INTELLIGENT STATES FOR AN INTERPOLATING ALGEBRA

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Intelligent states in the sense of field intensity dependent squeezing for a closed and symmetric algebra which interpolates between the Heisenberg Weyl algebra W_3 and $su(1,1)$ algebra are constructed. The explicit expressions of these states in terms of Laguerre polynomials are derived and their quantum statistical properties are investigated.

Keywords: Intelligent states; interpolating algebra.

1. Introduction

Intelligent states (IS) are quantum states which minimize Heisenberg uncertainty relation for noncommuting quantum observables.[1–3] In recent years there have been many studies concerning IS, mainly in the context of quantum optics.[4] One of the principal reasons for this interest is the close relationship between IS and squeezing.[5] In addition, the IS often show a variety of other nonclassical properties, such as antibunching effect, sub-Poissonian photon statistics,[6] and oscillatory photon count distributions.[7] In fact, the generalized intelligent states for two quantum observables can provide an arbitrarily strong squeezing in either of them.[8] A generalization of squeezed states for an arbitrary dynamical symmetry group leads to the intelligent states for the group generators.[8,9] In particular the concept of squeezing can be naturally extended to the intelligent states associated with the $su(2)$ and $su(1,1)$ Lie algebras.[10] An important possible application of squeezing properties of the $su(2)$ and $su(1,1)$ minimum uncertainty states is the reduction of the quantum noise in spectroscopy[11] and interferrometry.[12,13]

On the other hand there are a large number of states in quantum optics known as interpolating states which interpolate between two different states. A very important example of interpolating state is the Binomial state introduced by Stoler, Saleh and Teich.[14] In Ref. 15 interpolating states are obtained as the coherent states of closed and symmetric algebra which interpolates between the Heisenberg Weyl algebra W_3 and $su(1,1)$ algebra. It was shown in Ref. 15 that these states have a number of interesting properties.

In the present paper intelligent states in the sense of intensity dependent squeezing[10] corresponding to this algebra are introduced and their quantum statistical properties are investigated. The paper is organized as follows: In Section 2 we sketch the realization of the interpolating algebra and the eigenvalue equation for the intelligent states. In Section 3 we solve the eigenvalue equation and present the subset of its solutions which has a simple form. In Section 4 the properties of these states such as squeezing effects, sub/super-Poissonian statistics as well as their quasi-probability distribution (the Wigner functions) are discussed and finally Section 5 is devoted to concluding remarks.

2. Interpolating Algebra and Eigenvalue Equation

In this paper we shall restrict ourselves to the generalization of $su(1,1)$ algebra characterized by the set of generators

$$R_0 = ka^\dagger a + \frac{1}{2}\,, \quad R_- = \sqrt{ka^\dagger a + 1}\, a\,, \quad R_+ = a^\dagger \sqrt{ka^\dagger a + 1} \tag{1}$$

where k is a parameter which is nonnegative and less than or equal to unity.[15]

The generators $R_\pm$ and R_0 satisfy the following commutation relations

$$[R_0, R_\pm] = \pm k R_\pm\,, \quad [R_+, R_-] = -2R_0 \tag{2}$$

and the Casimir invariant is given by

$$C_0 = R_0^2 - \frac{k}{2}[R_-, R_+]_+ = \frac{1}{2}\left(\frac{1}{2} - k\right) \tag{3}$$

where $[R_-, R_+]_+$ denotes anticommutation of the two operators.

The algebra (2) interpolates between the Heisenberg Weyl algebra W_3 and $su(1,1)$ algebra as k takes the value zero and unity respectively. The method for obtaining one algebra from another algebra is known as "contraction" and the procedure to go from $su(1,1)$ to W_3 is known.[16] It is worth mentioning that algebra(1) is similar to Holstein–Primakoff realization of the $su(1,1)$ algebra:[17]

$$K_0 = a^\dagger a + j\,, \quad K_- = \sqrt{a^\dagger a + 2j}\,a\,, \quad K_+ = a^\dagger \sqrt{a^\dagger a + 2j} \tag{4}$$

with $j = \frac{1}{2}$ and the deformation operator is $\sqrt{ka^\dagger a + 1}$.

It is to be mentioned here that for the realization given in Ref. (1) the relevant interaction is given by the Hamiltonian

$$H_{\text{int}} = g\left(\sigma_- a^\dagger \sqrt{(ka^\dagger a + 1)} + \sigma_+ \sqrt{(ka^\dagger a + 1)} a\right). \tag{5}$$

In the absence of nonlinear coupling obtained by setting $k = 0$ in Eq. (1), this Hamiltonian describes the usual Jaynes–Cummings model.[23] The intelligent states for field intensity dependent squeezing are the solutions to the eigenvalue equation.[18]

$$(W_1 + iW_2\lambda)|\psi\rangle = \beta|\psi\rangle \tag{6}$$

where λ is real and β is complex. It can be shown[18] that

$$\langle\psi|W_1|\psi\rangle = \beta_r\,, \qquad \langle\psi|W_2|\psi\rangle = \frac{\beta_i}{\lambda} \tag{7}$$

where $\beta_r = \mathrm{Re}\,\beta$ and $\beta_i = \mathrm{Im}\,\beta$. Therefore β is directly related to the expectation values of W_1 and W_2 in minimum uncertainty states. Furthermore,

$$\begin{aligned}(\Delta W_1)^2 &= \frac{\lambda}{2}\langle\psi|N+\frac{1}{2}|\psi\rangle\,,\\ (\Delta W_2)^2 &= \frac{1}{2\lambda}\langle\psi|N+\frac{1}{2}|\psi\rangle\,.\end{aligned} \tag{8}$$

These equations confirm that λ is a squeezing parameter.

The quadrature operators W_1 and W_2 are[10]

$$W_1 = \frac{1}{2}(R+R_+)\,, \quad W_2 = \frac{i}{2}(R_+ - R)\,, \quad W_3 = N + \frac{1}{2}\,. \tag{9}$$

The commutator of W_1 and W_2 is

$$[W_1, W_2] = i\left(N + \frac{1}{2}\right) \tag{10}$$

where $N = a^\dagger a$ is the number operator. Consequently, they obey the uncertainty relation

$$(\Delta W_1)^2(\Delta W_2)^2 \geq \frac{1}{4}|\langle W_3\rangle|^2\,. \tag{11}$$

A state is said to exhibit field intensity dependent squeezing in the W_1 direction if

$$(\Delta W_1)^2 < \frac{1}{2}\langle W_3\rangle\,.$$

3. Solution of Eigenvalue Equation

To solve Eq. (6) we first express it in terms of R_+, R_- as

$$\left[\frac{1}{2}(1-\lambda)R_+ + \frac{1}{2}(1+\lambda)R_-\right]|\psi\rangle = \beta|\psi\rangle\,. \tag{12}$$

Now expanding $|\psi\rangle$ in terms of number states will lead to a three term recurrence relation for the expansion coefficients which will be difficult to solve. Instead it is simpler to introduce the state

$$|\psi'\rangle = S(\xi)^{-1}|\psi\rangle \tag{13}$$

which is related to $|\psi\rangle$ by the squeezing transformation $S(\xi) = e^{(\xi R_+ - \xi^* R_-)}$. The parameter ξ will be chosen later. The state $|\psi'\rangle$ can then be expanded in terms of photon number states. It will now be shown that for a proper choice of ξ, the recurrence relations determining expansion coefficients can be easily solved.

If we let $\xi = re^{i\theta}$, then $|\psi'\rangle$ satisfies

$$S(\xi)^{-1}\left[\frac{1}{2}(1-\lambda)R_+ + \frac{1}{2}(1+\lambda)R_-\right]S(\xi)|\psi'\rangle = \beta|\psi'\rangle$$

or

$$
\begin{aligned}
&[\{(1-\lambda)\cosh^2(r\sqrt{k})+(1+\lambda)e^{2i\theta}\sinh^2(r\sqrt{k})\}R_+ \\
&\quad+\{(1-\lambda)e^{-2i\theta}\sinh^2(r\sqrt{k})+(1+\lambda)\cosh^2(r\sqrt{k})\}R_- \\
&\quad-\{(1-\lambda)e^{-i\theta}+(1+\lambda)e^{i\theta}\}\frac{1}{\sqrt{k}}R_0\sinh(2r\sqrt{k})]|\psi'\rangle = 2\beta|\psi'\rangle\,,
\end{aligned}
\tag{14}
$$

where we have used the relations

$$
\begin{aligned}
S(\xi)R_-S(\xi)^{-1} &= R_-\cosh^2(r\sqrt{k})+R_+e^{2i\theta}\sinh^2(r\sqrt{k})-\frac{R_0}{\sqrt{k}}e^{i\theta}\sinh(2r\sqrt{k})\,,\\
S(\xi)R_0S(\xi)^{-1} &= R_0\cosh(2\sqrt{k}r)-\frac{R_-\sqrt{k}}{2}e^{-i\theta}\sinh(2\sqrt{k}r)-\frac{R_+\sqrt{k}}{2}e^{i\theta}\sinh(2\sqrt{k}r)\,,\\
S(\xi)R_+S(\xi)^{-1} &= R_+\cosh^2(r\sqrt{k})+R_-e^{-2i\theta}\sinh^2(r\sqrt{k})-\frac{R_0}{\sqrt{k}}\sinh(2r\sqrt{k})e^{-i\theta}\,.
\end{aligned}
\tag{15}
$$

We now want to choose ξ so that the coefficients of R_+ in Eq. (14) vanishes. The condition for this is

$$
\tanh^2(r\sqrt{k}) = \frac{\lambda-1}{1+\lambda}e^{-2i\theta}\,. \tag{16}
$$

For $\lambda \geq 1$, we choose $\theta = 0$, and for $0 < \lambda < 1$, we take $\theta = \frac{\pi}{2}$. The parameter r is then chosen so that $\tanh^2(r\sqrt{k})$ is equal to the absolute value of the right hand side. With these choices, we find that

$$
\begin{aligned}
\cosh(r\sqrt{k}) &= \sqrt{\left(\frac{1+\lambda}{2\lambda}\right)}\,,\\
\sinh(r\sqrt{k}) &= \sqrt{\left(\frac{1-\lambda}{2\lambda}\right)}\,,\quad 0<\lambda<1\,,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\cosh(r\sqrt{k}) &= \sqrt{\left(\frac{\lambda+1}{2}\right)}\,,\\
\sinh(r\sqrt{k}) &= \sqrt{\left(\frac{\lambda-1}{2}\right)}\,,\quad \lambda\geq 1\,.
\end{aligned}
\tag{18}
$$

These expressions can now be substituted into Eq. (14). The result for $0 < \lambda < 1$ is given by

$$
\left[R_- - \frac{iR_0}{\sqrt{k}}\sqrt{(1-\lambda^2)}\right]|\psi'\rangle = \beta|\psi'\rangle \tag{19}
$$

while for $\lambda \geq 1$ it is

$$
\left[R_-\lambda - \frac{R_0}{\sqrt{k}}\sqrt{(\lambda^2-1)}\right]|\psi'\rangle = \beta|\psi'\rangle\,. \tag{20}
$$

We now expand $|\psi'\rangle$ in terms of number states

$$|\psi'\rangle = \sum_{n=0}^{\infty} C_n |n\rangle \tag{21}$$

and substitute this expression into Eqs. (19) and (20). This leads to the recurrence relations

$$C_{n+1} = \left[\frac{i\sqrt{(1-\lambda^2)}(kn+\frac{1}{2}) + \beta\sqrt{k}}{\sqrt{k}\sqrt{(kn+1)}\sqrt{(n+1)}}\right] C_n\,, \qquad 0 < \lambda < 1 \tag{22}$$

and

$$C_{n+1} = \left[\frac{\sqrt{(\lambda^2-1)}(kn+\frac{1}{2}) + \beta\sqrt{k}}{\lambda\sqrt{k}\sqrt{(kn+1)}\sqrt{(n+1)}}\right] C_n\,, \qquad \lambda \geq 1\,. \tag{23}$$

These recurrence relations can be solved for any value of β and for $\lambda > 0$. Here, we wish to examine a particularly simple subset of solutions. This subset is found by noting that, if β and λ are related in the proper way, only a finite number of the coefficients C_n will be different from zero. In particular, if $0 < \lambda < 1$ and $\beta = -\frac{i}{2\sqrt{k}}(1-\lambda^2)^{1/2}(2kM+1)$ where M is a nonnegative integer, then we have

$$\begin{aligned} C_n &= 0\,, && n > M+1 \\ &= \frac{(i(1-\lambda^2)^{1/2}\sqrt{k})^n(-1)^n M!}{\sqrt{n![[k(n-1)+1]]!}(M-N)!} C_0\,, && n \leq M+1\,. \end{aligned} \tag{24}$$

Therefore

$$|\psi'\rangle = C_0 \sum_{n=0}^{\infty} \frac{(-i\sqrt{(1-\lambda^2)}\sqrt{k}R_+)^n M!}{[[k(n-1)+1]]!n!(M-n)!}|0\rangle \tag{25}$$

where

$$[[f(n)]]! = f(n)f(n-1)\cdots f(1)\,, \quad [[f(0)]]! = 1\,.$$

The constant C_0 is chosen so as to normalise the state.

It is possible to express this state in a relatively compact form in terms of Laguerre polynomial[19] defined by

$$L_M(x) = \sum_{n=0}^{M} (-1)^n \frac{M!x^n}{(n!)^2(M-N)!}\,. \tag{26}$$

The state $|\psi'\rangle$ can be re expressed as

$$|\psi'\rangle = C_M L_M\left(i\sqrt{(1-\lambda^2)}\sqrt{k}\frac{NR_+}{k(N-1)+1}\right)|0\rangle \tag{27}$$

where C_M is a normalization constant. Similarly for $\lambda \geq 1$

$$|\psi'\rangle = C_M L_M\left(\frac{\sqrt{(\lambda^2-1)}\sqrt{k}}{\lambda}\frac{NR_+}{k(N-1)+1}\right)|0\rangle\,. \tag{28}$$

Therefore

$$|\psi\rangle_{\lambda,M} = S(\xi)|\psi'\rangle$$
$$= \begin{cases} C_M e^{(\xi R_+ - \xi^* R)} L_M\left(\xi \dfrac{N}{k(N-1)+1} R_+\right)|0\rangle\,, \\ \qquad \xi = i\sqrt{(1-\lambda^2)}\sqrt{k} \quad \text{for } 0<\lambda<1\,, \\ C_M e^{(\xi R_+ - \xi^* R)} L_M\left(\dfrac{\xi}{\lambda}\dfrac{N R_+}{k(N-1)+1}\right)|0\rangle\,, \\ \qquad \xi = \dfrac{\sqrt{(\lambda^2-1)}\sqrt{k}}{\lambda} \quad \text{for } \lambda \geq 1\,. \end{cases} \tag{29}$$

4. Nonclassical Properties

We now examine if the minimum uncertainty state in the sense of intensity dependent squeezing also exhibits normal squeezing. For this the quadrature operators are

$$X = \frac{1}{2}(a + a^\dagger)\,, \quad Y = \frac{i}{2}(a^\dagger - a)\,. \tag{30}$$

They satisfy the commutation relation $[X, Y] = \frac{i}{2}$ and consequently their variances

$$(\Delta X)^2 = \langle X^2\rangle - \langle X\rangle^2\,,$$
$$(\Delta Y)^2 = \langle Y^2\rangle - \langle Y\rangle^2$$

obey the Heisenberg uncertainty relation

$$(\Delta X)^2(\Delta Y)^2 \geq \frac{1}{16}\,. \tag{31}$$

The field is said to be squeezed in the X(Y) quadrature if

$$(\Delta X)^2 < \frac{1}{4}\left((\Delta Y)^2 < \frac{1}{4}\right).$$

Now for $M = 1$

$$|\psi'\rangle = \begin{cases} C_0\left[|0\rangle - \left(\dfrac{\sqrt{(\lambda^2-1)}\sqrt{k}}{\lambda}\right)|1\rangle\right] & \lambda \geq 1 \\ C_0\left[|0\rangle - (i\sqrt{(1-\lambda^2)}\sqrt{k})|1\rangle\right] & 0 < \lambda < 1\,, \end{cases} \tag{32}$$

and

$$|\psi\rangle_{\lambda,1} = C_0 \frac{(\operatorname{sech}(r\sqrt{k}))^{\frac{1}{k}}}{\lambda} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^n}{k^{\frac{n}{2}}\sqrt{n!}}$$
$$\times \prod_{m=1}^{n\neq 0} \sqrt{(k(m-1)+1)}(2\lambda - 1 - 2kn\lambda)|n\rangle = |\psi\rangle_I\,, \quad \lambda \geq 1\,, \tag{33}$$

$$|\psi\rangle_{\lambda,1} = C_0(\operatorname{sech}(r\sqrt{k}))^{\frac{1}{k}} \sum_{n=0}^{\infty} \frac{(i\tanh(r\sqrt{k}))^n}{k^{\frac{n}{2}}\sqrt{n!}} \times \prod_{m=1}^{n\neq 0} \sqrt{(k(m-1)+1)}(2-\lambda-2kn\lambda)|n\rangle = |\psi\rangle_s\,, \quad 0<\lambda<1\,. \tag{34}$$

For $\lambda \geq 1$

$$(\Delta Y)_l^2 = \frac{1}{4} + \frac{1}{2}C_0^2 \frac{(\operatorname{sech}(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1}n!} \prod_{m=1}^{n\neq 0}(k(m-1)+1)4\lambda^2k^2 \tag{35}$$

$$\begin{aligned}(\Delta X)_l^2 = \frac{1}{4} &+ \frac{1}{2}C_0^2 \frac{(\operatorname{sech}(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1}n!} \\ &\times \prod_{m=1}^{n\neq 0}(k(m-1)+1)[(2\lambda-1-2\lambda k(n+1))^2 \\ &+ (2\lambda-1-2k\lambda(n+2))(2\lambda-1-2kn\lambda)] \\ &- \left[C_0^2 \frac{(\operatorname{sech}(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh\,(r\sqrt{k}))^{2n+1}}{k^{n+\frac{1}{2}}n!} \right. \\ &\left. \times \prod_{m=1}^{n\neq 0}(k(m-1)+1)(2\lambda-1-2kn\lambda)(2\lambda-1-2k\lambda(n+1))\right]^2\end{aligned} \tag{36}$$

where

$$C_0 = \left\{ \frac{(\operatorname{sech}(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n}}{k^n n!} \prod_{m=1}^{n\neq 0}(k(m-1)+1) \times [(2\lambda-1-2\lambda kn)^2] \right\}^{-1/2} \tag{37}$$

and for $0<\lambda<1$

$$(\Delta X)_s^2 = \frac{1}{4} + \frac{1}{2}C_0^2(\operatorname{sech}(r\sqrt{k}))^{\frac{2}{k}} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1}n!} \prod_{m=1}^{n\neq 0}(k(m-1)+1)4\lambda^2k^2 \tag{38}$$

$$(\Delta Y)_s^2 = \frac{1}{2}C_0^2(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1}n!} \prod_{m=1}^{n\neq 0}(k(m-1)+1)$$
$$\times [(2-\lambda-2k\lambda(n+1))^2 + (2-\lambda-2k\lambda n)(2-\lambda-2k\lambda(n+2))]$$
$$- \left[C_0^2(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+1}}{k^{n+\frac{1}{2}}n!} \prod_{m=1}^{n\neq 0}(k(m-1)+1)\right.$$
$$\left. \times (2-\lambda-2kn\lambda)(2-\lambda-2k\lambda(n+1))\right]^2 \tag{39}$$

where

$$C_0 = \left\{ \frac{(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n}}{k^n n!} \prod_{m=1}^{n\neq 0}(k(m-1)+1) \right.$$
$$\left. \times [(2-\lambda-2\lambda kn)^2] \right\}^{-1/2}. \tag{40}$$

From Eqs. (35) and (38) it can be easily seen that $|\psi\rangle_l(|\psi\rangle_s)$ does not exhibit squeezing in $Y(X)$ direction.

We have plotted X quadrature variance for $\lambda \geq 1$ given in Eq. (36) in Fig. 1 and Y quadrature variance for $0 < \lambda < 1$ given in Eq. (39) in Fig. 2 for different values of k. It is seen that $|\psi\rangle_s$ and $|\psi\rangle_l$ exhibits quite different squeezing behaviour. It is evident from Fig. 1 that except for $k = 0.25$, the X-squeezing behaviour of $|\psi\rangle_l$ is insensitive to the value of k whereas Fig. 2 shows that $|\psi\rangle_s$ exhibits Y-squeezing for all values of k.

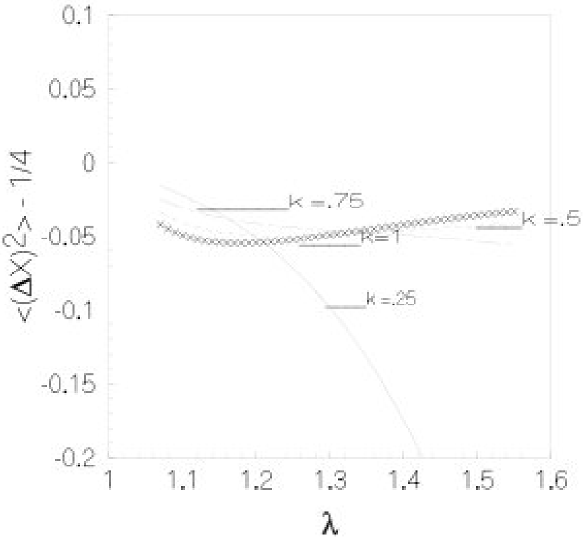


Fig. 1. Plot of X quadrature variance $\langle(\Delta X)^2\rangle - 1/4$ against λ (> 1) for different k.

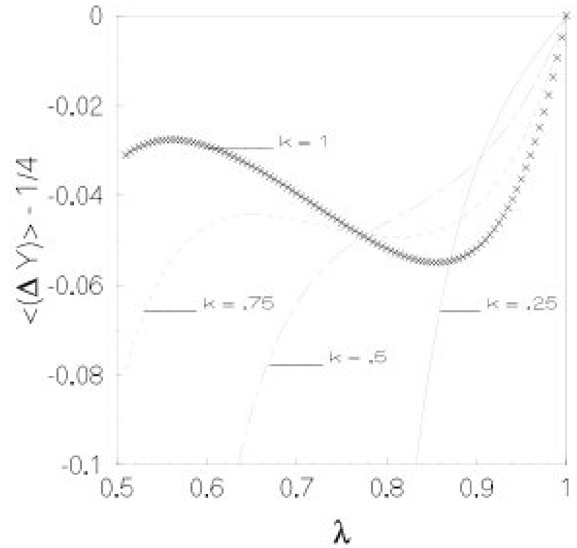


Fig. 2. Plot of Y quadrature variance $\langle(\Delta Y)^2\rangle - 1/4$ against $\lambda\,(<1)$ for different k.

In order to examine further nonclassical properties we shall now evaluate Mandel's Q parameter is defined by[6]

$$Q = \frac{\langle \Delta N^2\rangle - \langle N\rangle}{\langle N\rangle}\,, \quad N = a^\dagger a\,. \tag{41}$$

If $Q = 0$, the field is called Poissonian. If $Q > 0 (Q < 0)$ it is called super(sub) Poissonian respectively.

For $\lambda \geq 1$, the Mandel parameter is found to be

$$Q = \frac{A - (B)^2}{B} - 1 \tag{42}$$

where

$$\begin{aligned} A &= C_0^2 \frac{(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1} n!} (n+1) \prod_{m=1}^{n\neq 0} (k(m-1)+1) \\ &\quad \times (2\lambda - 1 - 2\lambda k(n+1))^2 \\ B &= C_0^2 \frac{(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1} n!} \prod_{m=1}^{n\neq 0} (k(m-1)+1) \\ &\quad \times (2\lambda - 1 - 2\lambda k(n+1))^2 \end{aligned} \tag{43}$$

where C_0 is given by Eq. (37).

On the other hand for $0 < \lambda < 1$, the Mandel parameter is given by

$$Q = \frac{C - (D)^2}{D} - 1 \tag{44}$$

where

$$C = C_0^2 \frac{(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1} n!} (n+1) \prod_{m=1}^{n\neq 0} (k(m-1)+1)$$
$$\times (2-\lambda-2\lambda k(n+1))^2$$
$$D = C_0^2 \frac{(\mathrm{sech}\,(r\sqrt{k}))^{\frac{2}{k}}}{\lambda^2} \sum_{n=0}^{\infty} \frac{(\tanh(r\sqrt{k}))^{2n+2}}{k^{n+1} n!} \prod_{m=1}^{n\neq 0} (k(m-1)+1)$$
$$\times (2-\lambda-2\lambda k(n+1))^2 \tag{45}$$

where C_0 is given by Eq. (40).

We have plotted Mandel's Q parameter for $0 < \lambda < 1$ given by Eq. (44) in Fig. 3(a) and $\lambda \geq 1$ given by Eq. (42) in Fig. 3(b) for different values of k. Fig. 3(b) shows that $Q > 0$ so that the state is superpoissonian. In Fig 3(a) it is interesting to note that for $0 < \lambda < 1$ the Q parameter is decreasing which for $k = 0.25$ is negative indicating that the state is partly subpoissonian and partly superpoissonian. So in this case parameter k plays a significant role. This behaviour may also be obtained for other values of the parameters.

Quasi-probability distributions provide insight into the nonclassical nature of radiation fields. Of these, the Wigner function[21] plays an exceptional role as it contains complete information about the state of the system. The Wigner function is the Fourier transform of the characteristic function, associated with the symmetrical ordering of the annihilation and creation operators. However, instead of the phase-space integration method, we would like to use the series representation of quasi-probabilities[20] in which the Wigner function reads

$$W(\beta) = \frac{2}{\pi} \sum_{k=0}^{\infty} (-1)^k \langle \beta, k|\rho|\beta, k\rangle \tag{46}$$

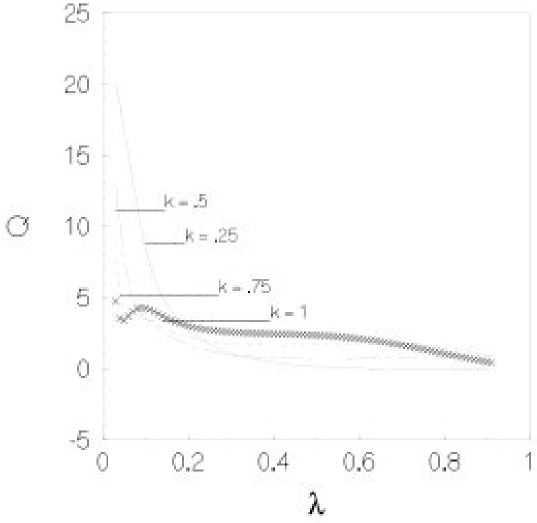
where $|\beta, k\rangle = D(\beta)|k\rangle$ are the so-called displaced number states and $D(\beta) = \exp(\beta a^\dagger - \beta^* a)$ is the Glauber's displacement operator.

If we now insert $\rho = |\psi\rangle_{\lambda,1}\langle\psi|$ where $|\psi\rangle_{\lambda,1} = \sum_{n=0}^{\infty} C_n|n\rangle$, given by Eqs. (33) and (34) for $\lambda \geq 1$ and for $0 < \lambda < 1$, respectively, we obtain

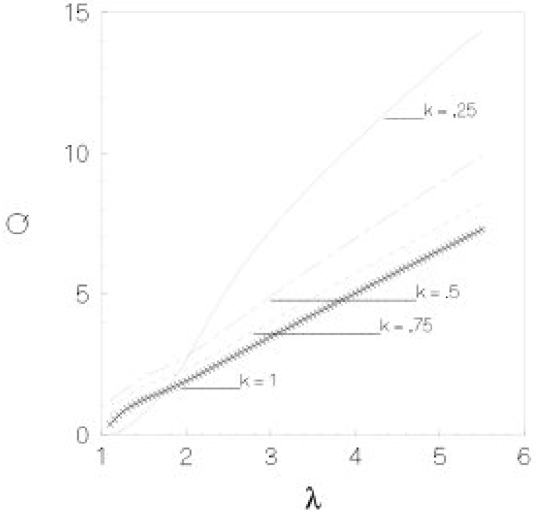
$$W(\beta) = \frac{2}{\pi} \sum_{k=0}^{\infty} (-1)^k \left| \sum_{n=0}^{\infty} C_n^* \langle n|\beta, k\rangle \right|^2 \tag{47}$$

where

$$\langle n|\beta, k\rangle = \chi_{nk}(\beta)$$
$$= \begin{cases} \sqrt{\frac{k!}{n!}} \exp(-|\beta|^2/2) \beta^{n-k} L_k^{n-k}(|\beta|^2) & \text{if } n \geq k \\ \sqrt{\frac{n!}{k!}} \exp(-|\beta|^2/2) (\beta^*)^{k-n} L_n^{k-n}(|\beta|^2) & \text{if } n \leq k \end{cases} \tag{48}$$



(a)



(b)

Fig. 3. Plot of Mandel's Q parameter Q against (a) $\lambda\,(<\ 1)$ for different k, (b) $\lambda\,(>\ 1)$ for different k.

where

$$L_m^\nu(x) = \sum_{l=0}^{m} \binom{m+\nu}{m-l} \frac{(-x)^l}{l!}$$

is the associated Laguerre polynomial. We have studied numerically the behaviour of the Wigner function $W(z)$ as a function of $z = x + iy$ for $\lambda = 1.5$ and for

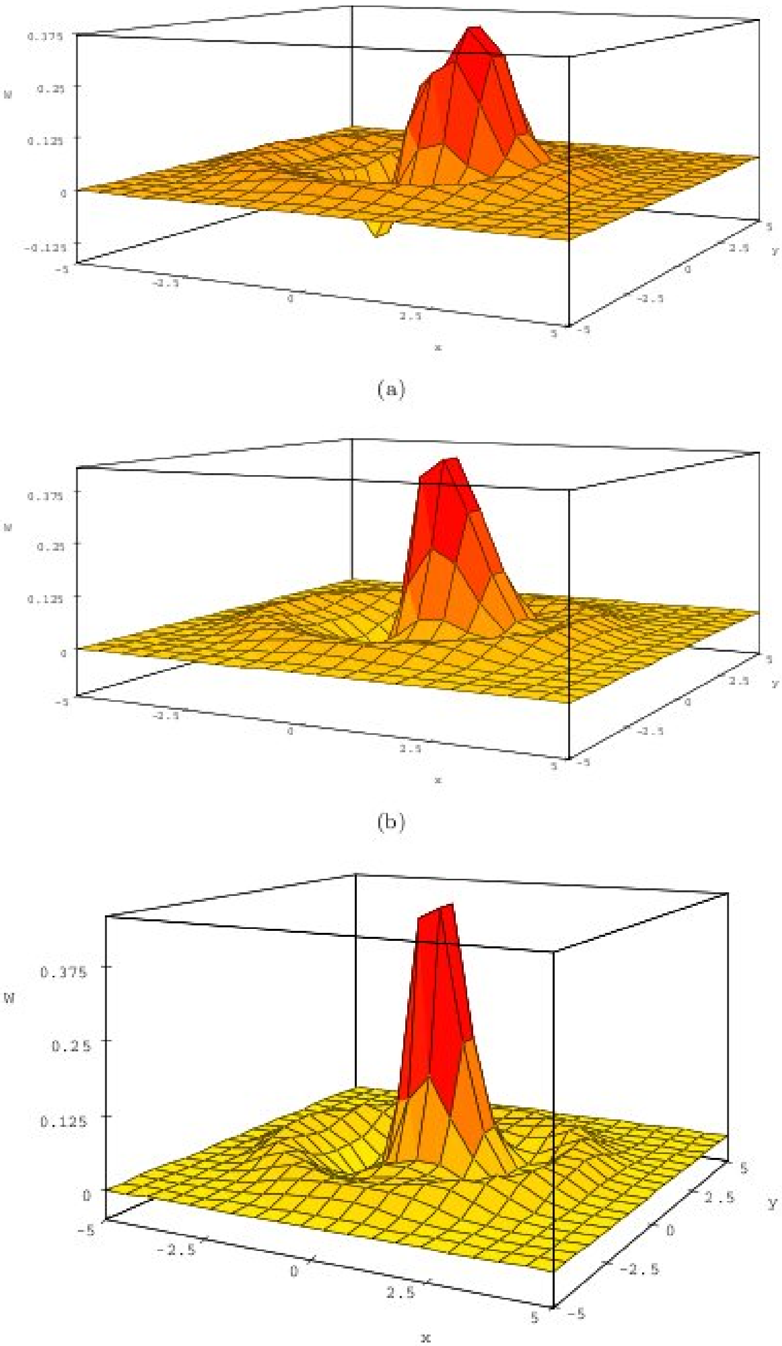


Fig. 4. Plot of Wigner function W for (a) $\lambda = 1.5$ and $k = 0.25$, (b) $\lambda = 1.5$ and $k = 0.5$, (c) $\lambda = 1.5$ and $k = 0.75$, (d) $\lambda = 1.5$ and $k = 1$.

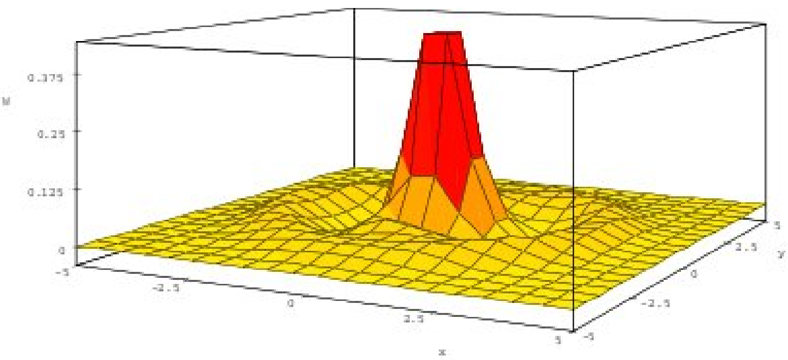


Fig. 4. (*Continued*)

different values of k. The results are shown in the Figs. 4(a)–4(d). The negativity of the Wigner function is prominent for $k = 0.25$. For other values of k there are valleys of negative values of the Wigner function.

5. Concluding Remarks

In this paper we have constructed IS in the sense of intensity dependent squeezing for an algebra interpolating between the Heisenberg Weyl W_3 and $su(1,1)$ algebra. The interpolation is made possible by the introduction of the real parameter k in the elements of the nonlinear realization of the $su(1,1)$ algebra. It is shown that these IS are squeezed in the normal sense and they exhibit sub or super Poissonian photon statistics depending on the parameter k. Regarding the possibility for the realization of these IS, it is worth mentioning[4] that in Ref. 21, for example, a method based upon a non-unitary collapse of the state vector of the cavity-field mode via atom ground state measurement is proposed for preparing a cavity-field mode undergoing a Jaynes–Cummings dynamics in any superposition of a finite number of Fock states in principle. The scheme in Ref. 22, however, uses a cavity quantum electrodynamics unitary time dependent interaction. The method proposed in Ref. 23 is an alternative method to construct a Hamiltonian which would allow the use of some kind of nonlinear interaction for the production of arbitrary pure states. In Ref. 24, it is shown that arbitrary pure quantum states can be realized by a succession of alternate state displacement and single photon adding. Based on these significant studies it can be hoped that the IS will be produced in the near future.

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