PRODUCTIVE CONSUMPTION AND ENDOGENOUS GROWTH: A THEORETICAL ANALYSIS

Manash Ranjan GUPTA

Economic Research Unit, Indian Statistical Institute, Calcutta, India

Abstract: A model of economic growth with endogenous accumulation of labour efficiency is developed in this paper. Luxury consumption is distinguished from productive consumption; and the rate of change of labour efficiency varies linearly with the productive consumption. The intertemporal equilibrium of the dynamic system is unstable; and hence the long run rate of growth of per capita income is endogenously determined. However, the equilibrium point itself is a 'No Growth' equilibrium.

Key words: Productive consumption, labour efficiency, intertemporal equilibrium, stability, endogenous growth

JEL Classification Number:

1. INTRODUCTION

In the existing literature on the theory of economic growth consumption is generally not considered as productive. However, in the literature on Development Economics, there exists substantial discussion[1] on the productivity of consumption and on the theoretical implications of 'Consumption Productivity Hypothesis'. Productive consumption raises the efficiency of labour. Gersovitz (1988) distinguishes three forms of productive consumption: (i) nutrition; (ii) health effort and (iii) education. The marginal utility of these forms of consumption is far lower than that of luxury consumption. However, these consumptions have either direct or indirect positive effects on the rate of growth of labour productivity.

A few dynamic models of productive consumptions are available in the literature. Banerji and Gupta (1997) and Jellal and Zenou (2000) develop dynamic efficiency wage models in which the representative firm maximizes discounted present value of profit over the infinite time horizon and the efficiency of labour accumulates over time. Jellal and Zenou (2000) also consider learning by doing effects in the efficiency accumulations. However, these models do not deal with consumption savings allocation problem

[1] For example, see the works of Bliss and Stern (1979), Dasgupta and Ray (1986), Gupta (1987, 1989), Leibenstein (1957), Mirrlees (1975), Stiglitz (1976) etc.

of the utility maximising individual and with the capital accumulation problem of the economy. Steger (2000) focuses on the Ramsey optimal capital accumulation path in the presence of productive consumption. However, in his model, physical capital and human capital are assumed to be perfect substitutes. Also entire consumption is assumed to be productive; and hence no distinction is drawn between luxury consumption and productive consumption.

The present paper also analyses the properties of capital accumulation and the growth of labour efficiency in a Cass-Ramsey framework removing the problems in Steger (2000). Utility of the representative individual is assumed to be a positive and concave function of luxury consumption only. The productive consumption only contributes to the rate of growth of labour efficiency. Physical capital and human capital (labour efficiency) are not perfect substitutes here. We get some interesting results from this model. The intertemporal equilibrium of the dynamic system is a 'No Growth' equilibrium of Solow (1956) type inspite of the existence of endogenous labour augmenting change resulting from productive consumption. Also, in this 'No Growth' equilibrium, the per capita income is independent of the rate of growth of population (number of workers). However, the intertemporal equilibrium is unstable; and hence we have endogenous growth in the long run.

The paper is organized as follows:

The model is described in section 2 and its working is described in section 3. The properties of the intertemporal equilibrium of the system are analysed in section 4. Concluding remarks are made in section 5.

2. THE MODEL

This is a one sector dynamic model with capital stock and efficiency (skill) of labour accumulating over time. Total output is allocated between consumption and investment of physical capital; and this allocation decision is taken by one infinitely lived economic agent maximizing the life time utility defined over luxury consumption.[2] Also total consumption is allocated between luxury consumption and productive consumption. The rate of growth of labour efficiency varies positively with the level of productive consumption. There is no depreciation of either physical capital or of labour efficiency. Number of individuals (workers) grow at a constant rate and all individuals are identical with respect to taste and endowment. Production function is linear homogenous in terms of physical capital and labour; and the inputs are not perfect substitutes.

We use the following notations to describe the model:

$t =$ Time – a non negative continuous variable.
$Y =$ Level of output at time t.
$K =$ Level of capital stock at time t.
$N =$ Number of workers at time t.
$h =$ Level of efficiency per worker at time t.

[2] For the sake of technical simplicity we assume utility to be independent of productive consumption.

$k = \dfrac{K}{hN}$ = Capital intensity.
C = Level of per capita consumption.
n = Constant rate of growth of workers over time.
λ = Proportion of luxury consumption.
σ = Elasticity of marginal utility with respect to luxury consumption.
α = Capital elasticity of output.
ρ = Rate of discount.
$u(\cdot)$ = Utility function of the representative individual.
W = Welfare.

Following equations describe the model:

$$Y = K^{\alpha} \cdot (hN)^{1-\alpha} \quad \text{with} \quad 0 < \alpha < 1 \tag{1}$$

is the CRS Cobb Douglas production function.

$$\dot{K} = Y - CN \tag{2}$$

implies that excess of output over consumption is invested and there is no depreciation of capital stock.

$$\dot{N} = nN \tag{3}$$

implies that the number of workers (individuals) grow at constant rate over time.

$$u = u(\lambda C) = \frac{(\lambda C)^{1-\sigma}}{1-\sigma} \quad \text{with} \quad 0 < \sigma < 1 \tag{4}$$

is the utility function. Here λC is luxury consumption; and utility is a positive and concave function of luxury consumption.

The dynamic efficiency function is given by the following:

$$(\dot{h}/h) = (1-\lambda)C\,. \tag{5}$$

This means that the rate of change in efficiency varies positively with the level of productive consumption yielding nutrition and/or education and/or health care. Here $(1-\lambda)C$ is the level of productive consumption. This is the dynamic version of the 'Consumption Efficiency Hypothesis'.

The welfare to be maximized is given by

$$W = \int_0^{\infty} N \cdot u(\lambda C) \cdot e^{-\rho t} \cdot dt\,. \tag{6}$$

This is the discounted present value of utility of all the workers (individuals) defined over the infinite time horizon, $[0, \infty)$.

The problem is to maximize W given by equation (6) subject to equations (1) to (5) with respect to C and satisfying the restrictions:

$$0 \leqslant C \leqslant (Y/N)\,; \quad \text{and} \quad 0 \leqslant \lambda \leqslant 1\,.$$

3. WORKING

Using the definition of k, we have

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{h}}{h} - \frac{\dot{N}}{N};$$

and then using equations (1), (2), (3) and (5) we have

$$\dot{k} = k^{\alpha} - (C/h) - (1-\lambda)Ck - nk. \tag{7}$$

Also putting $N(0) = 1$ and using equations (3) and (4), equation (6) can be written as

$$W = \int_0^{\infty} u(\lambda C) e^{-(\rho-n)t} dt. \tag{8}$$

So the problem is to maximize W given by equation (8) subject to the equations of motion given by (5) and (7). Here C and λ are control variables, and k and h are state variables. Here $\rho > n$, by assumption.

This is an optimal control problem with two state variables. In Banerji and Gupta (1997) and in Jellal and Zenou (2000), human capital is the only state variable. In Steger (2000), capital is the only state variable because physical capital and human capital are assumed to be perfect substitutes, and additive. So the model presented in this paper is technically more complicated than the earlier models.

The current value Hamiltonian, denoted by $He^{(\rho-n)t}$ and to be maximized at each time point with respect to the control variables, is given by

$$He^{(\rho-n)t} = \frac{(\lambda C)^{1-\sigma}}{1-\sigma} + q_k \cdot \dot{k} + q_h \cdot \dot{h} \tag{9}$$

where $\dot{k}$ and $\dot{h}$ are given by equations (7) and (5) and q_k and q_h are two costate variables —functions of time.

The first order optimality conditions with respect to C and λ are given by the followings:

$$(\lambda C)^{-\sigma}\lambda - (q_k/h) - (1-\lambda)q_k \cdot k + q_h \cdot (1-\lambda)h = 0 \tag{10}$$

and

$$(\lambda C)^{-\sigma} C + q_k \cdot C \cdot k - q_h \cdot C \cdot h = 0. \tag{11}$$

The canonical differential equations showing the time behaviour of costate variables are given by

$$\dot{q}_k = (\rho - n)q_k - q_k(\alpha k^{\alpha-1} - n - (1-\lambda)C) \tag{12}$$

and

$$\dot{q}_h = (\rho - n)q_h - q_k(C/h)(1/h) - q_h \cdot (1-\lambda)C. \tag{13}$$

The transversality conditions to be satisfied by the optimal solution are given by the followings:

$$\lim_{t\to\infty} q_k k e^{-(\rho-n)t} = 0 \tag{T1}$$

and

$$\lim_{t\to\infty} q_h h e^{-(\rho-n)t} = 0. \tag{T2}$$

Using equations (10) and (11) we have

$$(\lambda C)^{-\sigma} = (q_k/h) \tag{14}$$

and using equations (14) and (11) we have

$$(q_k/h)(1 + kh) = q_h \cdot h\,. \tag{15}$$

Here, kh is the capital-worker ratio. Now using equations (13) and (15) we have

$$(\dot{q}_h/q_h) = (\rho - n) - \frac{C}{1 + kh} - (1 - \lambda)C\,.$$

or

$$(\dot{q}_h/q_h) + (\dot{h}/h) = (\rho - n) - \frac{C}{1 + kh}\,. \tag{16}$$

From equation (12) we have

$$(\dot{q}_k/q_k) = (\rho - n) - \alpha k^{\alpha - 1} + n + (\dot{h}/h)\,.$$

or

$$(\dot{q}_k/q_k) - (\dot{h}/h) = (\rho - n) - \alpha k^{\alpha - 1} + n\,,$$

or

$$(\dot{q}_k/q_k) - (\dot{h}/h) = \rho - \alpha(kh)^{\alpha - 1} \cdot h^{1 - \alpha}\,. \tag{17}$$

Differentiating both sides of (14) with respect to t, we have

$$\frac{(\dot{\lambda C})}{\lambda C} = -(1/\sigma)[(\dot{q}_k/q_k) - (\dot{h}/h)]$$

and then using equation (17) we have

$$\frac{(\dot{\lambda C})}{\lambda C} = \frac{\alpha(kh)^{\alpha - 1} \cdot h^{1 - \alpha} - \rho}{\sigma}\,. \tag{18}$$

This implies that the rate of growth of utility yielding (luxury) consumption is equal to the excess of marginal productivity of capital over the rate discount normalized with respect to the elasticity of marginal utility of luxury consumption.

Again we have, from equations (5) and (7),

$$\frac{(\dot{kh})}{(kh)} = k^{\alpha - 1} - (C/kh) - n\,,$$

or

$$\frac{(\dot{kh})}{(kh)} = (kh)^{\alpha - 1} \cdot h^{1 - \alpha} - (C/kh) - n\,. \tag{19}$$

So the rate of change of capital worker ratio, kh, is the function of per capita consumption, C, efficiency, h, and capital worker ratio, kh, itself.

Now we are to express h and C in terms of λC and kh. Differentiating both sides of equation (15) with respect to t, we have,

$$[(\dot{k}/k) + (\dot{h}/h)] \cdot (kh/(1 + kh)) = (\dot{q}_h/q_h) + (\dot{h}/h) - [(\dot{q}_k/q_k) - (\dot{h}/h)]\,.$$

and then using equations (16), (17) and (19) we have

$$[(kh)^{\alpha-1}\cdot h^{1-\alpha}-(C/kh)-n](kh/(1+kh))$$
$$=(\rho-n)-(C/(1+kh))-(\rho-\alpha(kh)^{\alpha-1}h^{1-\alpha});$$

or

$$(kh)^{\alpha-1}\cdot h^{1-\alpha}[(kh/(1+kh))-\alpha]=n[(kh/(1+kh))-1];$$

or

$$(kh)^{\alpha-1}\cdot h^{1-\alpha}=\frac{n\cdot\left[\dfrac{kh-1-kh}{1+kh}\right]}{\dfrac{kh-\alpha(1+kh)}{1+kh}};$$

or

$$(kh)^{\alpha-1}\cdot h^{1-\alpha}=\frac{-n}{-\alpha+(1-\alpha)kh};$$

or

$$h^{1-\alpha}=\frac{n\cdot(kh)^{1-\alpha}}{\alpha-(1-\alpha)kh}. \tag{20}$$

Using equations (18) and (20) we have

$$\frac{(\dot{\lambda C})}{\lambda C}=\frac{\dfrac{\alpha n}{\alpha-(1-\alpha)kh}-\rho}{\sigma};$$

or

$$\frac{(\dot{\lambda C})}{\lambda C}=(1/\sigma)\left[\frac{\alpha n}{\alpha-(1-\alpha)kh}-\rho\right]. \tag{21}$$

Now differentiating both sides of equation (20) with respect to t, we have

$$(1-\alpha)\cdot(\dot{h}/h)=(1-\alpha)\cdot\frac{(\dot{kh})}{kh}+\frac{(\dot{kh})}{(kh)}\left[\frac{kh(1-\alpha)}{\alpha-(1-\alpha)kh}\right];$$

or

$$\frac{\dot{h}}{h}=\frac{(\dot{kh})}{kh}\left[1+\frac{kh}{\alpha-(1-\alpha)kh}\right]; \tag{20A}$$

and then using equations (5) and (19) we have

$$(1-\lambda)C=\left[(kh)^{\alpha-1}\cdot h^{1-\alpha}-\frac{C}{kh}-n\right]\left[1+\frac{kh}{\alpha-(1-\alpha)kh}\right];$$

and then using equation (20) we have

$$(1-\lambda)C=\left[\frac{n}{\alpha-(1-\alpha)kh}-\frac{C}{kh}-n\right]\left[1+\frac{kh}{\alpha-(1-\alpha)kh}\right];$$

or

$$C+(C/kh)\left[1+\frac{kh}{\alpha-(1-\alpha)kh}\right]$$
$$=\lambda C+n\left[\frac{1}{\alpha-(1-\alpha)kh}-1\right]\left[1+\frac{kh}{\alpha-(1-\alpha)kh}\right];$$

or

$$\left[kh + 1 + \frac{kh}{\alpha - (1-\alpha)kh}\right](C/kh) = \lambda C + n\left[\frac{1}{\alpha - (1-\alpha)kh} - 1\right]\left[1 + \frac{kh}{\alpha - (1-\alpha)kh}\right];$$

or

$$(C/kh) = \frac{\lambda C}{\psi_1(kh)} + \frac{n \cdot \psi_2(kh) \cdot \psi_3(kh)}{\psi_1(kh)}. \tag{22}$$

where

$$\psi_1(kh) = kh + 1 + \frac{kh}{\alpha - (1-\alpha)kh},$$

or

$$\psi_1(kh) = 1 + kh \cdot \left[1 + \frac{1}{\alpha - (1-\alpha)kh}\right];$$

$$\psi_2(kh) = \frac{1}{\alpha - (1-\alpha)kh} - 1; \quad \text{and}$$

$$\psi_3(kh) = 1 + \frac{kh}{\alpha - (1-\alpha)kh}.$$

Now using (20), (22) and (19) we have

$$\frac{(\dot{kh})}{kh} = \frac{n}{\alpha - (1-\alpha)kh} - \frac{\lambda C}{\psi_1(kh)} - \frac{n\psi_2(kh)\psi_3(kh)}{\psi_1(kh)} - n,$$

or

$$\frac{(\dot{kh})}{kh} = n \cdot \psi_2(kh) - \frac{\lambda C}{\psi_1(kh)} - \frac{n\psi_2(kh)\psi_3(kh)}{\psi_1(kh)}, \tag{23}$$

Now equations (23) and (21) are two equations of motions. Equation (23) shows that the rate of change in kh is function of kh and λC. Equation (21) shows that the rate of change in λC is function of kh only. Here

$$\psi_3(kh) = \psi_1(kh) - kh;$$

and hence equation (23) can be written as

$$\frac{(\dot{kh})}{kh} = -\frac{\lambda C}{\psi_1(kh)} + \frac{n \cdot \psi_2(kh) \cdot kh}{\psi_1(kh)}. \tag{23A}$$

Here $\psi_1(kh)$ is a positive function of kh. Also $(\psi_2(kh) \cdot kh/\psi_1(kh))$ is a positive function[3] of kh. Hence $(\dot{kh})/kh$ is a positive function of kh.

[3] It is shown in the Appendix.

4. LONG RUN EQUILIBRIUM

The reduced form dynamic system consists of the differential equations (21) and (23) solving which we get the time path of λC and kh. First, we try to investigate the properties of the equilibrium of the system at which $(\dot{\lambda C}) = (\dot{kh}) = 0$. Also using equations (14) and (15), it can be easily shown that $q_k \cdot k$ and $q_h \cdot h$ are time independent in the equilibrium. Here

$$(\dot{kh}) = 0 \Rightarrow (\dot{K}/K) = (\dot{N}/N).$$

This is the balanced growth path with the capital stock and the number of workers growing at equal rates. This is the long-run growth path in Solow (1956) model. Equation (20) shows that h is constant when kh is constant. So k is also time-independent; and the same is true for the costate variables. Hence efficiency of labour does not change along this balanced growth path. Also

$$(Y/N) = (kh)^{\alpha-1} \cdot h^{1-\alpha};$$

and using equation (20) we find that

$$(Y/N) = \frac{n}{\alpha - (1-\alpha)kh}.$$

Hence along the balanced growth path—in the intertemporal equilibrium—per capita income is constant. So intertemporal equilibrium is a no growth equilibrium. We get this property in Solow–Ramsey–Cass model in the absence of technical progress.

We now try to analyse the stability property of the equilibrium. For this we are to draw the stationery locus.

$(\dot{\lambda C}) = 0$ stationary locus is a vertical straight line because solving equation (21) with $(\dot{\lambda C}) = 0$ we get $kh = M$ (constant) where

$$M = (\alpha(\rho - n)/(1-\alpha)\rho) < (\alpha/(1-\alpha)).$$

Also

$$\frac{\partial(\dot{\lambda C}/\lambda C)}{\partial kh} = \frac{\alpha n(1-\alpha)}{\sigma[\alpha - (1-\alpha)kh]^2} > 0.$$

Here $kh \gtrless M \Rightarrow (\dot{\lambda C}) \gtrless 0$. When capital worker ratio is higher (lower) than M, luxury consumption, λC, rises (falls) over time.

Next, we turn to draw the $(\dot{kh}) = 0$ stationary locus.

$$\frac{\partial(\dot{kh}/kh)}{\partial \lambda C} = -\frac{1}{\psi_1(kh)} < 0$$

at the equilibrium point because there $kh = M$ and hence

$$\psi_1(kh) = 1 + ((\rho - n)(\alpha n + \rho)/(1-\alpha)\rho n) > 0$$

for $\rho > n$ and $0 < \alpha < 1$.

Slope of $\dot{kh} = 0$ curve is given by

$$\frac{d\lambda C}{dkh} = -\frac{\partial(\dot{kh}/kh)\partial kh}{\partial(\dot{kh}/kh)\partial \lambda C}.$$

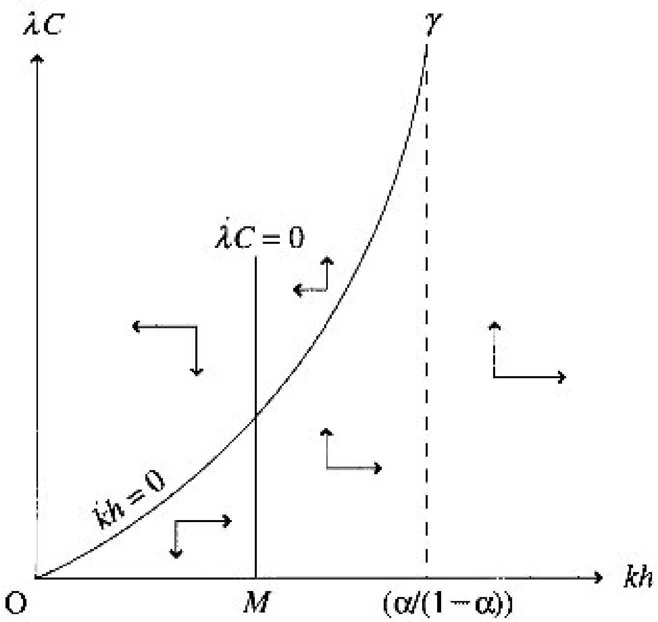

Figure 1.

Here $\partial(\dot{k}h/kh)/\partial kh$ is positive; and hence $(\dot{k}h) = 0$ locus slopes positively. Note that $(\dot{k}h) = 0$ curve passes through the origin because equation (23) shows that $kh = 0$ and $(\dot{k}h) = 0$ imply $\lambda C = 0$.

For $0 \leqslant kh \leqslant (\alpha/(1-\alpha))$, $\dot{k}h = 0$ locus slopes positively in the first quadrant; and, at $kh = (\alpha/(1-\alpha))$, $\lambda C = \infty$ along this locus.

$(\dot{k}h) = 0$ locus and $(\dot{\lambda}C) = 0$ locus intersect each others at $kh = M < (\alpha/(1-\alpha))$.

When $kh > (\alpha/(1-\alpha))$, then $\psi_1(kh) < 0$ and $\psi_2(kh) < 0$. So equation (23A) shows that $(\dot{k}h) > 0$ for $kh > (\alpha/(1-\alpha))$ and for $\lambda C \geqslant 0$. Along the $(\dot{k}h) = 0$ locus, λC is always negative for $kh > (\alpha/(1-\alpha))$. So, at any point in the first quadrant in this figure with $kh > (\alpha/(1-\alpha))$, we have $(\dot{k}h) > 0$. For $kh < (\alpha/(1-\alpha))$, at any point above (below) the $(\dot{k}h) = 0$ locus, $(\dot{k}h)$ is negative (positive).

The behaviour of the state-trajectories are described in the figure; and this behaviour is conditional on the initial conditions. If the initial point lies below the $(\dot{k}h) = 0$ locus, we find endogenous growth in the system with kh always rising and λC rising at least in the terminal phase of development.

Jacobian matrix is given by the following:

$$J = \begin{bmatrix} \dfrac{\partial(\dot{\lambda}C/\lambda C)}{\partial \lambda C} & \dfrac{\partial(\dot{\lambda}C/\lambda C)}{\partial kh} \\ \dfrac{\partial(\dot{k}h/kh)}{\partial \lambda C} & \dfrac{\partial(\dot{k}h/kh)}{\partial kh} \end{bmatrix} = \begin{bmatrix} 0 & + \\ - & \dfrac{\partial(\dot{k}h/kh)}{\partial kh} \end{bmatrix}.$$

Here

$$|J| > 0\,, \quad \text{trace} J > 0 \quad \text{because} \quad \frac{\partial(\dot{k}h/kh)}{\partial kh} > 0\,.$$

So the equilibrium is unstable.

In the case of stable equilibrium the system converges to the balanced growth path at which $kh = M$. Along this balanced growth path, we have exogenous growth because

$$(\dot{K}/K) = (\dot{Y}/Y) = (\dot{N}/N) = n\,.$$

However, we get unstable equilibrium in this model; and, in this case, kh either rises or falls over time indefinitely. It always moves away from the equilibrium point. We have unbalanced growth and this rate of growth is endogenously determined. Y/N rises (falls) as kh rises (falls) over time.

The equilibrium level of per capita income is given by

$$(Y/N) = \frac{n}{\alpha - (1-\alpha)M}$$

or

$$(Y/N) = (\rho/\alpha)\,.$$

So the per capita income in the 'No Growth' equilibrium is independent of the rate of growth of the number of workers. This result is interesting because, in Solow–Ramsey–Cass one sector model, per capita income in the long run equilibrium varies inversely with the rate of growth of the number of workers.[4]

From equation (2) we have

$$C = (Y/N) - (K/N)(\dot{K}/K)\,;$$

and then in the equilibrium

$$\begin{aligned} C &= (\rho/\alpha) - n \cdot M \\ &= (\rho/\alpha) - (n\alpha(\rho - n)/(1-\alpha)\rho)\,. \end{aligned}$$

So the per capita consumption is not independent of the rate of growth of the number of workers.

When the system is off the balanced growth path, the rate of growth of per capita income, (Y/N), is given by

$$\frac{(\dot{Y}/N)}{(Y/N)} = (\alpha - 1)\frac{(\dot{k}h)}{kh} + (1-\alpha)\cdot\frac{\dot{h}}{h}\,;$$

and then using equation (20A) we have

$$\frac{(\dot{Y}/N)}{(Y/N)} = (1-\alpha)\left(\frac{\dot{k}h}{kh}\right)\left[\frac{kh}{\alpha - (1-\alpha)kh}\right].$$

So the rate of growth of per capita income is increasing in kh. It does not converge to any exogenously given value.

[4] Obviously this result is not very important because the system does not converge to the long run equilibrium point.

Once we know the value of λC and kh at some t, we can also obtain the value of C from equation (22). Then the value of h is obtained from equation (20). Then we solve for q_h and q_k from equations (14) and (15).

How do we relate the results of unbalanced growth obtained here to the results existing in the literature? In our model, capital stock per worker, kh, and utility yielding consumption, λC, rises over time in the case of endogenous growth. However, there is no mechanism which ensures the equality between $(\dot{\lambda C}/\lambda C)$ and $(\dot{kh}/kh)$. In the models of Romer (1990), Lucas (1986) etc. endogenous growth takes place in a balanced manner. Per capita consumption and capital stock per worker grow at equal rates. Those models assume a steady state growth equilibrium and then explain the existence of a positive growth rate in the long run, because Solow–Swan–Cass model can not explain positive growth rate in the steady state equilibrium.

5. CONCLUSION

In this paper, we have analysed a dynamic model of a less developed economy where efficiency of labour and capital stock accumulate over time and the endogenous accumulation of labour efficiency is the source of endogenous growth. However, this increase in labour efficiency is explained by the productive consumption which is the result of sacrifice of utility yielding (luxury) consumption. This is a dynamic version of the consumption-efficiency hypothesis. The relative rate of growth of labour efficiency varies positively with the level of productive consumption. So this mechanism is different from that in Lucas (1988) where the relative rate of growth of labour efficiency (human capital) varies positively with the allocation of human capital to the human capital accumulation sector.

The long run equilibrium of the dynamic system implies a Solow type 'No Growth' equilibrium inspite of the positive role of consumption on the accumulation of labour efficiency. If we ignore the productive consumption hypothesis then the present model is reduced to a standard Ramsey–Cass model. Long run equilibrium in that model is stable and the long run rate of growth of per capita income is equal to zero. A positive long run rate of growth is obtained in such a model only introducing a labour augmenting technical progress. In the present model, labour augmentation takes place through the productive consumption hypothesis but the intertemporal equilibrium point implies a 'No Growth' equilibrium. Also the per capita income in the intertemporal equilibrium is independent of the rate of growth of population (number of workers)—a result not obtained in Solow–Ramsey–Cass model. However, the dynamic system does not converge to the intertemporal equilibrium point here because the unique equilibrium point is unstable.

In the model of Steger (2000), productive consumption is not distinguished from luxury consumption. Steger (2000) explains endogenous growth due to the assumption of AK technology of producing the product. With diminishing marginal productivity of capital one can not explain endogenous growth in his model. In this model, we assume diminishing marginal productivity of capital and can explain endogenous growth because we get the case of an unstable intertemporal equilibrium.

We have restrictively assumed that productive consumption does not yield any utility. In reality, marginal utility of productive consumption is positive but lower than that of luxury consumption. It would have been more meaningful to consider an utility function

$$u = \frac{(\lambda C)^{1-\sigma_1}}{1-\sigma_1} + \frac{((1-\lambda)C)^{1-\sigma_2}}{1-\sigma_2} \quad \text{with} \quad 0 < \sigma_1 < \sigma_2 < 1 .$$

However, technical complications prevent us from deriving meaningful results in this more realistic case.

APPENDIX

Define

$$g(kh) = \psi_2(kh)kh/\psi_1(kh) .$$

Using the expression of $\psi_2(kh)$ and $\psi_1(kh)$ we have

$$g(kh) = \frac{(1-\alpha)(1+kh)kh}{\alpha(1+kh)^2 - (kh)^2} = \frac{1-\alpha}{\alpha\left[1+\frac{1}{kh}\right] - \frac{1}{1+\frac{1}{kh}}} .$$

Hence $g(kh)$ is a positive function of kh.

REFERENCES

Banerji, S. and M. R. Gupta, "The efficiency wage given long run employment and concave labour constraint," Journal of Development Economics, 53 (1997), 185–195.

Barro, R. J. and X. Sala-i-Martin, Economic Growth, New York, McGraw-Hill (1995).

Behrman, R. and B. Deolalikar, "Health and Nutrition," in H. Chenery and T. N. Srinivasan (eds.), Handbook of Development Economics, Vol. I, Amsterdam: Elsevier Science (1988), 631–711.

Bliss, C. and N. Stern, "Productivity, Wages and Nutrition-1," Journal of Development Economics, 5 (1978), 331–362.

Dasgupta, P. and D. Ray, "Inequality as a determinant of malnutrition: theory," Economic Journal, 96 (1986), 1011–1034.

Fogel, R. W., "Economic Growth, Population Theory, and Physiology: The Bearing of Long Term Processes on the Making of Economic Policy," American Economic Review, 84 (1994), 369–395.

Gersovitz, M., "Savings and Nutrition at Low Incomes," Journal of Political Economy, 91 (1983), 841–855.

———, "Saving and Development," in H. Chenery and T. N. Srinivasan (eds.), Handbook of Development Economics, Vol. I, Amsterdam: Elsevier Science (1988), 382–424.

Giovannini, A., "Saving and the Real Interest Rate in LDCs," Journal of Development Economics, 18 (1985), 197–217.

Gupta, M. R., "A nutrition based theory of interlinkage," Journal of Quantitative Economics, 3 (1987), 189–202.

———, "Nutrition, dependents and the mode of wage payments," Oxford Economic Papers, 41 (1989), 737–748.

Hicks, N., "Growth vs. Basic Needs: Is there a Trade-Off?" World Development, 7 (1979), 985–994.

Jellal, M. and Y. Zenou, "A dynamic efficiency wage model with learning by doing," Economics Letters, 66 (2000), 99–105.

King, R. G. and S. Rebelo, "Transition Dynamics and Economic Growth in the Neoclassical Model," American Economic Review, 83 (1993), 908–931.

Koopmans, T. C., "On the Concept of Optimal Economic Growth," in the Econometric Approach to Development Planning. Amsterdam: North-Holland (1965), and Pontificiae Academia Scientiarum Scripta Varia, 28 (1965), 225–300.

Leibenstein, H., "The Theory of Underemployment in Backward Economies," Journal of Political Economy, April (1957), 91–103.

Lucas, R. E., "On the Mechanics of Economic Development," Journal of Monetary Economics, 22 (1988), 3–12.

Mirrlees, J., "A Pure Theory of Underdeveloped Economics," in Reynolds ed., "Agriculture in Development Theory," Yale University Press (1975).

Nelson, R. R., "A Theory of the Low Level Equilibrium Trap in Underdeveloped Economies," American Economic Review, 46 (1956), 894–908.

Nurkse, R., Problems of Capital Formation in Underdeveloped Countries, Oxford: Basil Blackwell (1962).

Ogaki, M., J. D. Ostry, and C. M. Reinhart, "Savings Behaviour in Low and Middle Income Developing Countries: A Comparison," IMF Staff Papers, 43 (1996), 38–71.

Rebelo, S., "Growth in Open Economics," Carnegie-Rochester Conference Series on Public Policy, 36 (1992), 5–46.

Romer, P. M., "Endogenous Technological Change," Journal of Political Economy, 98 (1990), 971–1002.

Solow, R. M., "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, 70 (1956), 65–94.

Steger, T. M., "Productive Consumption and Growth in Developing Countries," Review of Development Economics, 4 (2000), 365–375.

Stiglitz, J., "The Efficiency Wage Hypothesis, Surplus Labour and Distribution of Income in LDCs," Oxford Economic Papers, 28 (1976), 185–207.

Wheeler, D., "Basic Needs Fulfilment and Economic Growth: A Simultaneous Model," Journal of Development Economics, 7 (1980), 435–451.

Wichmann, T., "Food Consumption and Growth in a Two Sector Economy," Discussion Paper 1996/02, Technical University, Berlin (1996).