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Abstract

We identify a new route through which patent protection may affect R&D incentives, the *tournament effect*. It may decrease R&D incentives, in which case patent protection may either adversely affect the level of R&D, or may discourage licensing. In either case welfare may fall.

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1 Introduction

The main justification for the institution of patents is that it enhances R&D incentives by providing protection against imitation (by giving the inventor temporary monopoly rights to her invention), thus allowing the recovery of R&D costs. While the above argument has great explanatory power,[1] we argue that it does not exhaust the implications of the patent system.

We identify a new route through which patent protection may affect the R&D incentives, the *tournament effect* (henceforth T.E.). Suppose that several firms have claim to the same invention. In the presence of patent protection, at most one of these firms can obtain the technology, whereas in the absence of patent protection, all such firms can obtain the technology. Thus patents effectively turn R&D competition into tournaments.

2 The Basic Model Without Licensing

The market comprises two firms, 1 and 2 producing a single homogeneous commodity. Let q_i denote the output of firm i. The inverse market demand function $f(q)$, where $q = q_1 + q_2$, satisfies

Assumption 1. $f(q)$ is twice continuously differentiable, negatively sloped and satisfies the decreasing marginal revenue property i.e. $f'(q) + q_i f''(q) < 0$, $\forall q, q_i$.

Initially, the cost functions of both the firms are given by cq. However, both the firms, by investing an amount F (> 0) in R&D, can change its cost function to $c'q$, where $0 \leq c' < c$.[2]

The firms play a two stage game where, in stage 1, they simultaneously

[1] See Scherer (1980), chapter 16, for a thorough discussion of the patent system. For an incisive critique, see Boldrin and Levine (2003).

[2] We assume that the R&D technology is deterministic and involves process innovation. However, these assumptions can be relaxed without affecting the results qualitatively.

decide on whether to invest in R&D or not, while in stage 2, they simultaneously decide on their output level. Once the R&D decisions have been made, the subsequent outcome in stage 2 depends on whether there is patent protection or not.

First consider the case where there is no patent protection. There is spill-over of technology (caused, for example, by the movement of personnel across firms, etc). Thus if only firm i does R&D, it's cost function shifts down to $c'q$. However, because of spill-overs, the cost function of firm j also shifts down to $\tilde{c}q$, where $c' \leq \tilde{c} \leq c$. If $\tilde{c} = c'$, we say that there is complete spill-over. Whereas if there is no spill-over then $\tilde{c} = c$.

Next consider the case where there is patent protection.

In case only firm i does R&D, firm i obtains the patent. Hence the cost function of firm i shifts to $c'q$, whereas, because of the patent, that of firm j remains at cq.

Whereas if both the firms invest in R&D then the ownership of the patent is disputed. We shall focus on two possible scenarios.

In case the litigation costs of contesting a patent is very small, both the firms will contest the patent. For simplicity we assume that both the firms have an equal probability of being awarded the patent by the courts.

Whereas if litigation costs are relatively large,[3] then the outcome will involve one of the firms claiming the patent for the new invention, while the other firm will not contest it for fear of incurring litigation costs. Assuming that there is an equal probability of either firm relinquishing its claim, again the firms have an equal probability of obtaining the patent.

Given the preceding discussion, we work with a reduced form game which incorporates the features that if both the firms invest in R&D, then they both obtain the patent with probability half and there are no litigation costs.

[3] To quote Scherer (1980), "Between 1900 and 1941, 684 radio patents were entangled in a total of 1957 infringement suits..... A single lawsuit over petroleum cracking patents lasted 15 years, piling up court costs and legal fees exceeding 3 million."

Note that the above formulation implicitly assumes that licensing is not possible.[4] Of course, for licensing to take place it is necessary that it should lead to an increase in aggregate industry profits. However, even then technology transfer may involve costs that may make licensing infeasible. Suppose, for example, that the new technology is embodied in the R&D personnel, at least to some extent. Then licensing would require such research personnel to devote a substantial amount of time to the transfer process, with large opportunity costs for the transferring firm (see Boldrin and Levine (2003)). In fact, such costs are likely to be large in case of new technologies which are unlikely to be completely codified and standardized. Alternatively, licensing may make it easier for competing firms to invent around the patent, perhaps even develop superior products (see Tirole (1988)).

We solve for the set of pure strategy subgame perfect Nash equilibria of the above game.

Stage 2. Given the assumptions on the demand and the cost functions, it is standard to show that for every possible cost configuration the second stage game has a unique Cournot equilibrium. Moreover, the equilibrium is locally stable. Given that the equilibrium is unique, it must be symmetric if the firms have the same cost function.[5] Hence we can introduce the following notations for the equilibrium payoffs in stage 2 (gross of R&D costs):

$\pi(x, x)$ denotes the gross equilibrium payoff of both the firms when both the firms have the cost function xq, where $x \in \{c, c'\}$.

$\pi_i(x, y)$ denotes the gross equilibrium payoff of the i-th firm when the cost function of firm 1 is xq and that of firm 2 is yq, $x \neq y$, $x, y \in \{c, c', \tilde{c}\}$.

Given that the Cournot equilibrium is unique, $\pi_1(c', \tilde{c}) = \pi_2(\tilde{c}, c')$ and

[4]Firestone (1971) argues that most of the patents held by corporations are used exclusively by these corporations.

[5]See Vives (1999), Chapter 4.

$\pi_1(\tilde{c}, c') = \pi_2(c', \tilde{c})$. The following ranking is natural:

$$\pi_1(c', c) = \pi_2(c, c') > \pi(c', c') > \pi(c, c) > \pi_1(c, c') = \pi_2(c', c),$$
$$\text{and, for } \tilde{c} < c,\ \pi_1(c', c) > \pi_1(c', \tilde{c}),\ \pi_1(\tilde{c}, c') > \pi_1(c, c'). \qquad (1)$$

Straightforward calculations demonstrate that the above ranking is respected for linear demand functions.

Definition. Let both the firms invest in R&D. The *tournament effect* denotes the difference between the equilibrium payoff of the two firms in the presence and absence of patent protection i.e.

$$\frac{\pi_1(c', c) + \pi_1(c, c')}{2} - \pi(c', c'). \qquad (2)$$

Clearly, T.E. is positive whenever

$$\pi_1(c', c) + \pi_1(c, c') > 2\pi(c', c'). \qquad (3)$$

3 The Analysis

First consider the stage 1 payoff matrix without patent protection:

	R&D	No R&D
R&D	$\pi(c', c') - F,\ \pi(c', c') - F$	$\pi_1(c', \tilde{c}) - F,\ \pi_2(c', \tilde{c})$
No R&D	$\pi_1(\tilde{c}, c'),\ \pi_2(\tilde{c}, c') - F$	$\pi(c, c),\ \pi(c, c)$

where the strategies of firm 1 are written vertically and those of firm 2 are written horizontally. For every payoff vector the first and second entry represent, respectively, the net equilibrium payoff of firm 1 and firm 2.

Next, consider the stage 1 payoff matrix under patent protection:

	R&D	No R&D
R&D	$\frac{\pi_1(c',\, c)+\pi_1(c,c')}{2} - F,\ \frac{\pi_2(c',\, c)+\pi_2(c,c')}{2} - F$	$\pi_1(c',\ c) - F,\ \pi_2(c', c)$
No R&D	$\pi_1(c, c'),\ \pi_2(c, c') - F$	$\pi(c, c),\ \pi(c, c)$

We then compare the incentive for R&D with and without patent protection. These can be of two kinds, strategic and non-strategic.

Definition. The *non-strategic incentive* for R&D is firm i's payoff from R&D, net of its payoff from not doing R&D, when firm j does not do R&D. Let the non-strategic incentive for doing R&D in the absence of patent protection be denoted by $N(NP)$ and that in the presence of patent protection be $N(P)$. Clearly,

$$\begin{aligned} N(NP) &= \pi_1(c', \tilde{c}) - \pi(c, c) - F, & (4) \\ \text{and } N(P) &= \pi_1(c', c) - \pi(c, c) - F. & (5) \end{aligned}$$

Definition. The *strategic incentive* for R&D is firm i's payoff from R&D, net of its payoff from not doing R&D, when firm j invests in R&D. Let the strategic incentive for doing R&D in the absence of patent protection be $S(NP)$ and that in the presence of patent protection be $S(P)$. Clearly,

$$\begin{aligned} S(NP) &= \pi(c', c') - \pi_1(\tilde{c}, c') - F, & (6) \\ \text{and } S(P) &= \frac{\pi_1(c', c) + \pi_1(c, c')}{2} - \pi_1(c, c') - F. & (7) \end{aligned}$$

Given that $\pi_1(c', c) \geq \pi_1(c', \tilde{c})$ it is easy to see that $N(P) \geq N(NP)$, i.e. in a non-strategic context patent protection increases the incentive for R&D. This is the textbook justification for patent protection.

Next consider the effect of patent protection on the strategic incentive for R&D, i.e. the sign of $S(P) - S(NP)$. Note that

$$S(P) - S(NP) = [\frac{\pi_1(c', c) + \pi_1(c, c')}{2} - \pi(c', c')] + [\pi_1(\tilde{c}, c') - \pi_1(c, c')]. \quad (8)$$

The first term in square brackets represents T.E., whereas the second term in square brackets captures the effect of patents in ensuring that firms do not gain from spill-overs. This term is necessarily positive.

From equation (8), note that $S(P) > S(NP)$ if and only if

$$\pi_1(\tilde{c}, c') > \frac{2\pi(c', c') - \pi_1(c', c) + \pi_1(c, c')}{2}. \quad (9)$$

Observe that equation (9) is always satisfied if there is complete spill-over, i.e. if $\tilde{c} = c'$. Moreover, the L.H.S. of equation (9) is decreasing in $\tilde{c}$.

Our first proposition follows from the above observations.

Proposition 1. *(i) If T.E. is positive, i.e. if* $\pi_1(c', c) + \pi_1(c, c') > 2\pi(c', c')$*, then, for all* $\tilde{c} \in [c', c]$*, patent protection increases the strategic incentive for R&D.*

(ii) If T.E. is negative, i.e. if $\pi_1(c', c) + \pi_1(c, c') < 2\pi(c', c')$*, then there exists* $\hat{c}$ *such that patent protection increases the strategic incentive for R&D for all* $\tilde{c} < \hat{c}$ *and decreases it for all* $\tilde{c} > \hat{c}$*.*

Proof. (i) Follows from equation (8) and the fact that T.E. is positive.

(ii) Note that eqn. (9) is satisfied for $\tilde{c} = c'$ and violated for $\tilde{c} = c$. Given that the L.H.S. of eqn. (9) is decreasing in $\tilde{c}$, the result follows. ■

We then examine conditions under which T.E. is positive.[6] Suppose that $c'q$ is drastic compared to cq, i.e. $\pi_1(c, c') = \pi_2(c', c) = 0$ and $\pi_1(c', c) = \pi_2(c, c') = \pi_m(c')$, where $\pi_m(c')$ denotes the monopoly profit of a firm having the cost function $c'q$. Thus in this case equation (3) simplifies to $\pi_m(c') > 2\pi(c', c')$. Since the rent dissipation effect associated with Cournot competition is avoided under a monopoly, this is always true. Thus in this case T.E. is positive. However, T.E. may be be positive even if $c'q$ is non-drastic with respect to cq, e.g. consider the case where the demand function is $q = 14 - p$, $c = 10$ and $c' = 7$.

We then consider c' close to c. Since, from symmetry, $\pi_1(c, c') = \pi_2(c', c)$, equation (3) can be re-written as

$$\pi_1(c', c) + \pi_2(c', c) > 2\pi(c', c'), \tag{10}$$

where the L.H.S. of equation (10) represents the aggregate market profit when firm 1's cost function is $c'q$ and firm 2's is cq. We can then appeal

[6] Under price competition T.E. is necessarily positive.

to Tirole (1988), Chapter 10, Exercise 10.10, to claim that for c' close to c, equation (10) is necessarily violated.

Given the above discussion the next proposition follows from continuity.

Proposition 2. *(i) If, for* $\bar{c} = 0$*,* $\bar{c}q$ *is drastic compared to* cq*, then there exists* $0 \leq c'' < c$ *such that T.E. is positive for all* $c' \in [0, c'')$*.*

(ii) There exists $0 < c''' < c$ *such that T.E. is negative for all* $c' \in (c''', c)$*.*

Finally, we solve for the pure strategy subgame perfect Nash equilibria of the whole game under both the regimes.

Proposition 3. *(A) Suppose that T.E. is positive.*

(i) If $S(NP) \geq 0$*, then, irrespective of whether there is patent protection or not, both the firms investing in R&D constitutes an equilibrium.*

(ii) If $S(NP) < 0 \leq S(P)$*, then, both the firms investing in R&D constitutes an equilibrium under patent protection, but not in its absence.*

(B) Suppose that T.E. is negative. If $S(NP) \geq 0 > S(P)$*, then both the firms investing in R&D constitutes an equilibrium in the absence of patent protection, but not in its presence.*

Proof. Proposition 3(A)(i) follows since if T.E. is positive then $S(P) > S(NP) \geq 0$.

The proofs for the other parts of the proposition are straightforward.■

Proposition 3 demonstrates that if T.E. is positive, then patent protection has a positive effect on the equilibrium level of R&D. Whereas if it is negative, and $S(NP) \geq 0 > S(P)$, then patent protection may have a negative effect on equilibrium R&D. If, in addition, $\pi_1(c', c) - F > \pi(c, c)$, then, under patent protection, there are two equilibria both of which involve only one of the firms investing in R&D.[7] Whereas if $\pi_1(c', c) - F < \pi(c, c)$ then,

[7]Furthermore, there is a symmetric equilibrium in completely mixed strategies.

under patent protection, there is a unique equilibrium where neither of the firms invest in R&D.

Clearly the welfare effect in case patent protection adversely affects the equilibrium level of R&D, is ambiguous. While patent protection may reduce the level of R&D, and hence the level of competition in the market, it does reduce wasteful duplication of R&D. Depending on the parameter configuration (in particular the value of F), either effect may dominate.

4 Licensing

We next extend the analysis to allow for licensing. For simplicity we assume that technology transfer is costless. We consider a three stage game, where, in stage 1 the firms decide on whether to invest in R&D or not, in stage 2 there is licensing (possibly), and in stage 3 there is quantity competition.

Consider the stage 2 licensing game. At this stage the R&D cost F is sunk. Thus, in the absence of patents, such licensing would take place if and only if $2\pi(c',c') \geq \pi_1(c',\tilde{c}) + \pi_1(\tilde{c},c')$. Similarly, in the presence of patents, such licensing would take place if and only if $2\pi(c',c') \geq \pi_1(c',c) + \pi_1(c,c')$. We assume that in case of licensing, the outcome follows the asymmetric Nash bargaining solution, where the weight of the transferring firm is α, $0 \leq \alpha \leq 1$.

The case of interest is where there is licensing in the absence of patent protection, but not in the presence of patent protection (i.e. T.E. is negative). In that case we find that patents may discourage licensing, leading to a possible decrease in welfare.

Proposition 4. *Let* $\pi_1(c',c) + \pi_1(c,c') > 2\pi(c',c') \geq \pi_1(c',\tilde{c}) + \pi_1(\tilde{c},c')$. *Moreover, let* $\pi_1(c',c) - \pi(c,c) > F > \frac{\pi_1(c',c) - \pi_1(c,c')}{2}$ *and* $2\alpha\pi(c',c') - \alpha\pi_1(\tilde{c},c') + (1-\alpha)\pi_1(c',\tilde{c}) - \pi(c,c) > F > (2\alpha - 1)\pi(c',c') - \alpha\pi_1(\tilde{c},c') + (1-\alpha)\pi_1(c',\tilde{c})$. *Then patent protection leads to a reduction in efficiency,* *and possibly in welfare.*

The proof, which is straightforward, has been omitted. In the absence of patent protection, the equilibrium involves exactly one of the firms doing R&D. However, there is licensing, and both the firms obtain the technology. In the presence of patenting, the equilibrium again involves exactly one of the firms doing R&D. However, there is no licensing, so that only one of the firms has the new technology.[8] Given the decreasing marginal revenue property of $f(q)$, this causes a reduction in aggregate output and consumers' surplus, and hence possibly in welfare.

5 Conclusion

Depending on the efficiency of the new technology vis-a-vis the old one, T.E. may either increase or decrease R&D incentives. In case it decreases R&D incentives, patent protection may adversely affect the level of R&D, as well as welfare. Moreover, if T.E. is negative, then patenting may discourage licensing, thus leading to a reduction in efficiency, and possibly in welfare.

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[8] Note that this effect cannot arise in models with a competitive fringe (often employed in the literature), where licensing always takes place.

6 Appendix

Note 1. Formally modelling the firms' decision regarding whether to claim a patent or not when both the firms decide to do R&D:

Let the litigation costs, in case both the firms decide to claim a patent, be L. In case only one of the firms claim the patent, then there are no such costs. (Of course, even then there would be some administrative costs of claiming the patent. We, however, assume that these are relatively small.) Consider the game where both the firms simultaneously decide whether to contest the patent or not:

	Claim Patent	Do not Claim
Claim Patent	$\frac{\pi_1(c',c)+\pi_1(c,c')}{2} - L,\ \frac{\pi_2(c',c)+\pi_2(c,c')}{2} - L$	$\pi_1(c',c),\ \pi_2(c',c)$
Do not Claim	$\pi_1(c,c'),\ \pi_2(c,c')$	$\pi(c',c'),\ \pi(c',c')$

Note that if $L < \frac{\pi_1(c',c)-\pi_1(c,c')}{2}$, then the unique equilibrium involves both the firms claiming the patent. If, however, $L > \frac{\pi_1(c',c)-\pi_1(c,c')}{2}$, then there are two equilibria. Both these equilibria involve only one of the firms claiming the patent, and the other firm deciding not to claim the patent. Assume that there is some coordination device which allows the firms to coordinate on the above two outcomes with equal probability. Since the situation is symmetric, it is natural to use a symmetric coordination device. In that case the expected payoff of both the firms is $\frac{\pi_1(c',c)+\pi_1(c,c')}{2}$. Thus the expected payoff of both the firms is $\frac{\pi_1(c',c)+\pi_1(c,c')}{2}$ if either $L = 0$, or $L > \frac{\pi_1(c',c)-\pi_1(c,c')}{2}$.

Of course, for intermediate values of the litigation cost, i.e. $0 < L < \frac{\pi_1(c',c)-\pi_1(c,c')}{2}$, the equilibrium would involve both the firms claiming the patent and the litigation costs would be non-zero in equilibrium. However, we refrain from describing this case, since doing so does not in any way add to the economic content of this paper.

Note 2. Footnote 5:

Given Assumption 1, uniqueness follows from Kolstad and Mathiesen (1987), whereas stability follows from Hahn (1962). Finally, given uniqueness we can show that the outcome is symmetric whenever the costs are also symmetric. Suppose that both firms have the same cost function, but the equilibrium output vector (q_1^*, q_2^*) is such that $q_1^* \neq q_2^*$. In that case (q_2^*, q_1^*) also constitutes a Nash equilibrium of the above game, thus violating uniqueness.

Hahn, F., 1962, The stability of the Cournot oligopoly solution concept, Review of Economic Studies 29, 329-331.

Kolstad, C. and L. Mathiesen, 1987, Necessary and sufficient conditions for the uniqueness of a Cournot equilibrium, Review of Economic Studies 54, 681-690.

Note 3. Properties of eqn. (9):

(i) Note that in case of complete spill-over equation (9) simplifies to $\pi_1(c', c) > \pi_1(c, c')$, which is always satisfied.

(ii) Given Assumption 1, the fact that the L.H.S. of equation (9) is decreasing in $\tilde{c}$, follows from Dixit (1986).

Dixit, A., 1986, Comparative statics for oligopoly, International Economic Review 27, 107-122.

Note 4. Demonstrating that eqn. (10) is violated for c' close to c:

From Tirole (1988), Chapter 10, Exercise 10.10, observe that eqn. (10) is violated for c' close to c, whenever the Cournot equilibrium is locally stable and the industry marginal revenue is downward sloping. Given Assumption 1, both these conditions are satisfied. Note that the industry marginal revenue is decreasing if and only is $2f'(q) + qf''(q) < 0$, i.e. if $[f'(q) + q_1 f''(q)] + [f'(q) + q_2 f''(q)] < 0$. Given Assumption 1, this is always true.

Note 5. Example to demonstrate that Proposition 2 is not vacuous:

Let the demand function be $q = a - p$.

(i) Note that if $a = 11$, $c = 10$ and $c' = 7$, then $c'q$ is drastic compared to cq. This follows since $a - 2c + c' < 0$. Thus the tournament effect is positive i.e. Proposition 2(i) goes through.

(ii) Note that if $c'q$ is not drastic, then $[\pi_1(c', c) + \pi_1(c, c')] > 2\pi(c', c')$ if and only if $5c - 3c' > 2a$. It is easy to see that this is satisfied for $a = 14$, $c = 10$ and $c' = 7$, whereas it is violated for $a = 20$, $c = 10$ and $c' = 7$.

Note 6. Demonstrating that the tournament effect is positive under price competition (Footnote 6):

Let us abuse notation to use the same notations for the gross equilibrium profits under both quantity and price competition. Under price competition with linear cost functions and a homogeneous good we know that $\pi(c', c') = \pi_1(c, c') = 0$, whereas $\pi(c', c) = \min\{\pi_m(c'), f^{-1}(c)(c - c')\} > 0$. Hence equation (10) goes through.

Note 7. Example to demonstrate that Proposition 3b is not vacuous, as well as that depending on the parameter configurations, patent protection may either increase, or decrease welfare:

Consider the case where the demand function is $q = 20 - p$, $c = \tilde{c} = 10$, $c' = 7$, and $F = 11.5$. From the additional Note 5(ii) we know that in this case the tournament effect is negative. Moreover, it is easy to see that $\pi(c, c) = 11.11$, $\pi(c', c') = 18.78$, $\pi_1(c, c') = \pi_1(\tilde{c}, c') = 5.44$ and $\pi_1(c', c) = \pi_1(c', \tilde{c}) = 28.44$.

In the absence of patent protection the game matrix is:

	R&D	No R&D
R&D	7.28,7.28	16.94, 5.44
No R&D	5.44,16.94	11.11, 11.11

Thus the unique equilibrium involves both the firms investing in R&D. In equilibrium the aggregate output is 26/3. Thus the consumers' surplus is 37.56, and total welfare is 52.10.

Next consider the game matrix in the presence of patent protection:

	R&D	No R&D
R&D	5.44,5.44	16.94, 5.44
No R&D	5.44,16.94	11.11, 11.11

Thus there are two possible equilibria. In both these equilibria one of the firms invests in R&D, whereas the other firm does not. In equilibrium the aggregate output is 23/3. Thus the consumers' surplus is 29.39, and total welfare is 51.77.

Hence in the absence of patent protection the unique equilibrium involves both the firms investing in R&D, whereas in the presence of patent protection only one of the firms invests in R&D. Hence patent protection causes a reduction in the equilibrium level of R&D. Moreover, both consumers' surplus and total welfare is reduced as a result of patent protection.

We then modify the above example slightly so that the value of F is increased to 12. It is easy to check that this will not affect the equilibrium strategies regarding R&D investment, as well as the quantity levels. Thus, in the absence of patent protection, the outcome still involves both the firms investing in R&D. Thus aggregate welfare is 51.10. Note that the reduction in welfare is solely because of the increase in F. Next, note that in the presence of patent protection, the outcome again involves only one of the firms doing R&D. Thus the aggregate welfare is 51.27, which is strictly greater than that in the absence of patent protection.

Thus if the demand function is $q = 20 - p$, $c = \tilde{c} = 10$, $c' = 7$, and $F = 11.5$, then patent protection reduces both R&D and welfare. Whereas if, in the above example, the value of F is increased to 12, then patent protection reduces R&D, but increases welfare. ■

Note 8. Mixed Strategy Equilibrium under Patent Protection when there is no Licensing (Footnote 7):

Let us first consider the case where $\pi_1(c', c) - F > \pi(c, c)$. In this case, apart from the pure strategy equilibrium, there is a completely mixed strategy equilibrium where both the firms invest in R&D with probability $0 < r < 1$, where

$$r = \frac{2[\pi_1(c', c) - F - \pi(c, c)]}{\pi_1(c', c) + \pi_1(c, c') - 2\pi(c, c)}.$$

That $0 < r < 1$, follows from the fact that $S(P) < 0$ and $\pi_1(c', c) - F > \pi(c, c)$.

Whereas if $\pi_1(c', c) - F < \pi(c, c)$, then there is no equilibrium in completely mixed strategies. ■

Note 9. Sketch of Proof of Proposition 4:

First consider the case where there is no patent protection. Suppose that firm i has done R&D, whereas firm j has not. Given that $2\pi(c', c') \geq \pi_1(c', \tilde{c}) + \pi_1(\tilde{c}, c')$, there will be licensing. Since firm i's bargaining power is α, firm i's post-licensing payoff is

$$2\alpha\pi(c', c') + (1 - \alpha)\pi_1(c', \tilde{c}) - \alpha\pi_1(\tilde{c}, c') - F,$$

and that of firm j is

$$2(1 - \alpha)\pi(c', c') - (1 - \alpha)\pi_1(c', \tilde{c}) + \alpha\pi_1(\tilde{c}, c').$$

Whereas if both the firms invest in R&D, they both have a payoff of $\pi(c', c') - F$. Similarly, in case neither firm invests in R&D, their payoff is $\pi(c, c)$.

Hence, given that $2\alpha\pi(c',c') - \alpha\pi_1(\tilde{c},c') + (1-\alpha)\pi_1(c',\tilde{c}) - \pi(c,c) > F > (2\alpha-1)\pi(c',c') - \alpha\pi_1(\tilde{c},c') + (1-\alpha)\pi_1(c',\tilde{c})$, there are two equilibria both of which involve exactly one of the firms investing in R&D.

Next we consider the case where there is patent protection. Given that $\pi_1(c',c) + \pi_1(c,c') > 2\pi(c',c')$, there is no licensing. Thus the payoff matrix in stage 1 is the same as that in the absence of licensing (last paragraph, page 4). Given that $\pi_1(c',c) - \pi(c,c) > F > \frac{\pi_1(c',c)-\pi_1(c,c')}{2}$, there are two equilibria both of which involve exactly one of the firms doing R&D. Moreover, there is no licensing in equilibrium. Thus in the presence of patent protection only one of the firms will have the new technology. Given the decreasing marginal revenue property of $f(q)$, there is a reduction in aggregate output and hence in consumers' surplus. The argument is as follows. Summing up the Cournot first order conditions and differentiating, we obtain

$$\frac{dQ}{d(c+c')} = \frac{1}{2f'(Q) + Qf''(Q)},$$

where Q denotes the aggregate output level. From the decreasing marginal revenue property we have that $2f'(Q) + Qf''(Q) < 0$, hence the result.

We then use an example to show that there are parameter values satisfying the hypotheses of Proposition 4. Moreover, for this example patent protection leads to fall in both efficiency and welfare.

Let $q = 14 - p$, $c = 10$, $c' = 7$, $\tilde{c} = 9$, $F = 6$ and $\alpha = 1/2$ (symmetric Nash bargaining). It is easy to check that c' is not drastic with respect to either c or c'. Thus $\pi(c,c) = 16/9$, $\pi(c',c') = 49/9$, $\pi_1(c,c') = 1/9$, $\pi_1(c',c) = 100/9$, $\pi_1(\tilde{c},c') = 1$ and $\pi_1(c',\tilde{c}) = 9$. It is clear that these values satisfy the hypotheses of Proposition 4.

In the absence of patent protection the game matrix is:

	R&D	No R&D
R&D	49/9-F, 49/9-F	85/9-F, 13/9
No R&D	13/9, 85/9-F	16/9, 16/9

Thus there are two equilibria both of which involve exactly one of the firms doing R&D, followed by licensing. Note that in this case the aggregate output is 14/3, leading to a consumers' surplus of 98/9. Thus the aggregate welfare is $21.78 - F$.

Next consider the game matrix in the presence of patent protection:

	R&D	No R&D
R&D	101/18-F, 101/18-F	100/9-F, 1/9
No R&D	1/9, 100/9-F	16/9, 16/9

Thus there are two possible equilibria. In both these equilibria one of the firms invests in R&D, whereas the other firm does not. Moreover, there is no licensing in equilibrium. Next note that the aggregate output is 11/3, so that consumers' surplus is 121/18. Thus total welfare is $17.94 - F$. Note that this is strictly less than that in the absence of patent protection.

Another example which satisfies the hypotheses of Proposition 4, and where patent protection leads to fall in welfare is one where, $q = 11 - p$, $c = 10$, $c' = 7$, $\tilde{c} = 8$, $\alpha = 1/2$ and $F = 2$. However, it differs from the above example in that c' is drastic with respect to c.

Note 10. A few additional results for Section 4:

Let us define $N(NP, L)$, $S(NP, L)$, $N(P, L)$ and $S(P, L)$ in a manner analogous to that for $N(NP)$, $S(NP)$, $N(P)$ and $S(P)$ respectively, with the difference that we now allow for licensing possibilities.

Result. *Suppose* $\pi_1(c', c) + \pi_1(c, c') > 2\pi(c', c') \geq \pi_1(c', \tilde{c}) + \pi_1(\tilde{c}, c')$. *Then*

(i) $N(NP, L) \geq N(NP)$ *and* $S(NP, L) \leq S(NP)$.

(ii) $N(P, L) > N(NP, L)$ *and* $S(P, L) > S(NP, L)$.

The proofs, which are straightforward, have been omitted. Note that the second result shows that in the presence of licensing, patent protection increases both the strategic, as well as the non-strategic incentive for R&D. This makes Proposition 4 even more surprising.

Finally, it is straightforward to show that analogous results go through even if licensing takes place both in the presence, as well as in the absence of patent protection, i.e. if $2\pi(c', c') \geq \pi_1(c', \tilde{c}) + \pi_1(\tilde{c}, c')$ and $2\pi(c', c') \geq \pi_1(c', c) + \pi_1(c, c')$.

7 References

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