

**MEASURING INTER-INDUSTRY TRADE:
AN AXIOMATIC APPROACH**

Satya R. Chakravarty*, Andreas Pfingsten and Jacques Silber**

*** Economic Research Unit, Indian Statistical Institute, Calcutta, India.**

**** Westfälische Wilhlems-Universität, Münster, Germany.**

***** Department of Economics, Bar-Ilan University, Israel.**

January 2003

Abstract

This paper characterizes a general index of inter-industry trade using a set of intuitively reasonable axioms. The general index contains the Balassa and the Grubel-Lloyd indices as particular cases. The axiomatic approach developed in the paper gives us an insight of the indices in an elaborate way through the axioms employed in the characterization exercise.

Key words: Inter-industry trade, indices, axioms, Balassa index, Grubel-Lloyd index.

JEL Classification Numbers: F10, F12.

The authors would like to thank an anonymous referee for his very valuable comments. Satya R. Chakravarty wishes to thank the Bocconi University, Milan, and Jacques Silber, the Instituto de Análisis Económico, Barcelona, for supporting the final version of the paper.

I. Introduction

Various explanations have been given, during the past twenty years, to the existence and persistence of intra-industry trade. During a first phase, efforts were made to reconcile intra-industry trade with the theory of comparative advantage (e.g. Finger, 1975; Grubel and Lloyd, 1975; Lizondo et al. 1981; Chipman, 1986; Falvey and Kierkowski, 1987). Most of the international trade theorists however consider that it is impossible to explain trade in goods of similar factor content on the basis of conventional trade theory. They tend to argue that economies of scale are a kind of sine qua non condition of intra-industry specialization (see for example the work of Krugman, 1980; Helpman and Krugman, 1985). Nevertheless in a recent study Davis (1995) rejected the idea that intra-industry trade could be justified only by the existence of economies of scale and he showed that intra-industry trade may occur even with constant returns to scale and perfectly competitive markets.

The purpose of this paper however is not to propose an additional explanation for the existence of intra-industry trade but rather to derive axiomatically an index of inter-industry trade whose complement to one would evidently measure the degree of intra-industry trade. An exercise of this type provides a foundation for the characterized index. More precisely, we specify a set of properties for a measure of inter-industry trade that turns out to be necessary and sufficient for the exact identification of some index.

Knowing the properties of an index of inter-industry trade is important since the answer one gets by the use of an index is in general dependent on the choice of the index. Interestingly enough, the properties we consider in this paper for a measure of inter-industry trade characterize a general measure that contains the Grubel - Lloyd (1975) and the Balassa (1988) indices as special cases. By this we certainly do not

mean that other indices of inter-industry trade should be rejected. While the postulates we have formulated in this paper isolate the Grubel-Lloyd and the Balassa indices, it will be quite interesting to develop an alternative set of intuitively reasonable properties that will characterize other indices of inter-industry trade. This remains an important area of research to be explored.

We will proceed to the characterization of the general index in two steps. In section II, a “local” index of “unmatched” trade is derived, that is, a measure of the extent to which there is no matching between the exports and imports by a country of the goods belonging to a given industry, hence the name “local” index. The demonstration at this stage relies on the assumption of “Translation - Invariance” which we link to the recent proposal made by Davis (1995) to relate intra-industry trade to traditional Ricardian determinants of trade. The idea is that technical differences, when there is no rising marginal opportunity cost, allow production shifts between goods that are substitutes (“perfectly intra-industry” goods), thus inducing specialization.

In section III, the characterization is extended to the whole economy by aggregating a transformed version of the “local” indices into a “global” index of inter-industry trade. The other assumptions made at the two stages of the demonstration are quite intuitive. The axiomatic derivation presented here, though mathematical in its formulation, is therefore founded on both theoretical and empirical considerations. At the end of this study we also examine another index of inter-industry trade in the light of these axioms.

II. A “Local” Index of Inter-Industry Trade

Let n be the number of industries in the economy with $n \in N$, where N is the set of natural numbers. For any $n \in N$, the set $N(n) = \{1, 2, \dots, n\}$ will represent the set of industries in the economy.

The set of export and import levels (in value terms) for a typical industry i ($i \in N(n)$) is D^2 , where for all $n \in N$, D^n is the non-negative orthant of the Euclidean n -space with the origin deleted.

Analogously, the set of export and import levels (in value terms) for an economy consisting of n industries is D^{2n} .

A measure of inter-industry trade I for a typical industry $i \in N(n)$, $n \in N$, is defined as $I: D^2 \rightarrow R$, where R is the real line.

Such a measure determines the extent to which the exports of industry i (x_i) are not offset by the imports m_i of goods belonging to the same category i . Since the index I is defined for one particular industry, it will be called a *local* measure of inter-industry trade. As a first step towards our characterization theorems, we will propose the following axioms for an arbitrary local index of inter-industry trade $I(x_i, m_i): D^2 \rightarrow R$.

1. Homogeneity (HO): We will be concerned in the present study with relative measures of inter-industry trade. The index to be derived is therefore assumed not to depend on the units in which exports and imports are measured. More precisely, we require our measure to be homogeneous of degree zero, so that for all $n \in N$, for all $i \in N(n)$ and for all $(x_i, m_i) \in D^2$, we will have

$$I(x_i, m_i) = I(\lambda x_i, \lambda m_i)$$

where $\lambda > 0$ is any scalar.

2. *Boundedness (BD)*: Axiom *HO* ensures that the measure I depends on shares. But it does not guarantee boundedness of I . We therefore assume that a local index $I(x_i, m_i)$ of inter-industry trade should have natural bounds. The index value should be bounded below by zero, taking on this value when there is no trade imbalance (there is perfect trade overlap), that is, when exports x_i equal imports m_i in industry i . The index should also be bounded above by unity, which is achieved in the case when there is extreme trade imbalance (there is no trade overlap), that is, when either exports x_i or imports m_i are equal to zero.

More formally, we state that for all $n \in \mathbb{N}$, for all $i \in N(n)$ and for all $(x_i, m_i) \in D^2$, $0 \leq I(x_i, m_i) \leq 1$, where the lower and the upper bounds are obtained respectively when there is no trade imbalance and when there is extreme trade imbalance.

3. *Symmetry (SM)*: The index $I(x_i, m_i)$ should be symmetrical in the sense that the extent of trade imbalance between exports and imports should be considered as being the same as that existing between imports and exports. In other words, the index $I(x_i, m_i)$ should be insensitive to permutations of its arguments. We therefore write that, for all $n \in \mathbb{N}$, for all $i \in N(n)$ and for all $(x_i, m_i) \in D^2$, we will have

$$I(x_i, m_i) = I(m_i, x_i)$$

4. *Translation Invariance (TI)*: If we think of $I(x_i, m_i)$ as measuring the trade imbalance per dollar of trade (“one minus the extent of overlap per dollar of trade”), then the expression $(x_i + m_i) I(x_i, m_i)$ would be the corresponding measure when referring to the level $(x_i + m_i)$ of gross trade. An intuitively reasonable property for this gross measure is that it should be unaltered when both exports and imports

increase or decrease by the same amount. Such a property is referred to as translation invariance.

The justification for such an assumption which, a priori, may seem to be strong, is as follows. As indicated, $I(x_i, m_i)$ measures “one minus the extent of overlap per dollar of trade” (hence the upper bound of one implied by the boundedness assumption). Call $F(x_i, m_i)$ the amount $(x_i + m_i) I(x_i, m_i)$ of non-overlap for the gross trade $(x_i + m_i)$. The two arguments x_i and m_i of the function F are measured in dollars. If we represent these arguments on the straight line $[0, \infty]$, we see that the extent of overlap between them is simply m_i if $m_i < x_i$ (or x_i if $x_i < m_i$). Thus the gap $(x_i - m_i)$ (or $(m_i - x_i)$ in the converse case) is a simple measure of non-overlap and it remains invariant if both exports x_i and imports m_i increase or decrease by the same absolute amount. It is therefore reasonable to expect that the invariance property holds also for the gross measure F .

More rigorously, for all $n \in N$, for all $i \in N(n)$ and for all $(x_i, m_i) \in D^2$, we will have

$$[(x_i + c) + (m_i + c)] \cdot I(x_i + c, m_i + c) = (x_i + m_i) I(x_i, m_i)$$

where c is any scalar such that $(x_i + c, m_i + c) \in D^2$.

But, beyond these mathematical considerations, there seems to be an important economic justification for this Translation-Invariance. In a recent article, Davis (1995) has proposed a model which explains the existence of inter-industry trade without assuming increasing returns to scale and imperfect competition. The idea is that, with constant returns to scale in the production of goods which are close substitutes, cross-country technical differences in these goods, which lead to the expansion of one sector, will, at the same time, release factors in precisely the proportion used in the expanding sector, without raising marginal opportunity cost (these “perfectly intra-industry goods” have the same factor intensity). Therefore, if we consider industry i , the goods whose production expanded will induce a supplement c in exports, but, at

the same time, the local production of some substitute good will decrease so that the imports of the other goods are also likely to increase by an amount c . The level of inter-industry trade per dollar of trade will decrease, but the overall level of inter-industry trade will not change.

The following theorem shows that the above axioms are necessary and sufficient for unique identification of the local Balassa index of inter-industry trade.

Theorem 1: Axioms *HO*, *BD*, *SM* and *TI* hold together if and only if $I(x_i, m_i)$ is the local Balassa index of inter-industry trade, defined as

$$I(x_i, m_i) = |x_i - m_i| / (x_i + m_i) \quad (1)$$

The proof is given in the Appendix.

The Balassa index given by equation (1) is inversely related to intra-industry trade: an increase in the value of the index corresponds to a reduction in the amount of trade overlap in industry i , relative to total trade in this industry¹.

However, the global Balassa index as a simple sum of the local Balassa indices appears to be disadvantageous since it includes no consideration of the scale of the industry. To understand this more explicitly, consider a country with two industries, one being big with a large volume of output or trade, the other being small. Suppose that the inter-industry trade in the former (latter) is 0.8 (0.3). Then the global Balassa index is equal to 1.1. This global index would have assumed the same value, had the levels of inter-industry trade been reversed. It is, however, clear that the absolute amount of global inter-industry trade is larger in the first than in the second case. In

¹ It is important to stress that since, as indicated in (1), net trade $|x_i - m_i|$ is measured relative to gross trade $(x_i + m_i)$ and not to domestic production or sales, a high level of inter-industry trade, as measured by this index, does not necessarily imply a high level of inter-industry specialization.

order to take this factor into consideration, we need an element to express the scale of each industry into the global index. This becomes possible if we multiply the local Balassa index by $(x_i + m_i)/(X + M)$, where $X(M)$ is the total volume of export (import). However, this is only one possibility. Other possibilities may also exist. Therefore in the rest of the paper we consider the following transformed version of the Balassa local index as a local index of inter-industry trade:

$$\beta_i = a_i B_i \quad (2)$$

where B_i is the local Balassa index and a_i is a positive scalar.

III. An Axiomatic Derivation of a General Global Index of Inter-Industry Trade

The index given by β_i considered in the previous section is related to the extent of unmatched trade or inter-industry trade in a particular industry i . To get the picture of the extent of inter-industry trade in the whole economy, we have to consider all the industries together and derive a global index of inter-industry trade.

As a global index of inter-industry trade, we now propose the following:

$$P((x_1, m_1), \dots, (x_n, m_n), a_1, \dots, a_n) \quad (3)$$

where the a_i 's are the same positive real numbers considered in (2).

Next we specify a set of axioms for a global index P of trade imbalance where

$$P: D^{3n} \rightarrow R.$$

1. *Independence of Irrelevant Information (III)*: The principal property to be required seems, indeed, to be that the global index P should depend on the local

indices β_i defined in the first section ($i=1$ to n). More precisely, P will be assumed to be a function Q of the degrees of trade imbalance in the various industries and of industry-specific coefficients a_i . This will be written formally as follows:

The global index $P: D^{3n} \rightarrow R$ can be expressed as

$$P((x_1, m_1), \dots, (x_n, m_n), a_1, \dots, a_n) = Q(a_1 B_1(x_1, m_1), \dots, a_n B_n(x_n, m_n)) = Q(\beta_1, \dots, \beta_n)$$

where $Q: D^n \rightarrow R$ and $a_i B_i = \beta_i(x_i, m_i, a_i)$.

This property of independence of irrelevant information recognizes the dependence of Q on industry-wise export and import levels through the use of local measures as well as on industry-specific coefficients .

Although the assumption of *III* may look controversial, it is interesting to note that axioms quite similar to *III* can be found in many branches of economic theory. For example, in Welfare Economics it is argued that social utility is a function of individual utility levels. Similarly in Industrial Organization theory, market power is regarded as a function of firm level market powers.

2. *Continuity (CON)*: Q is continuous with respect to the functions β_i , $i=1$ to n .

3. *Symmetry (SYM)*: Symmetry (here) requires that an index of inter-industry trade should be insensitive to permutations of its arguments. In other words, the industries will be distinguished only by individual trade imbalance levels. Thus, the names or positions of the industries are completely irrelevant to the measurement process. For all $n \in N$, Q will therefore be a symmetric function. More precisely,

$$Q(\beta_1, \dots, \beta_i, \dots, \beta_k, \dots, \beta_n) = Q(\beta_1, \dots, \beta_k, \dots, \beta_i, \dots, \beta_n).$$

4. *Normalization (NOM)*: For all $n \in N$, $Q(\beta \mathbf{1}^n) = n\beta$ where β is any arbitrary nonnegative scalar and $\mathbf{1}^n$ is the n -coordinated vector of ones.

This normalization axiom says that if the functions β_i take the same value β for all the n industries considered, then the overall degree of inter-industry trade is $n\beta$. This axiom supports our intuitive reasoning that the overall index is an aggregate function of individual indices. Furthermore, we will notice that when there is only one industry, the global and local values will be the same.

In view of axiom *BD* of Section II, normalization guarantees that Q will take the value zero when there is no trade imbalance (there is perfect trade overlap) in all the n industries. Thus Q is bounded from below by zero. On the other hand, Q achieves its upper bound if there is extreme trade imbalance (no trade overlap) in all the n industries. This upper bound will depend on n and the positive scalars a_i .

5. *Strict Separability (SSP)*: According to this property, one should be able to calculate the overall amount of inter-industry trade by means of functions of the inter-industry trade levels faced by two subgroups of industries. In other words, the overall degree of inter-industry trade in the economy depends on functions of the degree of inter-industry trade in any two subgroups, one consisting of k industries ($1 \leq k \leq n$), the other consisting of the remaining $(n-k)$ industries.

One should keep in mind here that the definition of industries is, to some extent, arbitrary. It seems hence reasonable to require an index to be unaffected by replacing the values β_i and β_j by $Q^*(\beta_i, \beta_j)$, i.e. to be invariant with respect to industry aggregation. It will therefore be assumed that the overall inter-industry trade level

may be derived from the amount of inter-industry trade in two subgroups of industries which form a partition of n industries. Formally, we require that for all $n \in N$, with $n \geq 3$, Q is a strictly separable function, that is

$$Q(\beta_1, \dots, \beta_n) = \bar{Q}(\beta_i, i \in I(i), Q^*(\beta_j, j \in N(n) - I(i)))$$

where $Q: D^n \rightarrow \mathbb{R}$, $\bar{Q}: D^{i+1} \rightarrow \mathbb{R}$, $Q^*: D^{n-i} \rightarrow \mathbb{R}$, and Q , \bar{Q} and Q^* take the same functional form.

Finally, we have

6. Monotonicity (MON): Q is increasing with respect to $\beta_i, 1 \leq i \leq n$.

Having defined the various axioms to be used in the characterization of a global index of inter-industry trade, we can now state the following theorem.

Theorem 2: Axioms III, CON, SYM, NOM, MON and SSP hold together if and only if

$$P((x_1, m_1), \dots, (x_n, m_n), a_1, \dots, a_n) = Q(\beta_1, \dots, \beta_n) = \sum_{i=1}^n a_i B_i(x_i, m_i),$$

for all $n \in N, n \geq 3$, where $\beta_i = a_i B_i(x_i, m_i)$.

The proof is given in the Appendix.

Hence if we set $a_i = (x_i + m_i) / (\sum_{i=1}^n x_i + \sum_{i=1}^n m_i)$, $P((x_1, m_1), \dots, (x_n, m_n), a_1, \dots, a_n)$ becomes the Grubel and Lloyd index GL as the global index. Namely if we define $a_i^*(x_1, \dots, x_n; m_1, \dots, m_n) \equiv (x_i + m_i) / (\sum_{i=1}^n x_i + \sum_{i=1}^n m_i)$, then

$$P((x_1, m_1), \dots, (x_n, m_n), a_1^*(x_1, \dots, x_n; m_1, \dots, m_n), \dots, a_n^*(x_1, \dots, x_n; m_1, \dots, m_n))$$

becomes the Grubel - Lloyd index

$$GL = \sum_{i=1}^n |x_i - m_i| / (X + M).$$

If we set $a_1 = \dots = a_n = 1$, then the index P is expressed as

$P((x_1, m_1), \dots, (x_n, m_n), 1, \dots, 1)$, which is precisely the (global) Balassa index

$$B_A = \sum_{i=1}^n |x_i - m_i| / (x_i + m_i).$$

We have mentioned in the introduction that we start with a broader objective, that is, start with two sets of axioms. However, as observed in theorems 1 and 2, the Balassa and the Grubel - Lloyd indices satisfy these axioms under alternative specifications of the positive weight a_i . It is important to note that the choice of the weights a_i should be such that none of the axioms in theorem 2 gets violated.

We now examine another well-known index of inter- industry trade. It is the Michaely-Aquino index (see, Michaely, 1962, and Aquino, 1978) which is defined as

$$MA = (1/2) \left(\sum_{i=1}^n \left| \left(\frac{x_i}{\sum_{i=1}^n x_i} \right) - \left(\frac{m_i}{\sum_{i=1}^n m_i} \right) \right| \right)$$

If in *III* the global index P depends only on the industry-wise Michaely-Aquino indices $\left| \left(\frac{x_i}{x} \right) - \left(\frac{m_i}{m} \right) \right|$, then the Balassa and the Grubel - Lloyd indices do not satisfy this latter form of *III* and evidently the Michaely-Aquino Index fulfills this version of *III*. MA also satisfies *SYM*. *NOM* will be satisfied by the transformed index $2MA$ but not by MA . It will be an interesting exercise to axiomatise this index.

IV. Conclusion

Various indices have been proposed in recent years to measure the extent of inter-industry trade. Since empirical studies, whether based on cross-sections or times-series, may sometimes indicate that some of these indices move in opposite directions, an attempt has been made here to choose an index of inter-industry trade that would be derived axiomatically on some principles. We have proceeded in two steps, deriving first a local (at the level of the industry), then a global index (at the level of the entire economy). The axioms chosen took into account recent

developments in international trade theory as well as considerations that have to be taken into account in empirical work. This axiomatic characterization led us to derive, finally, a general global measure of inter-industry trade, which contains the widely used the Balassa and the Grubel - Lloyd indices as particular cases..

We have also discussed the Michaely-Aquino index of inter-industry trade in the light of the axioms considered in the paper. In view of the important role played by this index in the literature, it will be worthwhile to develop an economically interesting axiomatization of this index. This is left as an open research program.

Appendix

Proof of Theorem 1:

For all $(x_i, m_i) \in D^2$, define

$$g(x_i, m_i) = (x_i + m_i) \cdot I(x_i, m_i) \quad (\text{A-1})$$

Suppose that $x_i > m_i$. Then by axiom TI, we write

$$g(x_i, m_i) = g((x_i - m_i), 0) \quad (\text{A-2})$$

Since $I(x_i, m_i)$ is homogeneous of degree zero, g is homogeneous of degree one.

Consequently the right hand side of (A-2) becomes

$$g((x_i - m_i), 0) = (x_i - m_i) \cdot g(1, 0) \quad (\text{A-3})$$

Now let us assume that $x_i < m_i$. By axiom TI we get

$$g(x_i, m_i) = g(0, (m_i - x_i))$$

so that

$$g(x_i, m_i) = (m_i - x_i) \cdot g(0, 1) \quad (\text{A-4})$$

But the axiom of symmetry implies that g should be symmetric so that

$$g(1, 0) = g(0, 1) = k \quad (\text{A-5})$$

Combining (A-2) - (A-5), we derive

$$g(x_i, m_i) = k |m_i - x_i| \quad (\text{A-6})$$

From (A-1) and (A-6) we then obtain

$$I(x_i, m_i) = k (|m_i - x_i| / (x_i + m_i)) \quad (\text{A-7})$$

Suppose now that $m_i = 0$. Expression (A-7) is then written as

$$I(x_i, m_i) = k \quad (\text{A-8})$$

But by axiom BD, $I(x_i, m_i)$ should take the value 1 in such an extreme case. This

shows evidently that $k = 1$ so that $I(x_i, m_i)$ in (A-7) will be finally written as

$$I(x_i, m_i) = |m_i - x_i| / (x_i + m_i) \quad (\text{A-9})$$

Expression (A-9) shows that $I(x_i, m_i)$ is precisely equal to what we defined earlier as the local Balassa index of inter-industry trade. This conclusion establishes the necessity part of the theorem. The sufficiency is easy to verify.²

Proof of Theorem 2

The SSP axiom implies that any subset $\{i,j\}$ of the set of industries $N(n)$ is strictly separable from its complement $N(n) - \{i,j\}$. (We may note that here we require the assumption $n \geq 3$.) Let us assume that the pair of industries $\{i,i+1\}$ is separable. Then

$$Q(\beta_1, \beta_2, \dots, \beta_n) = \bar{Q}(\beta_1, \beta_2, \dots, \beta_{i-1}, Q^*(\beta_i, \beta_{i+1}), \beta_{i+2}, \dots, \beta_n) \quad (\text{A-10})$$

where, by the monotonicity axiom, \bar{Q} must be increasing in Q^* .

Since Q is symmetric, we have

$$Q(\beta_1, \beta_2, \dots, \beta_n) = \bar{Q}(\beta_1, \beta_2, \dots, \beta_{i-1}, Q^*(\beta_i, \beta_{i+2}), \beta_{i+1}, \beta_{i+3}, \dots, \beta_n) \quad (\text{A-11})$$

That is, the pairs of industries $\{i, i+1\}$ and $\{i, i+2\}$ are strictly separable from their respective complements in $N(n)$ and they overlap.

Then by Gorman's (1968) overlapping theorem,

$$Q(\beta_1, \beta_2, \dots, \beta_n) = \bar{Q}(\beta_1, \beta_2, \dots, \beta_{i-1}, f_i(\beta_i) + f_{i+1}(\beta_{i+1}) + f_{i+2}(\beta_{i+2}), \beta_{i+3}, \dots, \beta_n) \quad (\text{A-12})$$

where $f_j: R_+ \rightarrow R$, $j = i, i+1, i+2$, and R_+ is the nonnegative part of R .

By repeated use of the above argument we derive

$$Q(\beta_1, \beta_2, \dots, \beta_n) = h \left[\sum_{j=1}^n f_j(\beta_j) \right] \quad (\text{A-13})$$

where each f_j is a real valued function defined on R_+ and h is continuous and increasing in its arguments.

Symmetry of Q requires that the functions f_j , $j = 1, 2, \dots, n$, must be identical, say f .

Therefore

² We may notice that if (x_i, m_i) is interpreted as the distribution of income in a two - person society

$$Q(\beta_1, \beta_2, \dots, \beta_n) = h \left[\sum_{j=1}^n f(\beta_j) \right] \quad (\text{A-14})$$

Increasingness of Q in β_i requires that f is increasing in its argument.

Now suppose that $\beta_i = \alpha$ for all $i = 1, 2, \dots, n$. In such a case, using (A-14), we write

$$Q(\beta_1, \beta_2, \dots, \beta_n) = h \left[n f(\alpha) \right] \quad (\text{A-15})$$

But, by the normalization axiom in this particular case, the overall inter-industry trade level is given by $n\alpha$. Hence

$$h \left[n f(\alpha) \right] = n \alpha \quad (\text{A-16})$$

from which we have

$$H(n\alpha) / n = f(\alpha) \quad (\text{A-17})$$

where $H = h^{-1}$ is the inverse function of h . Since h is increasing, H is well defined and increasing. Continuity of h ensures continuity of H .

We will now show that (A-17) holds for all $\alpha \in R_+$ and for all $n \in N$ if and only if H and f are linear. We begin our demonstration by assuming that α is rational.

Choosing first $\alpha = 1$ in (A-17), we get

$$H(n) = nf(1) = nk \quad (\text{say}) \quad (\text{A-18})$$

If we now take any value of α (α rational) we may write α as p/q , where p and $q \in N$.

Then

$$H(p) = H(q(p/q)) = q f(p/q) \quad (\text{A-19})$$

Using (A-18) and (A-19), with $n = p$, we derive

$$f(p/q) = k(p/q) \quad (\text{A-20})$$

Combining now (A-17) and (A-20), we conclude that

$$H(n\alpha) = nk\alpha \quad (\text{A-21})$$

Increasingness of H requires that $k > 0$. Thus if α is rational, H and f are linear.

,then theorem 1 provides an axiomatization of the Gini index in such a society.

Next, suppose that α is irrational. We note from (A-17) that continuity of H ensures continuity of f . Now, using continuity of f , we can easily demonstrate that f (hence H) is linear for irrational values of α also.³

Conversely, if f and H are linear, then the relation expressed in (A-17) holds for all $n \in \mathbb{N}$.

Finally, noting that $f(\alpha) = k\alpha$, Q in (A-14) will be written as

$$Q(\beta_1, \beta_2, \dots, \beta_n) = h [k \sum_{j=1}^n \beta_j] \quad (\text{A-22})$$

Again, since $h = (h^{-1})^{-1} = H^{-1}$, we have

$$H(\beta) = \beta/k \quad (\text{A-23})$$

so that Q in (A-22) will be written as

$$Q(\beta_1, \beta_2, \dots, \beta_n) = \sum_{j=1}^n \beta_j \quad (\text{A-24})$$

This together with Axiom III establishes the necessity part of the theorem. The sufficiency condition is easy to verify.

³ The demonstration involves the construction of a sequence $\{\alpha_j\}$ of rationals such that $\alpha_j \rightarrow \alpha$. By continuity of f , $f(\alpha_j) \rightarrow f(\alpha)$. Thus $f(\alpha)$ can be approximated by $f(\alpha_j)$ for large j . But $f(\alpha_j)$ is linear for all j . Hence $f(\alpha)$ must be linear which implies that $H(n\alpha)$ is also linear.

Bibliography

- Aquino, A., "Intra-industry Trade and Inter-industry Specialization as Concurrent Sources of International Trade in Manufactures," *Weltwirtschaftliches Archiv*, 114(1978): 275-296.
- Balassa, B., "Trade Creation and Trade Diversion in the European Common Market: An Appraisal of the Evidence," *The Manchester School of Economic Studies* XLII (1974): 327-344.
- Balassa, B. and L. Bauwens, *Changing Trade Patterns in Manufactured Goods: An Econometric Approach*, Elsevier, New York, 1988.
- Chipman, J. S., "Intra-Industry Trade, Factor Proportions and Aggregation," Sonderforschungsbereich 178, Discussion Paper II-1, the University of Konstanz, 1986.
- Davis, D.R., "Intra-Industry Trade: A Heckscher-Ohlin-Ricardo Approach," *Journal of International Economics* 39 (1995): 201-226.
- Falvey, R. E. and H. Kierkowski, "Product Quality, Intra-Industry trade and (im)perfect Competition," in H. Kierkowski, ed., *Protection and Competition in International Trade*, Basil Blackwell, New York, 1987, 143-161.
- Finger, J. M., "Trade Overlap and Intra-Industry Trade," *Economic Inquiry* 13 (1975): 581-589.
- Gorman, W.M. , "The Structure of Utility Functions," *Review of Economic Studies* 35 (1968): 367-390.
- Greenaway, D. and C. Milner, *The Economics of Intra-Industry Trade*, New York, Basil and Blackwell, 1986.
- Grubel, H. G. and P. J. Lloyd, *Intra-Industry Trade: The Theory and Measurement of International Trade in Differentiated Products*, John Wiley, New York, 1975.

Helpman, E. and P. Krugman, *Market Structure and Foreign Trade*, M.I.T. Press, Cambridge, MA, 1985.

Krugman, P. R., "Scale Economies, Product Differentiation and the Pattern of Trade," *American Economic Review* 70 (1980): 950-959.

Lizondo, J. S., H. G. Johnson and Y. Yeh, "Factor Intensities and the Shape of the Production Possibility Curve," *Economica* 48 (1981): 199-202.

Michaely, M., "Multilateral Balancing in International Trade," *American Economic Review* 52(1962) : 685-702.

Weder, R., "Linking Absolute and Comparative Advantage to Intra-Industry Trade Theory," *Review of International Economics* 3 (1995): 343-354.