Cost Estimation Under Renewing Warranty Policy—An Application

Sandeep Mondal, Surajit Pal,* and D. K. Manna

SQC & OR Unit, Indian Statistical Institute, Aminjikarai, Chennai, India

ABSTRACT

This article deals with life analysis and warranty cost estimation of temperature switch-cum-sensor (TSS). It involves analysis of one-dimensional, renewing, and free-replacement warranty policy. Life distribution of TSS is identified from warranty failure data. Then, warranty cost is estimated for an extended time period using the bootstrap method.

Key Words: Warranty cost; Renewing warranty policy; Censoring; Bootstrap method.

INTRODUCTION

Warranty is generally viewed as a contractual agreement between the manufacturer and the customer in connection with the sale of a product. In broad terms, the purpose of a warranty is to establish liability in the event of a premature failure of an item or the inability of the item to perform its intended function. The contract specifies the promised product performance and, when it is not met, that redress is available to the customer as compensation for this failure. Warranty is also treated as a marketing strategy, which creates better customer satisfaction and finally helps to get hold of a bigger market share.

A warranty policy is a statement about the kind of compensation that would be provided by the manufacturer to its customers. A policy can be either simple or complex, depending on the type of product being covered by warranty. Different types of policies are in practice (Blischke and Murthy, 1994). Any warranty policy is generally characterized by a combination of the following elements:

a. *One-dimensional/two-dimensional policy*: The warranty limit(s) of a policy are stipulated by either or both the age and usage of the product.
b. *Free-replacement/pro-rata policy*: In free-replacement warranty (FRW), manufacturer agrees to repair or provide replacements for the failed items free of charge up to some specified period from the initial purchase. Under pro-rata policy, the manufacturer agrees to refund a fraction of the purchase price or to replace the failed unit with a new one at a prorated price if a unit fails under warranty.

*Correspondence: Surajit Pal, SQC & OR Unit, Indian Statistical Institute, 110 Nelson Manickam Road, Aminjikarai, Chennai-600029, India; E-mail: surajitpal@hotmail.com.

c. *Renewing/nonrenewing policy*: In a renewing policy, whenever an item fails under warranty, it is replaced by a new item with a new warranty replacing the old one. In contrast, in the case of a nonrenewing policy, replacement of the failed item does not alter the original warranty.

This article deals with cost estimation of a product that is presently being covered by a *one-dimensional renewing free-replacement* warranty policy. The product under consideration is a temperature switch-cum-sensor, abbreviated as TSS.

PRODUCT INFORMATION

The TSS is a temperature-sensing device fitted on the radiator body of heavy-duty trucks. Its function is to measure temperature of the coolant (used for cooling the engine) in the radiator. The switch mechanism gives an alert signal when the coolant temperature reaches 102°C. The operating range of TSS is from 40°C to 120°C. This is a nonrepairable item and, hence, is discarded if any fault occurs in it. The failure or absence of TSS is likely to cause severe damage to the engine due to overheating. When TSS fails, the temperature meter fitted on the dashboard shows no reading.

The product under reference is manufactured in India and is exported to Europe.

PROBLEM DESCRIPTION

The company is interested in extending the warranty period of TSS with the existing one-dimensional renewing free-replacement policy. Hence, it is of prime importance to evaluate the additional cost involved in the proposed revision.

The present warranty period of TSS is 365 days. That is, if a unit fails within a period of 365 days (1 year) from the date of its installation, free replacement will be given by the company. It means that whenever a unit is installed on the radiator, its warranty starts from that date, and free replacement will be provided in the event of its functional failure within 365 days. The associated cost is incurred by the company. When the total cost incurred toward the warranty service of a product is divided by the number of units sold, the *warranty cost per unit* is obtained. The present warranty cost of TSS is about \$1 per unit, against its unit selling price of \$15.

The study is taken up with the following objectives:

(a) Obtain the failure pattern and estimate of reliability measures for TSS, and
(b) Derive the warranty cost per unit for an extended period of (*i*) 547 days ($1\frac{1}{2}$ years) and (*ii*) 730 days (2 years).

FORMULATION OF THE PROBLEM

Note that we need to estimate the life distribution of TSS. Let us denote the life of TSS by T (days). We assume that Ts of different units of TSS are independently and identically distributed with cumulative distribution function $F(\cdot)$. Also, let the proposed warranty time be τ days.

Now, let $N(\tau)$ be the number of free replacements corresponding to a sold unit, during the time interval $(0, \tau]$. Hence $N(\tau)$ is a random variable with:

$$P[N(\tau) = 0] = 1 - F(\tau) = \bar{F}(\tau)$$

$$P[N(\tau) = 1] = F(\tau)\bar{F}(\tau)$$

$$P[N(\tau) = 2] = F(\tau)F(\tau)\bar{F}(\tau) = \{F(\tau)\}^2\bar{F}(\tau)$$

and so on (Blischke and Murthy, 1994). In general:

$$P[N(\tau) = k] = \{F(\tau)\}^k\bar{F}(\tau), \quad k = 0, 1, \ldots$$

so that $N(\tau)$ has geometric distribution. Therefore, the expectation of $N(\tau)$ is given by:

$$E[N(\tau)] = \frac{F(\tau)}{\bar{F}(\tau)}$$

It then follows that the expected warranty cost per unit, denoted by $W_c(\tau)$, is:

$$W_c(\tau) = C_s\left[\frac{F(\tau)}{\bar{F}(\tau)}\right] \tag{1}$$

where C_s is the unit selling price. (Strictly speaking, C_s in the above equation should exclude the profit.)

BASIC DATA

Warranty analysis mainly involves failure information (namely, life) of various units of the product under consideration. Since TSS is presently covered by warranty for more than 3 years, field failure data are available. It is obvious that these data provide realistic information on the life. Consequently, they are used to analyze the life characteristic of TSS.

As already mentioned, warranty cover on a unit starts from the time it is installed on the radiator. In the event of its failure within 365 days, a claim is made by the customer. The processing of this claim is based on the (i) identification code, (ii) production month, (iii) installation date, (iv) failure date, etc. If a unit fails within 365 days, a claim is made and its life is known. The company has no information on the life of a unit that has not failed within 365 days. Such cases are referred to as censored units, and the censoring time is 365 days.

The company maintains a complete database with regard to every claim made during the warranty period. All warranty claims for TSS that are made during the period July 1998 to March 2001 are collected. Apart from this, the monthly production figures of TSS corresponding to the above period are also collected.

PRELIMINARY SUMMARIZATION OF DATA

It is observed that there exists some amount of time lag from the date of production to the date of installation of a unit. This is because of transportation, shipment, inventory, etc. This time lag is found to be a maximum of 8 months. Thus, a unit of TSS will take a maximum of 20 months to cover the warranty phase from its production date (12 months warranty period and 8 months time lag). Thus, of the units that are produced in the month August 1999, all of them should complete the warranty phase by March 2001. Since warranty claim data are collected until March 2001, claim information (failed/not failed) of all the units produced beyond August 1999 is not complete. *Thus, all the units produced from July 1998 to August 1999 are selected for our study.* The total number of units produced during this selected period is 4406. From the warranty claim data, every claim is selected provided that it is produced during the selected production months. There are a total of 296 such claims (see Appendix A).

Each unit is observed over a maximum period of 365 days from the date of installation. The life of a unit is known provided that it has failed within 365 days. Otherwise, it is referred to as censored. The censoring time is 365 days. During the study period (July 1998 to August 1999), the total production (n) is 4406, and the total number (r) of failures is 296 within the censoring time. The observation on life T for i^{th} failed unit is denoted by t_i, $i = 1, 2, \ldots, r$. The values for t_i are obtained by subtracting installation date from failure date. Note that it is a time-censored nonrepairable case. All subsequent discussions will be in regard to the same.

DATA ANALYSIS

It is important to note that we are interested in the life characteristics of TSS at a time point (namely, proposed warranty time), which is beyond the present censoring time. Hence, it is necessary to fit a parametric model to the life data. Fitting of this model is generally done by assuming a standard parametric distribution and then estimating its parameters by the method of maximum likelihood. Further, the model is validated by a chi-square goodness-of-fit test.

MODEL IDENTIFICATION

Data Characteristics

In order to make some guess as to the probable life distribution, an empirical (observed) hazard rate plot is made with the given data (see Appendix B). The plot (see Fig. 1) shows an increasing trend in hazard rate. From the given data, it is also observed that the minimum life is 2 days. Therefore, the true value of minimum life is less than or equal to 2 days. We assume this to be zero. Further, it is well known that Weibull distribution exhibits varieties of hazard rate patterns—constant, increasing/decreasing, and linear/nonlinear. (Exponential distribution, a special case of Weibull distribution, describes only constant hazard rate.)

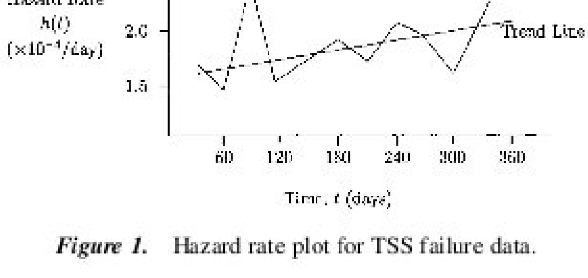

Figure 1. Hazard rate plot for TSS failure data.

Consequently, two-parameter Weibull distribution (zero-minimum life) is considered for modeling the life data, i.e., $T \sim W(\theta, \beta)$, where θ is the scale parameter and β is the shape parameter. The distribution function of T is given by:

$$F(t) = 1 - e^{-(t/\theta)^{\beta}}, \quad t \geq 0 \tag{2}$$

The density function $f(t)$, reliability function $R(t)$, and hazard rate $h(t)$ are also given as:

$$f(t) = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{\beta-1} e^{-(t/\theta)^{\beta}} \tag{3}$$

$$R(t) = 1 - F(t) = e^{-(t/\theta)^{\beta}} \tag{4}$$

$$h(t) = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{\beta-1} \tag{5}$$

Parameter Estimation

The method of maximum likelihood is followed for estimation of the model parameters. With nonrepairable failures and time-censored life data, the maximum likelihood estimates (MLEs) for Weibull distribution are obtained by solving the following two equations (Lawless, 1982):

$$\frac{1}{\beta}+\frac{1}{r}\sum_{i=1}^{r}\ln t_i - \frac{\sum_{i=1}^{r} t_i^{\beta}\ln t_i + (n-r)\tau^{\beta}\ln\tau}{\sum_{i=1}^{r} t_i^{\beta} + (n-r)\tau^{\beta}} = 0 \tag{6}$$

$$\frac{\sum_{i=1}^{r} t_i^{\beta} + (n-r)\tau^{\beta}}{r} = \theta^{\beta} \tag{7}$$

First, the β value is to be obtained from Eq. (6) using any iterative procedure. We have employed the Newton–Raphson method (Bazaraa and Shetty, 1979) for its simplicity in computation and obtained the value of β. Using this value of β, the estimate of θ from Eq. (7) is then obtained.

The MLEs are given by:

$$\hat{\beta} = 1.1081, \quad \hat{\theta} = 4048.5532\,\text{days}$$

Table 1. Test for goodness of fit.

Class (in days)	Observed frequency	Expected frequency	χ^2 value
0–30	22	19.17	0.36
30–60	19	22.05	0.49
60–90	32	23.21	2.42
90–120	20	23.95	0.78
120–150	23	24.47	0.09
150–180	24	24.86	0.03
180–210	22	25.17	0.46
210–240	27	25.41	0.09
240–270	25	25.60	0.01
270–300	20	25.76	1.66
300–330	27	25.88	0.05
330–365	35	30.30	0.63
> 365	4110[a]	4110.17	0.00
Total	4406	4406	7.07

[a] Number of units not failed.

Goodness-of-Fit Test

It is important to validate the Weibull distribution model for describing the life of TSS. This is carried out by the standard goodness-of-fit test based on the χ^2 method. The calculations are shown in Table 1 and the χ^2 value is found as 7.07. The critical value of χ^2, at a 5% level of significance and 10 degrees of freedom, is 18.31. Since the calculated χ^2 value is less than the critical value, it can be concluded that the life of TSS can be well represented by the assumed distribution, namely, Weibull.

Therefore, the estimated density function of T is given by:

$$\hat{f}(t) = 1.1151 \times 10^{-4} \times t^{0.1081} \times exp(-1.0063 \times 10^{-4} \times t^{1.1081})$$

RELIABILITY MEASURES

The associated measures are as follows:

1. Reliability:

$$\hat{R}(t) = exp(-1.0063 \times 10^{-4} \times t^{1.1081})$$

$$\hat{R}(365) = 0.9329, \quad \hat{R}(547) = 0.8969,$$

$$\hat{R}(730) = 0.8609.$$

2. Hazard rate (per day):

$$\hat{h}(t) = 1.1151 \times 10^{-4} \times t^{0.1081}$$

$$\hat{h}(365) = 2.1101 \times 10^{-4},$$

$$\hat{h}(547) = 2.2044 \times 10^{-4},$$

$$\hat{h}(730) = 2.2743 \times 10^{-4}.$$

We now proceed to obtain confidence intervals (CIs) for $R(\tau)$ when $\tau = 365, 547,$ and 730 days. Exact formulas for the same are not readily available. However, they can be derived using normal approximation, as discussed in Meeker and Escobar (1998) and Nelson (1982). On the other hand, we know that CI obtained through the bootstrap approach is expected to be more accurate than the normal approximation methods (Meeker and Escobar, 1998, p. 205). Therefore, we resort to the bootstrap approach in the present situation (Efron and Tibshirani, 1993).

The following steps are followed in this approach:

1. From the actual data set of size n (with r failures), a bootstrap sample of size n is generated by drawing sample from it with replacement.
2. Based on this bootstrap sample, $R(365)$, $R(547)$, and $R(730)$ are estimated using Eqs. (6), (7), and (4).
3. The above two steps are repeated 10,000 times.

Thus, we get 10,000 estimates for each of $R(365)$, $R(547)$, and $R(730)$. A 95% CI is determined for each of them using the percentile method (Efron and Tibshirani, 1993). They are as follows:

$$0.9253 \le R(365) \le 0.9401,$$

$$0.8853 \le R(547) \le 0.9089, \quad \text{and}$$

$$0.8447 \le R(730) \le 0.8768.$$

WARRANTY COST ESTIMATION

The expected warranty cost per unit is calculated for present warranty time (365 days) as well as for the proposed times (547 days and 730 days) using Eq. (1). Confidence intervals of 95% are also found and given in Table 2.

Thus, if the warranty time were revised to 547 days (730 days) from the present warranty time of 365 days, the percentage increase in warranty cost would be 59.77% (124.65%), respectively.

Table 2. Estimation of $W_c(\tau)$ (in \$).

τ (days)	365	547	730
$F(\tau)$	0.0671	0.1031	0.1391
$W_c(\tau)$	1.079	1.724	2.424
95% CI for $W_c(\tau)$	[0.956, 1.211]	[1.503, 1.943]	[2.108, 2.758]

Note: Unit price $C_s = 15$.

CONCLUSION

The one-dimensional renewing free-replacement warranty policy of TSS is studied. From the field data, life distribution of TSS is modeled as Weibull. Consequently, reliability and warranty cost for the proposed warranty times, along with their CIs, are obtained using the bootstrap method. It is estimated that the warranty cost per unit will increase by about 60% and 125% if the time is extended to $1\frac{1}{2}$ years and 2 years, respectively.

APPENDIX A: BASIC DATA (PRODUCTION AND CLAIM INFORMATION)

Sl. no.	Production month	Installation date	Failure date	Life (in days)
1	Jul-98	15/10/98	27/08/99	316
2	Jul-98	14/11/98	22/01/99	69
3	Jul-98	27/01/99	17/11/99	294
4	Jul-98	29/12/98	09/12/99	345
5	Jul-98	17/02/99	12/04/99	54
6	Jul-98	04/12/98	20/10/99	320
7	Jul-98	27/11/98	08/10/99	315
8	Aug-98	19/12/98	28/01/99	40
9	Aug-98	24/12/98	16/10/99	296
10	Aug-98	03/02/99	04/12/99	304
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…
292	Jul-99	29/12/99	03/12/00	339
293	Aug-99	05/02/00	09/11/00	277
294	Aug-99	26/12/99	05/11/00	314
295	Aug-99	31/12/99	30/11/00	334
296	Aug-99	26/01/00	17/11/00	295

APPENDIX B: HAZARD RATE CALCULATION

Time, t (in days)	Observed frequency	Hazard rate ($\times 10^{-4}$)
30	22	1.6644
60	19	1.4446
90	32	2.4437
120	20	1.5386
150	23	1.7776
180	24	1.8648
210	22	1.7190
240	27	2.1206
270	25	1.9761
300	20	1.5903
330	27	2.1572
365	35	2.8146

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ABOUT THE AUTHORS

Shri Sandeep Mondal has a Bachelor's degree in manufacturing engineering and an M.Tech. degree in quality, reliability, and operations research. He is studying in the SDP Fellowship programme in the Statistical Quality Control and Operations Research (SQC & OR) Unit of the Indian Statistical Institute, Chennai.

Shri Surajit Pal holds an M.Sc. degree in applied mathematics and has a postgraduate diploma in SQC & OR. He is working as a senior lecturer in the SQC & OR unit of the Indian Statistical Institute, Chennai. He is a faculty member of various courses offered by the institute.

Dr. D.K. Manna is a professor at the SQC & OR unit of the Indian Statistical Institute, Chennai. His current research interests are in the areas of scheduling and reliability analysis.

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