lambda m}} \tag{15}$$

and the derivation in detail is shown in Appendix A. Here $\frac{\partial g}{\partial m} > 0$ if $\lambda > \hat{\lambda}$ and $\frac{\partial g}{\partial m} < 0$ if $\lambda < \hat{\lambda}$ where

$$\hat{\lambda} = \frac{L_N - \frac{a_N \rho\alpha}{1-\alpha}}{L_N}$$

and it is obvious that $0 \leq \hat{\lambda} \leq 1$. Strengthening IPR lowers the rate of imitation, $m$. So we have the following proposition

**Proposition 1**

*Strengthening IPR raises (lowers) the innovation rate in the North if $\lambda$ is lower (greater) than $\hat{\lambda}$.*

This is the more general result; and the Helpman (1993) result is obtained as a special case when $\lambda = 1$. When $\lambda$ takes a very low value, i.e. when the South-based products contribute to the knowledge capital at a far lower rate compared to the rate of contribution of the North-based products, strengthening IPR leads to an increase in the rate of innovation. This is just the opposite of the result obtained in Helpman (1993).

We now turn to provide an intuitive explanation of this result. Here, equation (11) represents the supply (availability) of labour to the production sector in the North; and equation (12) represents the demand for labour from the production sector in the North. Both the demand for and the supply of labour in the production sector are expressed as functions of the innovation rate, $g$; and equating the supply to demand we determine the value of $g$. So the steady-state equilibrium rate of innovation, which is also equal to the rate of growth of wage-rate in each of the two countries, clears the Northern labour market. Equation (11) shows that the supply curve of labour slopes negatively and equation (12) shows that the demand curve for labour slopes positively. The labour market diagram is presented in Figure 1. The DD curve is the demand curve and the SS curve is the supply curve.[6]

In Helpman (1993), when $\lambda = 1$, the decrease in the imitation rate caused by strengthening IPR, causes an upward shift of the demand for labour curve. This is so because a stronger IPR protection leads to a reduction in both the effective cost of capital $(r + m)$ as well as the profit rate ($\frac{\pi_N}{v_N}$ where $v_N$ is the cost of developing a new variety) and its impact on the effective cost of capital is smaller in size than the impact on the profit rate. However, this does not cause any shift of the supply curve because with $\lambda = 1$, the labour

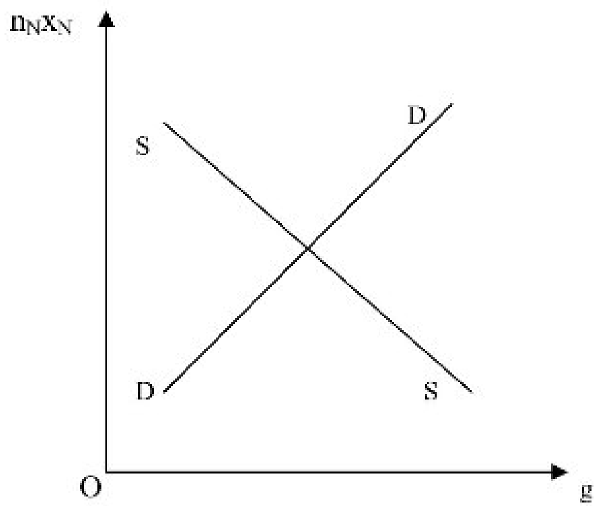


**Figure 1.** The labour market equilibrium

requirement in the R&D sector to maintain a given rate of innovation is independent of the rate of imitation. So $g$ falls in the new equilibrium. However, in the present case with $0 \leq \lambda \leq 1$, the decrease in $m$ causes an upward shift of the supply curve because with a lower threat of imitation the R&D sector can maintain a given rate of innovation with a smaller amount of labour. The demand curve may also shift downwards when $\lambda$ is very low, because when spillover from the imitated product is too low in the Northern R&D sector then the impact of the stronger IPR protection on the effective cost of capital may be higher than its impact on the profit rate. So $g$ may rise in the new equilibrium when $\lambda$ takes a very low value. In the extreme case, with $\lambda = 0$, there is no impact on the profit rate; and so only the reduction in the effective cost of capital matters, causing a downward shift of the demand curve. $g$ must rise in the new equilibrium in this case owing to the upward shift of the supply curve.

### Multinationalization and Imitation

In this section we consider multinationalization as the channel of production transfer from the North to the South; and this extended model is similar to that of Lai (1998). A Northern firm will decide in equilibrium whether to transfer production to the South to take the advantage of lower wage there. We will assume that the South can imitate a multinational (MNC) firm's product and direct imitation of the Northern based product is not possible. Then due to lower unit cost of production in the South, MNC's in the South earns a higher profit at each date compared with their Northern counterpart but they also face the extra risk of being imitated at the next instant by a Southern firm. Production technology for both the imitator and the MNC is the same – one unit of labour can produce one unit of the output. Once a MNC's product is imitated it is sold at a price equal to the marginal cost since both the imitator Southern firm and the South-based MNC face the same marginal cost of production and there is price competition between them. The MNC loses all its profits once its product gets imitated.

Here

$$n = n_N + n_S = n_N + (n_m + n_i)$$

where $n_N$, $n_m$ and $n_i$ stand for the number of North-based products, South-based MNC products, and South based imitated products respectively. Here

$$m = \frac{\dot{n}_i}{n_m}$$

is the imitation rate and

$$\omega = \frac{\dot{n}_S}{n_N}$$

is the multinationalization rate.[7]

Consumer behaviour is identical to that described in the earlier section. So equation (1) remains unchanged. The demand function for different types of products are given by the following:

$$x_N = E\frac{p_N{}^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}du} \tag{2E.1}$$

$$x_m = E\frac{p_m{}^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}du} \tag{2E.2}$$

$$x_i = E\frac{p_i{}^{-\varepsilon}}{\int_0^n p(u)^{1-\varepsilon}du} \tag{2E.3}$$

where $x_N$, $x_m$ and $x_i$ stand for the level of demand for the product produced by the North-based firm, MNC and the Southern imitator respectively; and $p_N$, $p_m$ and $p_i$ stand for their corresponding prices. In equilibrium

$$p_N = \frac{w_N}{\alpha} \tag{3E.1}$$

$$p_m = \frac{w_S}{\alpha} \tag{3E.2}$$

and

$$\text{p}_i = w_S \tag{4E}$$

The labour market equilibrium conditions in the North and South are given by

$$L_N = n_N x_N + L_R \tag{5E}$$

$$L_S = n_m x_m + n_i x_i \tag{6E}$$

respectively. The MNC products are produced by Southern labour.

Equation (7) in the basic model, which shows that the intertemporal growth of the number of varieties also remains unchanged here.[8] Equation (8) of the basic model is now modified as follows

$$\frac{a_N w_N}{n_N + \lambda n_S} = \frac{\pi_N}{r} \tag{8E.1}$$

and

$$\frac{a_N w_N}{n_N + \lambda n_S} = \frac{\pi_m}{r + m} \tag{8E.2}$$

The difference arises because it is the MNC not the Northern firm who faces the risk of imitation.

The maximum profits of a Northern monopolist and of a MNC are given by

$$\pi_N = \frac{1-\alpha}{\alpha} w_N x_N \tag{9E.1}$$

and

$$\pi_m = \frac{1-\alpha}{\alpha} w_S x_m \tag{9E.2}$$

respectively.

Steady-state equilibrium growth condition is given by

$$g = \frac{\dot{n}}{n} = \frac{\dot{n}_N}{n_N} = \frac{\dot{n}_m}{n_m} = \frac{\dot{n}_i}{n_i} = \frac{\dot{w}_N}{w_N} = \frac{\dot{w}_S}{w_S} = \frac{\dot{E}}{E} \tag{10E}$$

This model can solve for $g$ and $\omega$ simultaneously. It is otherwise identical to Lai (1998). The difference lies only in equation (7).

Using equations (8E.1) and (9E.1) we have

$$\frac{a_N}{n_N + \lambda n_S} w_N = \frac{\frac{1-\alpha}{\alpha} w_N x_N}{r}$$

Now we use equations (1), (10E) and the above-mentioned equation and obtain the following equation

$$n_N x_N = \frac{\alpha}{1-\alpha}(\rho + g) a_N \frac{g}{g + \lambda\omega} \tag{12E}$$

This shows the demand for labour in the production sector in the North. This is independent of the imitation rate, $m$, because the Northern based firms do not face the threat of imitation. MNCs who face the threat of imitation do not use the Northern labour. The supply of Northern labour in the production sector can be found from the Northern labour market equilibrium condition (5E) and equation (7). It is given by

$$n_N x_N = L_N - a_N g \frac{g + \omega}{g + \lambda\omega} \tag{11E}$$

Now using equations (11E) and (12E) we obtain the labour market clearing equation in the Northern production sector

$$\frac{g}{\omega} = \frac{\lambda L_N - a_N g}{\frac{\alpha}{1-\alpha} a_N(\rho + g) - L_N + a_N g} \tag{13E}$$

Next, using equations (8E.1) and (8E.2) we have

$$\frac{\pi_N}{\pi_m} = \frac{r}{r+m}$$

and then using equations (1), (9E.1), (9E.2), (2E.2),(2E.3), (3E.1), (3E.2), (4E), (5E), (6E) and (10E) we can express the above mentioned equation in the following form[9]

$$\left[\frac{L_N(g+\lambda\omega) - a_N g(g+\omega)}{(g+\lambda\omega)L_S} \cdot \frac{\omega}{g} \cdot \left(\frac{g}{g+m} + \frac{m}{g+m}\alpha^{-\varepsilon}\right)\right]^{\alpha} = \frac{\rho+g}{\rho+g+m} \tag{ME}$$

This is the multinationalization equilibrium condition. The policy of strengthening IPR affects this condition because the imitation rate, $m$, enters into it. The MNCs face the threat of imitation and so their discounted present value of profit varies inversely with the rate of imitation. This does not happen for Northern-based firms because imitation is not direct here.

If $\lambda = 1$, from equation (ME) we have

$$\left[\frac{L_N - a_N g}{L_S} \cdot \frac{\omega}{g} \cdot \left(\frac{g+m\alpha^{-\varepsilon}}{g+m}\right)\right]^{\alpha} = \frac{\rho+g}{\rho+g+m}$$

and this is identical to the multinationalization equilibrium condition as derived in Lai (1998). Using equations (13E) and (ME), we solve for $g$ and $\omega$. In Lai (1998), the multinationalization equilibrium condition gives a clear positive relationship between $\omega$ and $g$. However, for $0 \le \lambda \le 1$, this is not necessarily true. Indeed, the relationship is inverse when $g$ and/or $\omega$ take high values. In Lai (1998), i.e. with $\lambda = 1$, we find a positive relationship between $g$ and $\omega$ from equation (13E) for all $g < (\frac{L_N}{a_N})$. This is not necessarily true for $0 \le \lambda \le 1$. Indeed, in the other extreme case of $\lambda = 0$, equation (13E) gives a negative relationship between $g$ and $\omega$ for

$$g < \frac{(L_N - \rho a_N \frac{\alpha}{1-\alpha})}{\frac{a_N}{1-\alpha}}$$

From equation (13E) we solve $\omega$ in terms of $g$ and then, replacing this value of $\omega$ into (ME), we obtain the following equation

$$\left[\frac{\frac{\alpha}{1-\alpha} r}{L_S} \cdot \frac{\frac{\alpha}{1-\alpha} r a_N - L_N + a_N g}{\frac{\alpha}{1-\alpha}\lambda r + \lambda g - g} \cdot \frac{g+m\alpha^{-\varepsilon}}{g+m}\right]^{\alpha} = \frac{r}{r+m} \tag{14E}$$

This is one equation in one unknown, $g$. For $\lambda = 1$ in equation (14E) we get the equation of Lai (1998). We assume that

$$L_N > \rho a_N \frac{\alpha}{1-\alpha}$$

which implies that the North can sustain a positive rate of innovation as a closed economy. In Appendix C we show that there exists a unique equilibrium solution for $g$ for a given value of $\lambda$. For

$$0 \leq \lambda < \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{L_N(1 + \frac{\alpha}{1-\alpha})}$$

the equilibrium value of $g$ satisfies the following:

$$\frac{\rho\lambda\frac{\alpha}{1-\alpha}}{1 - \lambda(1 + \frac{\alpha}{1-\alpha})} < g < \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{a_N(1 + \frac{\alpha}{1-\alpha})}$$

Also $\frac{\partial g}{\partial m} \succ 0$ at the equilibrium point. In Appendix C, we also show that a positive equilibrium solution for $\omega$ exists for $\lambda$ satisfying

$$0 \leq \lambda < \frac{a_N g}{L_N}$$

Therefore for $\lambda$ satisfying

$$0 \leq \lambda < min\left\{\frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{L_N(1 + \frac{\alpha}{1-\alpha})}, \frac{a_N g}{L_N}\right\}$$

we have positive equilibrium solutions for both $g$ and $\omega$ with $\frac{\partial g}{\partial m} > 0$ and $\frac{\partial \omega}{\partial m} < 0$ at the equilibrium point. For example, if $\lambda = 0$ then equilibrium $g$ is positive and from equation (13E)

$$\frac{g}{\omega} = \frac{(-a_N g)}{(\frac{\alpha}{1-\alpha}(\rho + g)a_N - L_N + a_N g)}$$

is positive (since the denominator is negative for $g$ satisfying the above range) imply $\omega$ is positive. Then

$$\omega = \frac{L_N - \frac{\alpha}{1-\alpha}(\rho + g)a_N - a_N g}{a_N}$$

and a decrease in $m$ leads to a decrease in $g$ and this leads to an increase in $\omega$.

We also show in Appendix C that for $(1-\alpha) < \lambda \leq 1$ there exists a positive equilibrium value of $g$ in the range given by

$$\frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{a_N(\frac{1}{1-\alpha})} < g < \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{a_N \frac{\alpha}{1-\alpha}}$$

with the property that $\frac{\partial g}{\partial m} < 0$ at the equilibrium point. Equation (13E) shows that the existence of a positive equilibrium $\omega$ is guaranteed for $\lambda > (\frac{a_N g}{L_N})$. Therefore, for $\lambda$ satisfying

$$1 \geq \lambda > max\left\{(1 - \alpha), \frac{a_N g}{L_N}\right\}$$

we have positive equilibrium solutions for both $g$ and $\omega$ with $\frac{\partial g}{\partial m} < 0$ and $\frac{\partial \omega}{\partial m} < 0$ at the equilibrium point.

In the last two paragraphs we have seen that equilibrium solution for $g$ and $\omega$ depends on the choice of $\lambda$. If

$$0 \leq \lambda < min\left\{ \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{L_N(1 + \frac{\alpha}{1-\alpha})}, \frac{a_N g}{L_N} \right\}$$

then a decrease in $m$ leads to a decrease in the innovation rate and to an increase in the multinationalization rate. However, for

$$1 \geq \lambda > max\left\{ (1-\alpha), \frac{a_N g}{L_N} \right\}$$

a decrease in $m$ leads to an increase in the innovation rate and to an increase in the multinationalization rate. We state the main results in the following proposition.

**Proposition 2**

*Strengthening IPR in the South raises (lowers) the innovation rate in the North if $\lambda$ takes a value close to unity (zero). However multinationalization rate is always increased due to stronger IPR protection in the South.*

This is a generalization of the result of Lai (1998) where $\lambda = 1$ and strengthening IPR in South raises the innovation rate in the North. The intuition for the result is as follows. A stronger IPR in the South increases the expected PDV of profits from being a multinational and hence the rate of multinationalization, $\omega$, increases given $g$.[10] Now the changes in $\omega$ affects both the supply of labour to the production sector and the demand for labour from the production sector of the North given by the equations (11E) and (12E). This is so because an increase in $\omega$ causes the labour productivity of the R&D sector in the North to fall through its negative effect on the knowledge spillover term and hence more labour is required by the R&D sector to maintain the same innovation rate. The labour market clearing in the production sector in the North can be analysed using the same Figure 1, because here also the demand curve and the supply curve behave similarly as in the basic model without multinationalization. From equations (11E) and (12E) we find that the supply curve slopes negatively and the demand curve slopes positively as function of $g$. At the extreme case, when $\lambda = 0$, an increase in $\omega$ causes the more allocation of labour to the R&D sector (given $g$) and hence the supply curve of labour to the production sector shifts downward. However, the demand curve for labour does not change in this case. Hence, in equilibrium, the rate of innovation, $g$, is decreased. However, at the other extreme case when $\lambda = 1$, an increase in $\omega$ makes the demand curve for labour shift downward while the supply curve remains intact. This causes the equilibrium $g$ to increase. For values of $\lambda$ between 0 and 1 both the demand for and the supply of labour curves shift and in the new equilibrium $g$ may fall or increase depending on the value of $\lambda$.

## Conclusion

In this note, we have reanalysed the effects of strengthening IPR in the South on the steady-state equilibrium rate of growth in a North–South model in two cases. In the first case, our analytical framework is otherwise identical to that in Helpman (1993); and, in the second case, it is otherwise identical to that in Lai (1998). In both the cases, our definition of knowledge capital is different from theirs and this is the only modification introduced here. However, with this minor modification, we find major differences in the results. The nature of the qualitative effect of strengthening IPR on the growth rate may be completely opposite to what obtained in the original models of Helpman (1993) and of Lai (1998). So the existing theoretical results related to the policy of strengthening IPR are not robust with respect to the variations in the definition of knowledge capital. We do not claim that our definition of knowledge capital is the ideal one. We only claim that this note establishes the importance of further research in this line considering the more general definition of knowledge capital.

Obviously there are many other directions in which the basic model can be extended. We can introduce outsourcing of high technology jobs from the North to the South because in reality this outsourcing of 'the best of the West' jobs to the developing countries is a major issue in the US and in many other industrial countries. Outsourcing generates employment in the South, makes imitation easier and encourages innovation activities there. This leads to an increase in the South–North relative wage. In the absence of outsourcing, i.e. in the basic model of this paper, the profit rate of the Northern firm is independent of the South–North relative wage. However, the presence of outsourcing establishes a negative relation between this profit rate and the South–North relative wage. A policy of strengthening IPR in the South now has two conflicting effects on the profit rate. One is the direct effect obtained in Helpman (1993) and in our paper. The other is the indirect effect, which works through the change in the South–North relative wage. So the profit rate of the Northern firm may not necessarily be reduced when IPR is strengthened. However, the effect on the effective cost of capital remains the same as in the basic model. So this strengthening of IPR causes an additional downward shift of the demand curve in Figure 1; and, as a result, the equilibrium rate of growth will rise at a even higher rate in the presence of outsourcing when $\lambda = 0$. A complete analysis of the growth effects of outsourcing in this product-variety framework is beyond the scope of this paper. Glass & Saggi (2001) and Glass (2004) have analysed the growth effects of outsourcing using the quality ladder framework.

In this paper, the case of strictly positive effect of IPR protection on innovation rate advocates the traditional view. However, Saxanian (1994) has pointed out the importance of the relatively weaker property rights in driving the outstanding innovationin California and in Silicon Valley. The present paper does not consider the positive externalities of imitation. So the strengthening of IPR, which causes a reduction in the imitation rate does not generate any negative effect on the productivity of researchers by decreasing the flow of knowledge to the researchers. If knowledge capital is expanded through imitation and if this in turn makes labour more productive in the R&D sector, then the strengthening of IPR may produce a negative effect on the innovation rate.[11] In Silicon Valley, intense communications among researchers, high rates of inter-firm mobility, transfer of information regarding new firms and products fosters imitation. This ensures the faster diffusion of new knowledge, making labourers more productive. However, this positive external effect of imitation is strong when imitators are local firms. In this model, the Northern products are imitated by the Southern firms.

Knowledge capital is measured here as the sum of all products. This is similar to the Krugman (1979) assumption that returns to scale emerge when more similar products are created. However, in many cases, it is the mass products that create economies of scale and not a great many different and similar products. Thus, innovation should not necessarily be measured by a possible large number of rather similar products. However, all the models built on the Grossman & Helpman (1991b) and Helpman (1993) product variety framework suffer from this common limitation. Our exercise is nothing more than a contribution to the existing product variety literature.

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**Notes**

[1] Grossman and Helpman (1991a), Glass and Saggi (2002), Yang & Maskus (2001), etc have developed models based on the Quality ladder approach. Since our contribution is based on the product variety approach, we shall survey only works based on this approach.

[2] This production technology is the same for all Northern and Southern products.

[3] Price (quantity) of all the varieties produced in a country take the same equilibrium value because utility function is symmetric and technologies are identical.

[4] Wage rate is the marginal cost of production of a variety.

[5] This is equal to total production of all the Northern varieties.

[6] They may have different curvature for different values of $\lambda$. But their slopes and the nature of shifts remain the same.

[7] We follow Lai (1998) in defining $\omega$. If $\omega = \frac{n_m}{n_N}$ the result may be different.

[8] We may have an alternative specification of knowledge capital given by $(n_N + n_m + \lambda n_i)$ and the results may be different there. See Appendix D.

[9] The derivation of this equation is described in Appendix B.

[10] From equation (ME) we see that the left-hand side decreases and the right-hand side increases owing to a decrease in $m$. Given $g$, equilibrium can be restored only by increasing $\omega$ under the assumption that the relative labour supply of the North is sufficiently high.

[11] An analysis along these lines is available in Arnold (1995).



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## Appendix

*Appendix A*

$$g^2\left(\frac{a_N}{1-\alpha}\right) - g\left(L_N - \frac{a_N\rho\alpha}{1-\alpha} - \frac{ma_N}{1-\alpha}\right) - L_N\lambda m = 0$$

$$\Rightarrow g^2\left(\frac{a_N}{1-\alpha}\right) - gx - L_N\lambda m = 0, \quad \text{where } x = \left(L_N - \frac{a_N\rho\alpha}{1-\alpha} - \frac{ma_N}{1-\alpha}\right) > 0$$

$$\Rightarrow g = \frac{x \pm \sqrt{x^2 + 4(\frac{a_N}{1-\alpha})L_N\lambda m}}{2(\frac{a_N}{1-\alpha})}$$

Let us write, $g_1 = \frac{x+\sqrt{x^2+4(\frac{a_N}{1-\alpha})L_N\lambda m}}{2(\frac{a_N}{1-\alpha})}$ and $g_2 = \frac{x-\sqrt{x^2+4(\frac{a_N}{1-\alpha})L_N\lambda m}}{2(\frac{a_N}{1-\alpha})}$ clearly $g_1 > 0$ and $g_2 < 0$. Then,

$$g_1\left(2\frac{a_N}{1-\alpha}\right) = x + \sqrt{x^2 + 4\left(\frac{a_N}{1-\alpha}\right)L_N\lambda m}$$

Differentiating both sides with respect to $m$ we get,

$$2\frac{a_N}{1-\alpha}\frac{\partial g}{\partial m} = -\frac{a_N}{1-\alpha} + \frac{1}{2}\left[x^2 + 4\left(\frac{a_N}{1-\alpha}\right)L_N\lambda m\right]^{-\frac{1}{2}}(-2x + 4\lambda L_N)\left(\frac{a_N}{1-\alpha}\right)$$

$$\Rightarrow 2\frac{\partial g}{\partial m} = -1 + \frac{-x + 2\lambda L_N}{\sqrt{x^2 + 4(\frac{a_N}{1-\alpha})L_N\lambda m}}$$

Thus $\frac{\partial g}{\partial m} > 0$ imply $(-x + 2\lambda L_N) > \sqrt{x^2 + 4(\frac{a_N}{1-\alpha})L_N\lambda m}$.

The right-hand side of this inequality is always positive. But the left-hand side of this inequality is positive if $(-x + 2\lambda L_N) > 0 \Rightarrow \lambda > \frac{x}{2L_N}$.

Now for $\lambda > \frac{x}{2L_N}$ we get

$$(-x + 2\lambda L_N)^2 > x^2 + 4\left(\frac{a_N}{1-\alpha}\right)L_N\lambda m$$

$$\Rightarrow \lambda > \frac{L_N - a_N\rho\frac{\alpha}{1-\alpha}}{L_N}$$

Since $x < (L_N - a_N\rho\frac{\alpha}{1-\alpha})$ we get, $\lambda > \frac{L_N - a_N\rho\frac{\alpha}{1-\alpha}}{L_N} \Rightarrow \lambda > \frac{x}{2L_N}$.

Thus, for $\lambda > \frac{L_N - a_N\rho\frac{\alpha}{1-\alpha}}{L_N}$, $\frac{\partial g}{\partial m} > 0$. This is Helpman's result. Note that $\lambda \leq 1$.

For $\frac{x}{2L_N} < \lambda < \frac{L_N - a_N\rho\frac{\alpha}{1-\alpha}}{L_N}$, we get, $\frac{\partial g}{\partial m} < 0$. And for $\lambda \leq \frac{x}{2L_N}$, $\frac{\partial g}{\partial m}$ is clearly negative. Combining these two we have for $\lambda < \frac{L_N - a_N\rho\frac{\alpha}{1-\alpha}}{L_N}$, $\frac{\partial g}{\partial m} < 0$. This is our result. And for $\lambda = \frac{L_N - a_N\rho\frac{\alpha}{1-\alpha}}{L_N}$, $\frac{\partial g}{\partial m} = 0$.

*Appendix B*

We will derive equation (ME) here.

$$\frac{\pi_N}{\pi_m} = \frac{r}{r+m} \Rightarrow \left(\frac{x_N w_N}{x_m w_S}\right) = \frac{r}{r+m} \Rightarrow \left(\frac{x_N}{x_m}\right)^{\alpha}$$

$$= \frac{r}{r+m} \Rightarrow \left(\frac{L_N(g+\lambda\omega) - a_N g(g+\omega)}{(g+\lambda\omega)n_N} \cdot \frac{n_i\alpha^{-\varepsilon} + n_m}{L_S}\right)^{\alpha}$$

$$= \frac{r}{r+m} \Rightarrow \left(\frac{L_N(g+\lambda\omega) - a_N g(g+\omega)}{(g+\lambda\omega)L_S} \cdot \frac{\omega}{g} \cdot \frac{m\alpha^{-\varepsilon} + g}{m+g}\right)^{\alpha} = \frac{r}{r+m}$$

*Appendix C*

Replacing $\omega$ in terms of $g$ from equation (13E), equation (ME) can be written as

$$\Rightarrow \left(\frac{\frac{\alpha}{1-\alpha} r g a_N}{(g+\lambda\omega)L_S} \cdot \frac{\omega}{g} \cdot \frac{m\alpha^{-\varepsilon} + g}{m+g}\right)^{\alpha} = \frac{r}{r+m}$$

$$\Rightarrow \left(\frac{\frac{\alpha}{1-\alpha} r a_N}{L_S} \cdot \frac{\omega}{(g+\lambda\omega)} \cdot \frac{m\alpha^{-\varepsilon} + g}{m+g}\right)^{\alpha} = \frac{r}{r+m}$$

$$\Rightarrow \left(\frac{\frac{\alpha}{1-\alpha} r a_N}{L_S} \cdot \frac{1}{(\lambda + \frac{g}{\omega})} \cdot \frac{m\alpha^{-\varepsilon} + g}{m+g}\right)^{\alpha} = \frac{r}{r+m}$$

$$\Rightarrow \left(\frac{\frac{\alpha}{1-\alpha} r a_N}{L_S} \cdot \frac{1}{\left(\lambda + \frac{(\lambda L_N - a_N g)}{(\frac{\alpha}{1-\alpha} r a_N - L_N + a_N g)}\right)} \cdot \frac{m\alpha^{-\varepsilon} + g}{m+g}\right)^{\alpha} = \frac{r}{r+m}$$

$$\Rightarrow \left[\frac{\frac{\alpha}{1-\alpha} r}{L_S} \cdot \frac{\frac{\alpha}{1-\alpha} r a_N - L_N + a_N g}{\frac{\alpha}{1-\alpha}\lambda r + \lambda g - g} \cdot \frac{g + m\alpha^{-\varepsilon}}{g+m}\right]^{\alpha} = \frac{r}{r+m}$$

This is equation (14E).

*Existence of $g$ and $\omega$.* We write equation (14E) as follows

$$\left[\frac{\frac{\alpha}{1-\alpha} r}{\frac{\alpha}{1-\alpha}\lambda r + \lambda g - g} \cdot \frac{\frac{\alpha}{1-\alpha} r a_N - L_N + a_N g}{L_S} \cdot \frac{g + m\alpha^{-\varepsilon}}{g+m}\right]^{\alpha} \cdot \frac{r+m}{r} = 1 \qquad (14E.A)$$

$(\frac{\alpha}{1-\alpha} r a_N - L_N + a_N g) > 0$ if $g > \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{a_N(\frac{1}{1-\alpha})} = g^u$ (*say*), and $(\frac{\alpha}{1-\alpha}\lambda r + \lambda g - g) > 0$ if $\lambda > (1-\alpha)$ for all $g > 0$. Thus for $g > g^u$ and $\lambda > (1-\alpha)$ we get that the expression in the third bracket of the left-hand side of equation (14E.A) is positive.

The first term within the third bracket of the left-hand side of this equation, $\frac{\frac{\alpha}{1-\alpha}r}{\frac{\alpha}{1-\alpha}\lambda r+\lambda g-g}$, increases as $g$ increases for all $g > 0$ (and hence for $g > g^u$) and $\lambda > (1-\alpha)$.

To prove that the term $\left[\frac{\frac{\alpha}{1-\alpha}ra_N - L_N + a_N g}{L_S} \cdot \frac{g+m\alpha^{-\varepsilon}}{g+m}\right]^{\alpha} \cdot \frac{r+m}{r}$ (= A, say) increases as $g$ increases we take the log of this expression and then differentiate with respect to $g$. This gives,

$$\begin{aligned}\frac{\partial log(A)}{\partial g} &= \frac{\alpha}{\frac{\alpha}{1-\alpha}ra_N - L_N + a_N g}\left(\frac{\alpha}{1-\alpha}a_N + a_N\right) + \frac{\alpha}{g+m\alpha^{-\varepsilon}} \\ &\quad - \frac{\alpha}{g+m} + \frac{1}{r+m} - \frac{1}{r} \\ &= \left[\frac{\frac{\alpha}{1-\alpha}a_N}{\frac{\alpha}{1-\alpha}ra_N - L_N + a_N g} - \frac{1}{r}\right] + \left[\frac{1}{r+m} - \frac{\alpha}{g+m}\right] + \frac{\alpha}{g+m\alpha^{-\varepsilon}}\end{aligned}$$

The first term in the third bracket is positive if $g < \frac{L_N}{a_N}$ and the second term in the third bracket is positive if $g > \frac{\alpha\rho}{1-\alpha}$. Also, from equation (12E) we see $n_N x_N = \frac{\alpha}{1-\alpha}ra_N$ for $\lambda = 0$ which implies $L_N > \frac{\alpha}{1-\alpha}ra_N \Rightarrow g < \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{a_N(\frac{\alpha}{1-\alpha})} = g^s$(say). Now, $g > g^u \Rightarrow g > \frac{\alpha\rho}{1-\alpha}$ if $L_N > a_N\rho\frac{\alpha}{1-\alpha}(1+\frac{1}{1-\alpha})$ and $g < g^s \Rightarrow g < \frac{L_N}{a_N}$ if $\alpha \geq .5$. Also $g^s > g^u$ since $\alpha < 1$.

Thus we say that the left-hand side of equation (14E.A) is a monotonically increasing function in the range of $g \in (g^u, g^s)$ and $\lambda > (1-\alpha)$ under the sufficient condition $\alpha \geq 0.5$ and. $L_N > a_N\rho\frac{\alpha}{1-\alpha}(1+\frac{1}{1-\alpha})$. Again as $g \to g^{u(+)}$, the left-hand side of equation (14E.A) $\to 0$; and as $g \to g^{s(-)}$, the left-hand side of (14E.A) $\to$ (some value greater than 1) if $L_N > L_S\frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}a_N\rho$.

From equation (13E), $\frac{g}{\omega} = \frac{(\lambda L_N - a_N g)}{(\frac{\alpha}{1-\alpha}ra_N + a_N g - L_N)}$. For $g^u < g < g^s$, the denominator of this equation is positive. The numerator is positive for $\lambda > \frac{a_N g}{L_N}$.

Thus, the existence of an unique equilibrium solution for $g \in (g^u, g^s)$ and $\omega$ is ensured for $\lambda > max\{(1-\alpha), (\frac{a_N g}{L_N})\}$ under the sufficient condition $\alpha \geq 0.5$ and $L_N > max\{a_N\rho\frac{\alpha}{1-\alpha}(1+\frac{1}{1-\alpha}), L_S\frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}a_N\rho\}$.

Again $(\frac{\alpha}{1-\alpha}ra_N - L_N + a_N g) < 0$ if $g < \frac{L_N - \rho a_N \frac{\alpha}{1-\alpha}}{a_N(\frac{1}{1-\alpha})} = g^u$, and $(\frac{\alpha}{1-\alpha}\lambda r + \lambda g - g) < 0$ if $g > \frac{\rho\lambda\frac{\alpha}{1-\alpha}}{1-\frac{\lambda}{1-\alpha}} = g^l$ (*say*). Then we write equation (14E.A) as follows

$$\left[\frac{\frac{\alpha}{1-\alpha}r}{-\frac{\alpha}{1-\alpha}\lambda r - \lambda g + g} \cdot \frac{-\frac{\alpha}{1-\alpha}ra_N + L_N - a_N g}{L_S} \cdot \frac{g+m\alpha^{-\varepsilon}}{g+m}\right]^{\alpha} \cdot \frac{r+m}{r} = 1$$

For $g^l < g < g^u$, the bracketed term of the left-hand side of this above equation is positive. This range of $g$ is non-empty if $\lambda < \frac{(1-\alpha)L_N - \alpha\rho a_N}{L_N} = \lambda_1$

(*say*). Also, $g > 0$ implies $\lambda < (1-\alpha) = \lambda_2$ (*say*). But $\lambda < \lambda_1 \Rightarrow \lambda < \lambda_2$. Thus we say, for $\lambda < \frac{(1-\alpha)L_N - \alpha\rho a_N}{L_N} = \lambda_1$ and $g^l < g < g^u$, the bracketed term of the left-hand side of this above mentioned equation is positive and the left-hand side of this equation monotonically decreases as $g$ increases from $g^l$ to $g^s$. It monotonically decreases because $\frac{\frac{\alpha}{1-\alpha}r}{-\frac{\alpha}{1-\alpha}\lambda r - \lambda g + g}$ decreases as $g$ increases, $\frac{-\frac{\alpha}{1-\alpha}ra_N + L_N - a_N g}{L_S}$ decreases as $g$ increases, $\frac{g + m\alpha^{-\epsilon}}{g+m}$ decreases as $g$ increases and $\frac{r+m}{r}$ decreases as $g$ increases satisfying $g^l < g < g^u$, and $\lambda < \lambda_1$.

In addition, as $g \to g^{l(+)}$ then the left-hand side of this equation tends to $+\infty$ and as $g \to g^{u(-)}$ then the left-hand side of this equation tends to zero. So the left-hand side of this equation must intersect the right-hand side of this equation at some $g$ satisfying $g^l < g < g^u$. Hence the existence of a unique positive equilibrium $g \in (g^l, g^u)$ is guaranteed for $\lambda < \frac{(1-\alpha)L_N - \alpha\rho a_N}{L_N}$.

Again from equation (13E), $\frac{g}{\omega} = \frac{(\lambda L_N - a_N g)}{(\frac{\alpha}{1-\alpha}ra_N + a_N g - L_N)}$. For $g^l < g < g^u$, the denominator of this equation is negative. The numerator is negative for $\frac{a_N g}{L_N}$.

Thus the existence of an unique equilibrium solution for $g \in (g^l, gu)$ and $\omega$ is ensured for $\lambda < min\{(\frac{(1-\alpha)L_N - \alpha\rho a_N}{L_N}), (\frac{a_N g}{L_N})\} = min\{(\frac{L_N - \frac{\alpha}{1-\alpha}\rho a_N}{L_N(1+\frac{\alpha}{1-\alpha})}), (\frac{a_N g}{L_N})\}$.

*To find the sign of* $\frac{\partial g}{\partial m}$ *and* $\frac{\partial \omega}{\partial m}$. The left-hand side of equation (14E.A) decreases for a decrease in $m$ for $g \in (g^u, g^s)$ and $\lambda > max\{(1-\alpha), (\frac{a_N g}{L_N})\}$. In addition, we have already shown that the left-hand side of equation (14E.A) increases with an increase in $g$ for $g \in (g^u, g^s)$ and $\lambda > max\{(1-\alpha), (\frac{a_N g}{L_N})\}$. So, by the implicit function theorem, we say $\frac{\partial g}{\partial m} < 0$. In addition, we have $\frac{\partial \omega}{\partial m} = \frac{\partial g}{\partial m}\left[\frac{a_N g \frac{1}{1-\alpha} + \lambda L_N \frac{\omega}{g}}{\lambda L_N - a_N g}\right] < 0$ for $\lambda > max\{(1-\alpha), (\frac{a_N g}{L_N})\}$. Note that $\lambda$ has an upper bound equal to one.

The left-hand side of the rearranged form of equation (14E.A) decreases for a decrease in $m$ and decreases for an increase in $g$ for $g \in (g^l, g^u)$ and $\lambda < min\{(\frac{(1-\alpha)L_N - \alpha\rho a_N}{L_N}), (\frac{a_N g}{L_N})\}$. Thus, by the implicit function theorem, we say $\frac{\partial g}{\partial m} > 0$. In addiiton, we have $\frac{\partial \omega}{\partial m} = \frac{\partial g}{\partial m}\left[\frac{a_N g \frac{1}{1-\alpha} + \lambda L_N \frac{\omega}{g}}{\lambda L_N - a_N g}\right] < 0$ for $\lambda < min\{(\frac{(1-\alpha)L_N - \alpha\rho a_N}{L_N}), (\frac{a_N g}{L_N})\}$. Note that $\lambda$ has lower bound equal to zero.

## *Appendix D*

We extend the fourth section of our paper (Multinationalization and Imitation) to incorporate the following denition of knowledge capital $k_N = n_N + n_m + \lambda n_i$ with all other things remaining unchanged in our model.

With this definition, aggregate employment in the Northern research sector becomes $L_R = a_N g \frac{n_N + n_m + n_i}{n_N + n_S + \lambda n_i}$ and $L_N = n_N x_N + L_R$. These two together imply $x_N = \frac{L_N - a_N g \frac{n_N + n_m + n_i}{n_N + n_S + \lambda n_i}}{n_N}$. Then the Northern free entry implies

$$\frac{\alpha}{1-\alpha} a_N r = \frac{L_N(n_N + n_m + n_i\lambda) - a_N g(n_N + n_m + n_i)}{n_N} \quad (15E.A)$$

$$\Rightarrow \frac{\alpha}{1-\alpha} a_N r = L_N\left(1 + \frac{\omega}{g}\frac{g}{g+m} + \lambda\frac{\omega}{g}\frac{m}{g+m}\right) - a_N g\left(1 + \frac{\omega}{g}\frac{g}{g+m} + \frac{\omega}{g}\frac{m}{g+m}\right) \quad (16E.A)$$

$$\Rightarrow \frac{\omega}{g} = \frac{(g+m)(\frac{\alpha}{1-\alpha}a_N r + a_N g - L_N)}{L_N(g+\lambda m) - a_N g(g+m)}$$

The multinationalization condition implies

$$\left(\frac{x_N}{x_m}\right)^{\alpha} = \frac{r}{r+m} \Rightarrow \left[\frac{\frac{\alpha}{1-\alpha}a_N r}{n_N + n_m + \lambda n_i}\frac{n_m + n_i\alpha^{-\varepsilon}}{L_S}\right] = \frac{r}{r+m}$$

$$\Rightarrow \left[\frac{1}{L_S}\frac{(g + m\alpha^{-\varepsilon})(\frac{\alpha}{1-\alpha}a_N r + a_N g - L_N)}{(g+\lambda m) - \frac{g}{r}m(1-\lambda)\frac{1-\alpha}{\alpha}}\right]^{\alpha} = \frac{r}{r+m}$$

$$\Rightarrow \left[\frac{1}{L_S}\frac{(g + m\alpha^{-\varepsilon})(\frac{\alpha}{1-\alpha}a_N r + a_N g - L_N)}{g+m}\right]^{\alpha}\frac{r+m}{r} = \left[\frac{(g+\lambda m) - \frac{g}{r}m(1-\lambda)\frac{1-\alpha}{\alpha}}{g+m}\right]^{\alpha} \quad (17E.A)$$

Equation (17E.A) has been derived using equations (15E.A) and (16E.A). This is one equation with one unknown $g$. The left-hand side of this equation is exactly the same as Lai (1998). Lai (1998) has shown that the left-hand side of equation (17E.A) increases as $g$ increases and $m$ increases. We show that the right-hand side of equation (17E.A) decreases as we increase $g$ and $m$ under the sufficient condition $\frac{g^2+g\rho}{(g+\rho)^2} > \frac{\alpha}{(1-\alpha)\rho}$. Thus, $\frac{LHS}{RHS}$ of equation (17E.A) increases as $g$ increases and also increases as $m$ increases. Therefore by the implicit function theorem $\frac{\partial g}{\partial m} < 0$, implying that Lai's result is valid for all $\lambda \in [0,1]$. Note that this was not the case in the fourth section where we define the knowledge capital as $k_N = n_N + \lambda(n_m + n_i)$.