Magnus Force in High Temperature Superconductivity and Berry Phase

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Abstract

In the topological framework of high temperature superconductivity we have discussed the Magnus force acting on its vortices.

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In recent times, the debate on the problem of Magnus force gained a renewed interest. There are two conflicting points of view on the theory of transverse force. Volovik [1] has shown that the motion of the vortex with respect to the stationary condensate induces a spectral flow. A momentum transfer from the vortex system to a heat bath system is caused by a relaxation of the quasiparticles of the vortex bound states (i.e., the electronic states inside a vortex core). Therefore the vortex can apparently be moved without any external source of transverse momentum. In this spectral flow theory the coefficient of the transverse force k essentially depends on the electronic states inside a vortex core in combination of the relaxation time τ of the quasiparticles. On the contrary, Ao and Thouless [2] showed that the transverse force on a moving vortex is a robust quantity

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which does not depend on the details of the vortex bound states inside a vortex core but only on the superfluid density far from the core. Ao, and Thouless [2] calculated the Berry phase for the adiabatic motion around a closed loop at zero temperature and showed the existence of the Magnus force associated with Berry phase, as a general property of vortex line in a superconductor.

The purpose of the present note is to study the Magnus force in the vortex dynamics of high T_c superconductors. It is found to be consistent with the idea of the Ao Thouless theory of the robust Magnus force. In some recent papers [3,4] we have shown that due to certain features in the background lattice how Berry's topological phase plays an important role in describing high T_c superconductivity. Within this framework, we have studied here the Magnus force required for a vortex to move.

In a recent paper [4], from a topological approach we have shown the relevance of Berry phase in the understanding of pairing mechanism in high T_c superconductivity. We know that the system of correlated electrons on a lattice is governed by the Hubbard model which in the strong coupling limit and at half filling can be mapped onto an antiferromagnetic Heisenberg model with nearest neighbour interaction and is represented by the Hamiltonian

$$H = J \sum (S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z) \tag{1}$$

with J¿0. For a frustrated spin system on a lattice Wiegmann [5] has related two characterization operators of the ground state of an antiferromagnet, namely density of energy

$$\epsilon_{ij} = (\frac{1}{4} + \vec{S}_i.\vec{S}_j) \tag{2}$$

and chirality

$$W(C) = Tr \prod_{i \in C} (\frac{1}{2} + \vec{\sigma}.\vec{S}_i) \tag{3}$$

(where σ are Pauli matrices and C is a lattice contour) with the amplitude and phase Δ_{ij} of Anderson's resonating valence bond (RVB) through

$$\epsilon_{ij} = |\Delta_{ij}|^2 \tag{4}$$

and

$$W(C) = \prod_C \Delta_{ij} \tag{5}$$

This suggests that Δ_{ij} is a gauge field. The topological order parameter $W(C)$ acquires the form of a lattice Wilson loop

$$W(C) = e^{i\phi(c)} \tag{6}$$

which is associated with the flux of the RVB field

$$e^{i\phi(c)} = \prod_C e^{iB_{ij}} \tag{7}$$

B_{ij} represents a magnetic flux which penetrates through a surface enclosed by the contour C. This is essentially the Berry phase related to chiral anomaly when we describe the system in three dimensions through the relation

$$W(C) = e^{i2\pi\mu} \tag{8}$$

where μ appears to be a monopole strength. In view of this, we consider a two dimensional frustrated spin system on a lattice residing on the surface of a three dimensional sphere of a large radius in a radial (monopole) magnetic field and associate chirality with the Berry phase. In fact, the spherical geometry with a monopole at the center is *equivalent* to considering the effect of spin chirality in the RVB scenario of the high temperature superconductors.

In this geometry, we can consider a generalised Heisenberg-Ising Hamiltonian with nearest neighbour interaction

$$H = J \sum (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z) \tag{9}$$

where $J > 0$ and the anisotropy parameter $\Delta \geq 0$ and is given by $\Delta = \frac{2\mu+1}{2}$ [6]. It is noted that μ can take the values $\mu = 0, \pm 1/2, \pm 1, \pm 3/2$......... We observe that $\Delta = 1$ corresponds to $\mu = 1/2$. Indeed, the Ising part of the Hamiltonian corresponds to the near neighbour repulsion caused by free fermions and as $\mu = 1/2$ is related to a free fermion, which follows from the Dirac quantization condition $e\mu = 1/2$,the condition $\Delta = 1$ gives rise to a isotropic antiferrromagnetic Heisenberg model which is $SU(2)$ invariant. When $\Delta = 0$ ($\mu = -1/2$) we have the XX model. For a frustrated spin system, this effectively corresponds to a bosonic system of spin singlets which eventually leads to a RVB state.

To study the spinon and holon excitations, we consider a single spin down electron at a site j surrounded by an otherwise featureless spin liquid representing a RVB state.

We note that when the single spin down state characterised by $\mu = -\frac{1}{2}$ is coupled with the monopole in the background represented by $\mu = -\frac{1}{2}$ will give rise to a state with $\mu = -1$. Thus in this framework, the spinon is considered such that the elementary spin 1 excitation characterized by $|\mu| = 1$ is split into two parts, with one spin $\frac{1}{2}$ excitation in the bulk and the other part due to the *orbital spin* in the background characterized by the chirality of a frustrated spin system.

It may be mentioned here that the spin singlet state forming the quantum liquid are equivalent to FQH liquid with filling factor $\nu = 1/2$ [3]. Indeed, in some recent papers [7-9] we considered a 2D electron gas of N-particles on the surface of a three dimensional sphere in a radial (monopole) strong magnetic field and studied the behaviour of quantum Hall fluid from the view-point of the Berry phase which is linked with chiral anomaly . For the FQH liquid with even denominator filling factor *i.e.* for the state with $\nu = 1/2$, the Dirac quantization condition $e\mu = 1/2$ suggests that $\mu = 1$. Then in the angular momentum relation for the motion of a charged particle in the field of a magnetic monopole

$$\vec{J} = \vec{r} \times \vec{p} - \mu \vec{r} \tag{10}$$

we note that for $\mu = 1$ (or an integer) we can use a transformation which effectively suggests that we can have a relation of the form

$$\vec{J} = \vec{r} \times \vec{p} - \mu \vec{r} = \vec{r'} \times \vec{p'} \tag{11}$$

This indicates that the Berry phase which is associated with μ may be unitarily removed to the dynamical phase. This implies that the average magnetic field may be taken to be vanishing in these states. However, the effect of the Berry phase may be observed when the state is split into a pair of electrons each with the constraint of representing the state $\mu = \pm 1/2$. These pairs will give rise to the $SU(2)$ symmetry as we can consider the state of these two electrons as a $SU(2)$ doublet. This doublet of Hall particles for $\nu = 1/2$ FQH fluid may be taken to be equivalent to RVB singlets.

Now when a hole is introduced into the system by doping, the spinon will interact with the hole through the propagation of the magnetic flux and this coupling will lead to the creation of the holon which will have magnetic flux $|\mu_{eff}| = 1$. Eventually, the residual spinon will be devoid of any magnetic flux corresponding to $|\mu_{eff}| = 0$. This is realized when the single down spin in the RVB liquid will form a pair with another up spin having $\mu = +1/2$ associated with the hole following a spin pair. The holon having the effective Berry phase factor $|\mu_{eff}| = 1$ will also eventually form a pair of holes each having magnetic flux corresponding to $|\mu| = 1/2$ [4].

The Berry phase factor is associated with the chiral anomaly through the relation [10]

$$2\mu = -\frac{1}{2}\int \partial_\alpha J^5_\alpha d^4x \tag{12}$$

where J^5_α is the axial vector current $\bar{\psi}\gamma_\alpha\gamma_5\psi$. When a chiral current interacts with a gauge field we have the anomaly given by [11]

$$\partial_\alpha J^5_\alpha = -\frac{1}{8\pi^2} Tr\ {}^*\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta} \tag{13}$$

where $\tilde{F_{\alpha\beta}}$ is the field strength associated with the gauge potential B_α and from this we have the relation for the Pontryagin index

$$q \ = \ 2\mu = -\frac{1}{16\pi^2} Tr \int \ {}^*\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta} d^4x \tag{14}$$

In a frustrated spin system characterised by chirality this field can be associated with the background magnetic field. Actually, this gauge field is responsible for the spin-pairing and also for the hole pairing. Due to this interacting magnetic fluxoid the hole pair can overcome the bare Coulomb repulsion in high- T_c superconductivity. The superconducting phase order will be established when a spin pair with each spin having unit magnetic flux and a pair of holes with each hole having unit magnetic flux interacts with each other through a gauge force i.e., spin charge recombination comes into play. This helps us to infer of the topological aspect of pairing in high T_c superconductors and show that it is of magnetic origin [4]. We will show that this gauge field coupled with the vortex current will lead to the transverse force responsible for the motion of the vortices.

It is known that a vortex line is topologically equivalent to a magnetic flux. Thus in a cuprate superconductor the pair of charge carriers each having magnetic flux associated with it may be viewd as a quantized vortex line attached to each of them. These vortex lines lie along the $\hat{z}$ axis. To study this vortex dynamics we assume $T = 0$ and low magnetic field so that vortex-vortex interaction can be ignored. To move a vortex with respect to the superconducting flow requires a transverse lift force which is known as the Magnus force. The Magnus force acting on a vortex is proportional to the vector product of the velocity of the vortex relative to the superconducting system and a vector directed along the vortex core.

In our present formalism, we note that in the hole pair the associated flux quantum corresponding to $|\mu| = 1/2$ is derived from the bulk whereas the other flux quantum with $|\mu| = 1/2$ is due to the background related to the chirality of the frustrated spin system. In our model, we may assume that with the movement of the hole pair, the associated vortex line corresponding to the contribution from the bulk moves along with the centre of mass of the paired charge carriers and the condensate will experience an interaction with the background magnetic field. To study this interaction, we have to introduce the $\theta - term$ (last term in the Lagrangian (15)) as this corresponds to the vortex line representing magnetic flux quantum associated with the background magnetic field. The Lagrangian density of our model in spherical geometry, where the 2D surface is residing on the surface of a 3D sphere of large radius with a monopole at the centre, may be written as

$$\begin{aligned} L &= \frac{1}{2}[\phi^*(\partial_0 - ieA_0)\phi - \phi(\partial_0 + ieA_0)\phi^*] + \frac{1}{2m}|(\partial_a - ieA_a)\phi|^2 + \frac{\lambda}{2}(|\phi|^2 - \rho_0)^2 + \\ &\quad \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{4}{}^*\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta} \end{aligned} \tag{15}$$

Here ρ_0 corresponds to the stationary configuration with $|\phi|^2 = \rho_0$. The term $F_{\alpha\beta}$ corresponds to the electromagnetic field strength and $\tilde{F}_{\alpha\beta}$ corresponds to the background magnetic field. ${}^*\tilde{F}_{\alpha\beta}$ is the Hodge dual

$$^*\tilde{F}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\lambda\sigma}F_{\lambda\sigma}$$

It is noted that the P and T violating term ${}^*\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta}$ takes care of the chirality of the system. It is a four divergence and hence does not contribute to the equation of motion but quantum mechanically it contributes to the action. It is noted that there is a singularity at the z-axis and hence we can take the two dimensional formalism. To study the vortex dynamics, being inspired by Stone [12], we set $\phi = fe^{i\theta}$ so that we may write

$$L = if^2(\partial_0\theta - ieA_0) + \frac{f^2}{2m}(\partial_a\theta - ieA_a)^2 + \frac{\lambda}{2}(f^2 - \rho_0)^2 + \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \frac{k}{4\pi}\epsilon_{\alpha\beta\lambda}B_\alpha\partial_\beta B_\lambda \quad (16)$$

It is observed that the dimensional reduction suggests that the anomalous term ${}^*\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta}$ in 3+1 dimensions corresponds to the Chern Simons term $\epsilon_{\alpha\beta\lambda}B_\alpha\partial_\beta B_\lambda$ in 2+1 dimensions. We now introduce Hubbard-Stratonovich fields $\vec{J}$ with the relation $J_0 = f^2$ to obtain

$$L \to L' = iJ_\alpha(\partial_\alpha\theta - ieA_\alpha) + \frac{1}{8mJ_0}(\partial_a J_0)^2 + \frac{m}{2J_0}J_a^2 + \frac{\lambda}{2}(f^2 - \rho_0)^2 + gauge\ field\ terms \quad (17)$$

We set the vortex part of the phase $\theta = \bar{\theta} + \eta$ where $\bar{\theta} = \arg(\vec{r} - \vec{r_i}(t))$ is the singular part of the phase due to vortices at $\vec{r_i}$ and η is the non-singular part. Integration over η suggests the conservation equation $\partial_\alpha J_\alpha = 0$ indicating J_α as a current. So we can identify

$$J_\alpha = \epsilon_{\alpha\beta\lambda}\partial_\beta B_\lambda = \frac{1}{2}\epsilon_{\alpha\beta\lambda}\tilde{F}_{\beta\lambda} \quad (18)$$

such that the first term in expression (17) corresponds to the interaction with the background magnetic field. Indeed defining the vortex current

$$K_\alpha = \epsilon_{\alpha\beta\lambda}\partial_\beta\partial_\lambda\bar{\theta} \quad (19)$$

we note that the first term in expression (17) can be written as

$$iB_\alpha(K_\alpha - e\epsilon_{\alpha\beta\lambda}\partial_\beta A_\lambda)$$

This shows that the vortex current is coupled to the background gauge potential B_α. It is noted that J_0 has an equilibrium value ρ_0 even when the vortex is at rest. Motion with respect to this background field gives rise to a Lorentz force which is here just the Magnus force. So the Magnus force is generated by the background magnetic field when it interacts with the vortex current. In other words, the Magnus force is generated by the background magnetic field associated with the chirality of the system.

To calculate this Magnus force we may take resort to the Berry phase approach [2] . When the vortex moves round a closed loop, we can express the Berry phase $e^{i\phi}$ with

$$\phi \ = \ 2\pi N\mu \quad (20)$$

where N is the total number of flux quantum enclosed by the loop. In our approach each flux quantum in the background is associated with a hole pair and so the number of flux quanta N trapped is identical with the number of hole pairs enclosed by the loop. Thus we can identify N as the number of hole pairs and we can express ϕ as

$$\phi = 2\pi\mu\frac{n_s}{2} \tag{21}$$

where n_s is the charged superfluid number density far from the vortex core. The Magnus force is given by the vector product of the vorticity and the motion relative to the superconducting velocity

$$F_m = \pm 2\pi\frac{n_s}{2}\mu\hat{c} \times \vec{V}_{vortex} \tag{22}$$

Here +(-) corresponds to vortex parallel (antiparallel) to $\hat{c}$ axis and $\vec{V}_{vortex}$ is the velocity of the vortex with respect to the superconducting velocity. It is to be noted that the Magnus force explicitly depends on the number of carriers instead of their mass. This supports the Ao, Thouless theory of the origin of the Magnus force. As high T_c superconductors are type-II superconductors, in the presence of an external magnetic field, when some magnetic flux quanta penetrates the material, the number density n_s should be replaced by n, the total density of the fluid when the radius of the integration contour is much larger than the London penetration depth. This is a consequence of the Meissner effect [13].

As we know, Aharanov- Casher phase is generated when the flux moves through the mobile fluid charges. In the present situation, the phase arising out of the flux moving through the fluid charges will be cancelled by that coming from the flux motion through the static background ion charges. As the net charge in the macroscopic region is zero, the two Aharnov- Casher phases will cancel each other.

Here we can make a remark on the mysterios sign reversal of the Hall resistivity (conductivty) effect in the underdoped region in cuprate superconductors [14]. It is noted that in the underdoped region there will not be sufficient number of holes to form superconducting pairs. So in this case a holon characterised by $|\mu| = 1$ will not be able to share the magnetic flux with another hole and form the requisite pair. The integral value of μ will lead to the removal of the Berry phase to the dynamical phase as given by eqn.(11). Hence the Magnus force will be decreased. Besides, this in combination with the magnetic flux lines induced by the external magnetic field within the penetration depth may change the orientation of the vortices. Indeed, the interaction of this single holon with $\mu = 1$ with a magnetic flux line having $\mu = -1/2$(due to the external magnetic field) will correspond to $\mu = 1/2$ and as a result we will get a magnetic flux line with opposite orientation. This change in orientation of the magnetic flux line will change the sign of the Hall conductivity. The change of the electronic state due to doping could be related to the internal electronic structure inside vortex core so that it affects the dynamic property of vortices. Actually, some authors [15] have considered this many body effect between vortices and got results to support the Ao-Thouless theory. In our field theoretical analysis through Berry phase we got the same result by calculating the interaction of the background magnetic field with the vortex current .

References

1. G. E. Volovik, J. Phys. C **20**, 1488 (1986).
2. P. Ao, D. L. Thouless, Phys. Rev. Lett. **70**, (1993), 2158.
   P. Ao, D. L. Thouless, Q. Niu, Phys. Rev. Lett. **76**, (1996), 758
3. B. Basu, D.Pal and P. Bandyopadhyay, Int. J. Mod. Phys. **B, 13** (1999), 3393.
4. D.Pal and B. Basu, Europhys. Lett. **56**(1), 99, (2001).
5. P. Wiegmann, Prog. Theor. Phys. Suppl. **101**, 243, (1992)
6. P. Bandyopadhyay, Int. J. Mod. Phys. **A 15**, (2000), 1415.
7. B. Basu and P. Bandyopadhyay, Int. J. Mod Phys. B 11, (1997), 2707.
8. B.Basu, D.Banerjee and P.Bandyopadhyay, Phys.Lett. **A 236**, (1997), 125.
9. B. Basu and P. Bandyopadhyay, Int. J. Mod Phys. **B 12**. 2649, (1998)
10. D.Banerjee and P.Bandyopadhyay, J.Math.Phys. **33**, (1992), 990.
11. A. Roy and P. Bandyopadhyay, J. Math. Phys. **30**, (1989), 2366.
12. M. Stone, Magnus And Other Forces On Vortices In Superfluids And Superconductors : cond-mat/9708017.

13. M. R. Geller, C. Wexler and D. J. Thouless, Phys. Rev. **B 57**, (1998), R8119.
14. T. Nagaoka et al., Phys. Rev. Lett. **80**(1998), 3504
15. N. Hayashi et.al., J. Phys. Soc. Jpn. **67**,(1998), 3368