# Speed and shape of dust acoustic solitary waves

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Parametric conditions for the existence of dust acoustic solitary waves are investigated without dust charge variation, as well as with dust charge variation. The Sagdeev potential equation is derived in terms of the dust velocity $u_{d_0}$ in both cases. It is found that there exists, in both cases, a critical value of $u_{d_0}(\neq 0)$, the value of $u_d$ at which $(u_d')^2=0$, beyond which the soliton solution ceases to exist. This is demonstrated clearly in the plots given for $u_d$ against the traveling wave coordinate $\xi=x-Vt$.

## I. INTRODUCTION

Since the pioneering work by Ikezi, Taylor, and Baker,[1] who first discovered ion acoustic solitons (IAS) in a double layer, solitary waves in plasma have been studied quite extensively. At the lowest order, the nonlinear equation derived is the famous Korteweg–deVries (KdV) equation. The KdV soliton explains characteristics such as velocity and width of the solitons quite well.[2–7] The reductive perturbation technique (RPT) (Refs. 8–12) is the most popular perturbation technique used to derive the KdV and modified KdV (MKdV) equations in plasma. A few years ago, Malfliet and Wieers[13] reviewed the studies on solitary waves in cold collissonless plasma. It was found that as the RPT is based on the assumption of smallness of amplitude, the first order soliton would underestimate the amplitude of the solitary waves by as much as 20%. An exact solitary wave solution can be obtained in many cases by Sagdeev's pseudopotential method,[14] which has been applied successfully[15–18] in various cases, including multicomponent plasma. More recently, Johnston and Epstein[19] investigated nonlinear ion accoustic waves in a cold collissonless plasma by a direct analysis of the field equations; they derived Sagdeev's potential in terms of $u$, the ion accoustic speed, instead of the usual variable $\phi$, the electrical potential. They observed a dramatic difference in solutions for a change in initial condition of only one millionth, which illustrates the crucial dependence on the parameters for the existence of solitons. In the present paper our aim is to study the conditions for the existence of dust acoustic solitary waves by a similar analysis of the field equations as done in Ref. 19. Our motivation for studying dusty plasma is as follows. Dusty plasma exists in astrophysical bodies, cometary tails, interstellar clouds, and asteroid zones among various other space environments.[20–26] Also ion accoustic solitary waves and shocks were observed in dusty plasma recently.[27,28] In most of the theoretical studies on dusty plasma, dust charge variation was neglected. But, from the physical point of view, dust grains have variable charge due to fragmentation, coalescence,[29,30] etc. Hence, the effect of dust charge variation is of considerable importance in the study of dusty plasma waves of both small and large amplitudes. One of the motivations of the present study is to investigate the effect of dust charge variation in the formation of solitary waves.

Hence, we shall discuss here two cases. In the first case, the dust charge variation is neglected. In the second case, we take a popular model for dust charge variation. The organization of the paper is as follows. In Sec. II basic equations for collisonless dusty plasma in one dimension are written where dust charge variation is neglected. Sagdeev's pseudopotential is obtained analytically in terms of $u_d$, the dust velocity, and the dependence on $V$, the solitary wave velocity, for the existence of solitons is discussed. Section III is similar in content to Sec. II, except for the fact that dust charge variation is not neglected here. Section IV is kept for discussion, while Sec. V is kept for the conclusion.

## II. BASIC EQUATIONS NEGLECTING THE VARIATION OF THE DUST CHARGE

The one-dimensional normalized equations governing the dust dynamics in a dusty plasma consisting of warm dust particles and Boltzmann distributed electrons and ions are

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0, \tag{1}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}+\frac{\sigma}{n_d}\frac{\partial p_d}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_i-n_d, \tag{3}$$

where

$$n_e=\lambda \exp\left(\frac{\beta}{\mu+\lambda\beta}\phi\right), \tag{4}$$

$$n_i=\mu \exp\left(-\frac{\phi}{\mu+\lambda\beta}\right). \tag{5}$$

Here

$$\lambda=\frac{n_{e0}}{z_{d0}n_{d0}}, \tag{6}$$

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$$\mu=\frac{n_{i0}}{z_{d0}n_{d0}}, \tag{7}$$

$$n_{e0}=n_{i0}+n_{d0}z_{d0}, \tag{8}$$

and

$$\lambda-\mu=1. \tag{9}$$

$\beta$ is given by

$$\beta=T_i/T_e, \tag{10}$$

$T_i$, $T_e$ being the ion and electron temperatures. The densities of electrons and ions are normalized to $z_{d0}n_{d0}$. The dust density $n_d$, pressure $p_d$ is normalized to $n_{d0}$, $p_{d0}$, respectively, and the grain charge number $z_d$ is normalized to $z_{d0}$. The distance $x$, time $t$, dust particle velocity $u_d$, characteristic potential $\phi$ are normalized to $\lambda_{Dd}$, $\omega_{pd}^{-1}$, $c_d$, and $T_{\rm eff}$/e, respectively, where

$$T_{\rm eff}=\frac{T_iT_e}{\mu T_e+\lambda T_i}, \tag{11}$$

$$\lambda_{Dd}=(T_{\rm eff}/4\pi n_{d0}z_{d0}e^2)^{1/2}, \tag{12}$$

$$\omega_{pd}^{-1}=(m_d/4\pi n_{d0}z_{d0}^2e^2)^{1/2}, \tag{13}$$

$$c_d=(z_{d0}T_{\rm eff}/m_d)^{1/2}, \tag{14}$$

$$z_d=q_d/e, \tag{15}$$

$$\sigma=T_d/T_{\rm eff}z_{d0}. \tag{16}$$

Here $m_d$ is the mass of the dust particle and $T_d$ is its temperature. The equation of state is given by $p_d=n_d^\gamma$, where for the adiabatic process we take $\gamma=3$. In order to use quasipotential analysis, the dependent variables are made to be functions of a single variable $\xi=x-Vt$, $V$ being the solitary wave velocity. Equations (4) and (5) follow from the assumption of a Boltzmann distribution of electrons and ions, as can be verified by simplifying expressions (4) and (5) by replacing $\lambda$ and $\mu$ by their values given in (6) and (7). For example, (4) and (5) will then reduce to

$$n_e=n_{e0}\exp\left(\frac{e\phi}{T_e}\right) \tag{17}$$

$$n_i=n_{i0}\exp\left(\frac{e\phi}{T_i}\right) \tag{18}$$

after taking account of the normalization.

Equations (1)–(3) now reduce to

$$-V\frac{dn_d}{d\xi}+\frac{d}{d\xi}(n_du_d)=0, \tag{19}$$

$$-V\frac{du_d}{d\xi}+u_d\frac{du_d}{d\xi}+3\sigma n_d\frac{dn_d}{d\xi}=-\frac{d\phi}{d\xi}, \tag{20}$$

$$\frac{d^2\phi}{d\xi^2}=n_e-n_i-n_d. \tag{21}$$

From (19) we have

$$n_d=\frac{V}{V-u_d}. \tag{22}$$

From (21) eliminating $\phi$, $n_d$ in favor of $u_d$, we get

$$\frac{d^2u_d}{d\xi^2}=\frac{\partial\psi_d}{\partial u_d}, \tag{23}$$

where

$$\psi_d=\frac{(V-u_d)^6}{[(V-u_d)^4-3\sigma V^2]^2}g(u_d), \tag{24}$$

and $g(u_d)$ is given by

$$g(u_d)=\frac{\lambda}{C_1}\left[\exp\left\{C_1\left(Vu_d-\frac{u_d^2}{2}\right)\left(1-\frac{3\sigma}{(V-u_d)^2}\right)\right\}-1\right]$$
$$+\frac{\mu}{C_2}\left[\exp\left\{-C_2\left(Vu_d-\frac{u_d^2}{2}\right)\left(1-\frac{3\sigma}{(V-u_d)^2}\right)\right\}-1\right]$$
$$+g_1,$$

where

$$g_1=-Vu_d+\sigma\left[\frac{V^3}{(V-u_d)^3}-1\right]. \tag{25}$$

Thus

$$\frac{d^2u_d}{d\xi^2}=\frac{(V-u_d)^3[\lambda\exp(C_1V_1)-\mu\exp(-C_2V_1)]-V(V-u_d)^2}{(V-u_d)^4-3\sigma V^2}+\frac{(V-u_d)^9+9\sigma V^2(V-u_d)^5}{[(V-u_d)^4-3\sigma V^2]^3}2g(u_d),$$

$$V_1=(Vu_d-u_d^2/2)\left(1-\frac{3\sigma}{(V-u_d)^2}\right). \tag{26}$$

Here,

$$C_1=\frac{\beta}{\mu+\lambda\beta}, \tag{27}$$

$$C_2=\frac{1}{\mu+\lambda\beta}, \tag{28}$$

and

$$\psi_d=\frac{1}{2}\left(\frac{du_d}{d\xi}\right)^2. \tag{29}$$

## III. DERIVATION OF THE PSEUDOPOTENTIAL TAKING INTO ACCOUNT THE VARIATION OF DUST CHARGE

We consider a dusty plasma whose constituents are electrons, ions and dust grains. The electrons and ions are Boltzmann distributed. Taking into account the dust charge variation, we have the following equations for the dust fluid:

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0, \tag{30}$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}=-z_d\frac{\partial \phi}{\partial x}, \tag{31}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_i-n_d z_d, \tag{32}$$

$$\frac{\partial q_d}{\partial t}+u_d\frac{\partial q_d}{\partial x}=I_e+I_i, \tag{33}$$

where $I_e$ and $I_i$ are normalized by $q_{d0}\omega_{pd}$ and other normalizations are the same as in part I. Following Goertz,[23] we express $I_e$, $I_i$ in terms of the normalized quantities, so that

$$I_e=An_e\exp(az_d), \tag{34}$$

$$I_i=Bn_i\exp(-arz_d), \tag{35}$$

$$A=-\frac{\pi a_d^2 n_{d0}}{\omega_{pd}}\left(\frac{8T_e}{\pi m_e}\right)^{1/2}, \tag{36}$$

$$B=\frac{\pi a_d^2 n_{d0}}{\omega_{pd}}\left(\frac{8T_i}{\pi m_i}\right)^{1/2}, \tag{37}$$

$$a=\frac{e^2 z_{d0}}{a_d T_e}, \tag{38}$$

$$r=\frac{T_e}{T_i}. \tag{39}$$

The dust charge can reach local equilibrium on the hydrodynamic time scale at which $I_e+I_i=0$. Thus, we have a relation between the dust charge and the potential given by

$$z_d=1+b\phi, \tag{40}$$

where

$$b=-\frac{1+\beta}{(\mu+\lambda\beta)\ln\left[\left(\frac{\beta m_e}{m_i}\right)^{1/2}\frac{\mu}{\lambda}\right]}. \tag{41}$$

Introducing $\xi=x-Vt$, the transformed equations are

$$-V\frac{dn_d}{d\xi}+\frac{d}{d\xi}(n_d u_d)=0, \tag{42}$$

$$-V\frac{du_d}{d\xi}+u_d\frac{du_d}{d\xi}=-z_d\frac{d\phi}{d\xi}, \tag{43}$$

$$\frac{d^2\phi}{d\xi^2}=n_e-n_i-n_d z_d, \tag{44}$$

$$-V\frac{dq_d}{d\xi}+u_d\frac{dq_d}{d\xi}=I_e+I_i. \tag{45}$$

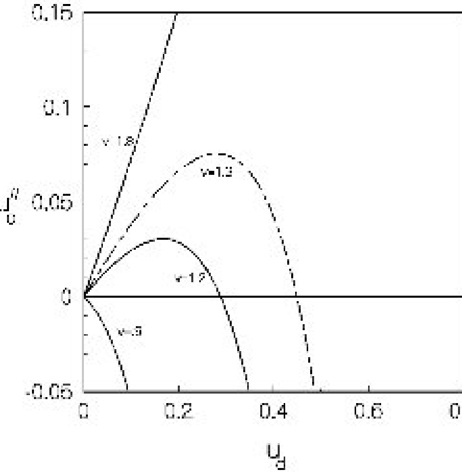


FIG. 1. $u_d''$ vs $u_d$ is plotted for different values of $V$, where the dust-plasma parameters are $n_{i0}=1.2\times10^{10}\ \mathrm{cm}^{-3}$, $n_{e0}=10^{10}\ \mathrm{cm}^{-3}$, $n_{d0}=1.61\times10^{6}\ \mathrm{cm}^{-3}$, $z_{d0}=-1239$, $T_e=1$ eV, $T_d=0.01$ eV, $T_i=1$ eV. Here the charge variation is neglected.

Eliminating $n_d$, $q_d$, $\phi$ in terms of $u_d$, Sagdeev's equation can be derived. It is given by

$$\frac{d^2u_d}{d\xi^2}=\frac{\partial\psi_d}{\partial u_d}. \tag{46}$$

$$\psi_d=\frac{(1+2bVu_d-bu_d^2)}{(V-u_d)^2}h_1(u_d), \tag{47}$$

where

$$h_1(u_d)=\frac{\lambda(\mu+\lambda\beta)}{\beta}[\exp\{D_1(-1+\sqrt{1+2bVu_d-bu_d^2})\}-1]+\mu(\mu+\lambda\beta)[\exp\{D_2(-1+\sqrt{1+2bVu_d-bu_d^2})\}-1]-Vu_d,$$

and

$$D_1=\frac{\beta}{b(\mu+\lambda\beta)}, \tag{48}$$

$$D_2=\frac{-1}{b(\mu+\lambda\beta)}. \tag{49}$$

## IV. RESULTS AND DISCUSSIONS

To find the region of existence of solitary waves, one has to study the nature of the function $\phi_1(u_d)$ defined by

$$\psi(u_d)=\frac{(u_d')^2}{2}, \tag{50}$$

where

$$u_d''=\frac{\partial\psi_d}{\partial u_d}=\phi_1(u_d). \tag{51}$$

For solitary waves $\phi_1(u_d)$ will have two roots, one being at $u_d=0$ and the other at some point $u_d=u_{d_1}(>0)$. Also, $\phi_1(u_d)$ should be positive on the interval $(0,u_{d_1})$ and negative on $(u_{d_1},u_{d_{\max}})$, where $u_{d_{\max}}$ is obtained from the nonzero

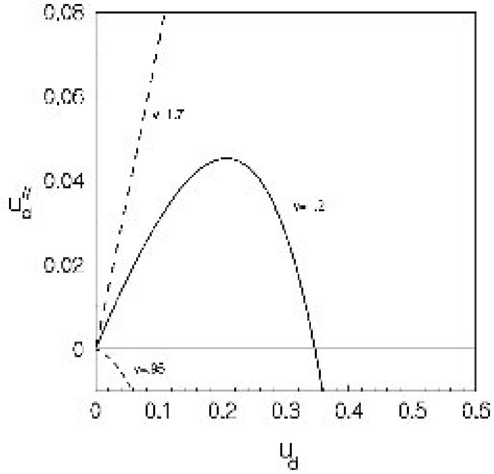


FIG. 2. Plot of $u_d''$ vs $u_d$ for different values of $V$ without neglecting the variation of charge; $T_d=0$ and other parameters remain the same as in Fig. 1.

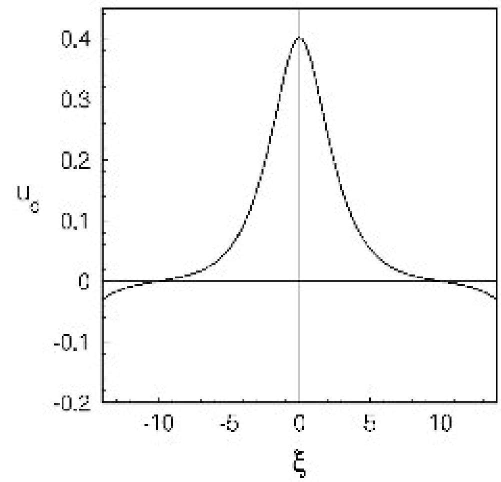


FIG. 5. Plot of $u_d$ vs $\xi$ for $V=1.2$ and $u_d(0)=0.400\,992$. Other parameters remain the same as in Fig. 1.

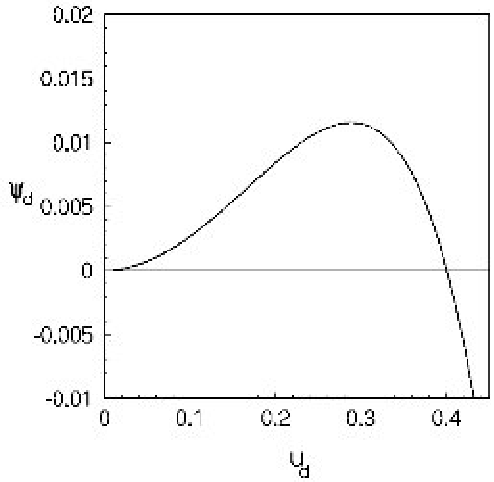


FIG. 3. Plot of $\psi_d$ vs $u_d$ for $V=1.2$. Other parameters remain the same as in Fig. 1.

root of $\psi(u_d)$. In Fig. 1, $\phi_1(u_d)$ is plotted against $u_d$ for different values of the solitary wave velocity $V$, where the dust charge variation is neglected (other parameters are given in figure caption). It is seen from the shapes of the respective figures that solitary waves would exist for $0.9<V<1.8$. Figure 2 shows the change of $u_d''$ with respect to $u_d$ for different values of $V$ when the dust charge variation is taken into account. In this case, the solitary waves exist for $0.95<V<1.7$. Figure 3 shows the plot of $\psi_d$ against $u_d$ with $V=1.2$, other parameters being same as those of Fig. 1. Figure 4 depicts the soliton solution $u_d(\xi)$ plotted against $\xi$. It is seen that $u_{d_0}=0.400\,991$ is the critical value $u_d$. For $u_d>u_{d_0}$, the solitary wave solution ceases to exist, as shown in Fig. 5. Figures 6–9 refer to the case in which dust charge variation was taken into consideration. Figure 6 shows the dependence of $z_d$ on $u_d$. It is seen that $z_d$ decreases slowly with $u_d$ for $u_d\leqslant 0.95$, and then increases gradually. Here $V=1.2$ and other parameters are shown in the figure caption. Figure 7 shows the plot of $\psi_d$ against $u_d$, showing the criti-

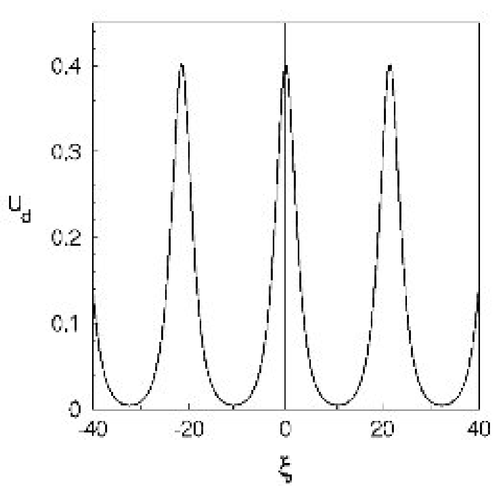


FIG. 4. Plot of $u_d$ vs $\xi$ for $V=1.2$ and $u_d(0)=0.400\,991$. Other parameters remain the same as in Fig. 1.

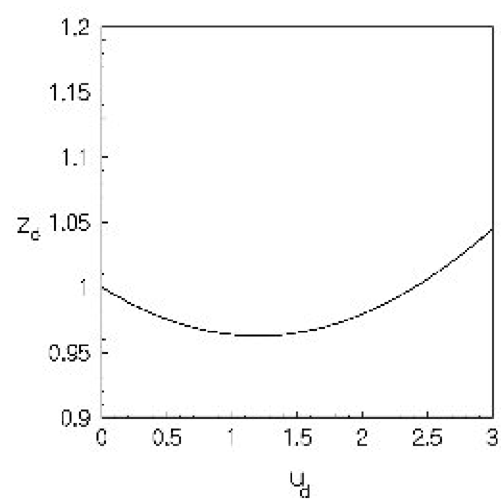


FIG. 6. $z_d$ vs $u_d$ is plotted for $V=1.2$, where the dust-plasma parameters are $n_{i0}=1.2\times10^{10}$ cm$^{-3}$, $n_{e0}=10^{10}$ cm$^{-3}$, $n_{d0}=1.61\times10^{6}$, $z_{d0}=-1239$, $T_e=1$ eV, $T_i=1$ eV.

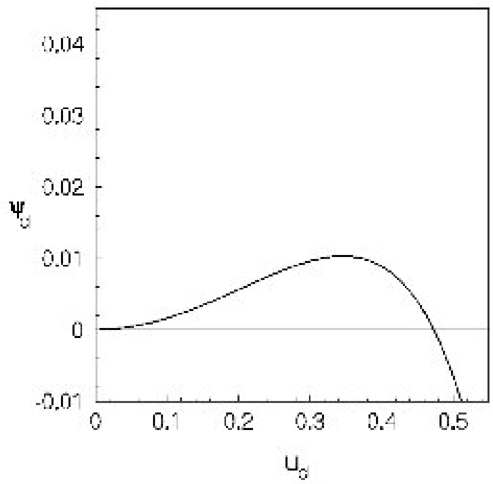


FIG. 7. Plot of $\psi_d$ vs $u_d$ for $V=1.2$. Other parameters remain the same as in Fig. 6.

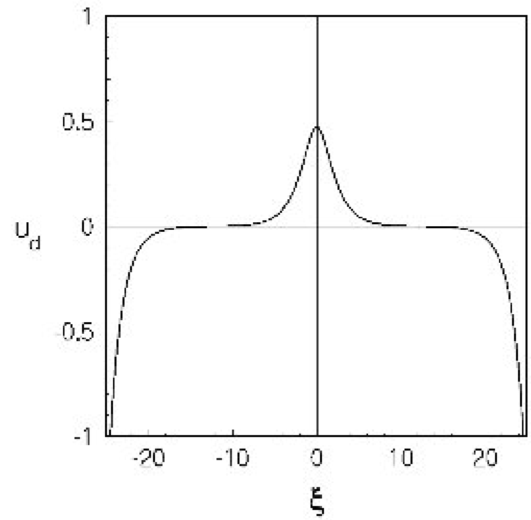


FIG. 9. Plot of $u_d$ vs $\xi$ for $V=1.2$ and $u_d(0)=0.473\,29$. Other parameters remain the same as in Fig. 8.

cal value of $u_{d_0}$ beyond which the soliton solution will not exist. Figure 9 shows that the solitary solution would cease to exist if $u_{d_0}$ is greater than the critical value. Also from Figs. 8 and 9, it is found that there is a transition in the behavior of $u_d$ from periodic to a divergent one as we take $u(0)>0.473\,28$. Figure 10 shows that if we keep the parameters the same for the cases with and without dust charge variation, qualitatively the behaviors of $\psi_d$ in these cases are similar, though the amplitude of the solitary wave is small in the case of the stationary dust charge. Also, the region of existence for solitary waves will be different for these two cases. The above result is not unexpected as the dust charge variation model we took gives a linear dependence of $z_d$ on $\phi$, the electric potential. A nonzero value of $I_e+I_i$, would give a more complicated relation between $z_d$ and $\phi$ and may significantly effect the behavior of solitary dust acoustic waves. This issue will be taken up in a future work.

## V. CONCLUSION

We have used Sagdeev's approach to study the speed and shape of dust acoustic solitary waves. Sagdeev's potential in terms $u_d$, the dust velocity, is derived for two cases. In the first case, the dust charge variation was ignored. In the second case, the dust charge has been determined from the current balance equation $I_e+I_i=0$, where $I_e$ and $I_i$ are, respectively, the electron and ion currents. It is found that there exists in both cases a critical value of $u_{d_0}(\neq 0)$, $u_{d_0}$ being the value of $u_d$ at which $(u_d')^2=0$, beyond which the soliton solution would not exist. The dust velocity $u_d$ has been plotted against $\xi$ to pinpoint the value of $u_{d_0}$. Our result can be compared with a similar study done recently by Xie *et al.*[30] When $\phi$ is not too large, our result is similar to the one by Xie *et al.* However, for large values of $\phi$ our result is different. This may be explained by the fact that Xie *et al.* considered two temperature ions, unlike the case we considered, apart from the fact their result is valid for small amplitude

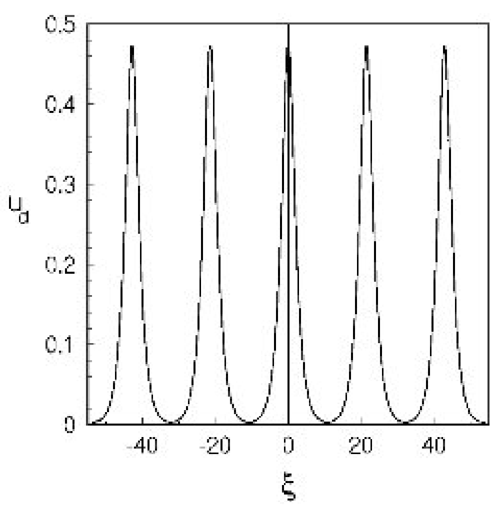


FIG. 8. Plot of $u_d$ vs $\xi$ for $V=1.2$ and $u_d(0)=0.473\,28$. Other parameters remain the same as in Fig. 6.

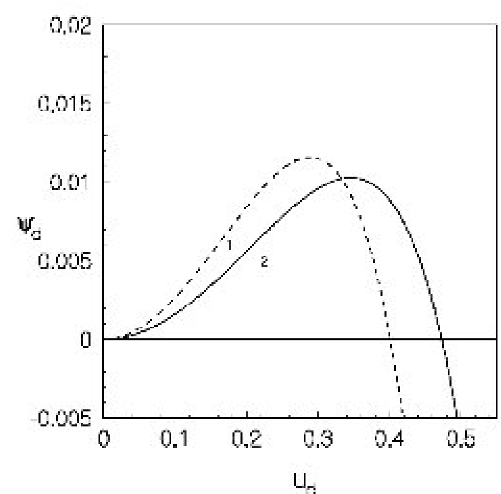


FIG. 10. Plot of $\psi_d$ vs $u_d$ for $V=1.2$ and $T_d=0$. The labels 1,2 corresponds to the cases without and with variation of charge. Other parameters remain the same as in Fig. 6.

solitary waves, whereas our result is valid for arbitrary amplitude solitary waves. Our technique can be extended to the study of the existence of dust acoustic solitary waves and shocks for other dust charging models. Work is in progress along these directions.

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