# Transverse Mass Distribution Characteristics of $\pi^0$ Production in $^{208}$Pb-induced Reactions and the Combinational Approach

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February 8, 2008

## Abstract

The nature of invariant cross-sections and multiplicities in some $^{208}Pb$-induced reactions and some important ratio-behaviours of the invariant multiplicities for various centralities of the collision will here be dealt with in the light of a combinational approach which has been built up in the recent past by the present authors. Next, the results would be compared with the outcome of some of the simulation-based standard models for multiple production in nuclear collisions at high energies. Finally, the implications of all this would be discussed.

**Keywords** : Relativistic Heavy Ion Collision, Inclusive Cross Section.

**PACS Nos.** : 25.75.-q, 13.60.Hb.

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# 1 Introduction

The reasons for interests in the specialized studies on *Pb*-induced reactions are modestly well-known and are, by now, somewhat commonplace for which we are not going to elaborate on them. In the recent past the WA98 Collaboration[1] presented a detailed study of neutral pion transverse mass spectra in the range $0.5 GeV/c^2 \leq m_T - m_0 \leq 4.0 GeV/c^2$ and $2.3 \leq y \leq 3.0$ for collisions of *PbPb* and *PbNb* at $158A$ GeV for different centralities. Besides, Aggarwal et al (WA98 Collaboration)[1] also pointed out how the data on the ratios of invariant multiplicities in nucleus-nucleus reactions to those in proton-proton reactions diverge from values predicted by nearly all the important standard versions of multiple production of particles in nucleus-nucleus collisions. Against this perspective, we would try to understand here the reported features of neutral pion production in two lead$(A = 208)$-induced reactions at $158A$ GeV and also try to explain the problematical ratios in the light of a combinational approach[2]. The present study is conducted obviously with the clear motivation of testing this newly projected approach for understanding the characteristics of particle production in heavy ion collisions.

We would concentrate here, in the main, on the following few aspects: (i) the nature of pionic inclusive cross-section, (ii) the property of average transverse momentum, (iii) the qualitative character of the multiplicity-behaviour and of their ratios, (iv) the nature of centrality-dependence of the ratios in the various $p_T$-intervals. Our results on these observables and the available data would also be compared with some of the most prominent models for multiparticle production in heavy ion collisions. The paper would be organized as follows: The section 2 offers the outline of the model and the sketch of the physical ideas which prompted us to proceed in the stated direction. In section 3 we present the calculational results and their graphical descriptions. The last section is, as usual, for our final comments on the totality of the work presented here.

# 2 The Model: A Sketch

Following the suggestion of Faessler[3] and the work of Peitzmann[4] and also of Schmidt and Schukraft[5], we propose here a generalized empirical relationship between the inclusive cross-section for pion production in nucleon(N)-nucleon(N) collision and that for nucleus(A)-nucleus(B) collision as given below:

$$E\frac{d^3\sigma}{dp^3}\left(AB \to \pi X\right) \sim (AB)^{\phi(y,\ p_T)} E\frac{d^3\sigma}{dp^3}\left(PP \to \pi X\right), \tag{1}$$

where $\phi(y,\ p_T)$ could be expressed in the factorization form, $\phi(y,\ p_T) = f(y)\ g(p_t)$; and the product, $AB$ on the right hand side of the above equation is the product of mass numbers of the two nuclei participating in the collisions at high energies, of which one will be the projectile and the other one the target.

While investigating any specific nature of dependence of the two variables ($y$ and $p_T$), either one of these is assumed to remain constant. In other words, more particularly, if and when the $p_t$-dependence is studied by experimental groups, the rapidity factor is treated to be constant and the vice-versa. So, the formula for studying the nature of $p_T$-spectra turns into

$$E\frac{d^3\sigma}{dp^3}\left(AB \to \pi X\right) \sim (AB)^{g(p_T)} E\frac{d^3\sigma}{dp^3}\left(PP \to \pi X\right), \tag{2}$$

The main bulk of work, thus, converges to the making of an appropriate choice of form for $g(p_T)$. And the necessary choices are to be made on the basis of certain premises and physical considerations which do not violate the canons of high energy particle interactions.

The expression for inclusive cross-section of pions in proton-proton scattering at high energies in Eqn.(2) could be chosen in the form suggested first by Hagedorn[6]:

$$E\frac{d^3\sigma}{dp^3}\,(PP \rightarrow \pi X)\ =\ C_1\,(\,1\,+\,\frac{p_T}{p_0})^{-n}\,, \tag{3}$$

where $C_1$ is the normalization constant, and $p_o$, $n$ are interaction-dependent chosen phenomenological parameters for which the values are to be obtained by the method of fitting.

The final working formula for the nucleus-nucleus collisions is now being proposed here in the form given below:

$$\begin{aligned} E\frac{d^3\sigma}{dp^3}\,(AB \rightarrow \pi X)\ &\propto\ (AB)^{(\epsilon\,+\,\alpha p_T\,-\,\beta p_T^2)}\,E\frac{d^3\sigma}{dp^3}\,(PP \rightarrow \pi X)\\ &\propto\ (AB)^{(\epsilon\,+\,\alpha p_T\,-\,\beta p_T^2)}\,(1\,+\,\frac{p_T}{p_0})^{-n}\,, \end{aligned} \tag{4}$$

with $g(p_T)\ =\ (\epsilon\,+\,\alpha p_T\,-\,\beta p_T^2)$, where this suggestion of quadratic parametrization for $g(p_T)$ is exclusively made by us and is called hereafter De-Bhattacharyya parametrization(DBP). In the above expression $\epsilon$, $\alpha$ and $\beta$ are constants for a specific pair of projectile and target.

Earlier experimental works[1,7,8] showed that $g(p_T)$ is less than unity in the $p_T$-domain, $p_T < 1.5$ GeV/c. Besides, it was also observed that the parameter $\epsilon$, which gives the value of $g(p_T)$ at $p_T = 0$, is also less than one and this value differs from collision to collision. The other two parameters $\alpha$ and $\beta$ essentially determine the nature of curvature of $g(p_T)$. However, in the present context, precise determination of $\epsilon$ is not possible for the following understated reasons:

(i) To make our point let us recast the expression for (4) in the form given below:

$$E\frac{d^3\sigma}{dp^3}(AB \rightarrow \pi X)\ \approx\ C_2\,(AB)^\epsilon\,(AB)^{(\alpha p_T-\beta p_T^2)}\,(\,1\,+\,\frac{p_T}{p_0}\,)^{-n} \tag{5}$$

where $C_2$ is the normalization term which has a dependence either on the rapidity or on the rapidity density of the pion and which also absorbs the previous constant term,$C_1$ as well.

Quite obviously, we have adopted here the method of fitting. Now, in Eqn.(5) one finds that there are two constant terms $C_2$ and $\epsilon$ which are neither the coefficients nor the exponent terms of any function of the variable, $p_T$. And as $\epsilon$ is a constant for a specific collision at a specific energy, the product of the two terms $C_2$ and $(A.B)^\epsilon$ appears as just a new constant. And, it will just not be possible to obtain fit-values simultaneously for two constants of the above types by the method of fitting.

(ii) From Eqn.(2) the nature of $g(p_T)$ can easily be determined by calculating the ratio of the logarithm of the ratios of nuclear-to-$PP$ collision and the logarithm of the product $AB$. Thus, one can measure $\epsilon$ from the intercept of $g(p_T)$ along y-axis as soon as one gets the values of $E\frac{d^3\sigma}{dp^3}$ for both $AB$ collision and $PP$ collision at the same c.m. energy. But, there is a problem that it will not be possible to get readily the data on inclusive spectra for $PP$ collisions at all c.m.energy values.

In order to sidetrack these difficulties and also to build up an escape-route, we have concentrated here almost wholly to the values of $\alpha$ and $\beta$ for various collision systems and the resultant effects of $C_2$ and $\epsilon$ have been absorbed into a single constant term $C_3$. Hence, the final expression becomes

$$E\frac{d^3\sigma}{dp^3}(AB \rightarrow \pi X)\ \approx\ C_3\,(AB)^{(\alpha p_T-\beta p_T^2)}\,(\,1\,+\,\frac{p_T}{p_0}\,)^{-n} \tag{6}$$

with $C_3 = C_2(AB)^\epsilon$.

The exponent factor term $\alpha p_T - \beta p_T^2$ obviously represents here $[g(p_T) - \epsilon]$ instead of $g(p_T)$ alone. The expression(6) given above is the physical embodiment of what we have termed to be the grand

combination of models(GCM) that has been utilized here. The results of $PP$ scattering are obtained in the above on the basis of eqn.(3) provided by Hagedorn's model(HM); and the route for converting the results of $NN$ to $NA$ or $AB$ collisions is built up by the Peitzmann's approach(PA) represented by expression(2). The further input is the De-Bhattacharyya parametrization for the nature of the exponent. Thus, the GCM is the combination of HM, PA and the DBP, all of which are used here.

And the choice of this form of parametrization for the power of the exponent in eqn.(4) is not altogether a coincidence. In dealing with the EMC effect in the lepton-nucleus collisions, one of the authors here(SB),[9] made use of a polynomial form of $A$-dependence with the variable $x_F$(Feynman Scaling variable). This gives us a clue to make a similar choice for both $g(p_T)$ and $f(y)$ variable(s) in each case separately. In the recent times, De-Bhattacharyya parametrization is being extensively applied to interpret the measured data on the various aspects[10] of the particle-nucleus and nucleus-nucleus interactions at high energies. In the recent past Hwa et. al.[11] also made use of this sort of relationship in a somewhat different context. The underlying physics implications of this parametrization stem mainly from the expression(4) which could be identified as a clear mechanism for switch-over of the results obtained for nucleon-nucleon($PP$) collision to those for nucleus-nucleus interactions at high energies in a direct and straightforward manner. The polynomial exponent of the product term on $AB$ takes care of the totality of the nuclear effects.

For the sake of clarity and confirmation, let us further emphasize a point here very categorically. It is to be noted that this model(GCM) containing all the Eqns.(4), (5) and (6) was described in some detail earlier and was made use of in analyzing extensive sets of data in the previous publicatioins[2, 14] by the same authors. And in verifying the validity of this model further, the purpose here is to apply the same model to some other problematical aspects of data which we would dwell upon in the subsequent sections.

Indeed, quite obviously, there are two phenomenological parameters in $g(p_T)$ which need to be physically explained and/or identified. In compliance with this condition we offer the following physical explanations for the occurrence of all these factors. The particle-nucleus or nucleus-nucleus collisions at high energies almost instantly gives rise, *ex hypothesi*, to an expanding blob or fireball with rising temperature. In real and concrete terms this stage indicates the growing participation of the already-expanded nuclear blob. As temperature increases at this stage, the emission of highly energetic secondaries(which are mostly peripheral nucleons or baryons) with increasing transverse momentum is perfectly possible. The coefficient $\alpha$ addresses this particularity of the natural event; and this is manifested in the enhancement of the nuclear contribution with the rise of the transverse momentum. Thereafter, there is a turnabout in the state of reality. After the initial fractions of seconds, the earlier-excited nuclear matter starts to cool down and there is a clear natural contraction at this stage as the system suffers a gradual fall in temperature. Finally, this leads to what one might call 'freeze-out' stage, which results in extensive hadronization, especially in production of hadrons with very low transverse momentum. In other words, the production of large-$p_T$ particles at this stage is lowered to a considerable extent. This fact is represented by the damping or attenuation term for the production of high-$p_T$ particles. The factor $\beta$ with negative values takes care of this state of physical reality. Thus the function denoted by $g(p_T)$ symbolizes the totality of the features of the expansion-contraction dynamical scenario in the after-collision stage. This interpretation is, at present, is only suggestive. However, let us make some further clarifications.

The physical foundation that has here been attempted to be built up is inspired by thermodynamic pictures, whereas the quantitative calculations are based on a sort of pQCD-motivated power-law formula represented by eqn.(4). This seems to be somewhat paradoxical, because it would be hard to justify the hypothesis of local thermal equilibrium in multihadron systems produced by high energy collisions in terms of successive collision of the QCD-partons(like quarks and gluons) excited or created in the course of the overall process. Except exclusively for central heavy ion collisions, a

4

typical parton can only undergo very few interactions before the final-state hadrons 'freeze out', i.e. escape as free particles or resonances. The fact is the hadronic system, before the freeze-out starts, expands a great deal – both longitudinally and transversally – while these very few interactions take place[12]. But the number of parton interactions is just one of the several other relevant factors for the formation of local equilibrium. Of equal importance is the parton distribution produced early in the collision process. This early distribution is supposed to be a superposition of collective flow and highly randomized internal motions in each space cell which helps the system to achieve a situation close to the equilibrium leading to the appropriate values of collective variables including concerned and/or almost concerned quantities. The parameter $\alpha$ in expression(4) is a measure of the ratio of the net binary collision number to the total permissible number among the constituent partons in the pre-freeze out expanding stage identified to be a sort of explosive 'detonation'[12] stage. This is approximated by a superposition of collective flow and thermalized internal motion, which is a function of hadronic temperature manifested in the behaviour of the average transverse momentum. The post freeze-out hadron production scenario is taken care of by the soft interaction which is proportional[1, 13] to the number of participant nucleons, $N_{part}$, according to almost any variety of wounded nucleon model. The factor $\beta$, we conjecture, offers a sort of the ratio of the actual participating nucleons to the total number of maximum allowable(participating) nucleons. In fact, this sort of physical explanations seems to have been acceded to by some of the physics community through their approval of some of our previous works[10, 14].

## 3    The Calculations and the Results

The graphical plots presented in the diagrams in Fig. 1 to Fig. 3 describe the measured data on pion production modestly well in the region of the moderate values of the transverse momentum. The Fig. 1 presents the fit for data on $PP$ collisions. As described in the previous section the parameter values to be inducted in calculations of $PP$ cross sections are to be obtained from Table-1. These are based on Hagedorn's model. Besides, in order to arrive at the theoretical values of invariant pion production cross section in $PbPb$ and $PbNb$ collisions at the measured energy with the help of this combinational approach, one has got to use the fit values of $\alpha$ and $\beta$ as provided in Table-2 and Table-3 for minimum bias events and for the various centralities of the nuclear collisions respectively. The average number of binary nucleon-nucleon collisions, $< N_{coll} >$ and that of participant nucleons, $< N_{part} >$ at different centralities are to be obtained from Table-4 which is an adaptation of a specific set of simulation results[1]. The statement on the nature of agreement with regard to the invariant multiplicity shown in Fig. 3 for pion production in $PbPb$ collisions at $158A$ GeV in the most peripheral and most central collisions, and also in the minimum bias events remains valid. These results obtained by the GCM are compared in Fig. 4 with some of the very popular models in the field, while the comparisons with the extracted data[1] as well lay in the background in all cases. Compatibility with data over a wide range of the $p_T$-values is modestly satisfactory. The graphs on the nature of the average transverse momentum vs. the number of participating nucleons which is a measure of the centrality of the collisions, depict faithfully (Fig. 5) the expected behaviours. The average transverse momentum values here are normally defined by:

$$< p_T > \quad = \quad \frac{\int_{p_T^{min}}^{p_T^{max}} p_T \, \frac{dN}{dp_T} \, dp_T}{\int_{p_T^{min}}^{p_T^{max}} \frac{dN}{dp_T} \, dp_T} \tag{7}$$

where $p_T^{min}$ and $p_T^{max}$ are the lower and upper limits of the transverse momentum, $p_T$ over which the integration has been performed. The values of average transverse momenta, $< p_T >$ have been

calculated in three $m_T - m_0$ intervals; and they are $0.5 \ GeV/c^2 \le m_T - m_0 \le 2.0 \ GeV/c^2$, $1.0 \ GeV/c^2 \le m_T - m_0 \le 2.0 \ GeV/c^2$ and $2.0 \ GeV/c^2 \le m_T - m_0 \le 3.0 \ GeV/c^2$. Here $m_T$ and $m_0$ are the transverse mass and the rest mass of secondary pions respectively; and they are related with $p_T$ by the equation $m_T^2 = p_T^2 + m_0^2$.

However, for a specific nucleus-nucleus collision, the nature of dependence of any observable on the center of mass(c.m.) energy per basic colliding pair of nucleons, i.e.$\sqrt{s_{NN}}$, is to be obtained from the same for what it is in nucleon-nucleon($PP$) reactions at high energies.

The ratio-behaviours of the $p_T$-spectra and the nature of another observable as would be defined later are studied here in the rest of the diagrams. The ratio-values are of two types. In Fig. 6 the ratio-values of the inclusive $p_T$-spectra for various degree of centrality of collisions to the same for minimum bias events have been studied. The other set of ratios are for the multiplicities for various centrality-degrees of collisions. The diagrams shown in Fig. 7(a)-7(d) demonstrate the plots of the ratios of invariant multiplicity distributions of neutral pions normalized to the number of binary collisions versus the transverse momentum. In Fig. 7(a) the ratio of the invariant cross section of neutral pions produced in least central $PbPb$ collision($N_{Coll} \approx 10$) normalized by the number of binary collisions to that of neutral pions produced in $PP$ collisions has been plotted as a function of transverse momentum; and the ratio-values, in this particular case, are given by

$$Ratio\left(\frac{Pb + Pb}{P + P}\right) \equiv \frac{\frac{E \frac{d^3 N}{dp^3}}{N_{coll}}|_{N_{coll}=10}}{E \frac{d^3 N}{dp^3}|_{PP}} \tag{8}$$

In the figure(Fig. 7(a)) the extracted data shown by the filled squares were obtained from Aggarwal et al[1] who assumed some parametrization for $PP$ reaction. In case of GCM, in order to obtain the values of $E\frac{d^3 N}{dp^3}|_{PP}$, we have first utilized the parameter values given in Table-1 for the $E\frac{d^3 \sigma}{dp^3}|_{PP}$ and then normalized them by the total inelastic cross section $\sigma_{in} = 43.6 \pm 4.0$ mb [15], according to the conventional rule of conversion arising out of the definitions.

The diagrams in Fig. 7(b)-7(d) depict the ratios of invariant cross sections at different centralities with a normalization of the corresponding number of binary collisions. The ratios in these cases can be written in the form

$$Ratio\left(\frac{N_{coll} = X}{N_{coll} = Y}\right) \equiv \frac{\frac{E \frac{d^3 N}{dp^3}}{N_{coll}}|_{N_{coll}=X}}{\frac{E \frac{d^3 N}{dp^3}}{N_{coll}}|_{N_{coll}=Y}} \tag{9}$$

where $X$ and $Y$ denote two different number of binary collisions at two different centralities.

The observable plotted in all the diagrams of Fig. 8(a)-8(d) and denoted by $R_{bin}$, is defined by the relation ,

$$R_{bin} \equiv E\frac{d^3 N}{dp^3}/N_{coll} \tag{10}$$

The extracted data[1] on the observable related to neutral pion yield per binary collisions are for different centralities in the separate $p_T$-intervals.

Both the sets of figures(Fig. 7(a)-7(d) and Fig. 8(a)-8(d)) provide roughly satisfactory description of the data. So, the present work could be presented as a successful continuation of one of our previous work[2]. Thus all this lends substantial degree of credence to the combinational approach built up and completed by us. And in comparison with even the most accepted version of the simulation-based standard models, i.e. the HIJING model, the present approach works quite well.

# 4  Concluding Remarks

Based on the analyses made by the chosen approach here, we make the following particular observations:

(i) The modestly successful reproduction of the measured data on $PP$ reaction, $PbPb$ and $PbNb$ collisions in the minimum bias events confirm that the used basic models have good degree of reliability.

Besides, the efficacy of the model is exhibited in a somewhat tangible way for the various degree of centrality of nucleus-nucleus collisions demonstrated by Fig. 3. In fact, the Fig. 4 is a summary statement in favour of the chosen model.

(ii) The average $p_T$ behaviour is also explained by the present approach. But for any specific nucleus-nucleus collision, the $s$-dependences of the $< p_T >$ values of the various secondaries are to be obtained mainly from the analysis of nucleon-nucleon collision in a model-based manner.

(iii) The ratios of the collisions with various degrees of centrality of collisions to the minimum bias events have been obtained with a fair degree of consistency.

(iv) The ratios of the most central to semi-central or least central(peripheral) nuclear collisions have also been computed by the used approach in conformity with the extracted available data[1] on pion production alone.

But, the above points cover no comments on the comparison of the performances of the few chosen models which are under consideration here and offer no insights into the state of physical reality. Regarding the former, the Fig.7 illustrates somewhat convincingly that the GCM performs much better to accommodate data than both the PQCD and HIJING versions of the standard multiparticle production models separately do. And, concerning the latter, firstly, the status of anomalous nuclear enhancement(ANE) is still ambiguous. In so far as pion production in high energy $PbPb$ collision is concerned, this effect is seen (Fig.7b) to be perceptible for the relatively peripheral collisions. And for these non-central collisions the GCM describes the observed effect modestly well. But the scenarios are different for the other sets of the medium central to most central collisions(Figs.7c and 7d), wherein the enhancement effect ceases to exist and the reversals of nature occur quite prominently with observations of the gradual and puzzling diminution of the effects related with transverse mass. Thus, in any of such cases, the concept of scaling with system size is not corroborated by the extracted data obtained by WA98 collaboration[1]. True, all these data-trends are captured by the GCM somewhat automatically. But the reasons for this spectacular success in terms of collision dynamics, i.e. the number of collisions or the number of participants, are still not very clear to us. Secondly, yields of neutral pions per binary collisions could increase with the number of collisions only upto a certain degree of 'hardness'($p_T \geq 1$ GeV/c - 2 GeV/c) of the multiple collisions. For large $p_T$-ranges, the validity of the physics of parton saturation at larger transverse momentum region is quite manifest at values of $N_{coll} \approx 200$(Figs.8c and 8d) from both the extracted data-sets and GCM-based results.

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Table 1: Different parameter values for $\pi^0$ production in $P + P$ collision at 200 GeV

| $C_1$ | $p_0(\text{GeV/c})$ | $n$ |
|---|---|---|
| $270 \pm 5$ | $3.0 \pm 0.2$ | $23 \pm 1$ |

Table 2: Necessary parameter values for neutral pion production in the minimum bias $Pb + Nb$ and $Pb + Pb$ collisions at 160A GeV.

| Collision | $C_3$ | $\alpha(\text{GeV/c})^{-1}$ | $\beta(\text{GeV/c})^{-2}$ |
|---|---|---|---|
| $Pb + Nb$ | $(3.0 \pm 0.5) \times 10^5$ | $0.17 \pm 0.02$ | $0.033 \pm 0.002$ |
| $Pb + Pb$ | $(8.6 \pm 0.8) \times 10^5$ | $0.17 \pm 0.02$ | $0.032 \pm 0.001$ |

Table 3: Various parameter values for $\pi^0$-production in $Pb + Pb$ collisions at 160A GeV for different centrality-values.

| $N_{Coll}$ | $C_3$ | $\alpha(\text{GeV/c})^{-1}$ | $\beta(\text{GeV/c})^{-2}$ |
|---|---|---|---|
| 10 | $19 \pm 1$ | $0.13 \pm 0.02$ | $0.042 \pm 0.002$ |
| 30 | $33 \pm 2$ | $0.15 \pm 0.02$ | $0.035 \pm 0.003$ |
| 78 | $130 \pm 4$ | $0.14 \pm 0.02$ | $0.038 \pm 0.003$ |
| 207 | $200 \pm 6$ | $0.17 \pm 0.01$ | $0.036 \pm 0.003$ |
| 408 | $250 \pm 3$ | $0.16 \pm 0.02$ | $0.030 \pm 0.003$ |
| 570 | $495 \pm 5$ | $0.16 \pm 0.01$ | $0.032 \pm 0.004$ |
| 712 | $746 \pm 3$ | $0.14 \pm 0.02$ | $0.035 \pm 0.003$ |
| 807 | $705 \pm 8$ | $0.15 \pm 0.01$ | $0.040 \pm 0.003$ |

Table 4: Used values of $< N_{part} >$ and $< N_{coll} >$ for various centrality-classes[1] of $PbPb$ collisions. The values in column 'Class' indicate vertically downwards gradual transitions of the collisions from the-lowest-to-the-highest centrality.

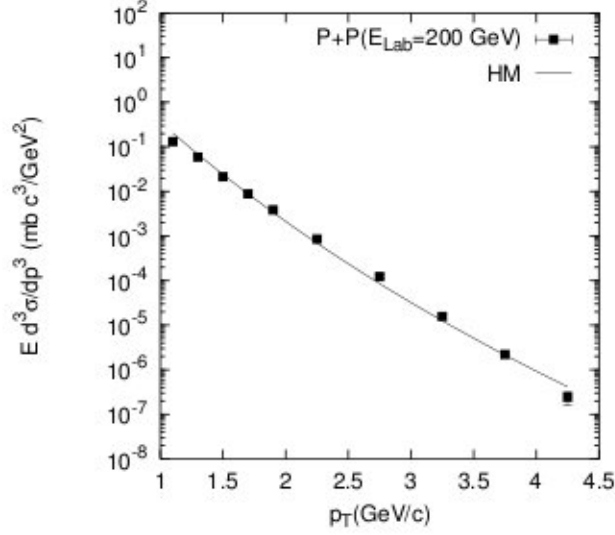| Class | $E_T(\text{GeV})$ | $< N_{part} >$ | $< N_{coll} >$ |
|---|---|---|---|
| 1 | $\leq 24.35$ | $12 \pm 2$ | $9.9 \pm 2.5$ |
| 2 | $24.35 - 55.45$ | $30 \pm 2$ | $30 \pm 5$ |
| 3 | $55.45 - 114.85$ | $63 \pm 2$ | $78 \pm 12$ |
| 4 | $114.85 - 237.35$ | $132 \pm 3$ | $207 \pm 21$ |
| 5 | $237.35 - 326.05$ | $224 \pm 1$ | $408 \pm 41$ |
| 6 | $326.05 - 380.35$ | $290 \pm 2$ | $569 \pm 57$ |
| 7 | $380.35 - 443.20$ | $346 \pm 1$ | $712 \pm 71$ |
| 8 | $> 443.20$ | $380 \pm 1$ | $807 \pm 81$ |

Figure 1: Plot of $E\frac{d^3\sigma}{dp^3}$ as a function of transverse momentum, $p_T$ for production of secondary neutral pions in $PP$ collision at $E_{Lab} = 200$ GeV. The experimental data are taken from [16]. The solid curve represents the fit obtained on the basis of eqn.(3).
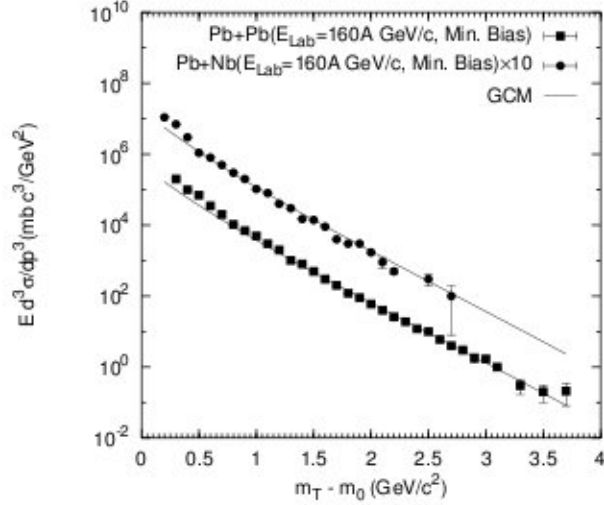


Figure 2: The inclusive spectra of secondary neutral pions produced in two $Pb-$induced reactions at $E_{Lab} = 160A$ GeV(minimum bias). All the experimental data are taken from [1]. The GCM-based results(eqn.(6)) are shown by the solid curves.

10
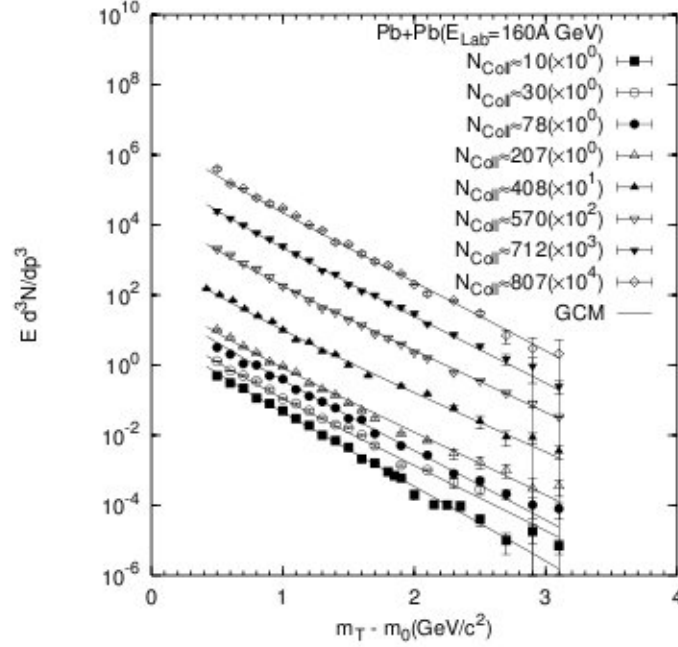
Figure 3: Invariant multiplicities of $\pi^0$ produced in $Pb + Pb$ collisions of different centralities at $E_{Lab} = 160A$ GeV as a function of $m_T - m_0$. The experimental data-points for various centrality bins are taken from [1]. The solid curves provide the GCM-based fits(eqn.(6)).
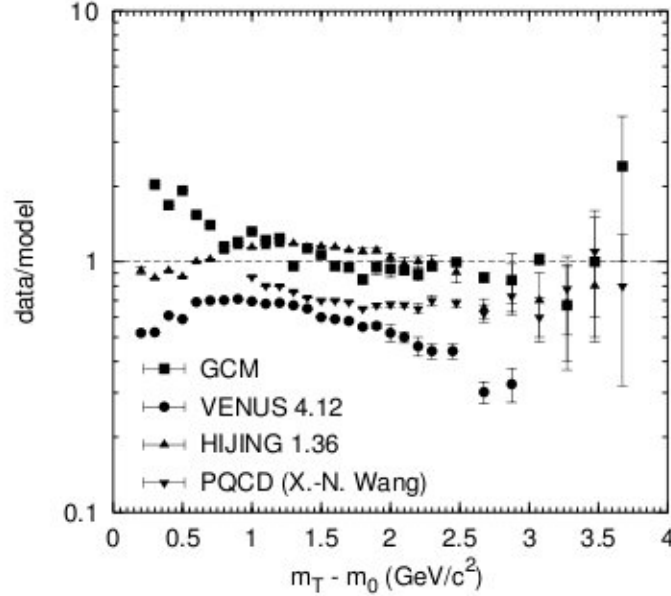


Figure 4: A comparison of the performance of different models via the ratio of the minimum bias data for $Pb + Pb$ collision to mode. based results. The solid squares provide the GCM-based results. Other model-based results are taken from [1].
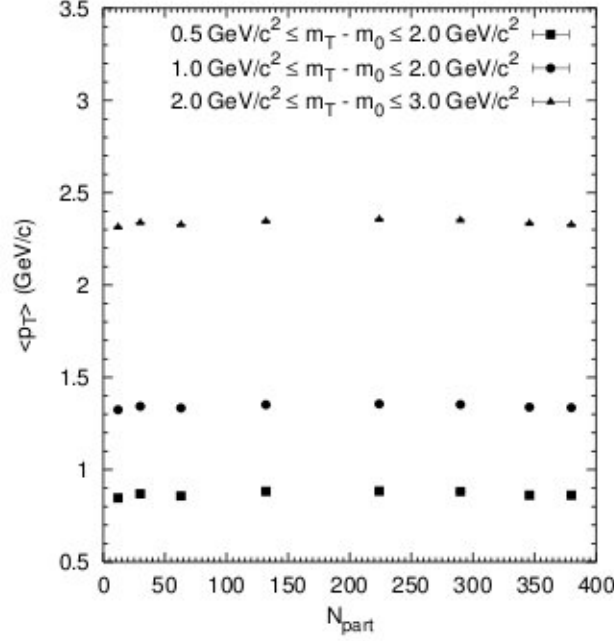
Figure 5: Plot of $< p_T >$, obtained on the basis of the present model(GCM), as a function of the no. of participant nucleons, $N_{part}$.
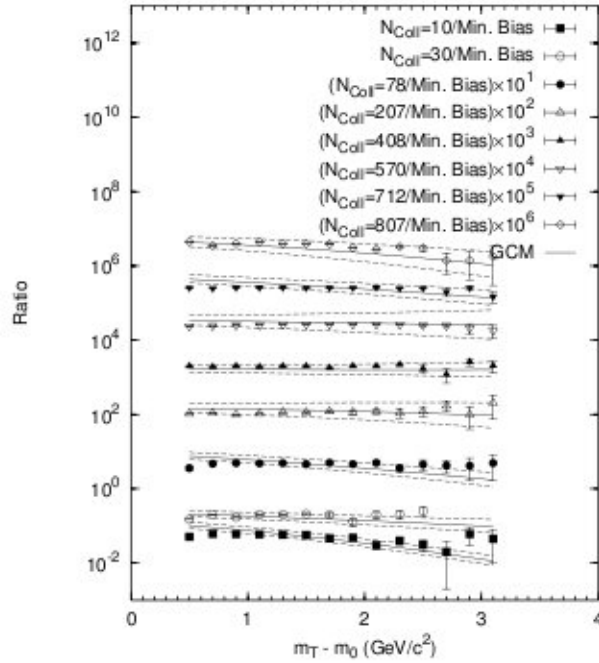


Figure 6: Plot of ratios of invariant multiplicities of neutral pions for $Pb + Pb$ collisions of different centralities to minimum bias events. The data-type points represent the experimentally measured values while the solid curvilinear lines show the GCM-based results. The dashed curves are for uncertainties arising out of the error-ranges of all the parameter values.
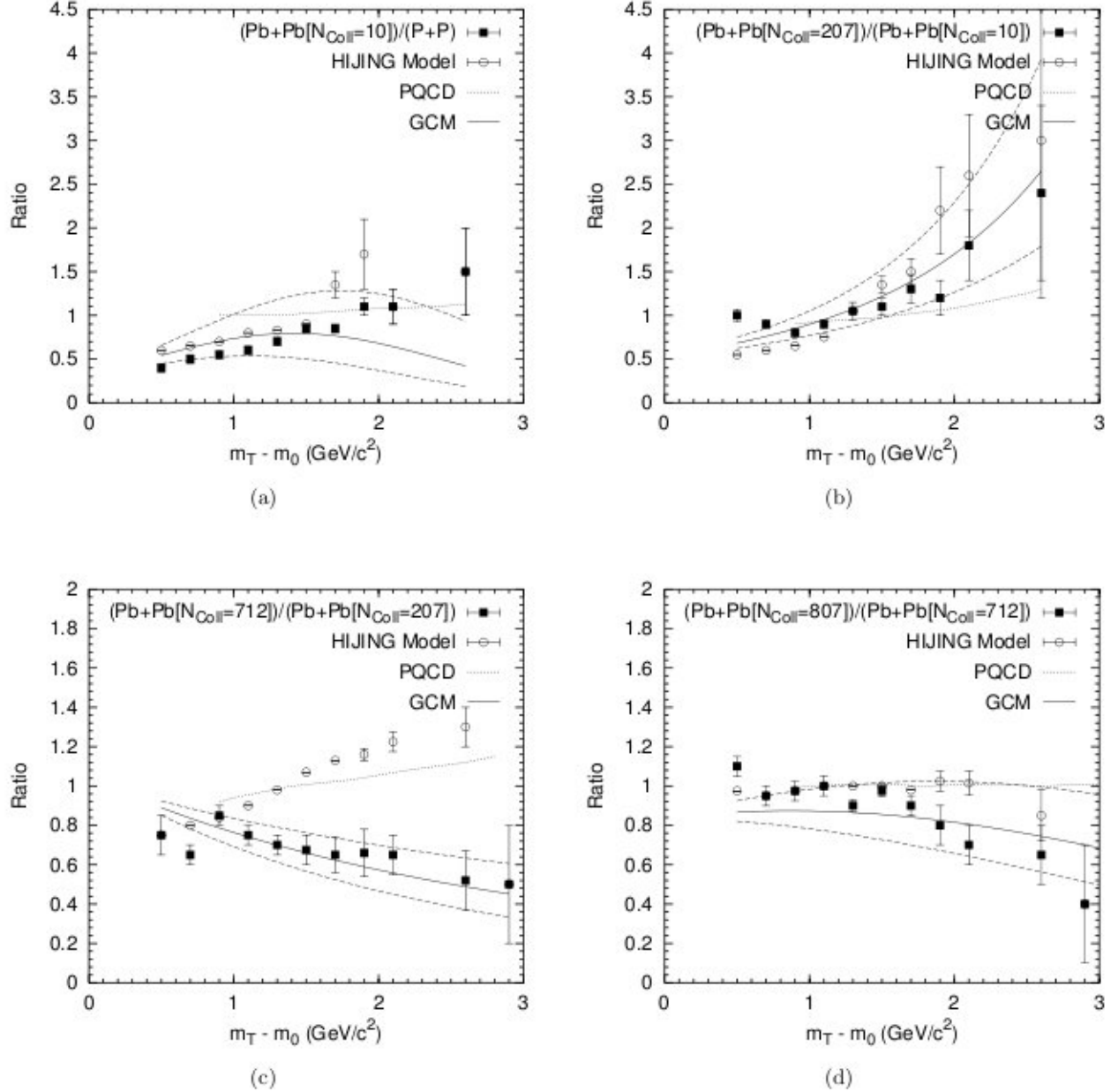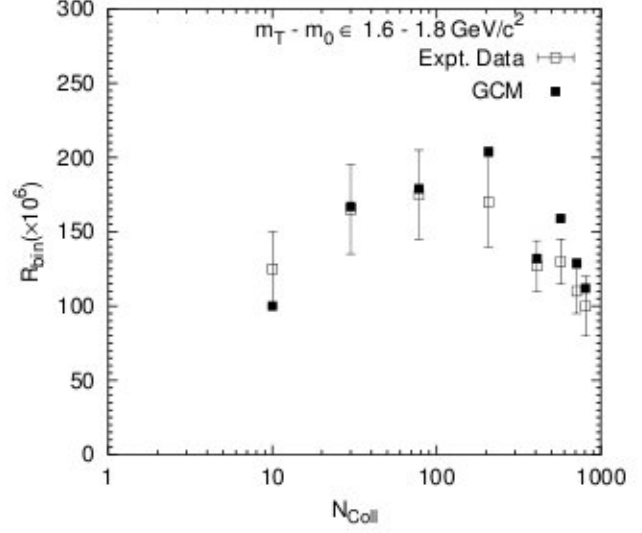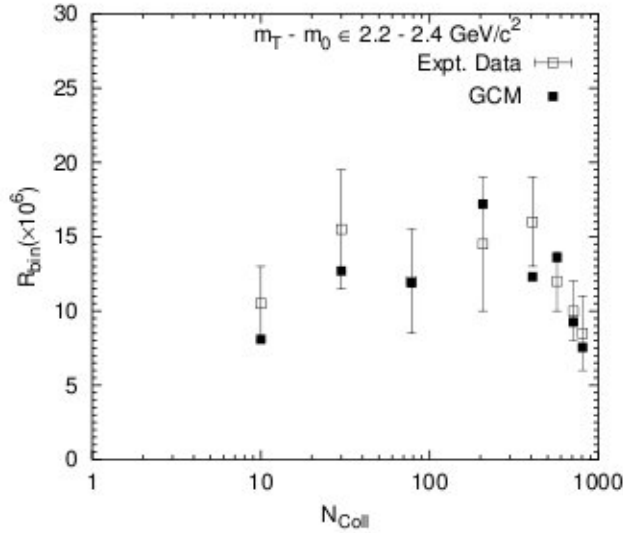
Figure 7: Ratios of $E\frac{d^3N}{dp^3}$ of neutral pions for different centralities normalized to the number of binary collisions. The GCM-based results are depicted by the solid curves. The dashed curves represent the uncertainties arising due to the errors in $C_3$, $\alpha$ and $\beta$. Other model-based results have been obtained from [1].

13
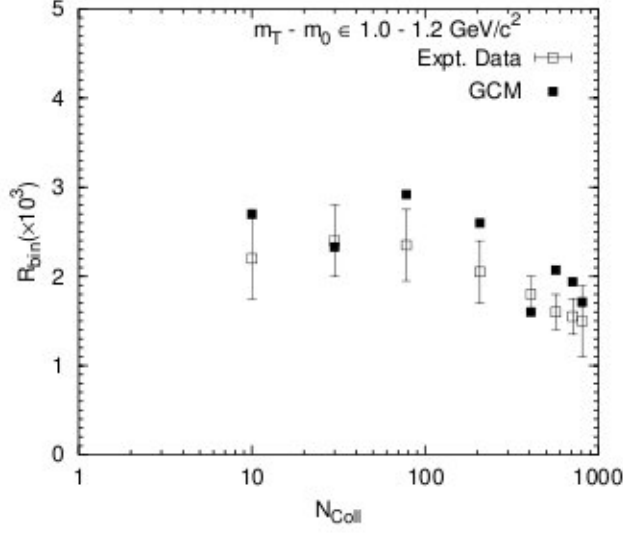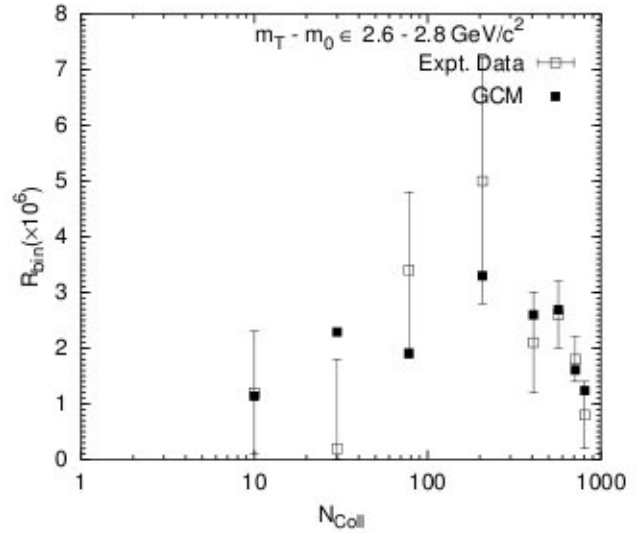
Figure 8: Invariant multiplicities of neutral pions normalized to the number of binary collisions(eqn.(10)) as a function of $N_{Coll}$ for different $m_T - m_0$ intervals. The open squares represent the experimentally extracted values, while the fully filled ones indicate the GCM-based results.