Permutation Routing in Optical MIN's with Minimum Number of Stages

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Abstract

In hybrid optical Multistage Interconnection Networks (MIN's), optical signals are routed by electronically controlled switches using directional couplers. One important problem of these hybrid optical MIN's is the path-dependent loss of the optical signal, which is directly proportional to the number of couplers, i.e., the number of switches through which the signal passes. In general, given the network size and the type of the MIN, the number of stages of a MIN is constant. Hence any input signal has to pass through a fixed number of couplers to reach the output.

In this paper, we propose that instead of using a fixed-stage $N \times N$ MIN, we may route any arbitrary $N \times N$ permutation P with minimum delay and minimum path-dependent loss, if we know the minimum number of stages of the MIN necessary to route P. Here, we present an $O(Nn)$ algorithm ($N = 2^n$) that checks whether a given permutation P is admissible in an m stage shuffle-exchange network (SEN), $1 \leq m \leq n$, and determines in $O(Nn \log n)$ time the minimum number of stages m of shuffle-exchange, required to realize P. Furthermore, for $n < m \leq 2n-1$, we present a necessary condition for permutation admissibility, in general, which is also a sufficient condition for BPC (bit-permute-complement) class permutations. Hence we find the minimum number of shuffle-exchange stages $m_{\min}$ required to make any arbitrary BPC permutation admissible, $1 \leq m \leq 2n-1$, in $O(Nn \log n)$ time. By this technique, a BPC permutation can be routed through minimum number of shuffle-exchange stages, that enables us to minimize the path dependent loss of the signal, as well as the communication delay in the optical MIN.

Keywords: Multistage Interconnection Network (MIN), hybrid optical MIN's, permutation admissibility, path-dependent loss, BPC (bit-permute-complement) permutations, shuffle-exchange networks.

1 Introduction

To meet the demands of increasing bandwidth and low communication latency, optical interconnection networks are felt to be perhaps the only feasible interconnection technology

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for high-performance computing/communication applications. With the advances in optical technology, optical Multistage Interconnection Networks (MIN's) have emerged as a promising networking choice. An optical MIN can be implemented with either free space optics or guided wave technology. In the hybrid optical networks using guided wave technology, optical signals are switched, but both the switch control and routing decisions are carried out electronically at a speed much lower than that of the optical signal. The other option is all-optical switch which would potentially overcome the speed mismatch problem. However, such systems are yet to come up in reality [7, 15]. Two key performance metrics for a hybrid optical MIN are its path dependent loss or attenuation and cross talk. In [15], the concept of *semi-permutations* has been introduced that avoids the problem of cross-talk by routing any permutation in two passes through the Benes network. In this paper, we consider the other problem, that is the path dependent loss. In optical interconnection networks, the path dependent loss of the optical signal depends on the number of stages of the MIN, or more specifically, on the number of switches, an input signal has to traverse to reach the desired output. Now, given the size of the network and the architecture of the MIN, the number of stages of a MIN is constant. Hence any input signal has to pass through a fixed number of couplers to reach the output.

A typical $N \times N$ blocking multistage interconnection network (MIN), with n-stages, where $N = 2^n$, is a minimal full-access unique-path structure, since it provides exactly one path between any pair of input-output, e.g., omega, baseline, cube, reverse-baseline etc. [14, 8]. However, to connect more than one input-output pairs simultaneously, a single link may be required by two or more paths, causing conflicts. An $N \times N$ permutation from the set of N inputs $(0, 1, \ldots, N-1)$ to the set of N outputs $(0, 1, \ldots, N-1)$ is said to be *admissible*, if N conflict-free paths (one for each input-output pair) can be set up simultaneously in the MIN [8, 9, 10].

Shuffle-exchange network (SEN) is a well-studied interconnection network with wide application in parallel processing and communication networks [12, 6, 13, 2]. To enhance the set of admissible permutations, and also to have some fault-tolerance, k-extra stage SEN's are proposed [8, 10]. Recent studies on the permutation admissibility problem of $N \times N$ shuffle-exchange network (SEN) reported that a permutation is admissible in a k-extra-stage SEN, i.e., with $(n+k)$-stages, if and only if the conflict graph is 2^k-colorable [11]. Although the c-coloring problem in graphs, for $(c > 2)$, is NP-complete, the complexity of determining permutation admissibility in a k-extra-stage MIN, for $k \geq 2$, is not known. For $0 \leq k \leq 1$, $O(Nn)$ algorithms were reported for checking *permutation admissibility*, but in general, the problem remains open [9, 10]. In [11, 5], $O(N \log N)$ algorithms have been developed for optimal routing of BPC (bit-permute-complement), and LC (linear-complement) permutations respectively, on a k-extra-stage SEN, $1 \leq k \leq (n-1)$, in multiple passes. This optimal technique results a transmission delay $O(n+k)p$, where p is the required number of passes, and $1 \leq p \leq 2^{\lfloor n/2 \rfloor}$ [14, 8].

In this paper, we address the following more general problem: given a $N \times N$ permutation P, find the minimum number of stages $m_{\min}$ of SEN required to make P admissible. For optical MIN's this technique will be especially helpful, because it will enable us to reduce the path-dependent loss of the signal by minimizing the number of stages of the MIN. For $1 \leq m \leq n$, we propose an $O(Nn \log n)$ algorithm to find $m_{\min}$. For $n < m \leq 2n-1$, we formulate a necessary condition that a permutation P must satisfy for being admissible on an m stage SEN.

Next we have shown that this necessary condition is sufficient for the permutation admissibility of BPC (bit-permute-complement) permutations. It enables us to find out the minimum number of shuffle-exchange stages required to make any arbitrary BPC permutation admissible in $O(Nn \log n)$ time. Therefore, given any arbitrary $N \times N$ BPC permutation P, now we can find the minimum number of stages of SEN, $m_{\min}$, required to make P admissible, $1 \leq m_{\min} \leq (2n-1)$. Hence employing a $(2n-1)$-stage hybrid optical SEN, we can always route a BPC permutation using m stages only, that enables us to keep the path dependent loss of the optical signal to a minimum. Since, BPC permutation is an important class of permutations, frequently used in parallel processing, this technique will be useful for high performance computing using optical MIN's.

The paper is organized as follows. In Section 2, we analyze some properties of m-stage $N \times N$ SEN, $1 \leq m \leq 2n-1$, and hence we formulate the conditions for admissibility for a permutation on an m-stage SEN. It enables us to find out the minimum number of shuffle-exchange stages $m_{\min}$ to make a given permutation P admissible for $1 \leq m \leq n$ in $O(Nn \log n)$ time. For $n < m \leq 2n-1$, we present some necessary conditions for admissibility. In section 3, we show that these necessary conditions are also sufficient for BPC permutations, and hence we find the minimum number of shuffle-exchange stages ($m_{\min}$) required to make any BPC permutation P admissible, for $1 \leq m_{\min} \leq 2n-1$, in $O(Nn \log n)$ time. Conclusions and further discussions appear in Section 4.

2 Input-Output Groups and Permutation Admissibility

A full-access unique-path $N \times N$ multistage interconnection network, consists of n stages of 2×2 switches, $(N = 2^n)$, which is essentially a minimal structure that provides full-accessibility with exactly one path between any input-output pair. Now, to provide some fault-tolerance, as well as to enhance the set of permutations admissible in the MIN, k extra stages, $1 \leq k \leq n-1$ are added to it. Here, given a $(2n-1)$-stage hybrid optical SEN, we find out the minimum number of stages $m_{\min}$ required to route a given BPC permutation P without conflict. Then P is routed using first $m_{\min}$ stages of the SEN, keeping the path dependent loss of the signal minimum. A possible configuration of such an SEN with 8 inputs is shown in Fig. 1. Each switch is of size 2×4, having an additional stage-wise control C_s. For $C_s = 0$, one pair of outputs are selected through which the signal is forwarded to the input of the next stage. Whereas $C_s = 1$ selects the other pair of outputs for each switch of the stage, through which signals appear at the final output passing through a passive optical concentrator.

Since, in a SEN, the connection patterns between adjacent stages are always the same, the sets of permutations admissible in an m-stage SEN, for $1 \leq m \leq 2n-1$ exhibit some elegant

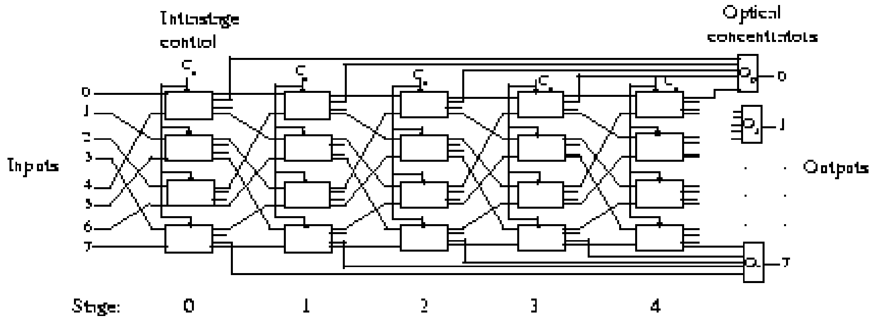

Fig. 1 : An 8x8 hybrid SEN with variable number of stages

properties.

2.1 Group Structures

The concept of input (output) group structures for MIN's was introduced earlier in [1, 3]. Here, we will extend those ideas of *group structures* for analyzing the relations between input (output) groups and permutation admissibility of a SEN. For completeness, we give a brief description of the input (output) group structures for SEN. In an m stage $N \times N$ SEN, $1 \leq m \leq 2n-1$,

- inputs (or outputs) are labeled as: $0, 1, \ldots, N-1$ respectively, from top to bottom, and each is represented uniquely by an n bit binary string;
- the stages are labeled as: $0, 1, \ldots, (m-1)$, from the input side towards the output side;
- the output links of each stage are labeled as: $0, 1, \ldots, (N-1)$, from top to bottom.

An SEN with $N = 8$ and $m = 2$ is shown in Fig. 2. Now let us consider an input represented in binary as: $(x_{n-1}x_{n-2}\ldots x_1x_0)$. If we keep the sub string $(x_{n-j-1}x_{n-j-2}\ldots x_1x_0)$ fixed, say it is of value p, $0 \leq p < 2^{n-j}$, and take all possible combinations of the remaining j bits in the sub string $(x_{n-1}x_{n-2}\ldots x_{n-j})$, $0 \leq j \leq n$, we will get the 2^j elements of an *input group at level* j, denoted by $g_i(j,p)$. The formal definition is given below:

Definition 1 *For an $N \times N$ SEN, an input group $g_i(j,p)$, at level j, $0 \leq j \leq n$, defines a set of 2^j inputs given by $\underbrace{**\ldots*}_{j \text{ times}}x_{n-j-1}x_{n-j-2}\ldots x_1x_0$, where $x_{n-j-1}\ldots x_1x_0$ is the binary representation of p, i.e., $0 \leq p < 2^{n-j}$, and $* \in \{0,1\}$.*

Definition 2 *Similarly, for an $N \times N$ SEN, an output group $g_o(j,p)$ at level j, $0 \leq j \leq n$, defines a group of 2^j outputs at level j, given by $x_{n-1}x_{n-2}\ldots x_j\underbrace{**\ldots*}_{j \text{ times}}$, where $(x_{n-1}\ldots x_j)$ is the binary representation of p i.e., $0 \leq p < 2^{n-j}$, and any $* \in \{0,1\}$.*

Example 1 *For a* 16×16 *SEN an input group*

$$g_i(2,3) = (**11) = (0011, 0111, 1011, 1111) = (3, 7, 11, 15).$$

Similarly, an output group

$$g_o(2,3) = (11**) = (1100, 1101, 1110, 1111) = (12, 13, 14, 15).$$

For an 8×8 SEN, the input (output) group structure is shown in the form of a tree in Fig. 3.

In next section, we describe how this idea of group structures help in deciding the *permutation admissibility* problem in an m-stage SEN, $n \leq m \leq 2n-1$. We have studied the two cases separately: one for $1 \leq m \leq n$ and the other with $n < m \leq (2n-1)$.

2.2 m-stage $N \times N$ SEN, $1 \leq m \leq n$

In this case, the network is not a full-access one, except when $m = n$. Starting from any input one can reach only a particular set of $2^m (\leq N)$ outputs. However, given any input-output pair, if a path exists, it is always unique.

Definition 3 *For an* m*-stage* $N \times N$ *SEN,* $1 \leq m \leq n$*, the set of outputs which can be reached from an input* x *is referred to as the reachable set of the input* x*.*

Lemma 1 *In an* m*-stage* $N \times N$ *SEN,* $0 < m \leq n$*, for any input* $x \in g_i(m,p)$*, the reachable set is the output group* $g_o(m,p)$*, where* $0 \leq p < 2^{n-m}$*.*

Proof : Let us consider an $N \times N$ SEN with m stages, $0 < m \leq n$. Now starting from any input, say $x = x_{n-1}x_{n-2}\ldots x_1x_0$, at each stage it may follow either a shuffle or a shuffle-exchange. Therefore, after the first stage, x may reach any output represented by $(x_{n-2}\ldots x_1x_0*)$, where $* \in \{0,1\}$. After m successive stages x may reach any output of the set $(y = x_{n-m-1}\ldots x_1x_0\underbrace{**\ldots*}_{m\text{ times}})$. But this set is the output group $g_o(m,p)$, where $p = (x_{n-m-1}\ldots x_1x_0)$.

This shows that for any input in the group $g_i(m,p) = (\underbrace{**\ldots*}_{m\text{ times}}x_{n-m-1}\ldots x_1x_0)$, the reachable set will be the same output group $g_o(m,p)$, for $0 \leq p < 2^{n-m}$. Hence the proof. □

Corollary 1 *In an* m *stage* $N \times N$ *SEN,* $0 < m < n$*, an output* $y = y_{n-1}y_{n-2}\ldots y_1y_0$ *is reachable from an input* $x = x_{n-1}x_{n-2}\ldots x_1x_0$*, if and only if* $y_{n-j} = x_{n-m-j}$*, for all* j*,* $1 \leq j \leq n-m$*.*

Proof : Follows directly from Lemma 1. □

Definition 4 *In an m-stage $N \times N$ SEN, $0 < m \le n$, if an output y is reachable from an input x, the path $x \to y$ is said to be a realizable path.*

Remark 1 *In an m-stage $N \times N$ SEN, $1 \le m \le n$, if an output $y = y_{n-1}y_{n-2}\dots y_1y_0$ is reachable from input $x = x_{n-1}x_{n-2}\dots x_1x_0$, the path $x \to y$ can be represented by the string of $(n+m)$ bits: $x \to y : x_{n-1}x_{n-2}\dots x_1x_0y_{m-1}y_{m-2}\dots y_1y_0$, where the window $(x_{n-j-2}x_{n-j-3}\dots x_1x_0y_{m-1}y_{m-2}\dots y_{m-j-1})$, is the binary representation of the link label, the path follows at any stage j, $0 \le j < m$.*

By Corollary 1, we can represent the same path as $x \to y : x_{n-1}x_{n-2}\dots x_{n-m}y_{n-1}\dots y_1y_0$, since $y_{n-j} = x_{n-m-j}$, for all j, $1 \le j \le n-m$.

The above result conforms with similar results established in [11], for $m = n$ only.

Example 2 *In a 3-stage 16×16 SEN, a path $3 \to 13$ is shown in Fig. 4a. The path is represented as $3(0011) \to 13(1101) : 0011101$, the four bits from the left represents the input 3, and the four bits from the right represents the output 13, the two overlap in the middle bit. The links followed by the path in stages $0, 1, 2$ are $7(0111)$, $14(1110)$, and $13(1101)$ respectively, as given by the respective windows, are shown in Fig. 4b.*

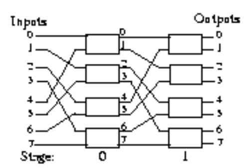

Fig. 2 : A 2-stage 8x8 SEN with link labels

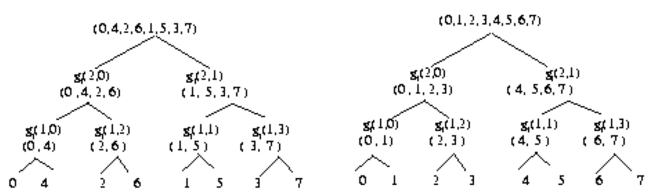

Fig. 3 : a) Input and **b)** output group structures of an 8x8 SEN

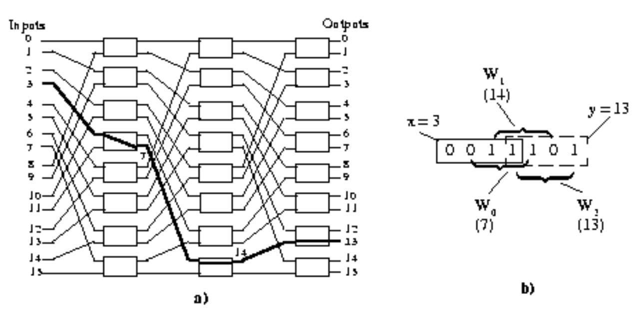

Fig. 4 (a): The Path $3 \to 13$ on a 3-stage 16x16 SEN **b)** the windows in each stage

Definition 5 *For an $N \times N$ SEN, with m stages, for $1 \le m \le n$, given an $N \times N$ permutation P, in which all the input-output paths are realizable, we may construct an $N \times (n+m)$ binary matrix M, where, each row $x_{n-1}x_{n-2}\ldots x_1x_0y_{m-1}y_{m-2}\ldots y_0$ represents one realizable input-output path $x \to y$ such that $x_{n-1}x_{n-2}\ldots x_1x_0$ is the input x, and $x_{n-m-1}x_{n-m-2}\ldots x_1x_0y_{m-1}y_{m-2}\ldots y_0$, is the corresponding output y.*

The matrix M is defined as the path matrix of P, for an m-stage SEN, $1 \le m \le n$.

The notion of *path matrix* has been introduced earlier by Shen, in [10], for m-stage SEN, $n \le m \le (2n-1)$. We have generalized it for $1 \le m \le (2n-1)$.

Definition 6 *A window $W_j, 1 \le j < m$, of the path matrix M, is defined as the set of n consecutive columns of M, $\{x_{n-j-2}\ldots x_1x_0\ y_{m-1}y_{m-2}\ldots y_{m-j-1}\}$.*

Definition 7 *A $p \times q$ matrix is called to be an independent matrix, if all its rows are distinct.*

Example 3 *The path matrix M for the permutation*
$P : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 3 & 0 & 1 & 2 & 5 & 6 & 7 & 4 \end{pmatrix}$, *on a 2-stage 8×8 SEN is given below:*

$$M : \begin{pmatrix} x_2 & x_1 & x_0(y_2) & y_1 & y_0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Note that all the windows $W_j, 0 \le j \le 1$ are independent.

Definition 8 *The window containing the leftmost n columns of a path matrix M, for any permutation on an m-stage SEN, i.e. the columns $x_{n-1}x_{n-2}\ldots x_1x_0$, is called the source window.*

Note that the source window is always an independent matrix.

Remark 2 *By virtue of the shuffle interconnection between stages, each row of W_j is the link, the corresponding path $x \to y$ follows at the output of stage j.*

Note that for each input-output pair in P, the path matrix M contains just a unique path for $1 \le m \le n$.

Therefore, for an m-stage $N \times N$ SEN, $1 \leq m \leq n$, given any permutation P, if all input-output paths are realizable, the set of N paths is represented by an $N \times (n+m)$ *path matrix* M, where each row stands for one input-output path, as explained earlier.

Now P will be admissible in the SEN, if and only if all the rows in every window W_j, $1 \leq j < m$, are distinct, i,e., no two paths at any stage need the same link, or, in other words there is no conflict.

Definition 9 *In an $N \times N$ SEN, two inputs (outputs) x_1 and x_2 are said to be covered by an input (output) group $g_{i(o)}(j,p)$ if x_1, x_2 belong to the same input (output) group at level j, $1 \leq j \leq n$, but to different groups at level $(j-1)$.*

Example 4 *For a 16×16 SEN, two inputs $8(1000)$ and $6(0110)$ are covered by $g_i(3,0)$, i.e., the group $(***0)$.*

Remark 3 *In an $N \times N$ SEN, two inputs (outputs) x_1 and x_2 covered by an input (output) group at level j, must differ in bit $x_{n-j}(x_j)$.*

Lemma 2 *In an m-stage $N \times N$ SEN, $(1 \leq m \leq n)$, two realizable paths $x \rightarrow y$ and $x' \rightarrow y'$ are conflicting if and only if for x, x' covered by an input group at level j, $1 \leq j \leq m$, y, y' belong to the same output group at level $(m-j)$.*

Proof : Let $x = x_{n-1}x_{n-2}\ldots x_1x_0$. Then $x' = x'_{n-1}\ldots x'_{n-j+1}\overline{x}_{n-j}x_{n-j-1}\ldots x_1x_0$, since they are covered by an input group at level $j, 1 \leq j \leq m$.

Since y is reachable for x, by Corollary 1, $y = x_{n-m-1}x_{n-m-2}\ldots x_1x_0y_{m-1}y_{m-2}\ldots y_1y_0$. Similarly, $y' = x_{n-m-1}x_{n-m-2}\ldots x_1x_0y'_{m-1}y'_{m-2}\ldots y'_1y'_0$.

Now the paths $x \rightarrow y$ and $x' \rightarrow y'$ can be represented as:

$$\begin{aligned} x \rightarrow y &: x_{n-1}\ldots x_{n-m-1}\ldots x_1x_0y_{m-1}y_{m-2}\ldots y_1y_0 \\ x' \rightarrow y' &: x'_{n-1}\ldots x'_{n-j+1}\overline{x}_{n-j}x_{n-j-1}\ldots x_{n-m-1}\ldots x_0y'_{m-1}\ldots y'_0. \end{aligned}$$

It is evident that these two paths will never conflict at any stage k, for $0 \leq k \leq j-2$, since $x_{n-j} \neq x'_{n-j}$.

If part: Let the corresponding outputs y and y' be in the same output group at level $(m-j)$, $y' = x_{n-m-1}x_{n-m-2}\ldots x_1x_0y_{m-1}\ldots y_{m-j}y'_{m-j-1}\ldots y'_1y'_0$. The two paths under consideration become:

$$\begin{aligned} x \rightarrow y &: x_{n-1}\ldots x_{n-m-1}\ldots x_1x_0y_{m-1}\ldots y_{m-j}\ldots y_1y_0 \\ x' \rightarrow y' &: x'_{n-1}\ldots x'_{n-j+1}\overline{x}_{n-j}x_{n-j-1}\ldots x_{n-m-1}\ldots x_0y_{m-1}\ldots y_{m-j}y'_{m-j-1}\ldots y'_0 \end{aligned}$$

Now it is obvious that these two paths will be always conflicting in stage $(j-1)$, where they both need the same link $(x_{n-j-1}\ldots x_{n-m-1}\ldots x_1x_0y_{m-1}\ldots y_{m-j})$. They may also conflict in a following stage $(j+k-1)$, $1 \leq k < m-j$, if $y'_{m-j-r} = y_{m-j-r}$, for all r, $1 \leq r \leq k$.

Only if: Let y and y' belong to two different output groups at level $(m-j)$, say $g_o(m-j,p)$, and $g_o(m-j,p'), p \neq p'$. Let $p = (y_{n-1} \dots y_{m-j})$, and $p' = (y'_{n-1} \dots y'_{m-j})$; they differ at least in one bit position. The two paths won't conflict at any stage k, for $0 \leq k < m$. □

Theorem 1 *In an m-stage $N \times N$ SEN ($1 \leq m \leq n$), a permutation P is admissible if and only if*

i) each input is mapped to a reachable output, and

ii) the 2^j inputs of any input group at level j, $1 \leq j \leq m$ are mapped to outputs so that no two of them belong to the same output group at level $(m-j)$.

Proof : Follows directly from Lemma 2. □

Definition 10 *For an $N \times N$ SEN, the i-admissible set of permutations π_i is defined as the set of all permutations admissible in i stages, where $1 \leq i \leq n$.*

Example 5 *In the path matrix for P, in a 2-stage SEN shown in Example 3, note that in each window, all the rows are distinct, proving that all the paths are conflict-free, i.e., the permutation is admissible in 2-stage SEN.*

Corollary 2 *In any m-stage $N \times N$ SEN, $1 \leq m \leq n$ all the i-admissible sets of permutations are disjoint.*

Proof : Follows directly from Theorem 1. □

In the next section we propose an $O(Nn)$ algorithm to determine whether or not an arbitrary $N \times N$ permutation P is admissible in an m-stage $N \times N$ SEN ($1 \leq m \leq n$). Formerly, an algorithm with the same complexity was reported only for an n-stage SEN [9].

2.3 Admissibility Algorithm

Here we present an algorithm for checking the admissibility of a given permutation P in an m-stage $N \times N$ SEN, $1 \leq m \leq n$. The permutation is given as an array *out* of length N, such that $out(i)$ stores the output corresponding to the input i, $0 \leq i \leq N-1$.

Algorithm : Admissibility Check

Input: *n, m, out(N)*
Output: *success*
Step 1: *success := 0*
Step 2: *for* $x = 0 \dots N-1$,
  if $x \pmod{2^{n-m}} \neq out(x) \text{ div } 2^m$
  then terminate
Step 3: *for* $i = 1 \dots m$
  for $p = 0 \dots (2^{i-1}-1)$

for $x = p2^{n-i+1} \ldots (p2^{n-i+1} + 2^{n-i} - 1)$
if i *is the minimum number such that* $out(x)$ *and* $out(x + 2^{n-i})$ *differ in bit* x_{m-i},
then if $out(x) > out(x + 2^{n-i})$,
exchange them in array out;
next x
else terminate;
next p
next i
Step 4: success:= 1, terminate;

The above algorithm is of time complexity $O(Nn)$.

Now given any permutation P, admissible on an i-stage SEN, $1 \leq i \leq n$, in order to find the minimum number of stages m necessary to make P admissible, we can perform a binary search over the interval 1 to i by invoking the above algorithm at most $\log n$ times. Therefore, m can be determined in $O(Nn \log n)$ time.

2.4 m-stage $N \times N$ SEN, $n \leq m \leq 2n - 1$

In this case, the network is *full-access*, i.e., each input can reach any output, but instead of a *unique-path* there exist 2^{m-n} paths between any pair of input-output. Now we may represent an input-output path $x \to y$, as a sequence of $(n + m)$ bits, $(x_{n-1}x_{n-2} \ldots x_1x_0 *_1 \ldots *_{n-m}\, y_{n-1} \ldots y_1y_0)$, where $(x_{n-1}x_{n-2} \ldots x_1x_0)$ is the binary representation of the input x, and $(y_{n-1} \ldots y_1y_0)$ represents the output y; the $(m-n)$ bits between the two, represented as $(*_1 *_2 \ldots *_{m-n})$ are arbitrary. In fact, each possible combination of these bits, represents a path from x to y.

In this case the path matrix may be redefined in the following way [10]:

Definition 11 *For an* $N \times N$ *SEN, with* m *stages, for* $n < m \leq 2n - 1$*, given any* $N \times N$ *permutation* P*, we construct an* $N \times (n + m)$ *binary matrix* M*, where, each row* $x_{n-1}x_{n-2} \ldots x_1x_0 *_1 *_2 \ldots *_{m-n}\, y_{n-1}y_{n-2} \ldots y_1y_0$ *represents one input-output path* $x \to y$*, such that* $x_{n-1}x_{n-2} \ldots x_1x_0$ *is the input* x*,* $y_{n-1}y_{n-2} \ldots y_0$ *is the corresponding output* y*, and each* $*_j \in \{0, 1\}$ *for* $1 \leq j \leq m - n$*.* M *is defined as the path matrix of* P*, for an* m*-stage SEN,* $n < m \leq 2n - 1$

Note that for $n < m \leq 2n-1$, there exist 2^{m-n} paths for each input-output pair, represented by the all possible combinations of the bits $*_1 *_2 \ldots *_{m-n}$, whereas for $1 \leq m \leq n$, there exist a unique path, if it is realizable.

Therefore, a given permutation P is admissible in an m stage $N \times N$ SEN, $n < m \leq 2n-1$, if and only if there exists an assignment for all the bits $(*_1 *_2 \ldots *_{m-n})$ of each row, such that each window W_j, for $0 \leq j < m$, of the path matrix M is independent.

Example 6 *A possible path matrix for the permutation*

$P : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 1 & 0 & 2 & 3 & 7 & 5 & 6 & 4 \end{pmatrix}$, *on a* 5*-stage* 8×8 *SEN is given below:*

$$M : \begin{pmatrix} x_2 & x_1 & x_0 & *_1 & *_2 & y_2 & y_1 & y_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Note that all the windows W_j, $0 \leq j \leq 4$, *are independent i.e.,* P *is admissible in a* 5*-stage SEN.*

Theorem 2 *If an* $N \times N$ *permutation* P *is admissible in an* m*-stage* $N \times N$ *SEN,* $n \leq m \leq 2n-1$*, then in* P*, the* 2^j *inputs of each input group at level* j*,* $m-n < j \leq n$ *are mapped to outputs so that exactly* 2^{m-n} *of them belong to the same output group at level* $m-j$*.*

Proof : Let us consider a permutation P, such that there is at least one input group at level j, $m-n < j \leq n$, of which more than 2^{m-n} inputs are mapped to outputs belonging to the same output group at level $(m-j)$.

Now let us consider the window W_{j-1}, consisting of the bit string $x_{n-j-1} \ldots x_0 *_1 \ldots *_{m-n}$ $y_{n-1} \ldots y_{m-j}$ for each path of P. For any input group at level j, all the inputs consist of an identical bit string $x_{n-j-1} \ldots x_0$. Now if more than 2^{m-n} inputs of the group are mapped to the outputs in the same output group at level $(m-j)$, they will have identical bit string $y_{n-1} \ldots y_{m-j}$. This will result in more than 2^{m-n} rows of W_{j-1} with identical bit strings for the part $x_{n-j-1} \ldots x_0$, as well as for $y_{n-1} \ldots y_{m-j}$. Now, by using the bits $*_1, \ldots, *_{m-n}$, we can make at most 2^{m-n} rows different. Hence for P, in W_{j-1}, there will be more than one identical rows for any possible assignments of $*_1 \ldots *_{m-n}$. □

The above Theorem states just a necessary condition for a permutation P to be admissible in an SEN with m stages, $n \leq m \leq (2n-1)$. Now let us study the admissibility problem for BPC permutations.

3 Admissibility of BPC Permutations

Given an $N \times N$ permutation P, the problem of partitioning P into minimum number of sets, such that all the paths in each set can be realized on a k-extra-stage SEN without conflict, is referred as the MP (minimum number of passes) problem in [11]. For SEN, with $k = 0$, the MP problem for BPC permutations was solved in [8]. Later it was extended to cover a larger class of permutations, namely the BPCL (bit-permute-closure) class of permutations in [3]. An $O(Nn)$ algorithm for solving the MP problem is proposed in [11], which realizes any BPC permutation on a k-extra-stage SEN, in multiple passes, $1 \leq k \leq (n-1)$. Though it is an optimal algorithm, still it may need a transmission delay $O(n2^{\lfloor n/2 \rfloor})$ [8], at maximum. By our technique, instead of using a fixed-stage SEN, given any BPC permutation P, we find the minimum number of stages $m_{\min}$ of an SEN required to make P admissible, and route P accordingly. In case of optical MIN's this technique enables us to keep the path dependent loss of the optical signal minimum by limiting the number of stages to be traversed by the signal.

Definition 12 *An $N \times N$ BPC (bit-permute-complement) permutation P is defined as $P : x_{n-1}x_{n-2}\ldots x_1x_0 \rightarrow y_{i_{n-1}}y_{i_{n-2}}\ldots\ldots y_{i_1}y_{i_0}$, where $(i_{n-1}i_{n-2}\ldots i_1i_0)$ is a permutation of $\{(n-1),(n-2),\ldots,1,0\}$, and $y_j \in \{x_j, \overline{x}_j\}$, for $0 \leq j \leq (n-1)$.*

The permutation is called a BP permutation, if $y_j = x_j$, for all j, $0 \leq j \leq (n-1)$.

Example 7 *$P_1 : x_2x_1x_0 \rightarrow x_0x_2x_1$ is a BP permutation given by*

$$P_1 : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 0 & 2 & 1 & 3 & 4 & 6 & 5 & 7 \end{pmatrix}$$

Similarly, $P_2 : x_2x_1x_0 \rightarrow x_0x_2\overline{x}_1$ is a BPC permutation, given by:

$$P_2 : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 1 & 3 & 0 & 2 & 5 & 7 & 4 & 6 \end{pmatrix}$$

Note that for an $N \times N$ system, there are $n!$ distinct BP's, and from each BP, we can generate 2^n BPC's. Therefore, $|BPC| = 2^n n!$.

Definition 13 *The unique BP permutation P, that generates the set of 2^n BPC permutations, (including the BP itself), by complementing some input bits in the BP-rule for P, is called the generator BP for any BPC permutation of the set.*

Example 8 *In Example 7, the permutation P_1 is the generator BP of the BPC permutation P_2.*

3.1 BPC Permutations for $1 \leq m \leq n$

Lemma 3 *In an m-stage $N \times N$ SEN, $1 \leq m \leq (n-1)$, all the input-output paths of a BPC permutation P will be realizable if and only if in P, $y_j = x_{j-m}$, for all j, $m \leq j \leq (n-1)$.*

Proof : By Lemma 1, given any BPC permutation P, any path realizable in an m-stage SEN, for $1 \leq m \leq n$ is represented as, $(x_{n-1}x_{n-2}\ldots x_1x_0y_{m-1}y_{m-2}\ldots y_0)$, where, $(x_{n-1}x_{n-2}\ldots x_1x_0)$ is the input and $(x_{n-m-1}x_{n-m-2}\ldots x_0y_{m-1}\ldots y_0)$ is the corresponding output. Therefore, if all the paths in P are realizable, P will be defined by a BPC rule where, $y_j = x_{j-m}$, for all j, $m \leq j \leq (n-1)$. □

Theorem 3 *In an m-stage $N \times N$ SEN, $1 \leq m \leq n$, a BPC permutation P is admissible if and only if:*

i) $y_j = x_{j-m}$*, for all* j*,* $m \leq j \leq (n-1)$*, and*

ii) $y_j \in \{x_{n-m+j}, \overline{x}_{n-m+j}\}$*, for all* j*,* $0 \leq j \leq (m-1)$*.*

Proof : By Corollary 1, the first condition is necessary, to make all the paths of P realizable in a m-stage SEN, $1 \leq m \leq (n-1)$.

The path matrix and the window W_0 consist of the columns $\{x_{n-1} \ldots x_0 y_{m-1} \ldots y_0\}$ and $\{x_{n-2} \ldots x_0 y_{m-1}\}$, respectively. The source window is $\{x_{n-1} x_{n-2} \ldots x_0\}$, and W_0 can be obtained from it by just deleting the column x_{n-1}, and adding the column y_{m-1}.

Since P is a BPC, it is defined by a unique BPC rule. Therefore, in P, for $0 \leq j \leq (n-1)$, y_j must be a unique bit of the set $\{x_{n-1}, x_{n-2}, \ldots, x_{n-m}\}$, or its complement. It is evident that W_0 will be independent, if and only if $y_{m-1} \in \{x_{n-1}, \overline{x}_{n-1}\}$. By the same argument, we get $y_j \in \{x_{n-m+j}, \overline{x}_{n-m+j}\}$, for all j, $0 \leq j \leq (m-1)$. □

Remark 4 *Note that Theorem 3 also follows from Corollary 1, as a special case.*

Example 9 *A BPC permutation* $P_1 : x_2 x_1 x_0 \rightarrow x_0 x_2 \overline{x_1}$*, given by*
$P_1 : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 1 & 3 & 0 & 2 & 5 & 7 & 4 & 6 \end{pmatrix}$*, satisfies Theorem 3, for* $m = 2$*.*

The path matrix M_1 *for* P_1*, on a 2-stage SEN is shown below:*

$$M_1 : \begin{pmatrix} x_2 & x_1 & x_0(y_2) & y_1(x_2) & y_0(\overline{x}_1) \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Note that all the windows W_j*,* $0 \leq j \leq 1 (= m-1)$ *are independent, i.e., the BPC permutation* P_1 *is admissible in a 2-stage SEN.*

Example 10 *A BPC permutation* $P_2 : x_2 x_1 x_0 \rightarrow x_0 \overline{x}_1 x_2$*, given by:*
$P_2 : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 2 & 3 & 0 & 1 & 6 & 7 & 4 & 5 \end{pmatrix}$*, satisfies Corollary 1, for* $m = 2$*, which implies that all paths are realizable. But* P *does not satisfy Theorem 3, i.e., the condition for admissibility, for* $m = 2$*.*

The path matrix M_2 for P_2, on a 2-stage SEN is shown below:

$$M_2 : \begin{pmatrix} x_2 & x_1 & x_0(y_2) & y_1(\overline{x}_1) & y_0(x_2) \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Note that there are conflicts in the window $W_0(x_1x_0y_1)$. Hence, though all the paths of P_2 are individually realizable, the permutation is not admissible on a 2-stage SEN.

Corollary 3 *In an n-stage $N \times N$ SEN, a BPC permutation P is admissible if and only if $y_j \in \{x_j, \overline{x}_j\}$, for all j, $0 \leq j \leq (n-1)$.*

Proof : Follows directly from Theorem 3. □

Corollary 4 *In an m-stage $N \times N$ SEN, where $1 \leq m \leq n$, only a unique BP is admissible for a given value of m.*

Proof : From Theorem 3, it is evident that in an m-stage SEN, with $1 \leq m \leq n$, if a BP permutation P is admissible, then any particular output bit y_j, in P, $0 \leq j \leq (n-1)$ is allowed to be a unique input bit x_k, $k = j - m$, for $m \leq j \leq (n-1)$, and $k = n - m + j$, for $0 \leq j \leq (m-1)$. Hence given any value of m, $1 \leq m \leq n$, the admissible BP is unique. □

Remark 5 *The unique BP admissible in an n-stage $N \times N$ SEN is the identity permutation.*

Corollary 5 *If any BP permutation P is admissible in an m-stage SEN, $1 \leq m \leq n$, only 2^m BPC permutations generated from P, are admissible in the m-stage SEN.*

Proof : Follows directly from Theorem 3, since complements are allowed only in the bits y_j, $0 \leq j \leq (m-1)$. Therefore only 2^m BPC permutations out of 2^n, generated from P are admissible, in the m-stage SEN, $1 \leq m \leq n$. □

3.2 BPC Permutations for $n < m \leq 2n - 1$

For an m-stage SEN, $n < m \leq 2n - 1$, given any permutation P, the path matrix is an $N \times (n + m)$ binary matrix, as it is explained in section 2.4.

Theorem 4 *A BPC permutation P is admissible in an m-stage SEN, where $m = n + k$, $1 \leq k \leq (n-1)$, if and only if $y_j(\overline{y}_j) \notin \{x_0, x_1, \ldots, x_{j-k-1}\}$, for all j, $(n-1) \geq j \geq (k+1)$.*

Proof : First, let us assume that a BPC permutation P is admissible in an $m = n+k$-stage SEN, for $1 \le k \le (n-1)$, and in P, some output bit $y_i = x_r$, where $(n-1) \ge i \ge (k+1)$, and $0 \le r \le (i-k-1)$.

Consider the window $x_{i-k-1} \dots x_r \dots x_0 *_1 *_2 \dots *_k y_{n-1} \dots y_i$. This window is an $N \times n$ binary matrix, with two identical (or, complement to each other) columns, since $y_i(\overline{y}_i) = x_r$. It is evident that this matrix is not independent, i.e., there is a conflict, which is a contradiction. Therefore, for any admissible BPC, the above condition is necessary.

To prove the sufficiency, let us assume that a BPC permutation P, satisfies the above condition. We show that it is always admissible in an $m(= n+k)$-stage SEN, $1 \le k \le (n-1)$. In this case the path matrix M consists of the columns $\{x_{n-1} x_{n-2} \dots x_0 *_1 *_2 \dots *_{m-n} y_{n-1} \dots y_0\}$.

Consider the window W_0 containing the columns $\{x_{n-2} x_{n-3} \dots x_0 *_1\}$. W_0 is obtained from the source window, just by deleting the column x_{n-1} from the left, and adding the column $*_1$ on the right. Therefore, if we make $*_1 = x_{n-1}$, the window W_0 remains independent, since the source window was independent.

If we repeat this procedure for each window W_j, $1 \le j \le k-1$ obtained from an independent window W_{j-1}, and in each step assign $*_{j+1} = x_{n-j-1}$, all the windows W_j, $0 \le j \le k-1$ remain independent.

Next, let us consider the window W_k containing the columns $\{x_{n-k-2} \dots x_0 *_1 \dots *_k y_{n-1}\}$, with $*_j = x_{n-j}$, for $1 \le j \le k$. By our assumption, $y_{n-1}(\overline{y}_{n-1}) \notin \{x_0, x_1, \dots, x_{n-k-2}\}$.

Since P is a BPC, y_{n-1} must be an input bit or its complement. Now if $y_{n-1} = x_{n-k-1}$, the present window remains independent as the previous window W_{k-1} containing the columns $\{x_{n-k-1} x_{n-k-2} \dots x_0 *_1 \dots *_k\}$ was independent.

But if $y_{n-1} \in \{x_j, \overline{x}_j\}$ where, $(n-k) \le j \le (n-1)$, there must be an $*_i = x_j$, $1 \le i \le k$ in W_k. Now reassign the column $*_i = x_j \oplus x_{n-k-1}$, where x_{n-k-1} is the column deleted from W_{k-1}, and $x_j(= y_{n-1})$ is the column inserted in W_k, and $\oplus$ denotes the exclusive-or operation. Now, for the rows in which the bit x_{n-k-1} is 0 in the source window, $*_i$ remains same as column x_j, but for the rows with $x_{n-k-1} = 1$, it is complemented, and vice versa. Hence the window will be an independent one, keeping the previous windows independent as well. In W_k, the column $*_i$ plays the role of the column x_{n-k-1}. But in the previous windows the column acts as bit x_j (see Example 11).

If we repeat this procedure for each window W_j, considering each column of the previous window to be equivalent with a single column of the source window for $k \le j < m$, we get all the windows independent. Therefore, the permutation P is admissible in m-stage SEN, which completes the proof. □

Example 11 *A BPC permutation* $P : x_2 x_1 x_0 \to x_1 \overline{x}_2 x_0$, *given by*
$P : \begin{pmatrix} 0 & 4 & 2 & 6 & 1 & 5 & 3 & 7 \\ 2 & 0 & 6 & 4 & 3 & 1 & 7 & 5 \end{pmatrix}$, *satisfies Theorem 3, for* $m = 4$.

The path matrix M for P, on a 4-stage SEN is shown below:

$$M : \begin{pmatrix} x_2 & x_1 & x_0 & *_1(x_2(\oplus)x_0) & y_2(x_1) & y_1(\overline{x}_2) & y_0(x_0) \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Note that each window is an independent one, hence P is admissible in a 4-stage SEN.

Remark 6 *For an m-stage SEN, $n < m \leq (2n-1)$, the necessary and sufficient condition for the admissibility of a BPC permutation, stated in Theorem 4, is the same as the necessary condition for the admissibility of a permutation, in general, presented in Theorem 2.*

Corollary 6 *If a BP permutation P is admissible in an m-stage SEN, $n \leq m \leq (2n-1)$, all the 2^n BPC permutations generated from P, (by complementing one or more bits in the same bit-permutation rule), are also admissible in the m-stage SEN.*

Proof : Follows directly from Theorem 4. □

In the next subsection we present an algorithm for finding out the minimum number of stages $m_{\min}$ of SEN, to make a given BPC permutation admissible.

3.3 Algorithm for finding $m_{\min}$ of SEN for BPC

Given a BPC permutation P, the following algorithm finds the minimum number of stages $m_{\min}$ of SEN, required to make P admissible.

The $N \times N$ permutation P is represented as an array A of length n, such that $A(i) = +j$ if $y_i = x_j$ and $A(i) = -j$ if $y_i = \overline{x}_j$ for $0 \leq j \leq (n-1)$.

Algorithm : Admissibility of BPC permutation

Input: $n, A(n)$
Output: M
Step 1: M := 0
Step 2: if $A(n-1)$ is negative, or $A(n-1) = (n-1)$ go to step 6
Step 3: if A(n-1) = k, scan A starting from A(n-2)
  to find the first A(j), which is negative; p:= j;
  if no A(j) is negative, go to step 4
  if $p > (n-k-2)$, go to step 7
Step 4: for j= 1 to k
    if A(n-j-1) = (k-j) then next j

else go to step 7
Step 5: M:= n-k-1; terminate
Step 6: k:= n-1
Step 7: M:= 2n-k-1
Step 8: for i= (n-2) to (M-n+1)
if $|A(i)| \leq (i - m + n - 1)$ *then* $M := M + 1$ *if* $M < (2n - 1)$ *then go to Step 8*
else terminate
else next i
Step 9: terminate

Given any BPC permutation P, the above algorithm finds the minimum number of stages ($m_{\min}$) to make P admissible. In the worst case, it may check for $(2n - 1)$ stages, and in each stage it will check n output bits. Hence the time complexity of the algorithm is $O(n \log n)$.

4 Conclusion

In hybrid optical MIN's, a serious problem is the path-dependent loss of the optical signal which depends on the number of stages of the MIN, the input signal has to traverse. In this paper, we have attempted to keep this path-dependent loss of the input signal to a minimum, by limiting the number of stages of the MIN to be traversed by each path.

In this paper, for permutations P admissible in m-stage SEN, $1 \leq m \leq n$, we find the minimum number of stages m, necessary to make P admissible, with a complexity $O(Nn \log n)$. For $n < m \leq 2n - 1$, we establish a necessary condition that a permutation P must satisfy for being admissible in an m stage SEN. It gives a lower bound on the number of stages required for making P admissible. For BPC permutations the condition is found to be necessary and sufficient.

Previously, the problem of realization of BPC permutations has been considered in fixed-stage shuffle-exchange networks (SEN) only [8, 11, 1]. The problem was referred as MP (minimum-pass) problem, where the given BPC permutation is partitioned in minimum number of non-conflicting parts, and each part is routed in a separate pass. For an n-stage SEN, it will result a transmission delay $O(n2^{\lfloor n/2 \rfloor})$ in the worst case. Whereas, by our technique, given any BPC permutation P, we find the minimum number of stages ($m_{\min}$) of SEN required to make P admissible, $1 \leq m_{\min} \leq (2n-1)$, and then we realize P, by routing the paths through ($m_{\min}$) stages only, with a transmission delay $O(n)$ only. By limiting the number of stages, i.e., the number of switches, the signal traverses to reach the output, this technique enables us to keep the path dependent loss of the optical signal minimum. Though BPC permutation has been proved to be an important class of permutations, that includes many frequently used permutations in parallel processing, we are to explore other classes of permutations, such as LC, BPCL (bit-permute-closure) etc. [5, 3, 4] as well, to extend the applicability of this technique.

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