Statistical process control procedures for controlling the weight of packets of biscuits

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ABSTRACT *The thickness and weight of a biscuit are its two most important quality characteristics. Although packets are not sold by weight, the declared weight is considered as the lower specification. Any deviation of these characteristics from their respective nominal values leads to a variation in the weight of packets of biscuits. High pack weight variation is a problem with considerable implications, because underweight packets lead to customer dissatisfaction and excess weight results in loss of profit. Conventionally, the weight and thickness of biscuits are monitored to control the weight of packets. This study describes how the variation in weight of packets was reduced by setting appropriate target values for dough weight and rise per gram (RPG) and controlling these values using simple statistical tools. This led to a substantial reduction of underweight packets from 12.2% to 2.0% and excess weight packets from 2.1% to 0.9% of the declared weight. The statistical techniques used for the purpose of controlling these characteristics are the test for equality of two means, and X-charts for controlling dough weight and RPG on a regular basis.*

Introduction

Biscuits are sold to consumers in packets of some declared weight. Each of these packets is also expected to contain a specified number of biscuits. To satisfy a customer, the weight of a packet should not fall below its declared weight. In order to eliminate the chance of packets with less weight reaching the customer, the manufacturing target is set above the declared weight. However, if the variation in pack weight during manufacturing is higher, the required targeted weight will be higher. This will result in loss of profit.

This study demonstrates that profitability as well as customer satisfaction can be increased substantially through

(i) identification of suitable characteristics for control purposes,
(ii) setting appropriate target values for these characteristics and
(iii) maintaining the process at target values taking into account the existing variability of the process.

For the above purposes, simple statistical tools and Shewhart type control charts are found to be useful.

This study was carried out in an Indian biscuit manufacturing unit, where packets containing biscuits in three stacks are sold in the market. The declared weight of a packet is 400 g with 69 biscuits.

The process

Different ingredients are mixed as per the recipe and dough is produced. Then a continuous dough sheet of the required thickness and width is formed through a series of operations. This dough sheet moves over a conveyor for further processing. The side of the conveyor facing the operator is known as the working side (WS) and the opposite side is known as the non-working side (NWS). The thickness of the dough sheet depends on the final gauge roller setting, where sidewise adjustment is also possible. Dough pieces of desired shapes are then cut from the dough sheet by identical dies mounted on a rotating roller. Another conveyor carries the dough pieces forward where a milk solution is sprayed on the top surface of the dough pieces. Subsequently, the dough pieces are baked in ovens and the biscuits of required thickness (also called height) and weight, are formed at the oven end. The biscuits are carried to the packing section through another conveyor allowing sufficient time for cooling prior to packing. In packing, a long row of biscuits is maintained on the inclined chute through which biscuits are fed to the packing machine. The pusher fitted into the packing machine pushes a stack of biscuits, which can be accommodated within the pusher length, into the channel. The width of this channel is 124 mm. The stack is dragged towards the packet assembly unit by the moving separators inside the channel.

The thickness of biscuits should be such that a stack of a specified number of biscuits can be accommodated within the channel width. In this case, the specified number of biscuits, i.e. count, is 23. If the pusher pushes fewer biscuits, the packing operator makes the necessary correction by inserting biscuits during the movement of the stacks through the channel. Three consecutive stacks are moved together to the packet assembly unit, where such sets of stacks are wrapped to form a packet. The pack weight is mainly dependent on the count per stack and the weight of individual biscuits. As the channel width is set according to the specified packet length the count per stack is dependent only on the thickness of individual biscuits. Thus, a biscuit's thickness and weight are the only two characteristics that affect the pack weight. Although the specified count per packet is 69 (3×23) at present, due to variations in count per stack this value is observed to be between 67 and 73.

In order to assess whether the specified count and weight could be achieved in packets, the following procedure is in use in the factory. The number of biscuits that can be stacked in a wooden gauge box of 228.7 mm length is counted. This number is called '*Gauge*'. Then the total weight of these biscuits is measured and, if this weight differs from 250 g, adjustment is made to discover how many biscuits are required to get a weight of 250 g. This number is used as a measure of weight and is simply called '*Weight*'. The target values for *Gauge* and *Weight* for this particular variety of biscuit are 43.5 and 43 respectively. The sample biscuits are collected randomly from the whole width of the conveyor for these *Gauge* and *Weight* measurements.

Approval of process setting

The process setting is approved in two stages. Initially, the weight of a set of ten dough pieces collected from each side of a conveyor is measured. If the supervisor judges from his experience that these two values are OK, the sidewise gauge roller setting is approved. Otherwise sidewise adjustment of the gauge roller is performed. Once the sidewise gauge roller setting is approved, one set of measurements on *Gauge* and *Weight* is made after baking, following the existing procedure. If the supervisor judges that the target values for *Gauge* and *Weight* have been achieved, the process is approved for regular production. Otherwise, necessary adjustments are made. It is important to note that although the thickness of biscuit is a critical factor with respect to variation in count per stack the existing practice does not consider the possibility of differences in thickness of biscuits during the setting approval.

Online process control

Subsequent to setting approval, the weight of a set of 13 dough pieces collected randomly from the whole width of the conveyor is measured. This weight is considered as the target value of weight for a set of 13 dough pieces. Such samples are collected on a routine basis and dough weights are measured for monitoring purposes. The *Gauge* and *Weight* of biscuits after baking are also measured on a routine basis and the observed values are compared with their respective target values. If the observed *Gauge* and *Weight* are considered by the supervisor/operator to be away from the target values action is initiated. The sample dough weights are also taken into consideration while deciding upon the corrective action. This is because both the *Gauge* and the *Weight* are dependent on the dough weight.

The problem

(1) The present target values for *Gauge* and *Weight* of biscuits (43.5 and 43 respectively) have been fixed on an ad hoc basis based on experience. The gauge of 43.5 cannot be counted and it is estimated subjectively by visual examination of the remaining gap in the gauge box after stacking 43 biscuits. Obviously, measurement error is high in the present measurement system leading to a high chance of inappropriate process setting. The problem is further compounded because of:

- not knowing what should be the target dough weight to achieve a particular weight of biscuits, taking into consideration the loss in weight due to baking;
- not utilizing statistical methodology for testing the equality of dough weight between the two sides. This may result in improper sidewise adjustment in the final gauge roller, leading to an increase in variability of a biscuit's weight;
- non-consideration of the requirement of achieving equality of thicknss of biscuits. A detailed investigation has revealed that there was a significant difference in thickness of biscuits. This would lead to an increase in variability of thickness.

(2) No control charts are used to identify whether process adjustments are required. Clearly, this results in frequent unnecessary process adjustments. It is to be noted that as both the thickness and weight of biscuits are affected by dough weight, the two characteristics are related. Monitoring by T^2 statistic, where the covariance between the characteristics is taken into consideration, could be ideal to detect the control state of the process. However, shop floor computation of this statistic is a serious problem.

(3) The thickness of a biscuit depends on its weight (which in turn depends on dough weight) as well as on some other factors, such as amount of milk solution spray, oven heat and recipe. Because of this dependence of thickness on multiple factors, identification of action to be taken often becomes difficult. Consequently, the time needed for corrective action is usually high.

Objectives of the study

The objectives of this study are therefore,

(1) To suggest a measurement system for a biscuit's thickness and weight so that measurement error is reduced.
(2) To set a target value for the weight of biscuits so that the proportion of cases when the total weight of 69 biscuits is less than the declared weight will not be more than a specified value; and accordingly to set the target value for the weight of dough pieces at the earlier stage based on the loss of weight during baking.
(3) To set a target value for a biscuit's thickness, i.e. height commensurate with the channel width in the packing machine, such that the number of packets containing 69 biscuits is maximized.
(4) To identify the appropriate characteristics for controlling the height and weight of biscuits, such that identification of corrective action becomes easier.
(5) To suggest a methodology for approving process setting and online control of the identified characteristics based on statistical principles.

Approach

(1) Instead of expressing the thickness and weight of biscuits by count, if the total thickness and weight of a particular number of biscuits are measured, the measurement error will be less. Therefore, the best proposition is to measure the overall height and weight of 23 biscuits, i.e. the target number of biscuits per stack, and to control these parameters.
(2) The target weight of packets containing 69 biscuits is to be determined statistically based on the inherent variation of the total weight of 69 biscuits. Obviously the target total weight for 23 biscuits should be a third of the target weight of the packet. The corresponding target dough weight should be decided based on the loss of weight during baking. A one-to-one correspondence between samples of dough pieces and samples of biscuits is to be ensured using the observed time lag based on the conveyor speed. Regression analysis is thought to be useful here.
(3) The number of packets containing 69 count will be maximized if the number of stacks with 23 count within the channel width in the packing machine is maximized. 23 biscuits will be accommodated in a stack as long as the overall height of 23 biscuits lies between the channel width and the channel width minus the thickness of a biscuit. As the overall height of 23 biscuits follows a normal (i.e. symmetric) distribution, the targeted height of 23 biscuits should be equidistant from the channel width and the channel width minus the thickness of a biscuit (See Fig. 1). This yields the following equation:

$$W - H = H - \left(W - \frac{H}{23}\right) \tag{1}$$

where W is the channel width, H is the mean height of 23 biscuits and $(H/23)$ is the mean thickness of individual biscuits.
(4) In order to make the identification of action easier, a quantity called rise per gram (RPG) is introduced. The RPG is defined as the quantity obtained by dividing the overall height of 23 biscuits by the total weight of the same 23 biscuits. Thus, RPG is a ratio of two correlated normal variables. The RPG approaches normality if $\mu_2\sigma_2^{-1} \to \infty$, where μ_2 and σ_2 are the mean and standard deviation (SD) of the total

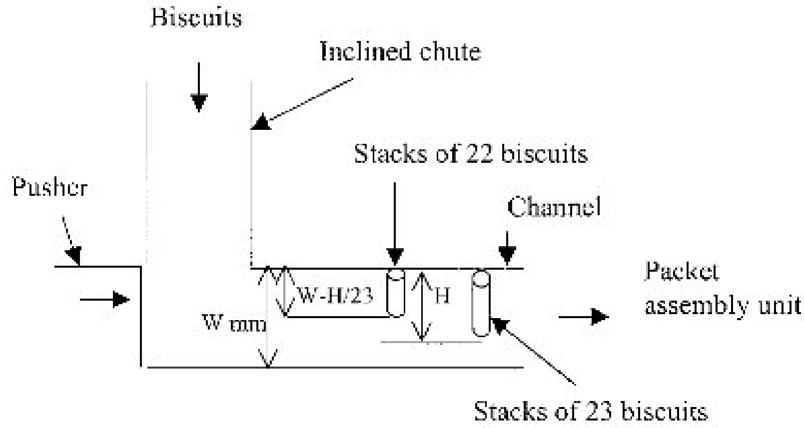

Figure 1. *Formation of stacks of biscuits before packing.*

weight of 23 biscuits respectively. This result is due to Hinkley (1969). It is very important to note that the total weight of 23 biscuits and RPG are expected to be independent since a biscuit's weight (or, equivalently, dough weight) depends solely on the final gauge roller setting, whereas RPG depends on factors other than the final gauge roller setting. Consequently identification of corrective action is expected to be easier leading to a shorter corrective action time. Note that,

$$\text{height of 23 biscuits} = \text{RPG} \times \text{weight of 23 biscuits}$$

Therefore control of the RPG and the total weight of 23 biscuits will ensure maintenance of appropriate thickness and weight of individual biscuits. However, since monitoring of a biscuit's weight at the oven end implies delaying the detection of the out-of-control state of the process by about 10 minutes, control should be exercised to achieve the target value for dough weight at an earlier stage. Under these circumstances, two separate control charts—one for RPG and the other for dough weight—will be appropriate.

It is important that the inherent variation of the above two characteristics should be estimated after ensuring that no significant differences exist between the two sides of the conveyor with respect to dough weight as well as RPG. For the online control of the characteristics, a sampling interval of 20 minutes will be adequate to ensure that one sample of dough pieces and one sample of biscuits can be collected from every mixing batch. This sampling method generates one observation (X) for each characteristic per batch. Therefore, use of the moving range (MR) control chart will be suitable for the estimation of the inherent variation of these characteristics. Roes *et al.* (1993) have shown that displaying a moving range chart has no real added value and so only X-charts will be maintained for online control purposes. For effective online control, the control charts for the dough weight and RPG should be used and maintained by the operators.

Establishment of statistical process control procedure

Comparison of dough weight and RPG between WS and NWS

Twelve sets of ten dough pieces were collected and weighed and 12 measurements on RPG were obtained from each side of the conveyor. The test of equality of means of two

Table 1. *Results of t-test*

Sl. No.	Characteristics	Mean WS ($\bar{X}_1$)	Mean NWS($\bar{X}_2$)	SD WS (s_1)	SD NWS (s_2)	SD Pooled (s_p)	Calculated 't' value
1	Weight of 10 dough pieces (g)	75.47	75.34	0.79	0.62	0.71	0.45
2	RPG (mm/g)	0.900	0.888	0.011	0.008	0.0096	3.06

independent normal distributions was carried out using a t-test for each of the characteristics. The results are summarized in Table 1.

Since $t_{0.025,22} = 2.074$, it was concluded that no significant difference was present in dough weight between WS and NWS, but a significant difference was present in RPG between WS and NWS. The root cause of the sidewise difference in RPG was investigated and found to be the non-uniformity of oven heat between the two sides. It might be worth noting here that this finding was contrary to the popular belief in the factory that a sidewise difference in a biscuit's characteristics is created by a sidewise difference in dough weight. The sidewise equality of RPG was tested again after initiating corrective action and no significant difference was observed.

Estimating inherent variability

Data on the total weight of 23 dough pieces and the total weight and height of 23 biscuits were collected within a shift at approximately 20 minute intervals, maintaining a one-to-one correspondence between samples of dough pieces and a sample of biscuits. Then, RPG values were obtained by dividing the total height of 23 biscuits by their total weight. During the entire data collection period, flour from the same lot was used for mixing. All subsequent calculations or estimations are made based on these data. A normal probability plot of RPG (shown in Fig. 2) confirmed that the RPG follows a normal distribution. The inherent variations of weight of 23 dough pieces, the weight of 23 biscuits and RPG were estimated by the

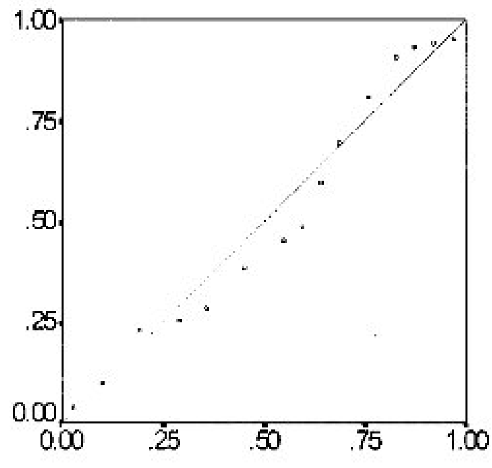

Figure 2. *Normal probability plot of RPG.*

Table 2. *Estimates of inherent variation of the characteristics*

Sl. No.	Characteristics	Average MR	$\hat{\sigma}$ (SD)
1	Weight of 23 dough pieces (g)	1.307	1.16
2	Weight of 23 biscuits (g)	1.140	1.01
3	RPG (mm/g)	0.0119	0.01

MR method. The moving ranges were based on two successive observations. The estimates of the inherent variations for these characteristics are given in Table 2. The *X*-MR charts for these characteristics showed that the process was stable during the data collection period.

The standard deviation of weight of a set of 69 biscuits was estimated from the estimated SD of weight of 23 biscuits, and was found to be 1.75 g. Under the present system, weights of a set of 13 dough pieces are used for controlling the overall dough weight. So the SD of the weight of 13 dough pieces was obtained from the estimated SD of the weight of 23 dough pieces and it was found to be 0.87 g.

Studying the relationship between dough weight and RPG

A scatter diagram of dough weight versus RPG was plotted (see Fig. 3). The diagram did not exhibit any relationship between these characteristics. This also confirms that the distribution of dough weight is independent of the distribution of RPG, which was earlier justified technically. This implies that the use of two separate control charts for the purpose of monitoring dough weight and RPG will be statistically valid.

Determining target values for the characteristics

Target value of weight of a set of 13 dough pieces. As a first step towards continuous improvement, the management proposed to set the mean weight of 69 biscuits at $2 \times$ SD above the declared weight. This implied that the target weight of 69 biscuits should be $400 + 2 \times 1.75 = 403.5$ g. It is estimated that, with this target, in 2.27% of cases, packs of 69 biscuits will be less than the declared weight. Accordingly, the target value for the weight of 23 biscuits is obtained as 134.5 g.

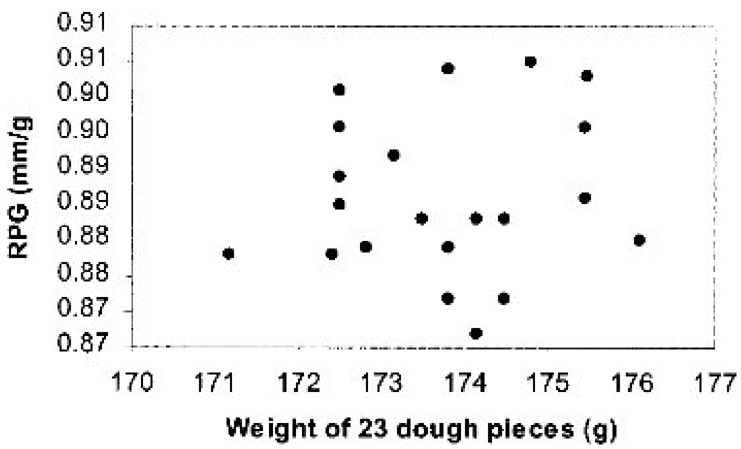

Figure 3. *Scatter plot of weight of 23 dough pieces versus RPG.*

The correlation coefficient between the weight of a biscuit and the weight of a dough piece was found to be 0.85 for the 21 pairs of observations. The regression analysis yielded the following equation:

$$\text{Weight of a biscuit} = 0.761 \times \text{Weight of a dough piece} \tag{2}$$

It is interesting to note that the above finding matches the existing standard of 24% loss due to baking. Based on equation (2), the required overall weight of 23 dough pieces to achieve an overall weight of 134.5 g for 23 biscuits was obtained as 176.74 g, implying that the target weight for a set of 13 dough pieces should be 99.9 g.

Target value of RPG. In the present case, the value of W is 124 mm. Putting this value of W in equation (1), the value of H is obtained as 121.35 mm. Therefore the target value of RPG should be $121.35/134.5 = 0.902$ mm/g.

Control limits for the weight of a set of 13 dough pieces and the RPG

The target values of the characteristics are chosen as the central lines (CL) of the respective control charts. A distance of three times the inherent variation from the central line yielded the control limits for the characteristics as shown in Table 3.

In order to ensure adequate confidence that the present level of process variation is maintained, the equality of sidewise dough weight and RPG should be tested twice during a shift as well as at the time of process setting.

Procedures for approval of sidewise gauge roller setting

The procedure for approval of sidewise gauge roller setting is recommended, taking into consideration the pooled estimate of sidewise SD of weight of sets of ten dough pieces, which was found to be 0.71 g (refer Table 1). The procedural steps are as follows.

Step 1. Collect samples of five sets of ten dough pieces from WS as well as NWS. Measure the weight of these samples and compute the average weight ($\bar{x}_1$) at WS and ($\bar{x}_2$) at NWS. Find the difference of averages, ($\bar{x}_1 - \bar{x}_2$).

Step 2. If the value of ($\bar{x}_1 - \bar{x}_2$) lies beyond the range $[-0.88, 0.88]$ (95% confidence level) make a sidewise adjustment in the gauge roller. Otherwise no sidewise adjustment is required.

Step 3. Whenever any adjustment in gauge roller is made, verify whether the adjustment is made correctly. For this purpose, after every adjustment collect samples, measure the samples' weight and make a computation following Step 1. If the value of ($\bar{x}_1 - \bar{x}_2$) obtained after making adjustment lies within the range $[-0.45, 0.45]$ (68.27% confidence level) consider that the adjustment has been made correctly. Otherwise continue readjustment and re-verification until the value of ($\bar{x}_1 - \bar{x}_2$) is found to be within the range $[-0.45, 0.45]$.

Table 3. *Control limits for the characteristics*

Sl. No.	Characteristics	Target value (CL)	Lower control limit (LCL)	Upper control limit (UCL)
1	Weight of 13 dough pieces	99.9 g	97.3 g	102.5 g
2	RPG	0.902 mm/g	0.872 mm/g	0.932 mm/g

Procedure for the approval of uniformity of oven heat and/or amount of milk solution spray over the two sides

The procedure for approval of uniformity of oven heat and/or amount of milk solution spray over the two sides is recommended, taking into consideration the pooled estimate of sidewise SD of RPG, which was found to be 0.0096 mm/g (see Table 1). The procedural steps are as follows.

Step 1. Collect samples of five sets of 23 biscuits from WS as well as NWS. Measure heights and weights of these samples and obtain the values of RPG. Compute the average RPG ($\bar{y}_1$) and ($\bar{y}_2$) at WS and NWS respectively. Find the difference of averages, ($\bar{y}_1 - \bar{y}_2$).

Step 2. If the value of ($\bar{y}_1 - \bar{y}_2$) lies beyond the range $[-0.012, 0.012]$ (95% confidence level) take appropriate corrective action to ensure uniformity of oven heat and/or quantity of milk solution spray over both sides. Otherwise no corrective action should be taken.

Step 3. Whenever any corrective action is taken, verify whether the action is taken correctly. For this purpose, after every action, collect samples, take measurements and make a computation following Step 1. If the value of ($\bar{y}_1 - \bar{y}_2$) obtained after corrective action lies within the range $[-0.006, 0.006]$ (68.27% confidence level) consider that the adjustment is made correctly. Otherwise continue readjustment and re-verification until the value of ($\bar{y}_1 - \bar{y}_2$) is obtained within the range $[-0.006, 0.006]$.

Approval of both the sidewise final gauge roller setting and oven heat and milk solution spray setting are to be made by the supervisor.

Procedure for approval of overall process setting

For this purpose, samples of a set of 13 dough pieces and a set of 23 biscuits are to be collected randomly from the whole width of the conveyor. The rules for approval of the overall process setting are expressed in terms of green zones (defined in a subsequent section) of the two control charts, which indicate the 68.27% confidence intervals of the respective target values.

Step 1. If the first observation, or two out of three consecutive observations, on dough weight fall within the green zone marked in the control chart for dough weight, approve the settings of the gauge roller.

Step 2. If the first observation or two out of three consecutive observations on RPG fall within the green zone marked in the control chart for RPG, approve the settings of oven heat, milk solution spray and the recipe adjustment.

Designing the control chart format and its implementation

The control charts are to be used and maintained by the operators. So the control charts are formatted in the form of checksheets, as given in the appendix. The check sheet is further colour coded as a target zone ($\pm 1\sigma$ limits) by the colour green, action zone (above $\pm 3\sigma$ limits) by red and no-action zone (between $\pm 1\sigma$ and $\pm 3\sigma$ limits) by white to facilitate easy decision making by the operators. In this form of control chart format, the operator will measure the values of the characteristics and put cross marks ($\times$) in the appropriate box. The decision regarding whether action is needed can be taken by examining in which colour-

Table 4. *Results of implementation*

Sl. No.	Characteristics	Before implementation	After implementation
1	Average pack weight	408.2 g	403.6 g
2	SD of pack weight	7.71 g	2.06 g
3	Range of count	67–73	68–70
4	Under-weight packet %	12.2	2
5	*Excess weight %	2.1	0.9
6	% Packets with 69 count	46.9	95.5

*The difference of average pack weight from the declared weight is considered as excess weight and is expressed as a percentage of the declared weight.

coded region the cross mark (×) falls. During implementation, this form of control chart was found to be highly acceptable to the operators.

No adjustment in the process should be made as long as the cross mark (×) falls in the green zone or the white zone. The purpose of the green zone is to approve the setting or adjustment. The adjustment will be made only when any cross mark (×) falls in the red zone.

In case the observation on dough weight falls in the red zone, the adjustment is to be made in the final gauge roller setting. If the observation on RPG falls in the red zone, the adjustment is to be made either in the oven heat setting or the air pressure in the milk solution spray system or in the recipe as decided by the operator/supervisor based on experience/expertise.

Since all adjustments are made on a trial and error basis, approval of the adjustments is very important. For this purpose the following guidelines are provided.

After every adjustment, check the value of the particular characteristic. If the value falls in the green zone the adjustment made for the characteristics will be considered appropriate. Otherwise continue readjustment and re-verification until the observation after adjustment falls in the green zone. Plot the value obtained after the final adjustment in the same column where the out of control point was indicated and then go on plotting observations at regular intervals as usual.

Training programmes were organized for the management, supervisors and operators of this factory to explain the concept of natural variability, its relevance to the control limits and the interpretation of different zones. In the training programmes, the effects of taking actions with and without the help of control limits were demonstrated through real-life data collected within the factory. In addition, the operators were given hands-on training on rounding off of data, plotting of cross marks (×) and interpretation of control charts.

Comparison of pack weight variation before and after implementation

The characteristics of the pack weight and count variation and their effects assessed before and after implementation, based on 196 and 200 samples respectively are given in Table 4.

References

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Roes, K.C.B., Does, R.J.M.M. & Schurink, Y. (1993) Shewhart-type control charts for individual observations, *Journal of Quality Technology*, 25, pp. 188–198.

Appendix. Control chart for individual observation on RPG

Variety : ***Date :*** ***Shift :***
Operator : ***Supervisor:***

RPG	Sample Time Point															
(mm/gm)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
0.95																
0.94																Red zone
0.93																
0.92																White zone
0.91																
0.90																Green Zone
0.89																
0.88																White zone
0.87																
0.86																Red zone
0.85																